rate the reciprocating parts at any points corresponding to the positions A′A″ &c. of the crank will be represented by the lengths of the horizontal lines A′*a*′, A″*a*″ &c. While the crank is moving from A to B the pressures go to accelerate the reciprocating parts, and must therefore be subtracted from the indicator diagram pressures for the corresponding points, if we wish to arrive at the true turning effort on the crank. On the other hand, as the crank moves from B to C the opposite effect takes place, the motion of the reciprocating parts is being retarded, and imparts pressures to the crank-pin, which for any given positions A^2, A^1 &c. are

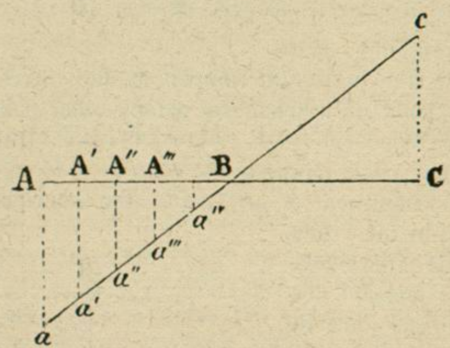

Fig. 36.

measured by the horizontal lines A^2a'', A^1a' &c. These pressures must therefore be added to the pressures shown by the indicator diagram for the corresponding points.

A simpler diagram for use with the indicator diagrams is made as follows: Let AC, fig. 36, represent the stroke of the piston or diameter of crank-pin circle, to the same scale as the indicator diagram is drawn. Calculate the centrifugal force per square inch of piston area from the known weights of the reciprocating parts, radius of the crank circle, and number of revolutions per minute, by the formula given, p. 161, and draw a perpendicular A*a* to the same scale as the pressure scale of the indicator diagram, to represent this

force. Erect an equal perpendicular from C. Join *ac*. Divide the line AC into ten equal parts corresponding to the ten divisions of an indicator diagram, then will the ordinates A*a*, A′*a*′, A″*a*″, &c. between A and B represent the pressures to be subtracted from those given by the indicator diagram, while the corresponding ordinates between B and *c* represent pressures to be added to those shown by the diagram.

Influence of the weights of the reciprocating parts in vertical engines.—If the engine were of the vertical type, we should also have to take account of the pressures required merely to overcome the force of gravity acting on the weights of the piston, &c. During the up stroke the weights act against, and during the down stroke in the same direction, as the motion of the piston. Hence we should have to modify fig. 36 as follows. Add to A*a* a portion *aa*′, fig. 37, representing the weight of the reciprocating parts per square inch of piston, to the same scale as A*a* represents the pressure required to accelerate the parts. At the other end of the stroke the action of the weights is to reduce the pressure restored by the retardation of the reciprocating parts to the crank pin. Set off therefore *cc*′=*aa*′ and join *a*′*c*′; then the ordinates of the line *a*′B′ represent the pressures to be subtracted from those given by the indicator diagram, while the ordinates of B′*c*′ represent pressures to be added to those shown by the diagram.

In the reverse stroke the action of the weights is in the same direction as the steam pressure, and aids the acceleration of the reciprocating parts at the commencement of the stroke, and increases the pressures on the crank-pin at the end. This is clearly shown by the ordinates of the line *a*′*c*′ in fig. 38. The influence of the direct weights of the reciprocating parts becomes of great practical importance in the case of the large low pressure cylinders of quick running compound engines in which the average steam pressures are low, and the weights often reach as much as 3·5 lbs. per square inch of piston.

It is impossible to exaggerate the benefit to be derived by testing the proposed indicator diagrams of any engine under design, in the manner described, before finally settling the weights of the moving parts, the pressure and distribu-

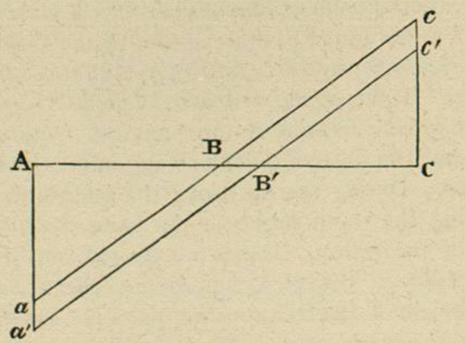

Fig. 37.

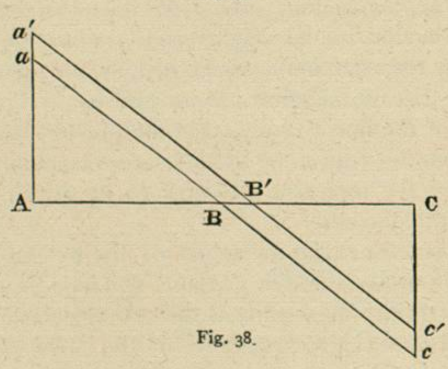

Fig. 38.

tion of the steam in a quick-running engine. This will be clearly shown from an example taken from actual practice, after we have considered the last remaining complicating circumstance, viz. the effect of the length of the connecting rod on the pressures required to accelerate and retard the reciprocating parts.

Effect of the connecting rod in modifying the influence of the reciprocating parts.—A finite connecting rod, instead of moving always parallel to the axis of the engine, vibrates from side to side, and is always inclined at an angle to the axis of the engine, except when the crank is on the dead centres. The result of this is that during the first quarter of a revolution the piston moves through more than half the stroke, and its average velocity is therefore greater than when the connecting rod is infinite, and *vice versâ* during the second quadrant the piston has to move through less than half stroke and its average velocity is therefore less. These changes are illustrated by fig. 39, in which the length of the connecting rod *ab* is three times the length of the

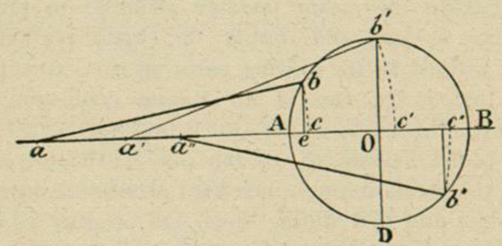

Fig. 39.

crank. While the crank has been moving from A to b' the piston has moved through a distance $Ac' = AO + Oc'$, where $Oc' = a'b'$ versin $Oa'b'$. While the crank moves from b' to B the piston moves through a distance$= c'B = OB - a'b'$ versin $Oa'b'$. During the return stroke the opposite effect takes place, while the crank moves from B to D, the piston moves through Bc', and while the crank moves from D to A, the piston moves through $c'A$. Generally speaking, if n be the ratio of the length of the connecting rod to that of the crank, then for any position of the crank, say b, in the first and second quadrants, the distance moved through by the piston$= Ae +$ $ec =$versin angle of crank $+ \, n$ versin angle of connecting rod. On the other hand for any position, say b'', in the second or third quadrants the distance traversed by the piston= versin angle of crank $- n$ versin angle of connecting rod.

The consequence of the increased velocity during the first part of the forward stroke is that more of the steam pressure will be required to accelerate the moving parts than in the case of an infinite connecting rod. The amount of this pressure varies from point to point, and is proportional to the amount of the acceleration at each point. During the latter part of the stroke, the retardation is less sudden, and consequently the moving parts never exert as great a pressure on the crank pin in coming to rest as they would do were the connecting rod infinite. During the return stroke the converse takes place. During the first portion of this stroke the steam pressure required to produce acceleration is less, and during the latter portion the pressure exerted by the moving parts on the crank pin is greater, than in the case of an infinite connecting rod. As, however, the length of the connecting rod cannot alter the *area* of the diagram, fig. 36, but only its shape, we shall find that the period during which high pressure is being exerted is less, and that during which low pressure is being exerted is greater than in the case of the infinite rod.

To calculate the exact amount of these pressures for every point of the stroke would be rather a tedious process. It is, however, easy to ascertain the pressures for three positions of the piston, and a curve drawn through these points will enable us to measure the pressures expended in accelerating the reciprocating parts, or given out during their retardation. The relative velocities of the crank-pin and the piston may easily be ascertained geometrically in the following manner.

Let AB, fig. 40, be a portion of the path of the crank pin, and C*b* any position of the crank, and *ab* the corresponding position of the connecting rod. The velocities of the two ends of the connecting rod are in different directions, the cross-head end always moving in the fixed direction *a*C

with variable velocity, while the crank pin end always moves with fixed velocity at right angles to the momentary position of the crank arm. As the velocity of the crank pin is known, that of the cross-head, which is the same as that of the piston, can be ascertained. Produce *ab* till it cuts the perpendicular CB in *e*. From *b* draw *bf* perpendicular to C*b*. Let the velocity of the crank pin be represented by the radius C*b*. Make *bf*=C*b*. Then *bf* represents the

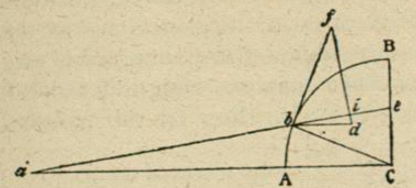

Fig. 40.

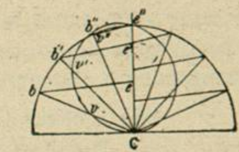

Fig. 41.

velocity of *b* both in magnitude and direction. From *f* let fall a perpendicular, *fi*, to the direction of the connecting rod, and produce the line *fi*. From *b* draw *bd* parallel to the direction of velocity of the point *a*. Then, since the connecting rod is of invariable length, the components of the velocities of each end resolved along the rod are equal. Now *bi* is this component for the velocity of *b*, therefore it is also the corresponding component for *a*, and therefore *bd* represents the magnitude as well as the direction of the velocity of *a*. Now, comparing the triangles C*be*, *bfd*, their angles are equal, because the sides of one triangle are perpendicular to those of the other; also the side *bf* equals the corresponding side C*b*, therefore the two triangles are equal, and therefore the line C*e*, cut off from CB by the prolongation of the line *ab*, represents the variable velocity of the point *a*, while C*b* represents the constant velocity of *b*.

We can now construct a very simple curve of piston velocities. For any positions of the crank, C*b*, C*b*′, &c. (fig. 41), set off C*v*, C*v*′, &c.=C*e*, C*e*′, &c. Through all the points v, v', &c., draw the closed curve $Cvv' \,.\,.\, e''C$, then the portion of the radius intercepted, at any position, between the centre C and the circumference of this curve represents the velocity of the piston for that position. It will be noticed that a portion of this curve travels outside the crank pin circle, showing that the piston velocity during a part of the revolution is greater than that of the crank pin. The maximum velocity and corresponding position of the crank can be obtained from the diagram. In practice, when the connecting rod is three or more times as long as the crank, the position for maximum velocity corresponds very nearly with the position when the connecting rod makes a tangent with the crank pin circle. Now for this position,

$$\frac{\text{length of connecting rod}}{\text{length of crank}} = \text{tan angle of crank.}$$

For instance, when the connecting rod is 3, 4, 5, or 6 times the length of the crank we have

$$\tan \phi = 3,\ 4,\ 5,\ \text{and } 6 \text{ respectively,}$$

which, by means of a table of natural tangents we find correspond with values of ϕ of $71^\circ\ 34'$, $75^\circ\ 58'$, $78^\circ\ 42'$, and $80^\circ\ 32'$ respectively.

Take the case of a piston rod four times the length of

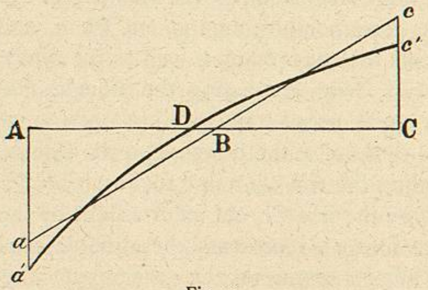

Fig. 42.

the crank. It is required to find three points on the curve $a'c'$, fig. 42 which corresponds with the straight line ac,

fig. 36, and the ordinates of which measure the pressures which have to be added to or subtracted from those of the steam diagram. The point D corresponds with the position which the piston occupies when the connecting rod makes a tangent with the crank circle—that is to say, when the crank is at the angle $75^\circ\ 58'$; for, at this point, as has been stated above, there is neither acceleration nor retardation. The points a' and c' of the curve may be found from the following considerations.

When the crank-pin gets into line with the axis of the cylinder—*i.e.* at the two dead centres, the weights of the reciprocating parts act with a full centrifugal effect on the crank. At the near dead centre they tend to pull the crank towards the cylinder, while at the far centre their action on the crank is reversed. The centripetal force in the crank at the near centre gives the measure and direction of the force absorbed in accelerating the reciprocating parts when the connecting rod is supposed infinite. When, however, the rod is of finite length another effect is produced; for on passing the near centre the crank-pin end of the connecting rod describes an infinitely small arc of a circle round the piston rod end as a centre, and a centrifugal effect is produced which increases that in the crank. On the other hand, when the far dead centre is being passed the centrifugal tendency in the connecting rod end diminishes that in the crank. The amount of the centrifugal force in the connecting rod circle may be derived from that in the crank, very simply, because the weights are common to both, also the velocities are the same, and nothing differs but the length of the radii. Now the radius of the connecting rod is four times that of the crank, therefore, as the centrifugal forces vary inversely as the radii when other things are equal, the centrifugal force in the connecting rod is one fourth of that in the crank. Add therefore to Aa, a piece $aa' = \frac{Aa}{4}$, and subtract from Cc a piece $cc' = \frac{Cc}{4}$ then will the points a' c' be the initial and terminal points on the curve. The point D has been already found. It may be proved that the curve to be drawn through the three points a', D, c' is a parabola, but for all practical purposes a circular arc is sufficiently accurate. The ordinates of this curve between a' and D represent pressures which are expended in accelerating the moving parts, which pressures must therefore be subtracted from the pressures as shown by the indicator diagram; on the other hand, the ordinates between D and c' represent pressures which are exerted by the moving parts on the crank-pin when they are being brought to rest, and which must therefore be added to the pressures as shown by an indicator diagram. For the return stroke the same diagram may be used, starting from C as the commencement of the stroke. The ordinates of c'D represent the pressures absorbed in accelerating, and those of Da' the pressures restored during retardation.

If the engine be of the vertical type a correction must be applied to fig. 42, similar to that already explained in the case of fig. 37.

An inspection of fig. 42 shows what a powerful influence on the working of the engine may be exerted by the action of the reciprocating parts. This influence may, according to circumstances, be either good or bad. Thus, take the case of a quick running single cylinder expansion engine. The steam pressure in such an engine would be high at the commencement of the stroke and low at the end, but the power required to impart motion to the reciprocating parts absorbs pressure at the beginning of the stroke, and thus relieves the pressure that would otherwise come on the crank; while at the end of the stroke the opposite effect takes place, and thus the inertia of the moving parts may tend to equalise the pressure throughout the stroke, and may consequently promote steady running. In this respect the action is similar to that of a fly-wheel. On the other hand, it may happen, if the speed at which the engine runs

is very high, or if the reciprocating parts are very heavy, that the whole of the steam pressure in the cylinder is insufficient to impart the requisite motion at the commencement of the stroke, and consequently the deficiency of force must be supplied by the fly-wheel, the result being that at the commencement of the forward stroke the connecting rod is actually being dragged by the crank-pin instead of turning the latter. As soon, however, as a certain amount of motion is imparted, the pressure of the steam begins to be felt on the crank-pin, and the strain on the rod changes from tension into compression, and a knock or jar is experienced at the joints which greatly tends to wear out the parts.

The following example, taken from the well-known Allen engine, has occasionally been adduced to illustrate this point. The essential particulars of this engine are given below:—

Diameter of cylinder	= 1 foot.
Stroke	= 24 inches.
Revolutions per minute	= 200
Weight of reciprocating parts	= 470 lbs.
Steam pressure	= 60 lbs. per sq. inch.
Ratio of length of connecting rod to crank	= 6·16 : 1.

ABC, fig. 43, shows an indicator diagram when steam is cut off at one twentieth of the stroke.

If the connecting rod were infinite, the pressure per square inch of piston area required to accelerate the moving parts at the commencement and end of the stroke would be obtained by the formula given on page 161, viz.:

$$F = \frac{wN^2r \cdot 00034}{\pi \frac{d^2}{4}} = \frac{470 \times 40000 \times 1 \times \cdot 00034}{113 \cdot 1}$$

$$= 56 \cdot 5 \text{ lbs. per square inch.}$$

Measure from A to C, and from D to C′, 56·5 lbs. to scale, and join CC′. Then the ordinates of the triangle

ACE represent pressures to be subtracted from the corresponding steam pressures as deduced from the diagram,

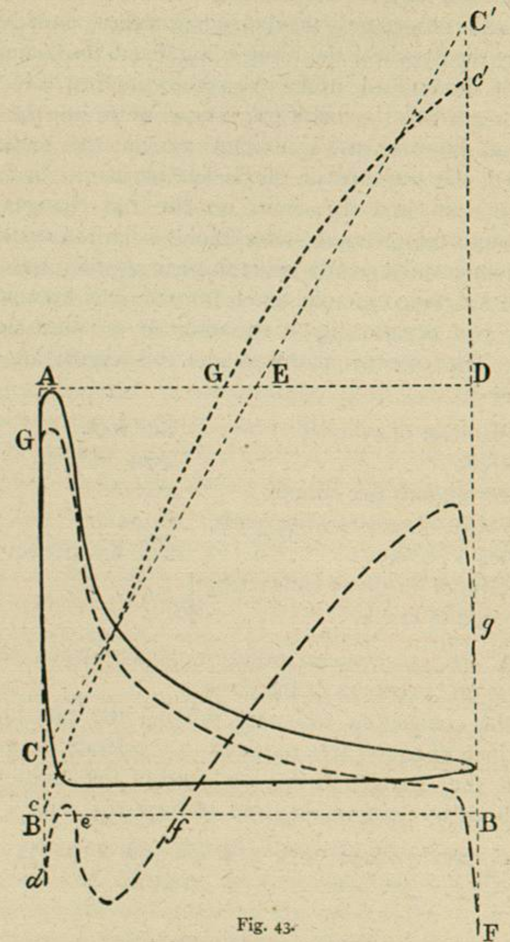

Fig. 43.

and the ordinates of the triangle DC′E represent pressures to be added to those of the diagram in order to arrive at

the true pressures transmitted through the connecting rod to the crank-pin, on the assumption that the length of the connecting rod is infinite. The effect of the connecting rod being 6·16 times the crank in length is to add $\frac{56{\cdot}5}{6{\cdot}16}$ $=9{\cdot}17$ to AC and to subtract the same quantity from DC′. Mark off therefore *c*C and *c*′C′ each $= 9{\cdot}17$ lbs. Ascertain the point G in the manner already described, and through *c*G*c*′ describe a parabolic curve. The ordinates of this curve above and below the line AD represent pressures to be respectively added to and subtracted from those deduced from the indicator diagram. We must next find from the indicator diagram the true net pressures of steam on the piston. This is done by deducting from the pressures, as shown by the diagram, the simultaneous back pressures as shown by another diagram, taken from the other cylinder cover. In the absence of such a diagram, we may make use of the back pressure line of the diagram on fig. 43, remembering, however, that from the steam pressure at A must be deducted the back pressure at the exhaust end. We thus obtain the true curve of pressures shown by the line GF beneath the expansion line of the diagram, and terminating below BB at the point F. Having now deducted from the true steam pressures the amounts given by the ordinates of the curve *c*G*c*′, we obtain the curve *defg*, the ordinates of which represent the true pressures transmitted through the connecting rod. The pressures above the line BB′ are positive, while those below are negative, and show that the steam pressure on the piston, wherever the curve falls below the line, are insufficient even to accelerate the moving parts. Thus, for instance, we have seen that at the commencement of the stroke $56{\cdot}5+9{\cdot}17=65{\cdot}67$ lbs. per square inch are required for this purpose, whereas only 60 lbs. are available, without even deducting for the back pressure. The deficiency has to be supplied by the fly wheel, and the consequence is that at the commence-