This is also the pressure of the point H in the diagram of the small cylinder, where

$$KD = \frac{OG}{R} = v - \frac{v}{r'} = v\left(1 - \frac{1}{r'}\right)$$

As soon as the steam is cut off in the low-pressure cylinder it expands to the end of the stroke, and as all the steam which is admitted to the high-pressure cylinder must eventually find its way to the low-pressure cylinder, its final pressure $PL = \frac{p}{E}$. At the same time the steam in the receiver is being compressed by the advancing piston of the small cylinder, and finally attains the pressure $p_R$ which was assumed to exist in it just before the small cylinder exhausted into it.

The value of $p_R$ can be easily expressed in terms of the known quantities. For, as we have seen, the volume of steam in the large cylinder when communication with the receiver was cut off was $\frac{V}{r'}$,

and its pressure

$$\frac{\frac{pv}{r} + p_R V_R}{\frac{V}{r'} + V_R + v\left(1 - \frac{1}{r'}\right)}$$

Now at the end of the stroke the volume is V and the pressure is $\frac{p}{E}$. Hence the products of these two quantities must be equal, or

$$\frac{\frac{pv}{r} + p_R V_R}{\frac{V}{r'} + V_R + v\left(1 - \frac{1}{r'}\right)} \times \frac{V}{r'} = V \times \frac{p}{E}$$

An expression from which $p_R$ may be obtained.

The above equation may be simplified by substituting for

$V_R$ its equivalent $\rho v$
$V$ „ $Rv$
$E$ „ $Rr$

And then reducing we get

$$p_R = \frac{p}{E}\left(\frac{r' - 1}{\rho} + r'\right)$$

An equation which gives the receiver pressure just before the small cylinder exhausts into it, in terms of the initial pressure, the total rate of expansion, the rate of expansion in the low-pressure cylinder, and the ratio which the volume of the receiver bears to that of the small cylinder.

In the diagram fig. 201 OE′ represents the pressure $p_R$, and the curve HE′ shows the line of compression as it affects the small piston.

In most cases the pressure in the receiver is less than the terminal pressure in the small cylinder, and consequently there is the sudden drop shown in the diagram by the line CE. It is obvious, however, from the above equation that the value of the receiver pressure can be varied by varying $r'$, the rate of expansion in the low-pressure cylinder. If it be wished to adjust the cut-off in this cylinder, so that the receiver pressure may just equal the terminal pressure in the small cylinder, we can easily find the required rate of expansion by equating the value just found for the receiver pressure with the terminal pressure of the small cylinder—viz. $\frac{p}{r}$, and solving for $r'$.

*Compound Engines with Receivers and Cranks at right angles.*—In tracing the diagrams of the above type of engines we must remember that, neglecting the effect of the length of the connecting-rod, the large piston is always at mid-stroke when the small piston is at either end of its cylinder; consequently the exhaust steam from the small cylinder enters the receiver when the large piston is at mid-stroke. Whether it also simultaneously enters the large cylinder depends upon the point of cut-off in the latter; if this be before half-stroke, the steam cannot enter, but if after half-stroke, it must enter the large cylinder as well as the receiver.

In the high-pressure cylinder the steam enters during the portion AB of the stroke depending on the point of cut-off, and expands to C, the end of the stroke. At C the exhaust takes place, and the pressure falls to E. The next step depends on the rate of expansion in the large cylinder. If this be after half-stroke the steam will expand in receiver and large cylinder together till the cut-off takes place. Suppose this to happen when the small piston has travelled through DG of its return stroke, and the pressure has then fallen to F. After this point the small piston continuing to advance compresses the steam in the receiver till it reaches the point I corresponding to half-stroke, the pressure having then risen to H. Now when the small piston is at mid-stroke the large one is just about to commence its stroke, and the large cylinder is just commencing to take steam from the receiver. Hence the pressure in the latter will fall till the end of the stroke of the small piston, finally attaining the value OK.

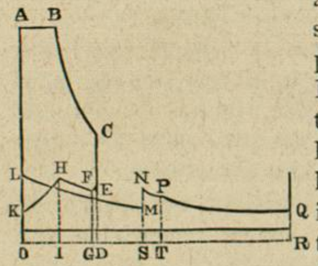

Fig. 202.

The diagram of the large cylinder must next be drawn, the line of volumes, as usual, corresponding with the ratio between the two cylinders. The initial pressure is the same as the receiver pressure when the small piston is at half-stroke, viz. HI. Set off, therefore, OL = HI to represent this pressure. From this point the steam expands in receiver and large cylinder till half-stroke, when the small cylinder exhausts into the receiver. At half-stroke, then, the pressure in the cylinder is the same as the pressure in the receiver when the small piston has returned to the end of the stroke, viz. OK. Erect therefore SM = OK. At this point the pressure suddenly rises to N, so that SN = DE, and then the steam again expands in receiver and large cylinder till the point of cut-off T is reached, when the pressure TP is the same as FG. From this point onwards the steam expands in the large cylinder till the end of the stroke R, while the steam in the receiver is compressed as already shown by the line FH. At the end of the stroke the pressure in the large cylinder falls to that of the condenser.

To express these various pressures in terms of the initial pressure and the volumes of the two cylinders and the receivers, we proceed precisely as in the first example. Let $k$ denote the portion DG of the stroke passed over by the small piston, when steam is cut off in the large cylinder; then $GO = (1-k)v$ is the fraction of the volume of the small cylinder which at that moment is open to the exhaust. The terminal pressures CD and QR are exactly the same as in the first example, viz. $\frac{p}{r}$ and $\frac{p}{E}$ respectively. Also the pressure in the large cylinder at the point of cut-off equals the terminal pressure multiplied by the rate of expansion; or $TP = \frac{pr'}{E}$, and this, as already shown, is the same as the pressure GF. At this point the volume occupied by the steam is made up of two parts, viz.

The contents of the reservoir $= V_R$
and the portion GO of the small cylinder $= (1 - k)v$
and consequently the total volume $= V_R + (1 - k)v$

As soon as steam is cut off in the low-pressure cylinder, it is compressed in the small cylinder and receiver along the line FH till the high-pressure piston is at half-stroke, when the volume occupied by the steam $= V_R + \frac{v}{2}$,

and the pressure

$$\mathrm{IH} = p_{\mathrm{R}} = \frac{pr'}{\mathrm{E}} \times \frac{\{\mathrm{V_R} + (1 - k)v\}}{\mathrm{V_R} + \frac{v}{2}} = \frac{pr'}{\mathrm{E}} \times \frac{\rho + 1 - k}{\rho + \frac{1}{2}}$$

This is also the value of OL, the initial pressure in the large cylinder. From thenceforward the steam enters the large cylinder and expands in it and the receiver till half-stroke, when its volume $= \mathrm{V_R} + \frac{\mathrm{V}}{2}$.

To obtain its pressure SM at this point we have

$$\mathrm{SM} \times \left(\mathrm{V_R} + \frac{\mathrm{V}}{2}\right) = \frac{pr'}{\mathrm{E}} \times \frac{\mathrm{V_R} + (1 - k)v}{\mathrm{V_R} + \frac{v}{2}} \times \left(\mathrm{V_R} + \frac{v}{2}\right)$$

$$\therefore \mathrm{SM} = \frac{pr'}{\mathrm{E}} \times \frac{\mathrm{V_R} + (1 - k)v}{\mathrm{V_R} + \frac{\mathrm{V}}{2}} = \frac{pr}{\mathrm{E}} \times \frac{\rho + 1 - k}{\rho + \frac{\mathrm{R}}{2}}$$

It now only remains to find the pressure SN = DE, which is that of the receiver when the small cylinder exhausts into it. The resultant pressure and volume is derived from two components, one being that of the steam in the small cylinder, whose volume is $v$ and pressure $\frac{p}{r}$, and the other that of the steam in the receiver and half the large cylinder, whose volume is $\mathrm{V_R} + \frac{\mathrm{V}}{2}$, and whose pressure SM is given above. Compounding these two we get

$$\frac{\frac{p}{r}v + \frac{pr'}{\mathrm{E}} \times \frac{\mathrm{V_R} + (1 - k)v}{\mathrm{V_R} + \frac{\mathrm{V}}{2}} \times \left(\mathrm{V_R} + \frac{\mathrm{V}}{2}\right)}{v + \mathrm{V_R} + \frac{\mathrm{V}}{2}}$$

Substituting for $\mathrm{V_R}$, V and $r$ their values $v\rho$, $v$R, and RE, reducing, we get

$$\mathrm{SN} = \mathrm{DE} = \frac{p}{\mathrm{E}} \frac{r'\{\rho + 1 - k\} + \mathrm{R}}{1 + \rho + \frac{\mathrm{R}}{2}}$$

Here, again, there is a considerable drop of pressure in the receiver, and an increase of pressure in the low-pressure diagram at half-stroke. In order that there should be no drop it would be necessary that the terminal pressure in the small cylinder should be equal to the receiver pressure. Then the resultant pressure would be the same as either of its components, and the line CD would equal DE. Equating the values previously given for these two pressures, and solving for R, we can if desired find out what must be the ratio between the two cylinders, with a given rate of expansion in the large cylinder, in order that the receiver pressure may be equal to the terminal pressure in the small cylinder.

When the cut-off in the low-pressure cylinder takes place *before* half-stroke the diagrams will differ somewhat from those explained in the preceding example.

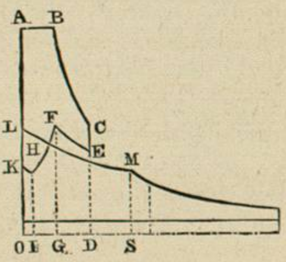

Fig. 203.

At C, fig. 203, the high-pressure cylinder exhausts into the receiver, the pressure falling to DE. Now when the small piston is at the end of its stroke the large piston is at half-stroke, and therefore the steam is already cut off, and consequently the exhaust steam from the small cylinder only enters the receiver, and not the receiver plus half the large cylinder, as in the previous example. When the small piston makes the return stroke it compresses the exhaust steam before it and in the receiver till mid-stroke, the line EF being the curve of compression. When the small piston is at mid-stroke the large piston is at one end of its cylinder, and consequently draws steam from the receiver till the cut-off is effected and the steam expands in the receiver and the large cylinder along the curve FH. When the small piston occupies the position I the steam is cut off in the large cylinder, and the steam in the receiver is then compressed by the small piston along the line HK till the end of its stroke.

The initial pressure OL in the large cylinder is of course equal to the receiver pressure GF. From the point L the steam expands in receiver and large cylinder together till the point of cut-off S is reached, and from this point expansion takes place in the large cylinder alone till the end of the stroke.

The values of the terminal pressures in the two cylinders are the same as in the previous example, viz. $\frac{p}{r}$ and $\frac{p}{\mathrm{E}}$. At the point of cut-off in the large cylinder, the pressure SM equals the terminal pressure, multiplied by the rate of expansion in the large cylinder ; therefore

$$\mathrm{SM} = \frac{p}{\mathrm{E}} \times r' = \mathrm{HI}$$

Immediately the cut-off is effected the volume of the receiver steam is made up of two parts, viz. the volume of the receiver $\mathrm{V_R}$ and the portion OI of the small cylinder which the piston has yet to travel at the moment of cut-off in the large cylinder. Now OI $= (1 - k)\,v$. Therefore the total volume occupied by the receiver steam is $\mathrm{V_R} + (1 - k)\,v$. By the time the small piston reaches the end of its stroke this volume is reduced to $\mathrm{V_R}$, and the pressure

$$\mathrm{OK} = \frac{pr'}{\mathrm{E}} \times \frac{\mathrm{V_R} + (1 - k)\,v}{\mathrm{V_R}}$$

The small cylinder next exhausts into the receiver, and a volume of steam having the above pressure and volume $= \mathrm{V_R}$ becomes mixed with the contents of the small cylinder having a pressure $\frac{p}{r} = \frac{p\mathrm{V}}{\mathrm{E}v}$ and volume $= v$.

The resulting pressure

$$\mathrm{DE} = \frac{\frac{pr'}{\mathrm{E}} \times \frac{\mathrm{V_R} + (1 - k)\,v}{\mathrm{V_R}} \times \mathrm{V_R} + \frac{p\mathrm{V}}{\mathrm{E}v} \times}{\mathrm{V_R} + v}$$

$$= \frac{\frac{pr'}{\mathrm{E}}(\mathrm{V_R} + (1 - k)\,v) + \frac{p\mathrm{V}}{\mathrm{E}}}{\mathrm{V_R} + v} = \frac{p}{\mathrm{E}}\,\frac{r'(\rho + 1 - k) + \mathrm{R}.}{\rho + 1}$$

This body of steam is compressed in the receiver by the advancing high-pressure piston till it reaches mid-stroke, when communication is opened with the large cylinder. Therefore, the volume is reduced at this point by half the contents of the small cylinder, and becomes $\mathrm{V_R} + \frac{v}{2}$.

While its pressure

$$\mathrm{GF} = \frac{p}{\mathrm{E}}\,\frac{r'(\mathrm{V_R} + (1 - k)\,v) + \mathrm{V}}{\mathrm{V_R} + \frac{v}{2}} = \frac{p}{\mathrm{E}}\,\frac{r'(\rho + 1 - k) + \mathrm{R}}{\rho + \frac{1}{2}}.$$

This is also the initial pressure OL in the large cylinder.

In order to avoid drop in the receiver when the small cylinder exhausts into it, we should have, as before, to equate the values of DC and DE, and solve for R, which would give us the necessary ratio of the two cylinders for the given rate of expansion.

The value of $k$, the fraction of the stroke traversed by the piston of the small cylinder when steam is cut off in the large cylinder, is not the same in the two examples given. Let CE be the position of the crank of the large cylinder when steam is cut off in it, in the case of the first example, i.e., after half-stroke. Then the corresponding position of the high-pressure crank is found by drawing FC at right angles to EC ; and H and G are the corresponding positions of the two pistons. Then $\frac{\mathrm{AG}}{\mathrm{AB}} = k$ is the fraction of stroke traversed by the small piston when steam is cut off in the large cylinder.

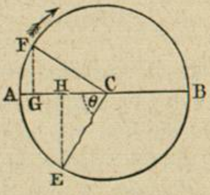

Fig. 204.

Calling the radius of the circle unity, we have—

$$AG = 1 - GC = 1 - \cos FCA = 1 - \sin\theta;$$

$$\text{and } \frac{AG}{AB} = k = \frac{1 - \sin\theta}{2}.$$

$$\text{Also } \frac{AB}{BH} = r' = \text{rate of expansion in large cylinder.}$$

$$\therefore \frac{2}{1 + \cos\theta} = r'.$$

$$\therefore \cos\theta = \frac{2 - r'}{r'}.$$

Deducing the corresponding value of $\sin\theta$ in terms of $r'$, we have—

$$k = \frac{r' - 2\sqrt{r' - 1}}{2r'}.$$

The case when steam is cut off in the low-pressure cylinder before half-stroke is represented by fig. 205, the same letters being used as in the previous example.

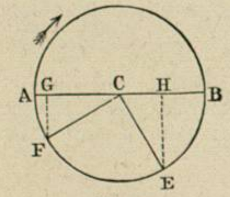

Fig. 205.

In this case it can easily be proved that

$$k = \frac{r' + 2\sqrt{r' - 1}}{2r'}.$$

Space will not permit of a full investigation being given for all the possible arrangements of cylinders, but the principles on which all such calculations proceed, having been fully illustrated in the three examples just given, the student will have no difficulty in applying them to the cases of ordinary compounds with two low-pressure cylinders and cranks at any given angles, or to the case of triple compound engines. In the case of ordinary compounds with two low-pressure cylinders and cranks set at angles of 120° with one another it is only necessary to bear in mind that three separate cases may occur. According as the small cylinder exhausts—

1. Into the receiver only; which it does when the cut-off in the large cylinder takes place before one-quarter stroke
2. Into the receiver and one of the large cylinders; which takes place when the cut-off in each low-pressure cylinder takes place between one-quarter and three-quarters stroke.
3. Into the receiver and both large cylinders; which takes place when the cut-off in the latter is after three-quarters stroke.

The latter case never occurs in practice, because the distribution of the steam would be very bad, as the high-pressure cylinder would discharge into one of the large cylinders when its piston was at one-quarter stroke, and into the other at three-quarters stroke, i.e. just before the cut-off took place. Hence, the amount of work done by each of the two large cylinders would be very unequal

*Actual Indicator-diagrams of Compound Engines.*—We give below a few specimens of indicator diagrams of various types of compound engines. The first is taken from a tandem, or direct expansion engine. The upper and larger diagram is from the small cylinder, while the lower one is from the low-pressure cylinder. The latter diagram, however, gives no idea to the eye of the relative work done by this cylinder, for it must be borne in mind that, though the pressures shown are low, the area of piston on which they act is, as a rule, from three to four times that of the high-pressure piston. In a subsequent example it will be shown how to combine the diagrams of the several cylinders of a compound engine, so that the work done by each may

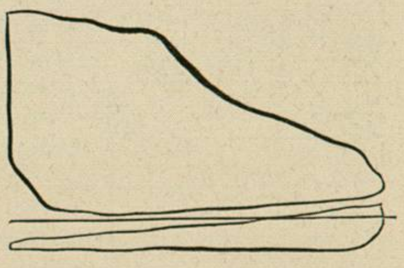

Fig. 206.