$\frac{6,211,100}{10} = 621,110$ lbs. Also as the diameter of the bore is six inches, its area is 28·27 square inches, and the average pressure per square inch $= \frac{621,110}{28\cdot27} = 21,970$ lbs., or a little less than ten tons.

Similarly take the case of the steam engine given in Ex. (5) p. 153. The moving parts which weigh 400 lbs. attain a maximum velocity towards the middle of the stroke, which is reduced to nothing at the end of the stroke. Required to find the work which the moving parts are capable of doing after having attained their maximum velocity, the length of stroke being $1\frac{2}{3}$ feet and the number of revolutions 200 per minute. The path described by the crank-pin in each rev. $= 1\frac{2}{3}\pi$ ft. $= 5\cdot236$ ft. and the velocity of the crank-pin per second

$$= \frac{5\cdot236 \times 200}{60} = 17\cdot5 \text{ ft.}$$

The energy stored up in the moving mass at this velocity is obtained from the formula $\frac{wv^2}{2g}$.

$$= \frac{400 \times 17\cdot5 \times 17\cdot5}{64\cdot4} = 1902 \text{ foot-pounds.}$$

This energy is given out while the piston is traversing half the stroke,[1] i.e. ten inches, and is consequently equivalent to a pressure of $\frac{1902 \times 12}{10} = 2282\cdot4$ lbs., acting through this space. As the area of the piston is 78·54 square inches the energy stored up in the moving parts is equivalent to an average pressure of $\frac{2282\cdot4}{78\cdot54} = 29\cdot06$ lbs. per square inch during the latter half of the stroke.

[1] This statement is only true when the connecting rod is infinitely long. It is also true for finite connecting rods if taken to apply to the mean of the forward and back strokes.

Motion of bodies in circles.—In all the cases hitherto considered, the motion has been in a straight line, but in dealing with the mechanics of the steam engine cases of great importance occur in which the motion takes place in a circular path. Such for instance is the motion of the fly-wheel, which is a wheel having a heavy rim. It is generally keyed to the crank axle of the engine, and is used for modifying the effects of any irregularity either in the driving power or in the resistance to be overcome. When, for instance, the driving power is in excess of the resistance to be overcome, the surplus is expended in increasing the velocity of the fly-wheel ; and, *vice versâ*, when the resistance is in excess of the driving power, the energy stored up in the fly-wheel is expended in helping to overcome the resistance, during which operation its velocity is lowered.

The consideration of the motion of bodies in circles is somewhat complicated by the fact that different parts of the bodies may be at different distances from the centres of the circles in which they are moving, and as the velocities necessarily vary directly with the distance from the centre, so also do the quantities of motion. Take, for instance, such a body as a fly-wheel represented by fig. 27. It is composed of a rim, a set of arms, and a central boss. The velocity of the rim is many times greater than that of the boss, and again the velocity of the exterior portion, *a*, of the rim is greater than that of the interior portion, *b* ; consequently it is usual in calculations respecting fly-wheels, to consider

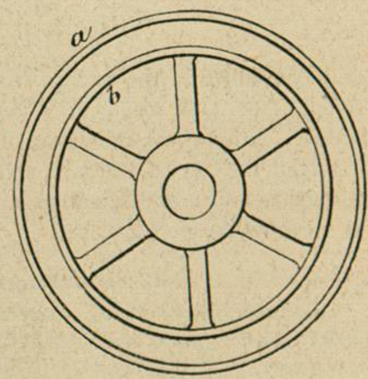

Fig 27

the whole of the weight as concentrated at a certain distance from the centre, where its effect will be the same as the sum of the effects of the various portions of the real wheel, each acting at its own distance from the centre. It is very often a complicated calculation to determine this distance with accuracy, but for all practical purposes we shall be sufficiently correct if we take the mean radius of the rim as the distance at which the whole of the weight is supposed to be concentrated.

The laws of motion, as already stated and illustrated, apply equally when the direction of the motion is in a circle. Thus, for instance, if a weight w move round a centre with a velocity v, the energy stored up in it$=\frac{wv^2}{2g}$. For a given number, N, of revolutions per second the velocity v varies with the length of the radius r, and equals $2\pi r$N. Substituting this expression for v in the above equation we have

$$\text{Energy}=\frac{w4\pi^2r^2N^2}{2g},$$

and consequently the energy varies as the square of the radius, that is of distance of the weight moved from the centre. A fly-wheel, therefore, of a given weight, the mean radius of the rim of which is five feet in length, is rather more than twice as efficient as a reservoir of energy as if its mean radius were 3·5 feet.

EXAMPLE (6).

How much energy is stored in a fly-wheel of 5,000 lbs. weight, the mean radius of the rim of which is 4 feet, and the number of revolutions 60 per minute? N.B.—The whole of the weight is, for simplicity, supposed to be concentrated at the end of the mean radius.

$$\text{The mean velocity per second, } v=\frac{2.\pi.4.60}{60}=25\cdot13 \text{ feet.}$$

$$\text{The energy}=\frac{w.v^2}{2g}=\frac{5{,}000\times631\cdot5}{64\cdot4}$$

$$=49{,}029 \text{ foot-pounds.}$$

EXAMPLE (7).

A fly-wheel weighs 5,000 lbs. and the mean rim moves with a maximum velocity of 35 feet per second. On account of the inequality of the force transmitted to the crank, the fly-wheel has, during a portion of the stroke, to expend 9,000 foot-pounds of energy; what will its velocity be after having done so?

$$\text{The maximum energy}=\frac{5{,}000\times35\times35}{64\cdot4}=95{,}108 \text{ foot-pounds.}$$

After expending 9,000 foot-pounds, the energy remaining

$$=95{,}108-9{,}000=86{,}108 \text{ foot-pounds.}$$

$$\therefore \frac{w.v^2}{2g}=86{,}108.$$

$$\therefore v^2=\frac{86{,}108\times2g}{5000}=1109.$$

$$\therefore v=33\cdot3 \text{ feet per second,}$$

being a loss of 1·7 foot per second from the maximum velocity, which is equivalent to a variation of 4·8 per cent. from the maximum or of 2·4 per cent. from the mean velocity.

Centrifugal force.—By the first law of motion a body will continue to move in *a straight line* unless compelled to do otherwise by impressed forces. When a body moves in a circle it is, however, changing its direction from instant to instant, and consequently must be continuously under the influence of some force. Suppose this force were removed, the body would no longer move in the circle, but would fly off in a straight line at a tangent to the circle from the point at which the force was removed. This is true of any and every point on the circumference, from which it is evident that the direction of the force which compels the body to move in the circle is always at right angles to the tangent at any point, and consequently always points to the centre. This force, which keeps a body moving in a circle, is, on account of its direction, always called the centripetal force. The resistance which the mass of the body opposes to being moved towards the centre, and which by the third law of motion is equal to the centripetal force, is called centrifugal force. This force may be measured as follows.

Let a body be supposed to start from the point a, fig. 28, and move in the circle represented, with the uniform velocity v feet per second. If the centripetal force F were removed, the body would during a very short time t move in a straight line over the space ab. By the second law of motion the effect of the centripetal force would therefore be to cause the body to move over the space bc during the time t.

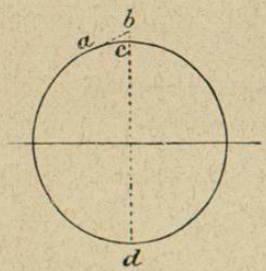

Fig 28.

By a well-known proposition in Euclid $bc \times bd = ab^2$. Calling $bc=x$ we have $x\ (2r+x)=ab^2$. As ab is supposed to be very small, and consequently also bc, we may neglect x^2 and put $ab=ac$.

$$\therefore 2rx = ac^2 \therefore x = \frac{ac^2}{2r}.$$

Also, since the motion in the circle is uniform, and since ac is the space moved in the time t, we have

$$ac = tv \therefore x = \frac{t^2v^2}{2r}.$$

Calling the weight of the body w, and f the velocity which the centripetal force F can generate in w in one second, we have

$$F : w :: f : g$$

$$\therefore F = \frac{wf}{g}.$$

We have next to express f in terms of v and r. Now x is the space bc which the body would move over from rest under the influence of the centripetal force in time t secs.

Therefore the velocity at the end of time $t'' = 2x$ per t''

$$= \frac{2x}{t} \text{ per sec.}$$

Therefore the velocity which would be acquired at the end of one second is

$$f = \frac{2x}{t} \div t = \frac{2x}{t^2},$$

and substituting the value of x as given above, we have

$$f = \frac{t^2v^2}{rt^2} = \frac{v^2}{r}.$$

$$\therefore F = \frac{wv^2}{gr}.$$

This important expression which is constantly made use of gives the centripetal force in terms of the weight of the body, its velocity, and the radius of the circle in which it moves.

If the velocity is given in revolutions per second, n, we have

$$v = 2\pi rn,$$

and the above formula becomes

$$F = \frac{w}{g} \times \frac{4\pi^2 r^2 n^2}{r}$$

$$= wn^2 r \times 1{\cdot}226.$$

If the revolutions are given as so many per minute, N, we have

$$n = \frac{N}{60}$$

$$\therefore F = w \left(\frac{N}{60}\right)^2 r \times 1{\cdot}226.$$

$$= wN^2 r \times 0{\cdot}00034.$$

Conversion of the Pressure of Steam on the Piston into Rotative Effect on the Crank Axle.

One of the most important applications of mechanical science to questions relating to the steam engine is, to ascertain the exact effect which the pressure of the steam on the piston has in causing the crank to rotate. In dealing with this question there are several points to consider :—

First of all, in the great majority of cases the pressure of the steam varies considerably at different parts of the stroke.

Secondly, this variable pressure is transmitted to the crank-pin through a connecting rod, which is constantly changing its angle of inclination to the axis of the cylinder, as it swings between its extreme positions on either side of this axis.

Thirdly, the varying pressure transmitted through the connecting rod meets the crank at an angle which is constantly changing. The pressure may be resolved at the crank-pin into two components, one in the direction of the crank, and the other at right angles to it, i.e. tangential to the circle described by the crank-pin. Of these the latter alone produces any turning effect on the crank, the former producing merely pressure on the bearing. The tangential component, or *turning effort on the crank*, as it may be called, varies in value continually, for it depends not only on the net pressure of the steam on the piston, but also on the varying angles of inclination of the connecting rod, and the crank.

Fourthly, the effective turning effort on the crank depends not only on the above-mentioned variables, but also on the weights and velocities of the reciprocating parts, viz. the piston, and piston and connecting rods; for, as we have seen before, p. 153, Ex. 5, a considerable proportion of the steam pressure may, during a portion of the stroke, be

absorbed in merely imparting motion to the reciprocating parts, and may consequently never reach the crank-pin at all; while on the other hand these parts as they come to rest may impart a considerable pressure to the crank-pin quite independently of the pressure due to the steam on the piston.

The problem will be investigated in the first instance freed from all possible complications. The pressure of the steam will be supposed to be uniform throughout the stroke. The connecting rod will be taken to be of infinite length, in other words it will be supposed to act always parallel to the axis of the cylinder. Lastly, the moving parts will be imagined to be without weight, or their velocity may be supposed to be so small that no appreciable part of the steam pressure is absorbed in imparting motion to them.

In the next instance the pressure of the steam will be supposed to vary during the stroke; then the angular vibration of the connecting rod will be taken into account, and finally the effects of the weights and velocities of the reciprocating parts will be considered. In every case graphical methods will be employed, in preference to analytical, to investigate the problems.

In the diagram, fig. 29, let the circle ABC represent the path of the crank-pin. Let AC represent the direction of the axis of the cylinder. Let the pressure of the steam on the piston throughout the stroke be P lbs. per square inch, and let the scale of the diagram be such that the length of the radius OA represents P lbs. The reason for so doing will soon become apparent. First assume that the crank lies in the position AO. The pressure transmitted through the crank at this moment acts radially through the centre O, and has no effect whatever in turning the crank. The same is true when the crank occupies the position OC hence the two positions OA, OC are called the dead centres. Next suppose the crank to occupy the position OB, at right angles to the dead centres. As the connecting rod is supposed to be of infinite length it acts in the direction B′B parallel to AC, and consequently the whole of the force

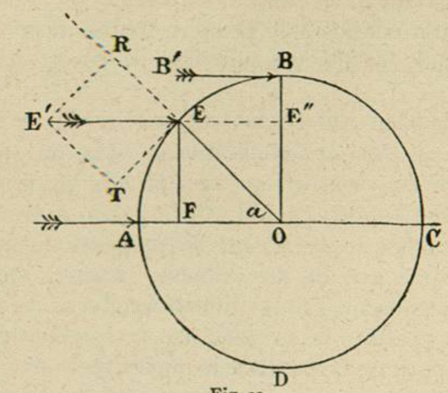

Fig. 29.

transmitted has the effect of turning the crank. The arm of the lever with which the force acts is BO, viz. the radius of the crank, and the turning moment per square inch area of piston $= P \times BO$. The same is true for the position D diametrically opposite to B. Hence we see that while the crank is at A and C the steam pressure has no effect whatever in turning it, at B and D, on the contrary, its whole effect is in turning. At any other point in any of the four quadrants the force of the steam is partly expended in turning the crank, and is partly transmitted through the crank as mere pressure on the main bearing at O.

Take, for instance, the point E. At this position, the force acts with a leverage measured by the length of the perpendicular let fall from the point O on the direction of EE′, viz. $E''O = EF = EO \sin \alpha$, and the turning moment consequently

$$= P \times EO \sin \alpha.$$

The tangential force which acts at the end of the crank, and tends to turn it round, as distinguished from the

turning moment is got by dividing the above quantity by the length of the crank arm. Calling this force P_T, we have

$$P_T = \frac{P \times EO \sin \alpha}{EO} = P \times \sin \alpha.$$

This expression is equally true for any point on the circumference of the circle. Hence, we see that the tangential pressure on the crank, when the connecting rod is infinitely long, is equal to the pressure on the piston multiplied by the sine of the angle of inclination of the crank to the axis of the cylinder.

The same result may be got by resolving the force P at the point E, into two components, viz. one acting radially, ER, and the other tangentially, ET. Of these, ER merely produces pressure on the main bearing, while ET alone tends to turn the crank.

Now, $ET = EE' \sin EE'T$.

Also, $EE' = P = EO$, the scale of the figure being such that EO represents P.

And the angle $EE'T = \alpha$, because E′T is parallel to EO and the angles ETE′ and EFO are both right angles. Therefore the two triangles are equal, and $ET = EF = P \sin \alpha$.

Thus we see that, though the pressure on the piston may be perfectly uniform throughout the stroke, the turning effort on the crank is very variable, and begins by being zero at the dead centre, increases to a maximum when the crank is at right angles to the axis of the cylinder, again decreases to zero by the time the other dead centre is reached, and so on during the return stroke, or second half of the revolution.

It may be here noted that it was this fact, that the tangential pressure on the crank is always less than the pressure on the piston, except for two positions of the crank, which led old writers on the steam-engine into the blunder of asserting that there is a loss in the employment of the crank as a means for converting reciprocating into circular motion. We now know, by the definition of work, that there is no such loss; for, although the average tangential pressure on the crank is much less than the pressure on the piston, on the other hand, the path traversed by the crank in a revolution is greater than that traversed by the piston in a double stroke, in the ratio of the circumference of a circle to its double diameter, i.e. $2 : \pi = 1 : 1{\cdot}57079$.

By the principle of work, the lesser average pressure on the crank, multiplied by the path described by the crank-pin, must equal the greater pressure on the piston multiplied by the space traversed by the latter.

Graphic representation of the tangential effort on the crank-pin.—The variable tangential pressure on the crank-pin

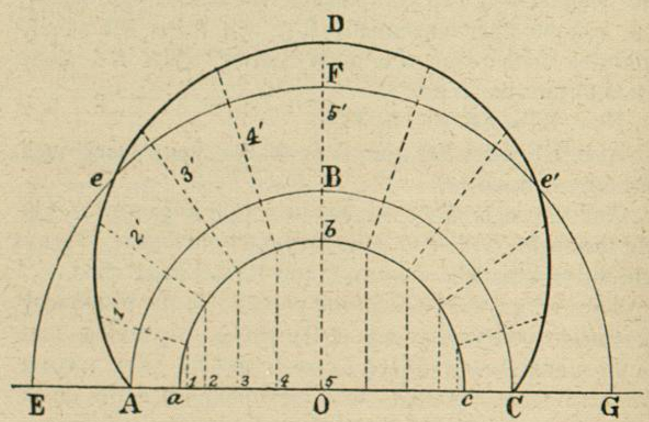

Fig. 30.

throughout a revolution can be very well shown, graphically, by means of a diagram. Let the semicircle ABC (fig. 30) represent the path described by the crank-pin during half a revolution. Draw O*a* to represent the uniform pressure on the piston to scale, and with centre O and radius O*a*, draw the inner semicircle *abc*. Divide the circumference of this semicircle into 10 equal divisions, for the sake of convenience, and draw radial lines through each point of division, intersecting the semicircle ABC. Then, at each position of the crank

represented by the points of division of the outer semicircle, the tangential force on the crank is equal to the pressure on the piston multiplied by the sine of the angle of the crank. As *a*O represents the pressure on the piston, the tangential forces are represented in magnitude by the perpendiculars 1, 2, 3, 4, 5, &c. let fall from the points of division of the inner circle on the line *a*O. On the prolongations of the radial lines beyond the outer circle set off the lines 1′, 2′, 3′, 4′, 5′, &c., equal, respectively, to 1, 2, 3, 4, 5. Join the extremities of these lines by the curved line ADC. Hence, ADC represents, graphically, the tangential pressure at every position of the crank; since, for any position, we have only to draw a radial line through the point in question, and the piece intersected between the outer circle and the curved line will represent the tangential force. If the tangential pressure were uniform all round the circle the curved line ADC would be a circle concentric with the path of the crank-pin. Its deviation from concentricity is the measure of its want of uniformity. The average tangential pressure on the crank-pin may be represented by drawing the circle EFG from the centre O, the line EA which represents this average pressure being obtained by the following proportion

$$EA : aO \text{ or } P :: 2 : \pi.$$

When the engine is running at a *uniform rate of speed*, this average tangential pressure on the crank, is, of course, exactly equal to the resistance which the work to be done offers to the motion of the crank-pin; for, if the resistance were greater, the speed would be reduced, and if the resistance were less, the speed would be increased, and in neither case would the engine be running uniformly. Consequently, this average tangential pressure circle may equally well be called the Resistance Circle.

The diagram (fig. 30) only shows the tangential pressures for one half of the revolution, but the other half is, of course, a precisely similar figure, and need not, therefore, be