shown. By inspection of the diagram, we see that there are four points during a complete revolution where the actual tangential pressure exactly equals the average, viz. the points $e\,e'$, &c.; where the resistance circle intersects the curves ADC, AD′C. Between the points $e\,e'$ and the corresponding points $e''e'''$ the pressure is in excess of the resistance, while between $e'''e$ and $e'e''$, the resistance is in excess of the pressure; consequently, during the two first intervals, the surplus work is poured into the fly-wheel, and during the last two intervals the deficiency in work is supplied by the energy of the fly-wheel being diminished. As the fly-wheel can only receive or restore energy by having its velocity increased or diminished, we see that the velocity of the crank-pin is not, strictly speaking, uniform, but it can be kept within any assigned limits of deviation from uniformity, by altering the weight of the fly-wheel.

The diagram shown on fig. 30 can also be drawn on a straight base instead of on the circumference ABC. This form of the diagram is more generally used in practice, because it is easier to test the accuracy of the work; but it is not so graphic to the eye as the circular form of fig. 30. To construct the diagram on a straight base, draw a straight line AC equal in length to the semicircumference of the circle described by the crank pin. Divide AC into

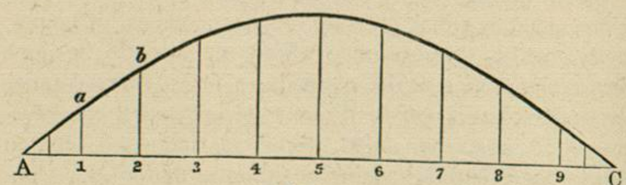

Fig. 30 A.

ten equal parts corresponding with the divisions of the semicircle ABC. From each of the points of division, 1, 2, 3, &c., erect a perpendicular, and mark off the lengths 1*a*, 1*b*, 1*c*, &c., equal to the lines 1′, 2′, 3′, &c., in fig. 30.

Through the points *a b*, &c., draw the curve A*ab*C; then the ordinates of this curve will give the tangential pressures on the crank for any position of the latter.

It is evident that the area bounded between the straight line AC and the curve measures the work done upon the crank; for the ordinates represent the effective pressures on the crank pin, and the abscissæ the spaces through which they are exerted. Now the amount of work done upon the crank is, as has been shown above, equal to the work done upon the piston. Hence the area of fig. 30A should be exactly equal to the area of the indicator diagram. By measuring the area and comparing it with that of the indicator diagram, we have a ready check of the accuracy of the work.

If we wish to show the diagram of tangential pressure for the whole revolution, we have only to prolong AC to double its original length and construct on the prolonged portion another curve precisely similar to A*ab*C.

We will now take the case of an expansion diagram and show the effect which the want of uniformity in the steam pressure acting on the piston has upon the form of the diagram which shows the tangential effort on the crank-pin. We will further suppose as before that the connecting rod is infinitely long, and that the moving parts possess no weight, or are moving at a very slow velocity.

The steam diagram is shown at the upper part of fig. 31. The cut-off is supposed to take place at $\frac{3}{10}$ of the stroke. The engine is non-condensing. The first thing to ascertain is the net pressure of the steam which urges the piston forward. In order to find this it will in most cases be necessary to construct from the ordinary indicator diagram a new diagram showing the actual pressures after deducting the corresponding back pressures (see page 340). In the present instance this step will not be necessary, because to avoid complication an indicator diagram has been chosen in which the back pressure is uniform throughout the stroke and the compression curve at the end of the return stroke is exactly similar to the exhaust curve at the commencement; consequently to obtain the net pressures on the piston we have only to measure the length of vertical ordinate bounded between the upper and lower boundary lines of the diagram.[1]

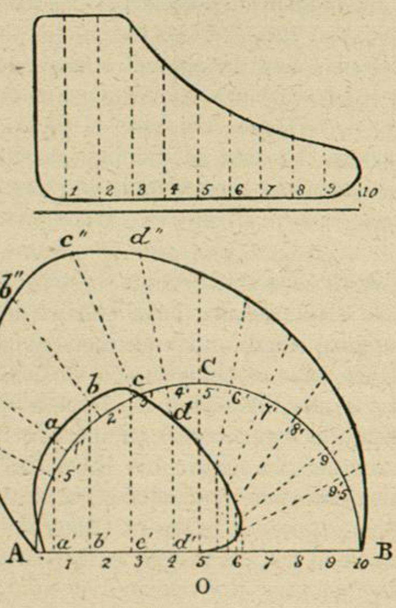

Fig. 31.

Divide the length of the diagram into ten equal parts, and from each point of division draw a vertical ordinate to the upper boundary of the diagram. Draw a line AB of the

[1] This is a case which would probably never occur in practice. In all ordinary cases two diagrams are required: viz. one from the top and the second from the bottom cover of the piston. The net pressures are then taken by deducting from the gross pressure as shown by one diagram the simultaneous back pressure as shown by the other (see page 340). This precaution is very frequently neglected, and has led to serious errors in the calculation of curves of twisting moments.

same length as the diagram to represent the diameter of the crank-pin circle. Divide the line AB into ten equal parts, and from each point 1 2 3 &c. erect a perpendicular 1,1′ 2,2′ 3,3′ &c., intersecting the circumference at the points 1′ 2′ 3′ &c. These lines are not actually drawn, so as to avoid complicating the diagram. Then at each position of the crank-pin 1′ 2′ 3′ &c., the direct pressure on the pin is represented by the corresponding ordinate taken from the indicator diagram. From the centre O draw radial lines O1′, O2′, O3′, &c. intersecting the circumference. It is only necessary to show the portions of these lines which are prolonged beyond the circumference ACB. On these lines measure off the parts O*a*, O*b*, O*c*, O*d*, &c., equal respectively to the ordinates of the steam diagram 1, 2, 3, 4, &c. Then, as in the first case, the actual *tangential* pressures on the crank-pin will be equal to these lines O*a*, O*b*, O*c*, O*d*, &c., multiplied by the sines of the angles which the crank makes with AB. In other words, the tangential pressures will be equal to the perpendiculars let fall from the points *a*, *b*, *c*, *d*, &c., on AB, i.e. *aa*′, *bb*′, *cc*′, *dd*′, &c. Produce the radial lines O*a*, O*b*, &c., and from the points 1′, 2′, 3′, 4′, &c. on the circumference set off the parts 1′*a*″, 2′*b*″, 3′*c*″, 4′*d*″, equal in length respectively to *aa*′, *bb*′, *cc*′, *dd*′, &c., then the curve drawn from A to B through the points *a*″, *b*″, *c*″, *d*″, &c., will be the diagram of tangential effort, or twisting moment on the crank-pin.

To ascertain the diameter of the circle of average tangential pressure, i.e. the resistance circle, we have only to compute the average steam pressure as shown by the indicator diagram and multiply the same by the fraction $\frac{2}{\pi}$ the product will give the radius of the resistance circle. The tangential effort on the return stroke is, of course, exactly similar to the curve A*a*″*b*″B, and is obtained in the same way.

The above method is purely graphical. In actual practice it will probably be found more expeditious to construct a curve of twisting moments, the radial ordinates of which are found by multiplying the steam pressure for any given position of the crank by the leverage at which it works, and then setting off the moment thus obtained to scale. Thus, for the position of the crank 2′, fig. 31A, we have a pressure of 18 lbs. to the square inch, according to the scale on which the diagram is drawn when the piston occupies the position 2 corresponding with the position 2′ of the crank. Also the leverage at which it acts is 2′*b*. If the radius of the crank be one foot, then to the same scale 2′*b*=·799 ft., and the product 18 × ·799 = 14·3 = the twisting moment. Draw a radial line through 2′ and set off 2′*b*′ = 14·3 to any convenient scale. Proceed in a similar manner for all other positions of the crank, and the curve A*b*′B drawn through the extremities of the radial lines is the curve of twisting moments.

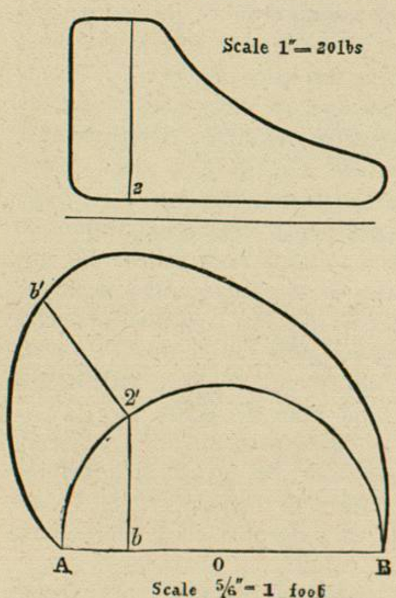

Fig. 31 A.

Influence of the connecting rod in modifying the curve of tangential effort on the crank-pin.—In actual steam engines the connecting rod is of course always of finite length, and consequently is always acting at an angle to the axis of the cylinder, except when the crank-pin is on the dead centres.

The size of the angle which the connecting-rod makes with the axis for any position of the crank-pin depends on the length of the rod compared to the length of the crank. The shorter the relative length of the rod, the greater the angle for any given position of the pin. To find the inclination of the connecting rod for any given position of the crank we have only to take the length of the rod as a radius, and

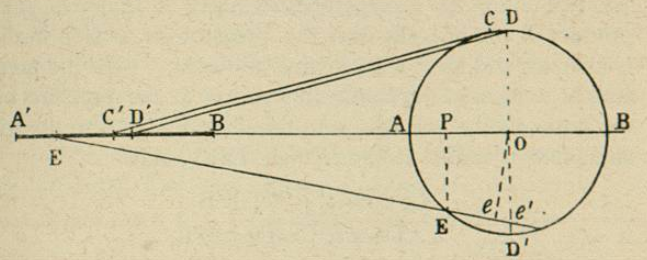

Fig. 32.

from the centre of the crank-pin to describe an arc intersecting the axis of the cylinder prolonged. The line joining the point of intersection with the centre of the crank-pin gives the angular position of the connecting rod and also the position of the piston for the given position of the crank-pin. Thus, let the length of the connecting rod be four times that of the crank, so that when the crank-pin is at A the piston rod end of the connecting rod will be at A′, and AA′=4AO. When the crank has moved round a quarter of a revolution to D, with centre D and radius =AA′ describe an arc intersecting the line AO. The point of intersection will be D′, the distance AD′ being greater than the half-stroke of the piston, which latter equals A′C′. When the piston is at half-stroke, the crank will occupy the position OC.

The tangential effort on the crank due to the piston pressure may be calculated by estimating the effect of this pressure in the direction of the connecting rod, and then resolving this tangentially and radially to the crank circle; or, it may be more conveniently computed geometrically by finding the leverage with which the connecting rod acts on the crank-pin. The amount of the tangential pressure will, as before, be proportional to the length of the arm of the lever. Take, for instance, the position E of the crank-pin, the connecting rod assumes the position EE′, and the leverage with which it acts, instead of being EP, as would be the case were the rod of infinite length, is eO, found by producing EE and letting fall a perpendicular upon it from O. The product of the line eO into the pressure or tension in the connecting rod gives the twisting moment. Now the pressure or tension in the connecting rod is to the pressure on the piston as the line EE′ is to the line E′P. Also the triangle Oee' is similar to the triangle EE′P, and

$$Oe : Oe' :: E'P : E'E$$
$$\therefore Oe' \times E'P = Oe \times E'E.$$

That is to say, the pressure on the piston multiplied by Oe' equals the force in the connecting rod multiplied by Oe, which latter product is the twisting moment.

Hence to obtain the twisting moment we have only to prolong the line of the connecting rod till it intersects the position DD′ of the crank, and the product of this line into the pressure on the piston gives the moment required. By proceeding in this way fig. 31A may be modified so as to take account of the influence of the connecting rod. By an inspection of fig. 32 it is evident that the new arms of the moments are greater than those obtained with infinite connecting rods until the axis of the connecting rod intersects the point D, after which they are less till the end of the stroke is reached. During the return stroke the opposite effect takes place, the arms of the levers being shorter during the first portion, and longer during the latter part of the stroke, than for the corresponding positions when the connecting rod is of infinite length. The arm of the moment is a maximum when the axis of the connecting rod makes a tangent with the circle described by the crank-pin.

An inspection of the diagrams, figs. 30 to 31A, shows how far from uniform is the tangential effort on the crank-pin, and consequently how irregular is the driving power, in the case of single cylinder engines, even when, as in the first case illustrated, the steam pressure is uniform throughout the stroke, and the angularity of the connecting rod is neglected. There are various methods of diminishing this irregularity of driving power. One plan is to fit on to the crank-shaft a fly-wheel of adequate weight and dimensions to overcome the irregularity. The principle of the action of fly-wheels has already been explained (page 157). Another and very usual plan is to use two or more cylinders with the cranks forming angles with each other. These are usually so arranged that the tangential effort on one crank is a maximum when it is a minimum on the other crank. This subject will be again referred to, and examples will be illustrated, when all the disturbing causes which influence the forms of twisting moment diagrams have been explained, but in the meantime it will be a useful exercise for the student to prepare such diagrams for the two cases illustrated in figs. 30, 31 (pp. 166 and 170), assuming that in each case a second cylinder of equal size with the first, and working under precisely similar conditions, is added, and that the cranks are set at right angles to each other.

The method of proceeding is as follows:—A precisely similar diagram to the curve $Aa''b''c''$ ..... B, fig. 31 (p. 170), must be constructed on the diameter COD which is at right angles to AOB. When complete, there will be four of these curves round the circle ACBD, viz. one for each stroke of each cylinder. The radial lines intercepted between the circumference and the curves for each position 01′, 02′, &c. of the crank, &c., must then be added together, and a resultant curve drawn through their extremities. This curve will be a diagram of the tangential effort for the two cylinders combined.

Influence of the weights and velocities of the reciprocating parts.—We must next consider the effects of the weights and velocities of the reciprocating parts on the form of the tangential effort diagram. In the first part of this chapter (p. 153) it was shown that a large portion or even the whole of the steam pressure during the first part of the stroke might be absorbed in generating the velocity in these parts; and consequently, that only a portion, or in some case not any of the steam pressure on the piston was available for transmission to the crank-pin; while, on the other hand, the effect of the velocities of the parts being reduced during the latter portion of the stroke would be to cause a greater pressure to appear on the crank-pin than is due to the steam pressure in the cylinder. It is evident, therefore, that to construct a tangential effort diagram for such cases we must first deduce from the indicator diagram a new diagram, by taking away from the steam pressure, during the first part of the stroke, such a portion of it as goes to accelerate the velocity of the moving parts, and *vice versâ*, adding to the steam pressure during the remainder of the stroke such a portion as represents the effect of their retardation.

If the steam pressure were free to act on the reciprocating masses in the same way that gravity acts on a falling body, or in the way that the pressure of powder-gases in a gun acts on a projectile, it would be easy to calculate the effects produced; but when the motion is controlled by a crank revolving uniformly, it is a little more difficult to calculate the increments of velocity imparted to the piston in successive intervals of time. The pressures required to impart these accelerations, as they are called, are of course proportional to the amounts of the increments.

We must again suppose for the sake of simplicity that the connecting rod is of infinite length. In such a case while the crank travels through successive equal angles corresponding to the positions A, A′, A,″ (fig. 33) the piston moves through successive spaces AA_1, A_1A_2, A_2A_3 &c. The

velocity of the piston at each point A_1 A_2 A_3 &c. may be calculated graphically as follows :—

Let the radius of the circle A B C represent to scale the linear velocity of the *crank-pin* in feet per second. At any point A″ for example, corresponding to the position A_2 of the piston, the velocity of the crank-pin may be resolved into two components, one horizontal and the other vertical. The horizontal component will be the velocity of the piston. From the point A″ draw a tangent A″T

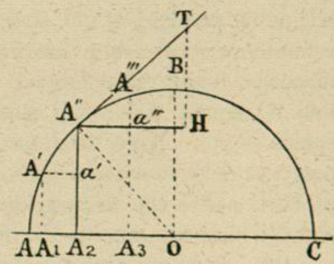

Fig. 33.

to the circle. Make A″T=the radius of the circle, then A″T represents the velocity of the crank-pin. Draw A″H horizontal and TH vertical, then will A″H and TH represent respectively the horizontal and vertical components of the velocity at the point A″, and A″H represents also the velocity of the piston when at a point in the stroke corresponding to the position A_2 in the line AC. In the same way the piston velocity may be obtained for any other point in the stroke. It may, however, be ascertained more simply as follows. The two triangles A″TH and $A''A_2O$ are in every respect equal to each other and A″H equals $A''A_2$. Now $A''A_2$ is the sine of the angle of the crank × A″O, therefore the velocity of the piston at any point A_2 is proportional to the sine of the angle of the crank for that position, and is in fact equal to the length of the perpendicular drawn from the given point, such as A_2, to meet the circumference of the circle, when the velocity of the crank-pin is represented by the length of the radius of the crank circle. Hence we see that the velocity of the piston at a series of successive positions A_1 A_2 A_3 &c. is represented by the vertical ordinates $A'A_1$ $A''A_2$ $A'''A_3$ &c.

As the crank-pin is supposed to travel at a uniform velocity, the crank-pin circle represents time, just as does the hour circle of a watch, and equal divisions of this circle, such as AA', $A'A''$, $A''A'''$, &c., represent equal divisions of time. Now, the difference between the velocity of the piston at the beginning and end of any such interval is the increment of velocity, or acceleration imparted to the piston during the interval. Thus the difference between the velocities at A' and A'' equals $A''a'$, and similarly between A''' and A'' equals $A'''a''$. Now, the force or pressure required to impart a velocity to a given mass in a given interval of time is proportional to the velocity imparted, and when this latter and the mass are known the force may be calculated (see p. 149). If we suppose the divisions AA' &c. to be taken so small that the force acting throughout the interval may be considered as uniform and the acceleration imparted uniformly, then in this case any division such as $A''A'''$ may be considered a straight line, and the two triangles $A''A'''a''$ and $A''A_2O$ are similar, because the angle $OA''A'''$ may be considered a right angle and $a''A''A_2$ is a right angle, and taking away the common angle the remainder $a''A''A'''=$ the remainder $OA''A_2$. Also the angles at a'' and A_2 are right angles, therefore the third angles in each triangle are equal and the two triangles are similar, therefore $\frac{A'''\,a''}{A'''A''}=\frac{A_2O}{A''O}=$cosine of angle of crank. Consequently the acceleration at any position of the crank equals the velocity of the crank-pin multiplied by the cosine of angle of crank, and the forces required to produce the accelerations are proportional to them.

The magnitudes of the forces may be found in the following manner. Suppose that the weight of the reciprocating parts is all concentrated round the crank-pin, the connecting rod being, as before, infinite. The weight is kept moving in the circular path by the action of centripetal force (see p. 159).

The centripetal force always acts radially, and at any point D, fig. 34, may be resolved horizontally and vertically. The horizontal component is the measure of the force which imparts motion to the reciprocating parts. The vertical component merely produces an upward or downward pressure on the bearings. At the dead centre A the force has no vertical component, and therefore the entire centripetal force produces acceleration of the reciprocating parts.

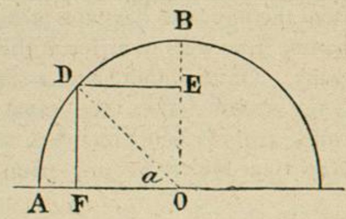

Fig. 34.

At B there is no horizontal component, and therefore there is at this point no acceleration, at any other point D, the horizontal component $=DE=FO=DO\cos a$. If the radius of the crank circle represent the centripetal force, then the horizontal component at any point=the centripetal force multiplied by the cosine of the angle of the crank.

The expression for the centripetal force in terms of the weight, the revolutions per minute, and the radius of the crank, is given on p. 161, and is $F=wN^2.\,r\times{}\cdot00034$. This may be expressed in pounds per square inch of piston area by dividing by the area of the piston. Draw a circle A B C D (fig. 35) of which the length of the radius represents the centripetal pressure per square inch of piston area. Then the pressures per square inch of piston area required to accele-

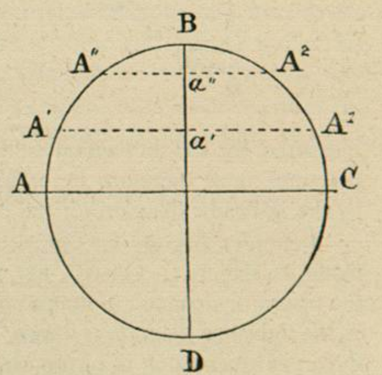

Fig. 35.