valve is at the end of its beat the port is still open to the same extent as the steam port. From G, therefore, set off the length GH equal to the half-travel of the valve *plus* the width of the steam port, then will FH represent the width of the exhaust port. The remaining dimensions are of course the same as those of the corresponding parts which have just been found.

Zeuner's Diagrams applied to Link Motion.—When a slide valve is driven by two eccentrics connected to a link, as

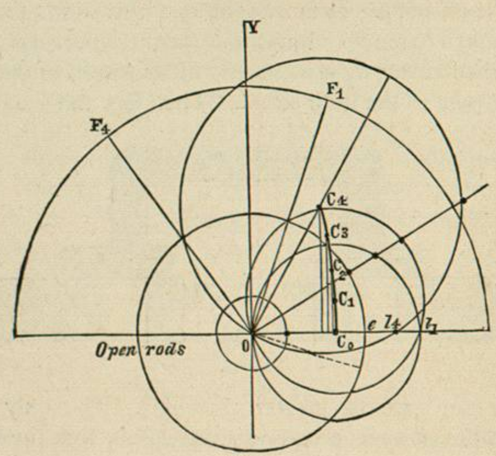

Fig. 117.

shown in fig. 94, the distance moved by the valve for any angle turned through by the crank depends upon the position of the block U in the link, and is also influenced by the manner in which the link is suspended. Neglecting for the moment the latter consideration, it is capable of analytical proof[1] that the distances moved by the valve from its central position may be represented by the chords of a pair of circles touching each other, precisely as in the case of a

[1] The proof is too long and too complicated for insertion in an elementary work. Those who wish to study it should refer to the English translation of Zeuner's work.

simple slide-gear. Moreover, to each new position of the block in the link corresponds a separate pair of valve circles, which differ both in their diameter and angle of lead from the circles corresponding to any other position of the block. It has also been found that the centres of these valve circles, figs. 117, 118, all lie upon a curve, which in the case of Stephenson's link motion with open arms is a parabola, $C_4 \ldots . C_0$, concave to the centre of the crank circle, fig. 117; while for the same motion with crossed arms the parabola is convex to the centre (see fig. 118).

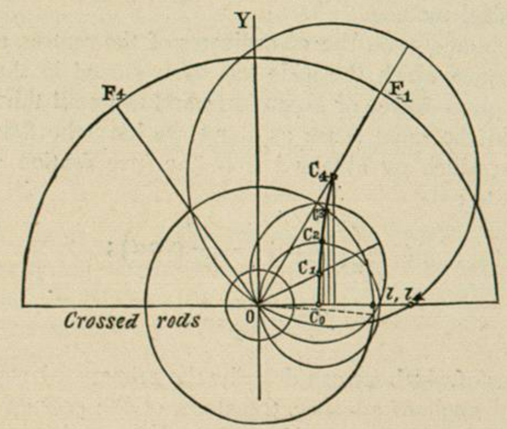

Fig. 118.

Calling the angle of advance α, the throw of the eccentric t, the length of the eccentric rod l, and the half-length of link $= k$, the parameter of this parabola $= \frac{lt \cos \alpha}{2k}$; while the distance between the vertex of the parabola and the centre O

$$= \frac{t}{2}\left(\sin \alpha \pm \frac{k}{l} \cos \alpha\right),$$

the *plus* or *minus* sign being used respectively according as the arms are crossed or open.

Analytical method of finding the centres of the valve circles in link motions.—One of the main problems in connection with the diagrams for link motion is to fix the positions of the centres of the primary valve circles, fig. 117, corresponding to the given positions of the sliding block in the link. These circles and the inside and outside lap circles being drawn in place, it is perfectly easy to trace the variations in the lead, release, compression, &c., corresponding to the varying points of cut-off. These points may be fixed either by analytical means, or with very approximate accuracy by a graphical method.

Let x and y be the co-ordinates of the centres, and u the distance which the slide has been moved in the link for the given degree of expansion; and using all the other letters in the same sense as above, we have the following formulæ, which are obtained from the investigation above referred to; viz.—

$$x = \frac{t}{2}\left(\sin\alpha + \frac{k^2 - u^2}{kl}\cos\alpha\right);$$

$$y = \frac{tu\cos\alpha}{2k}$$

by means of which we can describe the primary valve circles when the angle of advance, the throw of the eccentric, the lengths of link and eccentric rods, and the position of the block in the link are given.

Let, for instance, the link be of the sort illustrated in fig. 97, so that the block when fully raised or lowered in the link comes exactly opposite the ends of the eccentric rods.

Let the half-length k of the link be divided into say four divisions, called grades of expansion. Then u may be expressed as a fraction of k. Thus, at the third grade, $u = \frac{3k}{4}$.

At the fourth grade $u = \frac{4k}{4} = k$; and so on.

Substituting these values of u in the formulæ given above, we have, when the fourth grade of expansion is used,—

$$x = \frac{t}{2}\sin\alpha;$$

$$y = \frac{t}{2}\cos\alpha.$$

At the third grade, when $u = \frac{3k}{4}$, we have—

$$x = \frac{t}{2}\left(\sin\alpha + \frac{\frac{7}{16}k}{l}\cos\alpha\right);$$

$$y = \frac{3t}{8}\cos\alpha.$$

For the middle or dead point $u = 0$, and—

$$x = \frac{t}{2}\left(\sin\alpha + \frac{k}{l}\cos\alpha\right);$$

$$y = 0;$$

which shows that for this position of the slide block the centre of the valve circle lies in the straight line OX.

Let, for example, the angle of advance be 30°, the throw of the eccentric $1\frac{1}{4}$ inches, the half-length of link 3 inches, and the length of the eccentric rod 30 inches. Substituting these numerical values in the above formulæ, we obtain the centres of the primary valve circles as shown in fig. 117. Thus, for the largest circle,

$$x = \frac{1{\cdot}25}{2}\times\frac{1}{2} = {\cdot}312;\ y = \frac{1{\cdot}25}{2}\times{\cdot}866.$$

The ordinates of the centres c^4, c^3, &c., in fig. 117, are obtained in this way. In order to avoid unnecessary complications of the diagram, the valve circles for the second and third grades of expansion are omitted.

We can see at a glance from fig. 117 how completely all the critical points connected with the distribution of the steam are altered by the position of the sliding block. For instance, with the laps as given in the fig., and the link in full forward gear, the steam is cut off at F^4, and the lead equals el_4; while at the first grade of expansion the cut-off is earlier, viz. at F_1, while the lead is increased to el_1, and similarly the alteration in the points of compression and release may be ascertained by tracing the intersections of the primary valve circles with the inner lap circle.

The alteration of the lead with the rate of expansion is one of the peculiarities of the Stephenson link motion. In the example first given, in which the eccentric rods are open the lead increases with the rate of expansion; but if the rods are crossed, the contrary takes place, the lead decreasing. It will also be observed that the travel of the valve, which is represented by twice the diameter of the primary valve circles, varies with each rate of expansion, continually diminishing till it reaches a minimum, when the block occupies the middle of the link. Consequently, if in full forward gear, the valve completely uncovers the steam port and no more, for each succeeding rate of expansion the maximum opening of the port is reduced, till, at the central position, the maximum opening only equals the lead. As a consequence of this peculiarity it is necessary to make the ports of engines provided with link motion unusually broad, as otherwise the steam would be dangerously throttled at the higher rates of expansion.

The distribution of the steam when the block is in mid-gear is very peculiar. By reference to fig. 117 it will be clear that the steam is cut off at about a quarter-stroke, while it is released shortly after half-stroke, and admitted to the other side of the piston at about three-quarter stroke; also compression commences on one side of the piston very soon after expansion begins on the other. The consequence is that with the block at mid-gear it is impossible for the piston to make a stroke.

Geometrical method of finding the centres of valve circles in link motions.—It is found practically more convenient to fix the positions of the centres of the primary valve circles

by a graphic method of construction rather than by calculation. The method in common use, which will now be explained, though not theoretically exact, is quite accurate enough for all practical purposes.

As before, the throw and position of the eccentric and the lengths of the link and eccentric rods are supposed to be given. One of the results obtained by the analytical investigation of the subject is that the radius of the curvature of the link should always be equal to the length of the eccentric rod. This fact is made use of in the following construction. Let LL′ (fig. 119) represent the length of the link. Bisect LL′ in O, and through O draw OA at right angles to LL′. With centre L, or L′, and radius equal to the length of the eccentric rod, describe an arc intersecting OA in C. With centre C, and radius equal to the length of the eccentric rod, describe the arc LL′, which represents the centre line of the link. With centre C

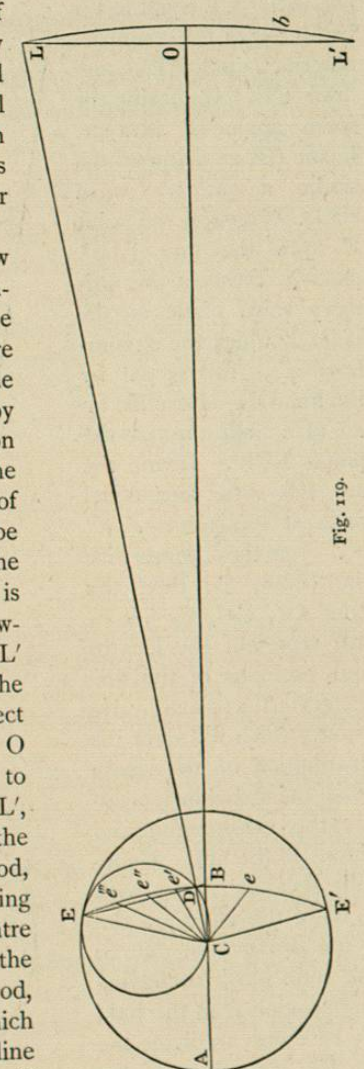

Fig. 119.

and radius CE equal to the half throw of the eccentrics describe a circle EE′A, and draw CE, CE′, having the given angle of advance. Upon CE as diameter describe a primary valve circle. Through the point E draw the line EDB, passing through the primary valve circle at the point D where the circumference is intersected by the line OL. Describe an arc of a circle through the points EBE′. Divide the arc EB into four equal parts at the points *e*′, *e*″, *e*‴. Join these points with the centre C; then the lines C*e*′, C*e*″, C*e*‴, CE, will represent the lengths and positions of the diameters of the primary valve circles, which will show the distribution of the steam when the slide block occupies the corresponding positions in the link. Generally speaking, if we wish to know the effect on the steam distribution due to the slide block occupying any position *b* in the link, we have only to divide the arc EE′ into two portions,

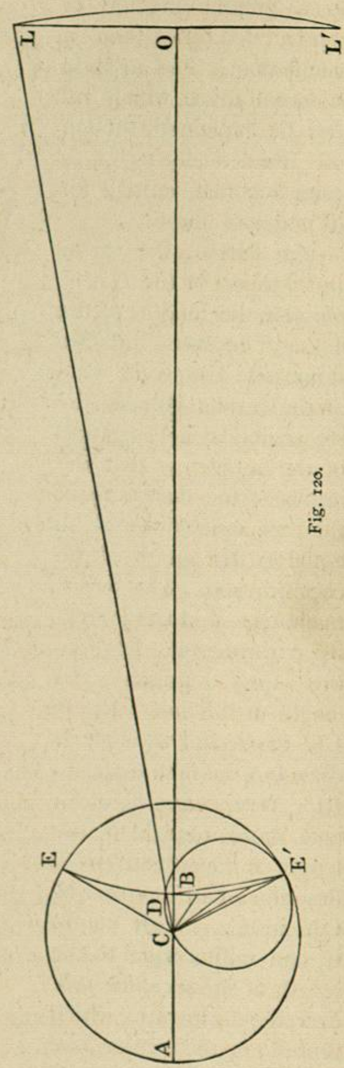

Fig. 120.

at *e* bearing to each other the same proportion which L*b* bears to *b*L′. By connecting the point *e* with the centre C we obtain the length and position of the primary valve circle, which will illustrate the distribution of the steam.

If the eccentric rods had been crossed instead of open we should have proceeded as above, except that to find the position of the point B we should have joined C with L; then the line joining E′ and the intersection of CL with the circumference of the primary valve circle will, at its intersection with the line CO give a point B, such that the curve drawn through EBE′ is convex to C, instead of being concave as in the former case. This is illustrated in fig. 120, which is merely a portion of fig. 119, the rods being supposed to be crossed.

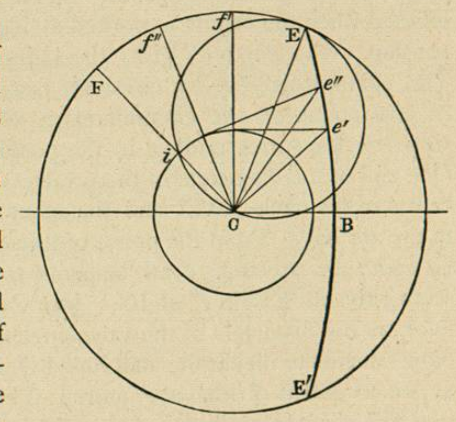

Fig. 121.

Supposing that, instead of having to find the distribution of the steam when the position of the block in the link is given, the problem be reversed, and we are given the positions of the crank at which the steam is to be cut off, and are required to find the proper method of dividing the quadrant of the reversing lever, so as to provide for the given rates of expansion, the other data being as above, we should proceed as follows. The link is supposed to be of the type illustrated in fig. 97, which admits of the block being put into full gear.

Let CE, fig. 121, be the radius and position of the eccentric. Let the arc EE′ be drawn in the manner already described from the known dimensions of link and eccentric rods. Let CF be the position of the crank when the steam is to be cut off, when the block is in full forward gear, and let f' f'' be the points where the cut-off is required to be effected when the engine is worked at higher grades of expansion. The length C*i* gives the radius of the lap circle. The points where the lap circle intersects the two radii *cf* ′ *cf* ′ give points on the circumferences of the two primary valve circles corresponding to the points of cut-off f'', f'. One end of the diameters of these valve circles is in O, the centre of the circle EAE′, and the other ends are situated in the arc EE′. From the points of intersection of Cf'' and Cf', with the lap circle draw tangents to this circle intersecting the arc EE′ in e'' and e'. Join Ce'' and Ce'. These lines are the diameters of the valve circles required. It now only remains to divide the half-link LO, in the same ratio as the arc EB is divided at e'' and e'. These points of division will give the positions of the slide block necessary in order to effect the required expansion ; and the half of the expansion lever quadrant, fig. 94, requires to be divided in the same ratio, in order to obtain the positions of the notches necessary to bring the slide block into the required positions.

If we had not originally been given both the length and position of CE, but had been furnished instead with some of the data given in Problems II. to VI., we could have proceeded to find the throw and angle of advance in the manner explained in those problems by means of a separate diagram, afterwards proceeding as above.

We have hitherto considered cases in which the data relate either to the position and throw of the eccentric, or else to the lead, cut-off, and opening of the valve when the link is in *full gear*. We might, however, have to solve a question, the data referring to, say, the lead, point of cut-off, and maximum opening of the valve when the slide block

occupied some position in the link *intermediate between full and central gear*, the lengths of link and eccentric rods being given as before. Find the centre C, fig. 122, and describe the curve of the link as explained. Next, with the aid of a separate diagram, find the position and throw of an eccentric which, if connected direct to the valve spindle, will give the required lead, cut-off, and maximum opening of the valve (see Problem VI.). Transfer the result to fig. 122, C*e* being the position and length of the throw as found. On C*e* as diameter describe a primary valve circle. Let *b* be the given intermediate point in the link which has to drive the valve in the manner required. Join *b*C, intersecting the circumference of the primary valve circle in D. Join *e*D, and prolong it to intersect the line CO in B. Describe an arc of a circle which shall pass through *e* and B, and be

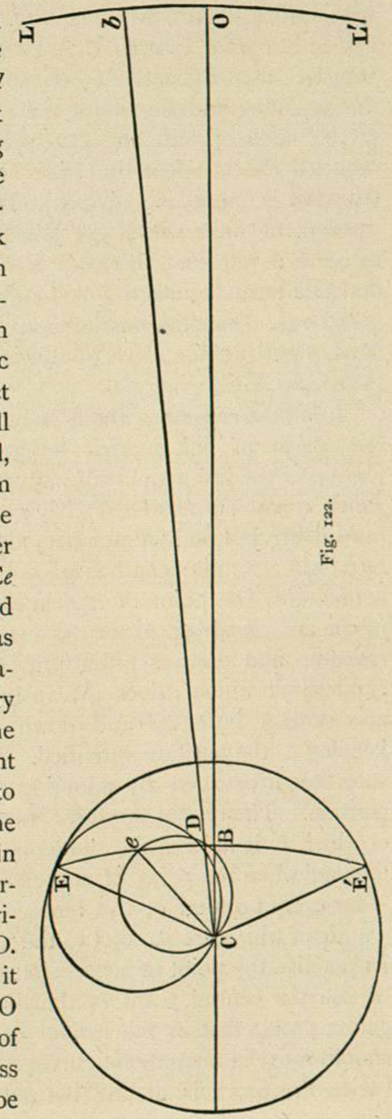

Fig. 122.

symmetrical about the line CO. Prolong this arc, and find in it a point E such that E*e* is to *e*B as *b*L is to *b*O. Join EC, then EC and the corresponding line E′C will give the positions and throws of the eccentrics, which with the given length of link and eccentric rods will produce the required effects when the slide block is at *b*. This construction is useful in solving problems connected with the working of links which are joined to the eccentric rods in such a way that the slide block can never be brought opposite to the connecting-rod ends. Such a link is shown in fig. 97A. Once the position and length of the line CE are fixed, other problems can be solved in the same manner as in the preceding example.

In all the diagrams which have been given to illustrate the action of link gearing it has been assumed that the motion of the link was in no way affected by the manner in which it was suspended. This assumption is, however, far from being justified by practice, for, the link being held up by a rod *l*′, of finite length (see fig. 94), which oscillates about a point M, the point of suspension P consequently moves in an arc of which M is the centre, and must therefore move up and down a little during each stroke. The point of the link which drives the valve spindle must therefore also slide a little up and down in the link, instead of keeping to the position intended. It is very easy to aggravate this irregularity by adopting a wrong method of suspension. The end of the rod *l*′ moves in an arc of a circle of which K is the centre. The object aimed at will be best attained when the point of suspension is made to oscillate —for every position of the block in the link—in arcs, the chords of which are parallel to the line of the valve spindle. In practice the point of suspension is either the lower end or else the central point of the link. Theoretical investigation proves that in the former case the point M, fig. 94, should move in a parabolic curve, the highest position of M (when the block is at the bottom of the link) being the

vertex of the parabola, and the parameter being equal to twice the length of the eccentric rods; the co-ordinates of the vertex referred to C, fig. 123, as origin and to CX and CY as axes, being C*x* = the length of the eccentric rod, and C*y* = the length of the lifting link *l*.

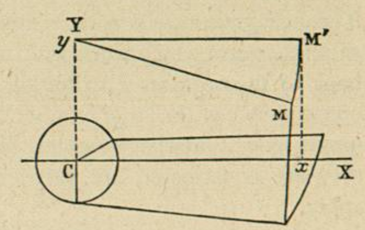

Fig. 123.

In practice, instead of the parabolic curve we may make use of a circular arc, the radius of which is equal to the length of the eccentric rod, and the centre of which is vertically above the point C at a distance $= l$.

When the link is suspended from its central point it is found that the point M, fig. 124, should also move in a parabola, the middle position of M corresponding with the vertex, and the parameter being twice the length of the eccentric rod.

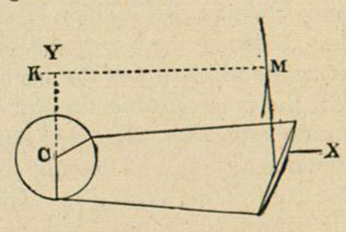

Fig. 124.

The axis of the parabola is parallel to the line CX, and the co-ordinates of the vertex are C*x* = the length of the eccentric rod $-\frac{k^2}{2l}$ (where k = the half-length of the link, and l = the length of the eccentric rod), and C*y* = the length of the lifting link *l*. In practice we may substitute for the parabola a circular arc of which the centre lies in a line KM parallel to CX, and at a distance above it equal to *l*, the central point K lying to the left of Y at a distance $=\frac{k^2}{2l}$.

In actual practice it is never possible to give the arm

KM a length equal to the eccentric rod, but it is always desirable to make it as long as possible.

Zeuner's Diagrams applied to Meyer's Valve Motion.—These diagrams are peculiarly applicable to the investigation of those gears which work with an expansion valve on the back of the ordinary distribution valve. It is true that in consequence of the large number of circles employed the diagrams look somewhat complex, but the principles on which they are constructed are very easy to understand and to apply.

It is perfectly obvious, from what has gone before, that a separate primary valve circle may be employed to show the distance which each valve circle has travelled from its central position, for every position of the crank. Consequently the difference between the lengths of the chords of the two circles got by drawing the direction of the crank in any position, gives the distance apart of the centres of the two valves for that position of the crank. Thus in fig. 125 let CE denote the length and position of the radius of the eccentric which drives the main or distribution valve, and CK the corresponding throw and position of the eccentric for the expansion valve. On each of these lines as diameter describe a circle, and describe the circle AEA′, to represent the path of the crank-pin. Then the circle described on CE shows in the usual way, in conjunction with the lap circle, the lead, cut-off, &c., as provided for by the distribution valve. Also for any position of the crank such as CD, the chord C*e* shows the distance moved by the main valve from its central position, while the chord C*e*′ shows the corresponding distance travelled by the expansion valve; therefore C*e* − C*e*′ = *e*′*e* shows the distance apart of the central lines of the two valves for the position CD of the crank.

We will next prove that it is possible to draw a third circle, viz. CG, the chords of which, such as Cϵ, will represent the differences C*e*−C*e*′, and which chords will consequently

show at a glance the distances apart of the centres of the two valves for any position of the crank.

Join EK, and from C draw CG parallel to EK, and from E draw EG parallel to CK; then CG will represent the length and position of the diameter of the third or resultant circle. The problem is to prove that Cϵ = *e*′*e*. Join Gϵ and K*e*′. From E let fall a perpendicular EB on K*e*′ produced. Then in the two triangles GCϵ and KEB we have the side GC equal to the side KE, being the opposite sides of a

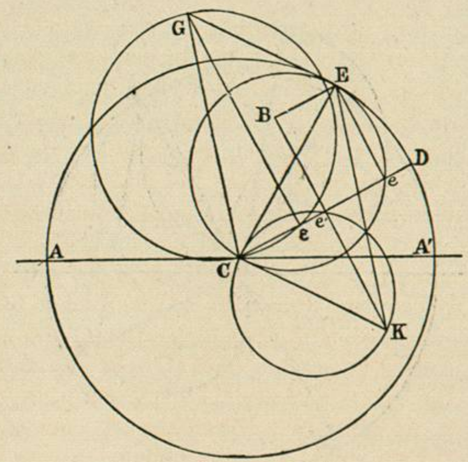

Fig. 125.

parallelogram; also the angle GCϵ equals the angle BEK since each of them is the sum of two alternate and consequently equal angles. Also the angle GϵC is a right angle, being the angle in a semicircle; and EBK is a right angle by construction; therefore the two triangles are equal, and Cϵ equals BE. But BE can easily be proved to be equal to *e*′*e*. For join E*e*; then in the quadrilateral B*e* we have the angle at B a right angle by construction; also the angle B*e*′*e* is equal to the vertically opposite angle C*e*′K, which being the angle in a semicircle is also a right angle. For