Graphical Analysis
OF
Roof Trusses

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Duchemin's formula

$n = \frac{p \cdot 2\sin^2\alpha}{1+\sin^2\alpha}$ = normal pres

$\angle\alpha$

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WORKS OF PROF. C. E. GREENE

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A MANUAL FOR DESIGNERS, AND A TEXT-BOOK FOR TECHNICAL SCHOOLS

TRUSSES AND ARCHES

ANALYZED AND DISCUSSED BY GRAPHICAL METHODS

BY

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IN THREE PARTS.

I.

ROOF-TRUSSES: Diagrams for Steady Load, Snow, and Wind.

II.

BRIDGE-TRUSSES: Single, Continuous, and Draw Spans; Single and Multiple Systems; Straight and Inclined Chords.

III.

ARCHES, in Wood, Iron, and Stone, for Roofs, Bridges, and Wall-Openings; Arched Ribs and Braced Arches; Stresses from Wind and Change of Temperature; Stiffened Suspension Bridges.

Part I.—ROOF-TRUSSES.

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PREFACE TO PART I.

The use of Graphical Analysis for the solution of problems in construction has become of late years very wide-spread. The representation to the eye of the forces which exist in the several parts of a frame possesses many advantages over their determination by calculation. The accuracy of the figures is readily tested by numerous checks. Any designer who fairly tries the method will be pleased with the simplicity and directness of the analysis, even for frames of apparently complex forms. Those persons who prefer arithmetical computation will find a diagram a useful check on their calculations. Being founded on principles absolutely correct, these diagrams give results depending for their accuracy on the exactness with which the lines have been drawn, and on the scale by which they are to be measured. With ordinary care the different forces may be obtained much more accurately than the several parts of the frame can be proportioned.

It is advisable to draw the figure of the frame to quite a large scale, as the lines of the stress diagram are drawn parallel to the several pieces of the frame. If it is objected by any that a slight deviation from the exact directions will materially change the lengths of some of the lines, and therefore give erroneous results, it may be suggested that just so much change in the form of the frame will produce this change in the forces; one is therefore warned where due allowance for such deformation should be made by the proper distribution of material. The comparison of different types of truss for the same locality can be made with ease, and the changes produced in all of the forces in any frame by a modification of a few of its pieces can be readily shown. By applying each new principle to a new form of truss, quite a variety of patterns have been treated without an undue multiplication of figures.

The method of notation used was introduced by Mr. Bow, in his "Economics of Construction." The diagrams, as here developed, are credited in England to Prof. Clerk-Maxwell, and the method is known by his name. The arrangement of the subjects, the application of the method, and the minor details have been carefully studied by the author. A very limited knowledge of Mechanics will enable the reader to understand the method of treatment here carried out.

NOTE TO REVISED EDITION.

THE reception of this Part at the hands of teachers and designers, since its first appearance as a reprint of a series of articles in "*Engineering News*," has been so hearty and sustained, that it has been thought best to put ROOF-TRUSSES in a uniform dress and agreement with BRIDGE-TRUSSES and ARCHES. The opportunity has been seized to arrange the material in a more systematic order, introduce some additional problems, and improve, as it is thought, in some matters of detail.

Quite a modification has been made in the way of regarding trusses which exert horizontal thrust, and Chapter VIII., Special Solutions, is new. The solution by *reversal of a diagonal* has been used in the author's class-room for several years. The concluding example of this chapter will afford a good test of the reader's mastery of the preceding principles.

C. E. G.

ANN ARBOR, MICH., March 11, 1890.

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ROOF-TRUSSES.

CHAPTER I.

GENERAL PRINCIPLES.

1. **Aim of the Book.**—It is proposed, in this volume, to explain and illustrate a simple method for finding the stresses in all of the pieces of such roof or other trusses, under the action of a steady load, as permit of an exact analysis; to show how the wind or any oblique force alters the amount of the stresses arising from the weight; to add a device for solving some systems of trussing which otherwise appear insoluble by the above method; and to conclude with such an explanation of bending moments and moments of resistance as will make this part reasonably complete for roof designing.

2. **Triangle of Forces.**—Taking it for granted that, if two forces, acting at a common point, are represented in length and direction by the two adjacent sides of a parallelogram *c a* and *c n*, Fig. 2, their resultant will be equal to the diagonal *c b* of the figure, drawn from the same point,—it follows that a force equal to this resultant, and acting in the opposite direction, will balance the first two forces. Hence, considering one-half of the parallelogram, we have the well-known proposition that, if three forces in equilibrium act at a single point, and a triangle be drawn with sides parallel to the three forces, these sides will be proportional in length, by a definite scale, to these forces. The forces will also be found to act in order

round the triangle, and must necessarily lie in one plane. If the magnitude of one force is known, the other two can be readily determined.

For example:—Let a known weight be suspended from the points 1 and 2, Fig. 1, by the cords 1–3, 3–2, and 3–4. Draw $c\,b$ vertically to represent the weight by any convenient scale of pounds to the inch. This line will then be parallel to, and will equal the tension in 3–4. Draw $c\,a$ parallel to 1–3, and $b\,a$ parallel to 3–2. Then will the sides of the triangle $c\,b\,a$ represent the forces which act on the point 3, and they will be found to follow one another round the triangle, as shown by the arrows.

3. **Notation.**—A notation will now be introduced which will be found very convenient when applied to trusses and diagrams. In the frame diagram write a capital letter in every *space* which is cut off from the rest of the figure by lines, real or imaginary, along which forces act. See Fig. 2 and following figures. Thus D represents the space within the triangular frame, A the space limited by the external forces acting at 1 and 2, B the space between the line to 2 and the line which carries the weight. Then let that piece of the frame or that force which lies between any two letters be called by those letters; thus, the upper bar of the triangle is A D, the right hand bar is B D, the cord to the point 1 is A C, that to the weight, or the weight itself, is C B, etc. In the diagrams drawn to determine the magnitude and kind of the several forces acting upon or in the frames the corresponding small letters will be used; thus $c\,b$ will be the vertical line representing the force in C B, $b\,a$ the tension of the cord B A, and $a\,c$ the pull on 1.

4. **External Forces.**—Returning to Fig. 1, let us suppose that a rigid, triangular frame is made fast to those cords, so that, as shown by Fig. 2, the cords are attached to the vertices of the triangle, while their directions are undisturbed. It is evident that the same stresses still exist in those cords, if the frame has no weight, and that the portion of the cords

within the triangle may be cut away without destroying the equilibrium of this combination. Hence we see that the equilibrium of this frame is assured, if the directions of these cords, or forces external to the frame, meet, if prolonged, at a common point.

The external forces C B, B A and A C, taken in the order C B A, or passing around the exterior of the triangle in a direction contrary to the movement of the hands of a watch, give the triangle of forces $c\,b\,a$, in which $c\,b$ acting in a known direction, i.e. downwards, determines the direction of $b\,a$ and $a\,c$ in relation to their points of application to the frame, since for equilibrium, by § 2, they must follow one another in order round the stress triangle.

5. **Stresses in the Frame.**—Consider the left-hand apex of the triangle. This point is in equilibrium under the action of three forces, viz., those in A C, C D, and D A, which we read around the point *in the same order as before;* we found the direction and magnitude of A C in the previous section, and the inclinations of the other two are known. The three forces at this joint must therefore be equal to the three sides of a stress triangle, as before.

Begin with A C, the fully known force, and pass from a to c, because that is the direction of the action of the force A C on the joint under consideration. Next, from c, draw $c\,d$ parallel to C D, prolonging it until a line from its extremity d, parallel to the piece D A, will strike or close on a. The stress $c\,d$ is found in C D, and the stress $d\,a$ exists in D A. The direction in which we passed around $a\,c\,d$, that is, from c to d, and then to a, shows that C D and D A both exert tension on the joint where they meet.

Next take the lowest joint. Remembering again to take the three forces in equilibrium here in the order in which the external forces were taken, and commencing with the first known one, we go, in the stress diagram, from d to c; because, since we have just found that $c\,d$ represents the pull of C D on the left-hand apex of the frame, $d\,c$ must be the equal and opposite pull of D C on the lowest joint. Next comes *c b*, along which we pass *down*, the direction in which the weight acts; and finally we draw from *b*, *b d* parallel to the piece B D. This last line will close on the point *d*, if the construction has been carefully made, and the direction in which we pass over it, from *b* to *d*, shows that the piece B D exerts tension on the lowest joint. If the reader will now run over the triangle *d b a*, which must belong to the right-hand joint, he will see that the directions just given are again complied with.

The reader can invert Fig. 2; then the weight will press down upon the upper apex of the triangle, and he will find, upon drawing the stress diagram, that the three external forces are thrusts, and that compression exists in each piece of the frame.

6. **Second Illustration: External Forces.**—In order to make these first principles more plain let us take another case. Suppose a triangular frame, Fig. 3, to rest against a wall by one angle, to have a weight of known amount suspended from the outer corner, and to be sustained by a cord attached to the third angle and secured to a point 2. Since this frame is at rest under the action of three external forces which are not parallel, their lines of action must, by § 2, meet at one common point; and since the known directions of two of these forces, A C and C B, will meet at 4, if prolonged, the force exerted on the frame by the wall at 1 must have the direction of the line 1–4. The magnitude and kind of the two unknown external forces therefore will be found by the following construction, observing the rules of interpretation already laid down:—

Draw *a c*, vertically down, equal to the known weight and force A C; next, from *c*, a line parallel to the cord and force C B, and prolong it until, from its extremity *b*, a line may be drawn parallel to B A, to strike *a*. As we went from *c* to *b*, and from *b* to *a*, C B must pull on, and B A must thrust against, the frame.

7. **Stresses in the Frame.**—Take whichever joint is most

convenient, for instance the one where the weight is attached; pass down *ac* for the external force and then, observing the order in which the triangle of external forces was drawn, draw *cd* parallel to C D and *d a* parallel to D A. Since *cd*, in the triangle *a c d* (made up of forces *a c*, *c d*, and *d a*), must represent a force acting upwards, C D exerts tension on this joint; and, similarly, *d a* (not *a d*) shows that D A thrusts against the same joint.

Take next the joint at 1. Here the reaction, as before ascertained, is *b a*; next comes *a d*, the thrust of the piece A D against this joint; and lastly *d b*, drawn parallel to D B, to close on *b* the point of beginning, shows that D B also thrusts with this amount at 1.

8. **Third Illustration.**—Once more, suppose that the triangular frame, Fig. 4, has a weight attached to its lowest angle and that the two other points are supported by inclined posts. The forces 1–4 and 2–4 must intersect 3–4 at the same point. Draw *a b* vertically downwards, and equal to the given weight; draw *b c* parallel to 2–4 or B C and *c a* parallel to 1–4 or C A. Hence *b c* and *c a* are thrusts. For the lowest joint, after passing down *a b* for the weight, draw *b d* parallel to B D and *d a* parallel to D A, thus finding that B D and D A both pull on the joint A B, and hence are tension members. As in former cases, find *d c*, which proves to be compression.

9. **General Application.**—Since, in Mechanics, the polygon of forces follows naturally from the triangle of forces, being simply a combination of several triangles, the same rules will apply when we have to deal with several external forces or a number of pieces meeting at one joint. 1°. Draw the polygon of external forces for the whole frame, taking them in order round the truss, either to the left or right, as may seem convenient. 2°. Take any joint where not more than two stresses in the pieces are unknown, and draw the polygon of forces for it. Treat the pieces and external forces which meet at the joint in that order, to the left or right, in which the external forces were taken, and begin, if possible,

with the first known force, so that the two unknown forces will be the last two sides of that particular polygon. 3°. The direction in which any line is passed over, in going round the polygon as above directed, shows whether the stress in the piece to which that line was drawn parallel acts towards or from the joint to which the polygon belongs, and hence is compression or tension. The reader must understand this principle in order to correctly interpret his diagrams.

10. **Reciprocal Figures.**—Prof. Clerk-Maxwell called the frame and stress diagrams *reciprocal* figures; for, referring to the figures already drawn, we see that the forces which meet at one point in the frame diagram give us a triangle or closed polygon in the stress diagram, and the pieces which make the triangular frame have their stresses represented by the lines which meet at one point in the stress diagram. The same reciprocity will exist in more complex figures, and it is one of the checks which we have upon the correctness of our diagrams.

The convenience of the notation explained in § 3 depends upon the above property.

CHAPTER II.

TRUSSES WITH STRAIGHT RAFTERS; VERTICAL FORCES.

11. **Triangular Truss; Inclined Reactions.**—Suppose that the roof represented in Fig. 5 has a certain load per foot over each rafter, and let the whole weight be denoted by W. It is evident that one-half of the load on the rafter C F will be supported by the joint B and one-half by the upper joint; the same will be true for the rafter D F; therefore the joint B will carry $\frac{1}{4}$ W, the upper joint $\frac{1}{2}$ W, and the joint at E $\frac{1}{4}$ W. The additional stress produced in C F by the bending action of the load which it carries is not considered at this time, but must be noticed and allowed for separately. (See Chap. IX.) Taking the external forces in order from right to left over the roof, lay off *ed*, or $\frac{1}{4}$ W, vertically, to represent the weight E D acting downward at the joint E, next *dc* equal to $\frac{1}{2}$ W, for the weight D C, and lastly *cb* for the weight at B. Call *eb* the *load line.*

Let the two reactions or supporting forces for the present be considered as a little inclined from the vertical, as shown by the arrows B A and A E. Since the truss is symmetrical and symmetrically loaded, the resultant of the load must pass through the apex of the roof, and, as the two supporting forces must meet this resultant at one point, the two reactions must be equally inclined. Then, to complete the polygon of external forces:—as we have drawn *ed*, *dc*, and *cb* in order, passing over the frame to the left,—draw next *ba*, up from the extremity *b* of the load line, and parallel to the upward reaction B A; and lastly a line *ae*, parallel to the other reaction A E, to close on *e*, the point of beginning.

12. **Triangular Truss: Stresses.**—While in this truss we might find the stresses at any joint, let us begin at B. Here

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12. **Triangular Truss: Stresses.**—While in this truss we might find the stresses at any joint, let us begin at B. Here

we have equilibrium under the action of four forces, of which the two external ones are known. Taking the latter in the same order as above, and beginning at c (§ 9, 2°), pass over cb downwards and ba upwards; then draw af parallel to A F, in such a direction that fc, drawn from f parallel to F C, will strike c, the point of beginning. Because we passed from a to f, A F will pull on the joint B, and as we then passed from f to c, F C will exert a thrust on B. (It is usual to draw af from a and fc from c till they meet at f; but to determine the *kind* of stress, one must pass over the lines in the directions noted.)

Passing next to the apex of the roof, and again taking the forces in the same order, pass down the line dc for the external force, thence up to f for the thrust cf, and finally draw fd parallel to F D, thus determining the thrust of that rafter against the top joint. If this line does not close on d, the drawing has not been made with care. As all the stresses are now found we need not examine the remaining joint. It may again be noted that we pass over a stress line in one direction when we analyze the stresses at the joint at one end of the piece to which the line is parallel, and in the reverse direction when we consider the joint at the other end of the same piece.

13. **Effect of Inclined Reactions.**—If the supporting forces had been more inclined from the vertical, the point a, of their meeting in the stress diagram, would have been nearer f, thus diminishing the tension in A F, but not affecting the compression in the rafters. The inclination might be so much increased that a would fall on f, when the piece A F would have no stress, the thrust of the rafters being balanced without it. If a fell to the right of f, af would be a thrust.

14. **Triangular Truss: Vertical Reactions.**—If the two reactions are vertical, as will be the case when the roof truss is simply placed upon the wall, B A and A E, Fig. 6, will each be $\frac{1}{2}$ W, and the point a will therefore be found at the middle of eb. The polygon of external forces has closed up and become a straight line, but in the analysis it must still be used. Thus we pass down $ed + dc + cb$ for the weights at the joints and back over $ba + ae$ for the reactions. The rest of the diagram follows from § 12.

The diagrams which the reader draws may be inked in black and red, one denoting compression, the other tension, or the two kinds of stress may be indicated by the signs + and −.

15. **King-post Truss.**—In the truss of Fig. 7 the rafters are supported at points midway between their extremities. Each point of junction of two or more pieces is considered a joint around which the pieces would be free to turn were they not restrained by their connections with other points. Whatever stiffness the joint may possess from friction between its parts, or from the continuity of a piece, such as a rafter, through the joint, is not taken into account, and may add somewhat to the strength of the truss.

In this example, therefore, half of the uniform load on C L will be carried at B, and be represented by the arrow B C; the other half together with half of the uniform load on D K will make the force C D, and so on, three of the joints carrying each one-quarter of the whole load, and the two extreme ones one-eighth each.

On a vertical line lay off $gf = \frac{1}{8}$ W, $fe = ed = dc = \frac{1}{4}$ W and $cb = \frac{1}{8}$ W; then $ba = ag = \frac{1}{2}$ W, the two supporting forces. In the order shown by the arrow, for the joint B we have cb external load, ba reaction; then draw al, tension, § 9, 3°, parallel to A L and lc, compression, parallel to L C. At the joint C D the unknown forces now are those in L K and K D. Begin with the load dc, following with cl, the stress just found in C L; then draw lk, compression, parallel to L K, and kd, compression, parallel to K D, to close on d. Passing next to the joint D E, ed is the load, dk the thrust of D K on this joint, ki the tension in K I,* and ie, to close on e, is the compression in I E. Take next the joint in the middle of the

* It will be seen that K I is a tension member or tie, and not a post as would be inferred from the name given to this truss by old builders.

lower tie; we know ik, kl, and la; the next stress lies in A H; as we have just arrived at a from l, we must pass back horizontally until a line from h parallel to H I will close on i, the point from which we started. The remaining line hf is easily determined by taking either the joint E F or the one at G.

It will be noticed that, since the truss is symmetrically made and loaded, the stress diagram is symmetrical; ki must be bisected by al; dk and ei must intersect on al. Attention to such points ensures the accuracy of the drawing.

A truss, Fig. 8, is now submitted, which the reader is advised to analyze for himself, as a test whether the principles thus far explained are clearly understood.

16. **Wooden Truss with Frequent Joints.**—The truss represented by Fig. 9, a simple extension of Fig. 7, is one well adapted for construction in timber, the verticals alone being made of iron. It can be used for roofs of large span. In any actual case, before beginning to draw the diagram, assume an approximate value for the weight of the truss, add so much of the weight of the purlins, small rafters, boards and slates, or other covering, as is supported by one truss, and divide this total weight by the number of equal parts, such as D I or E L, in the two rafters. We thus obtain the weight which is supposed to act at each joint where two pieces of the rafter meet. The weight at each abutment joint will be half as much. If the rafter is not supported at equidistant points, divide the total load by the combined length of both rafters, to obtain the load per foot of rafter, and then multiply the load per foot by the distance from the middle of one piece of the rafter to the middle of the next, to obtain the load on the joint which connects them. Numerical values will be introduced in later chapters.

Draw the vertical load line equal to the total weight, and beginning with bc as the load on B from one-half of C H, space off the weights cd, de, etc., in succession, closing at p with a half load as at b. The point of division a, at the middle of pb, marks off the two supporting forces pa and ab, which close the polygon of external forces. Beginning now at B, draw, as heretofore directed, § 9, $abcha$ for this joint. The order of these letters gives the directions of the forces on the joint B. Then for the joint C D we have $hcdih$; for H K we have $ahika$; for D E we have $kidelk$, etc. Observe that, by taking the joints in this order, first the one on the rafter, and then the one below it on the tie, we have in each case only two unknown forces, out of, at some joints, five forces. We repeat, also, the remark that it is expedient, when possible, first to pass over all the known forces at any joint, taking them in the order observed with the external forces when laying off the load line. The rest of the diagram presents no difficulty.

After the stress in N O is obtained, the diagram will begin to repeat itself inversely, the stress in O G being equal to that in F N. It is therefore unnecessary to draw more than one-half of this figure, except for a check on the accuracy of the drawing by the intersections which are seen on inspection of this diagram. Noting the stresses found in the several polygons, we see that all the inclined pieces are in compression, while the horizontal and vertical members are in tension.

17. **Superfluous Pieces.**—Sometimes a vertical rod is introduced in the first and last triangles, where dotted lines are drawn. It is evident that this rod will be of no service if all the load is assumed to be concentrated on the joints of the rafters, and this fact can be determined from the stress diagram as well. Thus, taking the joint below H, Fig. 9, we have three forces in equilibrium; begin at a in the stress diagram and pass to h along the line already found for A H; then we are required to draw a vertical line from h and, from its extremity, a horizontal line to close on the point a from which we started; the vertical line therefore can have no length. All that this vertical rod can do is to keep the horizontal tie from sagging, by sustaining whatever small weight is found at its foot.

Therefore, whenever there are at a joint but three pieces or lines along which forces can act, and two of these pieces lie in one straight line, it follows from the above that the third piece must be without stress, and that the first two pieces or lines will have the same stress. Thus, L K of Fig. 7 and H I of Fig. 9 would have no compression if the external load C D were removed. This fact will often prove of service in analysis.

18. **Problem.**—Draw the stress diagram for the truss illustrated by Fig. 10, which is supported on a shoulder at the wall and by an overhead tie running from the right end. It will be convenient to imagine that tie replaced by the inclined reaction shown by the arrow at the right, as thus the reaction is kept on the right of the load at that joint. The reaction at the wall will cut the tie where the resultant of the load cuts it; if the load is uniform over the rafter, that intersection is at the middle of the tie.

Next, try this problem with the two inclined diagonals reversed, so as to slant up to the right. Notice the upper left-hand joint. Compare the two cases, as to difference in magnitude and kind of stress.

19. **Joints where three Forces are Unknown.**—It appears impracticable to determine the stresses at any joint where more than two forces are unknown. In Fig. 9, we could not start with the joint C D or at D E; for we should know only the external force or load, and have three unknown stresses to find; therefore our quadrilateral, of which one side is known, might have the other sides of various lengths, but still parallel to the original pieces of the frame. When the joints were taken in the order observed this difficulty was not met with.

When, in some cases, we find three or more apparently unknown forces at a joint we may have some knowledge of the proportion which exists between one or more of them and a known force, and can thus determine the proper length of the line in the stress diagram. An example of such a case will be given in Fig. 11. In Chapter VIII. will be found a treatment

that is applicable to certain trusses which otherwise offer difficulties in solution.

20. **Polonceau or Fink Truss.**—Fig. 11 shows a truss which is often built in iron. The loads at the several joints of the rafters are found by the method prescribed in § 16. It will be unnecessary to dwell upon the manner of finding the stresses at the joints B, C D, and H K, for which the stresses will be ch, ha, ak, ki, hi and id. But when we attempt to analyze the joint D E, we find that, with the external load, we have six forces in equilibrium, of which those along E M, M L, and L K are unknown. If we try the joint L A we find four forces, three of which are at present unknown. We are therefore obliged to seek some other way of determining one of the stresses.

It will be seen, upon inspection, that the joint E F is like the joint C D; and it will appear reasonable that N M should have an equal stress with I H. We may then expect that there must be as much and the same kind of stress exerted by M L to keep the foot of the strut N M from moving laterally as is found necessary in K I to restrain the foot of I H.

Returning then to the joint D E, and beginning with ki, pass next over id, then de, then draw em, parallel to E M, to such a point m, that (having drawn ml until its extremity l comes in the middle of what will be the space between em and fn, or until ml equals in length ik), the line lk shall close on k whence we started. The ties and struts can be readily selected by the direction of movement over these lines in reference to the joint D E. The remaining joints when taken in the usual order of succession offer no difficulty, and the other half of the diagram need not be added, unless one desires a check on the results.

This truss will be treated again in § 74.

The polygon which we have just traced, $kidemlk$, affords a good illustration of the rule that the forces which meet at a joint make a closed polygon in the stress diagram. The symmetry of the triangles hik and mnl, and their resemblance to

klo, are worth noting, and will assist one in drawing diagrams for trusses of this type.

21. **Cambering the Lower Tie.**—Sometimes it is thought desirable to raise the tie A O, either to give more height below the truss or to improve its appearance. The effect on the stresses of such an alteration is very readily traced, and one then can judge how much change it is expedient to make. Let it be proposed to raise the portion A O of the tie to the position indicated by the dotted line, and thus to introduce such changes in the other members that they shall coincide with the other dotted lines in Fig. 11, while the load remains unchanged.

The line $c\,h$ for joint B now becomes $c\,h'$, being prolonged until $h'\,a$ can be drawn parallel to H A in its new position. Next come $h'\,i'$ and $i'\,d$; then we easily draw $i'\,k'$, $k'\,l'$, $l'\,m'$, $m'\,n'$, etc. The struts H I, K L, and M N are the only pieces in this half of the truss unaffected by the change; the amount of increase, and the serious increase, of the other stresses for any considerable elevation of the lower member can be readily seen.

22. **Load on all Joints.**—If one prefers to consider that a portion of the weight of the truss, or that a floor, ceiling or other load is supported at the lower joints, the load may be distributed as in Fig. 12. Here the joints Q R and R S carry their share of the weight of the pieces which touch these joints, as well as such other load as may properly be put there. Each supporting force, if the load is symmetrical, will still be one-half the total load, but the two will no longer divide the load line equally, nor can the load line be at once measured off as equal to the total weight.

Begin, if convenient, with the extremity H of the truss, and lay off $h\,i$, $i\,k$, $k\,l$, etc., downwards, ending with $o\,p$. Passing on, around the truss, lay off next the reaction $p\,q$ upwards, equal to one-half the *total* weight, then $q\,r$ and $r\,s$ downwards, and finally $s\,h$ upwards, for the other supporting force, to close on h. The polygon of external forces, therefore, doubles back

on itself as it were, and $h\,p$ is still the load on the exterior of the roof. The diagram can now be drawn, by taking three joints on the rafter in succession before trying the joint Q R; when taking that joint remember that there is a load upon it. The loads on the horizontal tie cause the stresses in its three parts to be drawn as three separate lines, instead of being superimposed as in the figures before given.

A diagram may now be drawn for Fig. 13. The upper part of the roof, dotted in the figure, throws its load, through the small rafters, on the upper joints of the truss.

23. **Stresses by Calculation.**—It is evident, from inspection of the preceding diagrams, that the stresses may be calculated by means of the known inclinations of the parts of the trusses. The degree of accuracy with which they can be scaled equals, however, if it does not exceed the approximation which designing and actual construction make to the theoretical structure.

24. **Distribution of Load on the Joints.**—In Unwin's "Iron Bridges and Roofs" the rafter is treated as a beam continuous over three or more supports, and the distribution of the load on the several joints is there determined by that hypothesis. That such an analysis may be true, it is necessary that all the points at which the rafter is supported shall remain in definite positions, usually a straight line. As slight deformations of the truss and unequal loading of the joints will prevent the realization of that assumption, a division of the load at any point of a rafter or other piece so that the joints at its two ends shall be loaded in the inverse ratio of the two segments into which the point divides the piece will best represent the case. Uniform loads will be distributed easily by § 16. A different distribution of the load, however, if one prefers it, will only require a corresponding division of the load line. (See Part II., Bridge Trusses, Chaps. VIII. and IX.)

CHAPTER III.

TRUSSES FOR FLAT ROOFS.

25. **Trapezoidal Truss; Equal Loads.**—A consideration of the trapezoidal, or queen-post, truss, represented by Fig. 14, will bring out two or three points which will be of use in the analysis of other trusses. In this case, let us suppose the load to be on the lower part, or bottom chord, of the truss. In order to separate the supporting forces from the small weights on the ends of the truss, and to permit them to come consecutively with the other weights in the load line, let us draw the supporting forces above the tie, instead of below as before. The rectangle formed by the two vertical and two horizontal pieces might become distorted; we will therefore introduce the brace H I, represented by the full line. The rectangle is thus divided into two triangles and movement prevented. The dotted line shows a piece which might have been introduced in place of the other.

If the truss is symmetrically loaded, or C D = D E, we shall get the first stress diagram. The stress in each vertical is here seen to be the load at its foot. The stress in the piece H I proves to be zero. If the load had been on the upper joints, no stress would have been found in the verticals also. (See § 17.) It is evident that a trapezoidal truss, when symmetrically loaded, requires no interior bracing. This fact might readily be seen if we considered the form assumed by a cord, suspended from two points on a level, and carrying two equal weights symmetrically placed.

26. **Trapezoidal Truss; Unequal Loads.**—The second stress diagram will be drawn when the weight C D is less than D E. Let us suppose that bc and ef are of the same

magnitude as in the first diagram, and let the span of the truss, or distance between supports, which we shall denote by l, be divided by the joints into three equal parts. The first step is to find the supporting forces. If each external force be multiplied by the perpendicular distance of its line of action from any one assumed point, which distance may be called its leverage, and all the products added together, those which tend to produce rotation about this point in one direction being called plus, and those tending the other way minus, it is necessary for equilibrium that the sum of these products shall be zero; otherwise the rotation can take place. A convenient point to which to measure the distances will be one of the points of support, for instance the right-hand one. Then we shall have

$$AF \,.\, l - FE \,.\, l - ED \,.\, \tfrac{2}{3}l - DC \,.\, \tfrac{1}{3}l - CB \,.\, 0 + BA \,.\, 0 = 0,$$

or

$$AF \,.\, l = FE \,.\, l + ED \,.\, \tfrac{2}{3}l + DC \,.\, \tfrac{1}{3}l;$$

therefore

$$AF = FE + \tfrac{2}{3}ED + \tfrac{1}{3}DC.$$

If E D be taken as 3 D C,

$$AF = FE + \tfrac{7}{9}ED.$$

It will be seen that the object in taking the point or axis at B is to eliminate B A, and have only one unknown quantity, A F. This method of determination is called *taking moments*, and is at once the simplest and most generally applicable. Lay off the above reaction at fa; ab will be the reaction at the right support. One cause of a diagram's failure to *close*, when drawn by a beginner, is carelessness in placing the reactions on the load line in the wrong order.

The point a being now located, we can proceed to draw the second diagram. The construction requires no explanation; but we will call attention to the fact that a compressive stress here exists in H I. If, in place of the diagonal represented by the full line, the one shown by the dotted line is now supplied, the reader can without difficulty trace out for himself the change in the diagram, which is denoted by the dotted lines and the letters marked by accents. The stress in this diagonal will be seen to be tensile. Changing the diagonal reverses its stress.

It is also worthy of notice that the only pieces affected by the substitution of one diagonal for the other are those which form the quadrilateral enclosing the diagonals. This fact will be of service later.

27. **Use of Two Diagonals.**—If, at another time, this excess of load might fall on C D in place of D E, the stress on either diagonal would be reversed: that is, if it sloped down to the right it would be a tie; if to the left, a strut. As a tension diagonal is likely to be a slender iron rod, which is of no practical value to resist a thrust, while the compression member, unless made fast at its extremities, will not transmit tension, a weight or force which may be shifted from one joint to another may require the designer to introduce two diagonals in the same rectangle or trapezium, or else to so proportion and fasten one diagonal as to withstand either kind of stress.

Where both diagonals occur the diagram can still be drawn. Determine which kind of stress, tension or compression, the two shall be designed to resist, and then, when drawing the diagram, upon arriving at a particular panel or quadrilateral, try to proceed as if only one of the diagonals existed. If a contrary kind of stress to the one desired is found to be needed, erase the lines for this panel only, and take the other diagonal. In the treatment for wind pressure, this method becomes serviceable, since the wind may blow on either side of the roof.

This truss can be used for a bridge of short span.

28. **Trusses for Halls.**—It is sometimes the case that, in covering a large building, it is desired to have the interior clear from columns or partitions, while a roof of very slight pitch is all that is needed. As it is not expedient to have a truss of much depth, since the space occupied by it is not generally available for other purposes, one of several types of

parallel-chord bridge trusses may be employed, for instance the "Warren Girder," of Fig. 15, which is an assemblage of isosceles triangles. In a public hall, galleries may be suspended from the roof, and the weight of a heavy panelled or otherwise ornamented ceiling may be added to what the truss is ordinarily expected to carry. The depth may be less than here drawn, but, for clearness of figure, we have not made the truss shallow.

If the roof pitches both ways from the middle of the span, the top chord may conform to the slope, making the truss deeper at the middle than at the ends; but a light frame may be placed above, as shown by the dotted lines, and supported at each joint of the top chord. The straight-chord truss is more easily framed. If the roof pitches slightly transversely to the trusses, it will be convenient to make them all of the same depth and put on some upper works to give the proper slope. The ends of the truss could readily be adapted to a mansard roof.

29. **Warren Girder.**—In Fig. 15, each top joint is supposed to be loaded with the weight of its share of roof, in which case the joint L M or P Q will have three-quarters of the weight on N O or O P, if the roof is carried out to the eaves as marked on the left; or practically the same as N O, if the roof follows the line I L. The bottom joints are supposed to carry the weight of the ceiling, and in addition the tension of a suspending rod to a gallery on each side. The load line will be equal to the weight on the upper part of the truss, and the polygon of external forces will overlap, as in Fig. 12, previously explained, § 22. We go from k to r, for the loads on the exterior in sequence, then up to s for the left-hand reaction, then down to w for the loads on the interior, and finally close on k with the right-hand reaction.

Upon drawing the diagram it will be seen that the stress is compression in the top chord and tension in the bottom chord; that the stresses in the chords increase from the supports to the middle; that the stresses in the braces decrease from the ends of the truss to the middle, and that alternate ones are in compression and in tension, those which slant up from the abutment towards the centre being compressed, and those which incline in the other direction being in tension. The tie-braces are, therefore, A B, C D, F G, and H I. A decrease of depth in the truss will increase the stresses in the chords.

30. **Howe Truss; Determination of Diagonals.**—A truss with parallel chords may be employed, in which the braces are alternately vertical and inclined. The designer will choose whether the verticals shall be ties and the diagonals struts, in which case the type is called the "Howe Truss," Fig. 16, or the verticals struts and the diagonals ties, when it is known as the "Pratt Truss." There is an advantage in having the struts as short as possible, but, if one desires to use but little iron, the Howe is a good form.

To decide which diagonal of the rectangle shall be occupied by the piece:—Start from the wall as a fixed point; it is evident that, to keep the load C D from sinking, C Q must be a strut. If we wish to put a tie in this panel, it must lie in the other diagonal, shown by the dotted line. C D now being held in place, P O as a strut will uphold D E. We thus may work out from each wall until we have passed as much load as equals the amount supported, or the reaction, at that wall. If the last load passed exactly completes the amount required to equal the reaction, no diagonal will be required in the next panel. We might draw diagonals, one in each panel, sloping in either direction as we pleased, and then construct the stress diagram. If we found a stress in any diagonal opposite to the stress we desired, § 27, we could then erase that diagonal and substitute the other, erasing also so much of the diagram as referred to the pieces in that panel. Were the chords not parallel, this method might be necessary (see Fig. 20), but in the present case it is better to draw the load line first, find the dividing point *a*, Fig. 16, for the two reactions, see what load it cuts, and then incline the diagonals from each wall either up or down, as preferred, towards that loaded joint.

31. **Howe Truss; Diagram.**—In the present example C D is supposed to be four times D E, etc. A tower on that end of the truss or some suspended load will account for the difference. Recalling the manner in which the supporting forces were found when the load was unsymmetrical, § 26, use a panel as a unit of distance, call a panel length p and the ordinary weight on a joint w. Then we shall have, taking moments about H,

$$w \,.\, p\,(1 + 2 + 3) + 4\,w \,.\, 4\,p + \tfrac{1}{2}\,w \,.\, 5\,p = \mathrm{R} \,.\, 5\,p, \quad \text{or} \quad \mathrm{R} = 4.9\,w,$$

the reaction at B, or ab. The two supporting forces will then be ha and ab. Draw the stress diagram as usual; the diagonals will all come in compression as intended, and the verticals will be ties. There will plainly be no stress in the dotted vertical O N. The stress in the chords is inversely proportional to the depth of the truss, and economy of material in the chords will be served by making the depth as much as possible, within reasonable limits. In bridge trusses this depth is seldom less than from one-sixth to one-eighth of the span.

32. **Moving Load.**—If the joint D E also might become heavily loaded, we could draw another diagram for that case, and, as the joints in succession had their loads increased, we might make as many diagrams. From a collection of diagrams for all positions of a moving load, we could select the maximum stress for each piece. A truss designed to resist such stresses would answer for a bridge. We should find that the greatest stresses in the chords occurred in all panels when the bridge was heavily loaded throughout, and that the greatest stress in a diagonal was found when the bridge was heavily loaded from this piece to one end only, that end generally being the more distant one. As we have more expeditious methods of analyzing a bridge truss, this one is not used. The graphical treatment of bridge trusses is found in Part II. of this work.

CHAPTER IV.

WIND PRESSURE ON PITCHED ROOFS.

33. **Action of Wind.**—The forces hitherto considered have been vertical; the wind pressure on a roof is inclined. It was once usual to deal with the pressure of the wind as a vertical load, added to the weight of the roof, snow, etc., and the stresses were obtained for the aggregate pressure. This treatment manifestly cannot be correct. The wind may be taken without error as blowing in a horizontal direction; it exerts its greatest pressure when blowing in a direction at right angles to the side of a building; it consequently acts upon but one side of the roof, loads the truss unsymmetrically, and sometimes causes stresses of an opposite kind, in parts of the frame, from those due to the steady load. Braces which are inactive under the latter weight may therefore be necessary to resist the force of the wind.

It will not be right to design the roof to sustain the whole force of the wind, considered as horizontal; nor will it be correct to decompose this horizontal force into two rectangular components, one perpendicular to the roof, and the other along its surface, and then take the perpendicular or normal component as the one to be considered; for the pressure of the wind arises from the impact of particles of air moving with a certain velocity, and these particles are not arrested, but only deviated from their former direction upon striking the roof. Yet the analysis applicable to a jet of water striking an inclined surface cannot be used here, for water escapes laterally against the air, a comparatively unresisting medium, while the wind particles, if we may so term them, deflected by the roof, are turned off against a stream of similar air, also in motion, which retards their lateral progress and thus causes

them to press more strongly against the roof. We are obliged, therefore, to have recourse to experiments for our data, and from them to deduce a formula.

34. **Formula for Wind Pressure.**—It appears that, for a given pressure exerted by a horizontal wind current on any square foot of a vertical plane, the pressure against a plane inclined to its direction is perpendicular to the inclined surface, and is greater than the normal component of the given horizontal pressure. Unwin quotes Hutton's experiments as showing that, if P equal the horizontal force of the wind on a square foot of a vertical plane, the perpendicular or normal pressure on a square foot of a roof surface inclined at an angle i to the horizon may be expressed by the empirical formula

$$P \sin i^{1.84 \cos i - 1}.$$

If, then, the maximum force of the wind be taken as 40 pounds on the square foot, representing a velocity of from 80 to 90 miles per hour, the normal pressure per square foot on surfaces inclined at different angles to the horizon will be:

Angle of Roof.	Normal Pressure.	Angle of Roof.	Normal Pressure.
5°	5.2 lbs.	35°	30.1 lbs.
10	9.6	40	33.4
15	14.0	45	36.1
20	18.3	50	38.1
25	22.5	55	39.6
30	26.4	60	40.0

For steeper pitches the pressure may be taken as 40 pounds.

Any component in the plane of the roof, from the friction of the air as it passes up along the surface, or from pressure against the butts of the shingles or slates, is too slight to be of any consequence.

The above maximum is a sufficient amount to be provided for, although wind gauges have been known to register somewhat higher pressures at rare intervals.

35. **Example: Steady Load.**—The truss of Fig. 17 is supposed to be under the action of wind pressure from the

left. If the truss is 67 feet span, and the height is 15 feet, the angle of inclination will be 24° 7′, and the normal wind pressure, interpolated from the table, will be 21.8 pounds per square foot. The rafter will be 36.7 feet long. If the trusses are 10 feet apart, the normal wind pressure on one side will be

$$36.7 \times 10 \times 21.8 = 8000 \text{ lbs.}$$

For steady load of slates, boards, rafters, purlins, and truss, let us assume 11 pounds per square foot of roof, or

$$36.7 \times 10 \times 2 \times 11 = 8074 \text{ lbs., total vertical load.}$$

The truss is here drawn to a scale of 30 feet to an inch, and both diagrams are drawn to a scale of 6000 pounds to an inch. In actual practice these figures should be much larger, the diagrams showing perhaps 1000 pounds or 800 pounds to an inch.

We will, in the present case, treat the two kinds of external force separately. The diagram on the right for steady load needs no description. Each supporting force will be 4037 pounds, and the weights at the joints of the rafters will be, 673 pounds for the end ones, and 1346 pounds for each of the others. The above weights are laid off on a vertical load line and the diagram then drawn. The stresses in the various pieces for half of the truss are given in the table to follow, the sign + denoting compression, and the sign −, tension.

36. **Wind Diagram; Reactions.**—The normal pressure of 8000 pounds distributed uniformly over the whole of the left side of the roof, and on that alone, will have its resultant, shown by the dotted arrow, at the middle of that rafter. To find the supporting force on the right we may take moments about the left-hand wall, remembering to multiply each force by the lever arm drawn perpendicular to its direction: or

$$AP \times HT = 8000 \times HK,$$

or

$$AP \times 61.15 = 8000 \times 18.35;$$

whence AP = 2400 pounds, and AH = 5600 pounds.

But since these arms, HT and HK, are proportional to the span and the left part of the horizontal tie cut off by the resultant, an easier way to get the supporting pressures due to an inclined force is to prolong this force until it cuts the horizontal line joining the two abutments, when the two reactions will be inversely proportional to the two segments into which the horizontal line is thus divided, the larger force being on the side of the shorter segment, or, for ordinary pitches, on the side on which the wind blows.

The pressures on the joints will be 2667 pounds each on IK and KL, and 1333 pounds each on HI and LM, as denoted by the arrows. Draw *mh* by scale, equal to 8000 pounds, so inclined as to be in the direction of the given forces, that is, perpendicular to the roof; divide the reactions of the supports by means of the point *a*, and lay off the joint forces in their proper order, *ml*, *lk*, *ki* and *ih*. Before going further be sure that the external forces and the reactions follow one another in their proper order, down and up the load line; for, through heedlessness, the reactions are sometimes interchanged.

37. **Wind Diagram; Stresses.**—Proceed with the construction of the diagram by the usual rules, remembering that wind alone is being treated. After the joint KL has given *lkcdel*, the joint EA gives *edafe*. Taking next the apex LM, and passing along *ml*, *le* and *ef*, we find that there will be no line parallel to FG, since *gm*, parallel to GM, will exactly close on *m*, the point of beginning. As no stress passes through FG, the remainder of the bracing on this side can experience no stress, and therefore the compression *gm* affects the whole of the right-hand rafter while the tension *af* is found in the remainder of the horizontal tie. The stress triangle for the point P will therefore be *mgam*. That the above result is true will be seen if we notice that the piece QR, having no wind pressure at its upper end, can, by § 17, have no stress. Then it follows that RS is now free from stress, and next SG and lastly GF, all by § 17. Further:

imagine all of the braces in the right half to be removed; it is evident that the right rafter is a sufficient support to the joint L M, conveying to the wall the stress *g m* which compresses its upper end, while the tie A F keeps the truss from spreading. If the lower tie or the rafter was not straight, some of the braces would come into action, as will be seen later.

38. **Remarks.**—At another time the wind may blow on the right side. Then the braces on the right will be strained as those on the left now are, and those on the left will be unstrained. The wind stresses are placed in the third column of the table. As in this truss they are all of the same kind, in the respective pieces, as those from the steady load, they are added to give the total or maximum stresses. The force *g m*, being smaller than, while it is of the same kind as *l e*, is of no consequence; for, with wind on the right, M G would have to resist a stress equal to *l e*.

A combination of the two components of the supporting forces at each end, as shown in the figure, by either the parallelogram or triangle of force, will give the direction and amount of each reaction from the combined load. Wind on the other side will exactly reverse the amounts and bring them on the opposite side of the vertical line.

TABLE OF STRESSES FOR FIG. 17.

Piece.		Steady Load.	Wind.	Total.
Tie	A B	− 7520 lbs.	10,440 lbs.	17,960 lbs.
	A D	− 6020	7,160	13,180
	A F	− 4520	3,900	8,420
Braces	E F	− 1830	3,990	5,820
	C D	− 1500	3,280	4,780
	B C	+ 1230	2,670	3,900
	D E	+ 1840	4,000	5,840
Rafter	I B	+ 8240	9,530	17,770
	K C	+ 7690	9,530	17,220
	L E	+ 5760	6,550	12,310

If the truss is simply placed upon the wall-plates, and either of the supporting forces makes a greater angle with the

vertical than the angle of repose between the two surfaces, the truss should be bolted down to the wall; otherwise there will be a tendency to slide, diminishing the tension in the tie, perhaps causing compression in that member, and changing the action of other parts of the truss. This matter will be treated of further.

If the weight of snow is also to be provided for, it may readily be done by taking the proper fraction of the stresses from the steady load and adding them to the above table.

39. **Truss with Roller Bearing; Dimensions and Load.**—We propose, in the example illustrated by Fig. 18, to consider the truss as supported on a rocker or rollers at the end T, where the small circle is drawn, to allow for the expansion and contraction of an iron frame from changes of temperature. It is therefore plain that the reaction at T must always be practically vertical. The truss is supposed to be 79 feet 8 inches in span, and 23 feet in height, which gives an angle of 30° with the horizon, and makes the length of rafter 46 feet. It would be proper usually to support the rafter at more numerous points; but our diagram would not then be so clear, with its small scale, from multiplicity of lines, and one can readily extend the method to a truss of more pieces.

This frame supports 8 feet of roof, and the steady load per square foot of roof is taken, including everything, as 14 pounds. The total vertical load will then be

$$14 \times 46 \times 2 \times 8 = 10{,}304 \text{ lbs.},$$

or 1717 lbs. on each joint except the extreme ones.

We find, from the table of § 34, that the normal pressure of the wind, for a horizontal force of 40 pounds on the square foot, may be taken as 26.4 pounds per square foot of a roof surface inclined at an angle of 30°. The total wind pressure, normal to the roof, will therefore be

$$26.4 \times 46 \times 8 = 9715 \text{ lbs.},$$

or 3238 lbs. and 1619 lbs. on the middle and end joints respectively of one rafter. The truss is drawn to a scale of 40 feet to an inch, and the diagrams to that of 8000 pounds to an inch.

40. **Diagram for Steady Load.**—The diagram for steady load, having a vertical load line, is the one above the truss, and a little more than one-half is shown. The only piece at all troublesome is G F. On arriving in our analysis at the apex of the roof, or at the middle joint of the lower member, we find three pieces whose stresses are undetermined: but as we have reached the middle of the truss, we know that the diagram will be symmetrical, and therefore that gf will be bisected by al. In the case of an unsymmetrical load we can recommence at the other point of support and close on the apex. The stresses caused by this load are given in the first column of figures in the table in § 44, compression being marked +, and tension −. If it is thought necessary to provide for snow, in addition to the stresses yet to be found for wind, make another column in the table, of amounts properly proportioned to those just found.

41. **Wind on the Left; Reactions.**—Upon turning our attention to the other diagrams, we shall find that the rollers at T cause something more than a reversal of diagram,—often a considerable variation of stress, when the wind is on different sides of the roof. Taking the wind as blowing from the left, we draw the diagram marked W. L. The line $q\,m$, 9715 lbs., § 39, is divided and lettered as shown for the four loads at the joints where arrows are drawn. The resultant of the wind pressure, at the middle point of the rafter, when prolonged by the dotted arrow, will divide the horizontal line or span in the proportion in which the load line should be divided to give the two parallel reactions, if there were no rollers at T. This proportion, for a pitch of 30°, is 2 to 1; it locates the point a', and gives $m\,a' = 6477$ lbs., and $a'\,q = 3238$ lbs.

But the reaction at T must be vertical, and consequently only the vertical component of $a'\,q$ can be found at T, while

the horizontal component of $a'\,q$ must come, through the lower member, from the resistance of the other wall. Therefore draw $a'\,a$ horizontally and we shall get $a\,q$ as the vertical reaction at T, while $m\,a$, to close this triangle of external forces, must give the direction and amount of the reaction at M.

42. **Verification.**—It may, at first sight, strike the reader that this analysis will not be correct; for, if only the vertical component is resisted at T, and if we decompose the resultant of the wind pressure at O, where it strikes the roof, into two components, we get results as follows:

Vert. comp. of 9715 lbs., for angle 30° = 8414 lbs.
Hor. " " " " " = 4858 lbs.

The vertical from the middle point of the rafter will divide the span at $\frac{1}{4}$ M T. Therefore, amount of vertical component carried at T = 2103 lbs., and the remainder is supported at M, with all of the horizontal component. But take next into account the moment, or the tendency of the horizontal component at O to cause the truss to overturn. It naturally decreases the pressure at M and increases that at T, or, in other words, the couple formed by the horizontal component at O and the equal horizontal reaction at M with an arm of half the height of the truss must be balanced by an opposite couple, composed of a tension at M and an equal compression at T, with a leverage of the span. Making the computation of this tension, or compression T, we have

$$4858 \times 11.5 = T \times 79\tfrac{2}{3}, \text{ or } T = 702 \text{ lbs.}$$
$$2103 + 702 = 2805 = \tfrac{1}{3} \text{ of } 8414 \text{ lbs.}$$

as obtained by the first process.

Still another way to find the supporting forces is to prolong the resultant until it intersects the vertical through T, then to draw a line from M to the point of intersection, and finally to draw $m\,a$ and $q\,a$ parallel to the lines from M and T. This method depends for its truth on the fact that the three external forces which keep the truss in equilibrium, not being parallel must meet in one point.

43. **Diagram for Wind on Left.**—Having completed the triangle of external forces, and laid off the pressures on the joints, we can readily draw the diagram. It will be found, as in Fig. 17, § 37, that braces on the right experience no stress, the lines gf and eq closing the polygon which relates to the joint P Q. If the lower tie were cambered to the joint D C, we should find a stress from wind in E F and C D, but not in B C or C E, as explained in § 37.

Upon combining with the inclined reaction ma the steady load reaction also marked ma, the direction of the resultant supporting force at M will be found; and it may be so much inclined to the vertical that provision against sliding on the wall-plate at M should be made. The stresses given by this diagram for wind on the left are found in the table to follow, in the column marked W. L. It will be seen that all of them agree in *kind* with those for steady load.

44. **Diagram for Wind on Right.**—This diagram is marked W. R. The supporting force at T, while still vertical,

TABLE OF STRESSES FOR FIG. 18.

Piece.		Steady Load.	W. L.	W. R.
Rafters	B S	+ 8570 lbs.	5600 lbs.	8480 lbs.
	C R	+ 6850	5600	6540
	E Q	+ 5700	5600	5880
	I P	+ 5700	5880	5600
	K O	+ 6850	6540	5600
	L N	+ 8570	8480	5600
Tie	L A	− 7440	11400	0
	H A	− 5450	7050	0
	D A	− 5450	4850	2150
	B A	− 7440	4850	6480
Braces	B C	+ 1720	0	3800
	C E	+ 1520	0	3300
	E F	− 1000	0	2150
	F G	− 2300	2500	2500
	G I	− 1000	2150	0
	I K	+ 1520	3300	0
	K L	+ 1720	3800	0

is greater in amount than before. If diagram W. L. has been already constructed, the reaction at T can be taken as that portion of the vertical component of the wind pressure not included in aq of that figure; that is, $aq + ta =$ vertical component of qm or pt. If this should be the first diagram drawn, find the supporting forces in one of the three ways given above. The reaction at M is rightly denoted by ap, for, when the wind is on the right, there is no external force to divide the space from M to P.

The point a is moved considerably from its place in diagram W. L., and this change affects the amounts of stress in the horizontal member, but not in those pieces which bear similar relations to the two sides of the truss; in other words, I P and E Q interchange stresses, etc. In some forms of truss, however, we find more material changes. In the present example it happens that the vertical fg strikes the point a, so that ip, the stress in the rafter, coincides with ap, the reaction at M; the wind on the right consequently causes no stress in L A and H A. The stresses from this diagram are found in the last column of the table.

45. **Remarks.**—There is no need to tabulate the stress in K H, if that in I G is given, nor gh, if ki is given. Notice that the joint K G or C F gives a parallelogram in each diagram, the stress in K I passing to G H without change, so that the diagonals which cross may be considered and built as independent pieces. It will be seen on inspection of the table that the combination of steady load with wind on the left gives maximum stresses in I P, K O, L N, L A, H A, D A, G I, I K, and K L, while the remainder, with the exception of F G, have maximum stresses for wind on the right. F G is strained alike in both cases.

These wind diagrams may be drawn on either side of the line of wind force, as in the case of steady load, by changing the order in which the supporting forces are taken, going round the truss and joints in the opposite direction. Although there exist two four-sided spaces C and K, the structure is sufficiently braced against distortion; for these spaces are surrounded by triangles on all sides but one.

It may perhaps not be amiss to suggest again how to determine the kind of stress in any member without retracing the whole polygon for any joint. Notice, from the load line, whether the forces were taken in right-hand or left-hand rotation. Read the letters of a piece in that order with reference to the joint at one end of it; then read the stress in the diagram in that same order, and it will show the direction of the stress in the piece, either to or from that joint. Thus diagram W. L. is written in left-hand rotation; K L is then the reading for that brace at its *lower* end, and kl reads downward or is thrust. If we read L K, it must apply to its upper end, and lk acts upwards or thrusts against the joint near N.

Wind diagrams for the truss of Fig. 21 can now be drawn. The apex of the roof can be treated first, and the stresses, obtained in the dotted lines, can then be transferred to the ends of the upper horizontal member. The truss proper goes no higher.

CHAPTER V.

WIND PRESSURE ON CURB (OR MANSARD) AND CURVED ROOFS.

46. **Truss for Curb Roof; Steady Load Diagram.**—To have a definite problem we will assume that the truss of Fig. 19, drawn to scale of 20 feet to an inch, is 50 feet in span, that the height to ridge is 20 feet, to hips $14\frac{1}{2}$ feet, and that C D is 14 feet. The sides K B and G E are practically $16\frac{2}{3}$ feet long, at an angle of 60° with the horizon, so that their horizontal projection is $8\frac{1}{3}$ feet. The upper rafters are $17\frac{1}{2}$ feet long, and therefore make an angle with the horizon of 18° 19′. The trusses are assumed to be 8 feet apart, and are loaded at the joints only. The rafters in a larger truss would commonly be supported at intermediate points; but more lines would make our diagrams less plain.

The steady load is taken at 12 pounds per square foot of roof surface, or

$$(2 \times 16\tfrac{2}{3} + 2 \times 17\tfrac{1}{2})\, 12 \times 8 = 6560 \text{ lbs., total load.}$$

The joint L will carry one-half the load on K B, or 800 pounds; the joint I K will carry one-half the load on K B and one-half of that on I C, or $800 + 840 = 1640$ pounds; $\text{I H} = 840 + 840 = 1680$ pounds, etc. These weights are laid off, in the diagram marked S. L., from l to f by a scale of 4000 pounds to an inch, and the diagram is drawn. It shows that the rafters are in compression, marked +, and all the braces in tension, marked −.

47. **Snow Diagram.**—In treating this truss for snow load, it is considered that K B and E G are too steep for any weight of snow to accumulate there, as whatever fell on them would

structure is sufficiently braced against distortion; for these spaces are surrounded by triangles on all sides but one.

It may perhaps not be amiss to suggest again how to determine the kind of stress in any member without retracing the whole polygon for any joint. Notice, from the load line, whether the forces were taken in right-hand or left-hand rotation. Read the letters of a piece in that order with reference to the joint at one end of it; then read the stress in the diagram in that same order, and it will show the direction of the stress in the piece, either to or from that joint. Thus diagram W. L. is written in left-hand rotation; K L is then the reading for that brace at its *lower* end, and *kl* reads downward or is thrust. If we read L K, it must apply to its upper end, and *lk* acts upwards or thrusts against the joint near N.

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The joint L will carry one-half the load on K B, or 800 pounds; the joint I K will carry one-half the load on K B and one-half of that on I C, or 800 + 840 = 1640 pounds; I H = 840 + 840 = 1680 pounds, etc. These weights are laid off, in the diagram marked S. L., from *l* to *f* by a scale of 4000 pounds to an inch, and the diagram is drawn. It shows that the rafters are in compression, marked +, and all the braces in tension, marked —.

47. **Snow Diagram.**—In treating this truss for snow load, it is considered that K B and E G are too steep for any weight of snow to accumulate there, as whatever fell on them would

soon slide off. Therefore a weight of 12 pounds per *horizontal* square foot, for the upper rafters only, is taken for the maximum snow load, and, as the horizontal projection of I C + D H is $33\frac{1}{3}$ feet, that load will be

$$12 \times 33\tfrac{1}{3} \times 8 = 3200 \text{ lbs.},$$

laid off from k to g, in the diagram marked S. The end portions, ki and hg, are each 800 pounds, and ih is 1600 pounds. The division into two equal reactions at the points of support gives a. This diagram much resembles the other, but there is one point worth noticing; the lines of stress, ic and hd, cross in the first diagram, but do not in the second; while the reverse is the case with ed and bc. The result is that the stress of C D is reversed by the maximum snow load, and, as this stress is greater in amount than the one for the weight of roof and truss, C D will be a compression member whenever such a load of snow falls on the roof; and will be in tension when that load is removed. The stresses from these two diagrams are marked on the truss above each piece on its left with the usual signs. This strain sheet is more convenient than the table of § 44.

48. **Wind from the Left; No Roller.**—When the rafters do not slope directly from the ridge to the eaves, but are broken into two or more planes of descent, we shall have wind pressures of different directions and intensities on the two portions, I C and K B. From the table of wind pressures, § 34, we see that the intensity of pressure on K B will be 40 pounds, and on I C 16.9 pounds, normally, per square foot of roof. The total pressure on K B therefore will be $40 \times 16\frac{2}{3} \times 8 = 5333$ pounds, of which one-half will be supported at the joint L, and the other half at the joint J, as indicated by the two arrows perpendicular to K B. The pressure on I C will be $16.9 \times 17\frac{1}{2} \times 8 = 2366$ pounds, or 1183 pounds on each joint.

If the truss has no rollers under it, the diagram marked W. L., I. is obtained. On a scale of 4000 pounds to an inch,

$hi = ij = 1183$ pounds; $jk = kl = 2667$ pounds. For ij and jk may be substituted ik, if desired, the resultant of these two components at J.

To find the supporting forces:—Prolong the resultants of the wind pressure from the middle point of each rafter to intersect the span L F. The resultant K will be resisted at L and F by two reactions parallel to it, and *inversely* proportional to the two segments into which this resultant divides L F, as shown in § 36. The same will be true for the resultant I. By scale, or from the known angles, it will be found that resultant K cuts L F at $16\frac{2}{3}$ feet, or one-third the span, from L, and that resultant I cuts it at 22.4 feet from the same end. Dividing jl at $\frac{1}{3}$ its length, we have la' for one component of the reaction at L and $a'j$ for one component of the reaction at F. If we divide hj at $\frac{22.4}{50}$ of its length, ja'' will be a component of the supporting force at L, and $a''h$ at F. By drawing the parallelogram $a'ja''a$ we shall bring the component reactions for each wall together, and shall have, for the supporting force at L, or L A, la' and $a'a$, or their resultant la; and for that at F, aa'' and $a''h$, which combined give ah, properly called A H in the truss, since the letters from F to H are not in use at present. Take care to lay off the component reactions on the proper ends of the wind-pressure lines.

The polygon of external forces, when there is no roller under the truss, is therefore hi, ik, kl, la, and ah. The completion of the diagram, by drawing lines parallel to the several pieces, will be easy without further explanation. That the point e should apparently fall on ik is accidental. The signs affixed to the lines will enable one to see readily that the stresses in B C and E A are now reversed, the pressure I K obliging us to use a strut to keep that joint in place. The resultant, however, from the combined stresses in E A is still tension. The amounts given by diagram W. L., I. have not been placed on the truss, as we prefer to treat it from another point of view. Had they been used, it would be unnecessary to draw a diagram for wind on the right, for the different members of the truss would exchange stresses symmetrically; that is, A B would have the stress of E A, and E A that of A B; D H of C I, etc., C D remaining the same.

49. **Wind from the Left; Roller at Left.**—If rollers are placed at L, to permit of movement resulting from change of temperature, the supporting forces will be modified, L A becoming vertical. The diagram marked W. L., II. shows the effect of this change. So far as drawing the lines of wind pressure *hijkl*, the polygon of external forces will be obtained in the same manner as before. We may then draw the parallelogram and locate the point here marked a'; then draw $a'a$ horizontally, and we shall get la, the vertical reaction at L, equal to the vertical component of la of the figure just preceding.

In case the former diagram has not been drawn, a readier way to determine la will be as follows:—Draw hl, plainly the resultant of hj and jl; then, having prolonged the dotted arrows at I and K until they meet, draw a line, parallel to hl, through their intersection. This line will give the position of the resultant of the wind pressures, and lh is now to be divided in the inverse ratio of the two segments into which the resultant divides the span L F. The point of division will fall at a'', from which draw horizontally $a''a$, and the reaction la is thus determined. This method will not answer for finding the supporting forces if they are both inclined, as it will make L A and A H parallel to one another. The reaction at L being la, the one at F is ah, requiring the resistance at F of the entire horizontal component of the wind pressure.

A comparison of the two W. L. diagrams will show that the stress in every piece is changed very decidedly in amount, and that in a number of pieces the stresses are reversed by rollers at L. These latter stresses are marked on the truss, at the right of each piece.

50. **Wind from the Right.**—When the wind blows from the right, the diagram marked W. R. will be obtained. The lines *ihgf*, representing the wind pressures, will correspond in value with *hikl* of the preceding figure, and, since the other diagram has been constructed, the vertical reaction at L will now be obtained by drawing the horizontal line $a'a$, from either the angle of the parallelogram or the proper point of division of the resultant if, so as to give ai, the smaller part of the vertical component of the wind pressure; that is, la from W. L., II., plus ai from W. R., equals the vertical projection of the polygon of external forces.

51. **Results.**—When this diagram is completed by the customary rules, a comparison of it with the one preceding will make clear the effect of wind on different sides. The stress in the rafters is much greater when the wind blows on the side farther from the rollers, but it is always compressive. The forces in the braces are all reversed.

The weight of the roof and truss may be the only external force, or snow may be added; and, in either case, the wind may also blow on one side or the other. Selecting then those stresses which may exist together, we find the maximum tension and compression marked below each piece. The rafters are always compressed, and A B is always in tension. The other pieces must be designed to resist both kinds of stress, although the compression in D E is quite insignificant.

52. **Curved Roof Truss: Example.**—If the truss has a curved exterior outline, the pressure of the wind will make a different angle with the horizon for every point. But there will be no sensible error if the pressure on each piece is assumed to be normal to the curve at its middle point, or, what is practically the same thing, perpendicular to the straight line joining its two extremities. Thus, in the truss of Fig. 20, the wind pressure on C T is taken as perpendicular to a straight line from B to the next joint in the rafter.

The span of this truss, drawn on a scale of 30 feet to an inch, is 60 feet; height at middle of rafters 15 feet, at middle of main tie 6 feet. The curves are arcs of circles, the radii of the upper and lower members being respectively $37\frac{1}{2}$ feet and 78 feet. The rafters are spaced off at intervals of $11\frac{1}{2}$ feet each way from the middle, and the tie is divided into $10\frac{1}{4}$ feet lengths. The end portions will differ slightly from these measures. The trusses are to be 10 feet apart. From the data, radius $37\frac{1}{2}$ feet, and half-chord or sine $5\frac{3}{4}$ feet, it is easy to calculate that the chord of the first piece of rafter from the middle will make an angle with the horizon of $8° 49\frac{1}{4}'$. The second piece will be inclined three times as much, or $26° 28'$, and the last five times as much, or $44° 6'$. The intensity of normal wind pressure will then be, when interpolated in the table, § 34, 8.6 pounds per square foot for the upper length, 23.7 pounds for the next length, and 35.6 pounds for the lowest piece. Multiplying these intensities by $11\frac{1}{2} \times 10$, we get 989 pounds, 2725 pounds, and 4094 pounds, respectively, represented by the small arrows, as if concentrated at the middle points of E, D, and C. The steady load is taken at a small figure, 2300 pounds per piece of rafter, to allow the disturbing effect of the wind to be more marked.

The diagonals in this truss are light iron rods, not adapted to resist compression, and therefore, if a compressive stress would occur in a particular diagonal, in case it were alone in a panel, we substitute the other diagonal, which will then be in tension. In lettering the figure, that *tie* which is required for a particular distribution of load is supposed to be present, and the other diagonal is not taken account of. Thus, in the panel through which the dotted arrow is drawn, if the brace which goes from the top of O P to the bottom of Q R is under stress, it will be called P Q, while the rafter will be Q E and the bottom tie P A. If the other diagonal is strained, the rafter will be called P E and the main tie Q A.

53. **Steady-Load Diagram.**—The diagram for weight of roof and truss is drawn on a scale of 8000 pounds to an inch. The vertical load line is *i b*, and the polygon for the point of support B is *c b a t c*. On passing to the next joint in the top

or bottom member we find three pieces whose stresses are unknown. Both diagonals R S cannot be in action as ties at once; therefore suppress one, for instance that which runs to the upper end of S T. We then shall have only two unknown stresses at the upper joint, and can draw ts' and $s'd$. The lower joint will then give $s't$, ta, ar', and $r's'$. But $r's'$ will be a compressive stress, as we read from r' to s', and this diagonal is not the desired one. Taking the other, and trying the lower joint first, we have *t a s t*, and the upper joint then gives *d c t s r d*, where *s r* is tension. Notice that change of diagonal affects the stresses in no pieces beyond those which bound the quadrilateral or panel in which the diagonal is changed. Analogy will rightly lead us to take the other diagonals which slope the same way, that is, down towards the middle. It is therefore easy, after the first attempt, to decide which diagonal to reject and which to retain.

54. **Remarks.**—If *d r* had been slightly more inclined, so as to strike *s*, no diagonal R S would have been required for this distribution of load. It will be seen that the stresses, all tensile, in the bracing are very small as compared with those in the main members, a fact due to the approximation of the rafter outline to the equilibrium curve or polygon for a load distributed as in this case. See § 88. If the outline of a truss coincides with the equilibrium polygon pertaining to a certain distribution of load, no interior bracing will theoretically be needed for such distribution; but if the distribution or direction of the external forces is at any time changed, bracing will be called into action. Further discussion of this subject comes in Parts II. and III.

The length of *h k*, etc., as compared with H K, etc., shows the necessity of drawing the truss skeleton on a large scale, to secure parallelism of the respective lines in each figure. As a slight change in the inclinations of the rafter and lower tie lines will change the magnitude of the stresses in those pieces quite materially, we are warned by the appearance of the diagram to provide, by an increase in size of these pieces, against such a change in the truss as would be caused by slight errors in construction or by deflection under the load. Stress diagrams are particularly serviceable in this way.

55. **Wind and Steady Load.**—We might analyze the effect of the wind separately upon the truss, but, as there is a likelihood that the wind will reverse the stress in some of the diagonals which experience tension from the steady load, and that we shall be obliged, therefore, to substitute the other diagonals in such panels, it seems better to draw the diagram for the wind and the weight of the roof in conjunction. Therefore the two diagrams marked W. R. and W. L. are drawn for the maximum force of wind on either side, combined with the weight of the roof, etc. The external load line bi of one case is the exact reverse of ib of the other. An explanation of the construction of W. R. will suffice for both.

When the wind blows from the right, there is only the steady load on the left half of the truss. Beginning therefore with the joint at I, lay off vertically $hi = 1150$ pounds, or one-half the load on H K; next $gh = 2300$ pounds, load at G H, and so on to F E, as in the steady-load diagram already discussed. At F E we find, in addition to 2300 pounds vertical pressure, an inclined force perpendicular to the tangent at E, or to the chord of the piece, and equal to one-half of 989 pounds, the wind pressure before computed for E. We thus get the inclined line as far as e in the diagram. The joint D E gives de, manifestly made up of the other half of 989 pounds, of the vertical 2300 pounds as usual, and finally of one-half of 2725 pounds from the next length of rafter, and perpendicular to it. The forces for the remaining joints C D and B C will be plotted in the same manner, and we therefore see that, commencing at B, as is proper for this load line, we lay off the vertical and inclined forces in regular succession from one side of the truss to the other. If one draws a straight line from c to d, it will be the resultant of the combined external forces at C D.

56. **Reactions and Diagrams.**—Connect b with i by the dotted line, which will be the resultant of all these forces. As the resultant of the dead weight, symmetrically distributed, acts in the line of the vertical O P, and hence through the centre of curvature of the rafters, and as the wind pressures all point to the same centre of the circle, the resultant, parallel to bi, must pass through the same point. Therefore draw the dotted arrow through the centre from which the rafter was struck, and parallel to bi. This arrow cuts the span B I, by measurement, at $25\frac{1}{4}$ feet from B, or $34\frac{3}{4}$ feet from I. The resultant bi scales 20,620 pounds. If the supporting force at B were parallel to this resultant, it would be found by taking moments about I, when we should have

$$B \times 60 = 20,620 \times 34\tfrac{3}{4}; \quad \text{or} \quad B = 11,942 \text{ lbs.}$$

Lay off this force from b to a'. If rollers are placed at B, that reaction will be vertical, and the horizontal component of $a'b$ must be resisted at I. Let fall ba vertically, determining the point a by drawing $a'a$ horizontally, and connect i with a. The two supporting forces will be ia and ab.

In the W. L. diagram the point a' comes nearer to b than to i,—that is, the quantity just obtained now applies to the point of support I,—and a falls very near to, but just outside of f, in the prolongation of the vertical line.

If there are no rollers under the truss, find the supporting forces for each oblique pressure separately, as in § 48. The same course must be pursued when the curve of the rafters is not circular, as the forces will not then meet at a common centre. Having thus completed, in either case, the polygon of external forces, the remainder of the construction will be made as in any example. After the first trial to ascertain the proper diagonal, it appears that, in each case, the diagonals all slant one way; so that, for wind on one side, one set of diagonals is in tension, and for wind on the other, all of the other set are strained.

57. Change of Diagonal.—The effect on the five pieces of a panel, top, bottom, two sides and the diagonal, of drawing the diagram so as to give compression in a diagonal, is shown anew in the W. L. figure for the panel P Q. Instead of op and qr, we get op' and $q'r$, considerably increased in amount but the same in kind; for ep and aq are substituted eq' and ap', unchanged in kind, but having practically what is taken from one added to the other; while the diagonal stress is, as we said, reversed, but very nearly the same in amount.

It might be practicable to deduce some rule for determining beforehand the diagonal which would have the desired kind of stress, but the tentative process seems easy. We find it convenient to draw the lines parallel to the rafter and main tie first, as ep and ap', then to sketch roughly two lines for the suspending piece and diagonal, see whether that diagonal comes in tension, and finally draw the right ones carefully.

58. Resultant Stresses.—It is not necessary to put the signs + and − on these lines, for it may be seen that all the rafter is compressed, the whole lower member extended, and all of the diagonals are in tension, as well as all the suspending pieces except O P and Q R, which are compressed a trifle when the maximum wind comes from the right. Such pieces are easily selected, if one notices that op and qr in the W. R. diagram are drawn in a direction opposite to the prevailing one.

The stresses are given in the following table. The lengths of rafter are denoted by a single letter. The pieces of the main tie, having the letter A in common, have also the letters which stand before the stresses in the proper columns. The inclination of the diagonal is shown by the sign prefixed to the stress. The effect of the wind on the roller side is to materially reduce the stress in a large portion of the main tie. The light bracing required is a marked feature of this type of truss, and the predominance of tensile members favors the use of iron bars. The two compressions, marked +, are too insignificant to require an increase of section.

Table of Stresses for Fig. 20.

		S. L.	W. R.	W. L.	Max.
Rafters	C	12,600	18,900	16,200	+ 18,900
	D	11,400	17,500	15,600	17,500
	E	10,800	15,000	16,200	16,200
	F	10,800	13,300	17,900	17,900
	G	11,400	12,700	20,100	20 100
	H	12,600	13,100	21,800	21,800
Main Tie.....A		K 9,600	K 5,500	K 19,500	− 19,500
		L 9,500	L 5,500	M 18,000	18,000
		N 10,400	N 7,200	O 16,000	16,000
		Q 10,400	P 9,000	Q 14,200	14,200
		S 9,500	R 10,900	S 12,300	12,300
		T 9,600	T 12,800	T 12,300	12,800
Diagonals	L M	╲ 900	╲ 1,800	╱ 1,800	
	N O	" 400	" 2,100	" 2,400	
	P Q	╱ 400	" 2,400	" 2,200	
	R S	" 900	" 2,200	" 2,100	
Suspenders	K L	1,200	700	1,200	− 1,200
	M N	1,000	200	900	1,000
	O P	900	+ 100	700	900
	Q R	1,000	+ 50	1,000	1.000
	S T	1,200	400	1,600	1,600

If the designer proposes to proportion the pieces with regard to minimum as well as maximum stresses, he can readily select the former from the table.

If a fall of snow is supposed to be uniformly distributed over the roof, the increased action of the several pieces can be easily obtained by proportion from column S. L. But, if it is thought that the inclination of the portions near C and H is too great to permit of snow accumulating there, a diagram for snow should be drawn. The horizontal projection of a piece of the rafter is properly taken when reckoning a snow load.

We think the reader will have no difficulty in drawing diagrams for a truss of similar outline, but with only a system of simple triangular bracing.

CHAPTER VI.

TRUSSES WITH HORIZONTAL THRUST.

59. **Scissor Truss.**—When it is desired to strengthen the rafters in a roof of moderate span by supporting them at their middle points, a simple means, often employed, is to spike on a piece from the lower end of one rafter to the middle of the other, as shown in Fig. 22. The two pieces may or may not be fastened together where they cross. At the first glance we should say that, to draw the diagram, we must lay off the load line ke, divide it as usual, and then, beginning at the joint E, draw $a'b'$ and $b'f$, parallel to A B and B F. Next, for the joint F G, we should get the lines $b'c'$ and $c'g$. For the apex we should have three lines, viz., hg, gc', and a line from c' parallel to C H to strike h. There is evidently something wrong here. If we start from the other point of support K, we obtain the remainder of the diagram in dotted lines, and find that we have two points marked c', some distance apart, which ought to come together; we also have two conspiring forces, gc' and hc', whose vertical components ought to balance hg.

Abandoning this diagram for the present, let us start at the apex of the roof, where we may feel sure that there are but two unknown forces. Taking the load hg at that point, draw the full lines gc and ch. Next for the joint G F, starting with cg, pass down gf and draw fb and bc. The joint H I will similarly give the figure $ihcdi$. Lastly, the joint A C will add ba and ad to the stresses dc and cb. To close the polygon for the joint E we must now supply to $abfe$ the line ea, which must be the inclined reaction at E, required to keep this truss

from sliding outwards on the wall-plates, on the supposition that the points of meeting of two or more pieces are true joints (ones about which the parts are free to turn). As ea may be decomposed into ea' and $a'a$, the force $a'a$ is called the horizontal thrust of the truss, which may be resisted by the wall or by a tie-rod from E to K. The pieces of this truss are all in compression.

60. **Horizontal Thrust or an Additional Member Necessary.**—That the truss is not in equilibrium without this inclined or horizontal reaction at the walls is seen, if we suppose that E and K are not prevented from sliding laterally; the joint A C will drop, the joints F G and H I will approach one another, and the angle at the apex will become sharper. This change will take place unless the above or some other restraining force is applied. The trouble arises from the four-sided space C, which is here free to change its form. A member added in either diagonal of this space will cure the evil. One from the apex to the joint C A will plainly act as a tie, and will be found to supply the missing line $c'c'$ in the dotted diagram first drawn. From this diagram we see that the stresses in most of the pieces will then be greater than when the resistance comes from the wall. A strut between the joints F G and H I will also make the truss secure; the reader can try such a diagram, and see what pieces have their stresses reversed by the change. Either of the above modifications puts the truss into the class having vertical reactions.

61. **Remarks.**—As these trusses are usually made, reliance against change of form, where little or no horizontal thrust is supplied by the walls, is placed upon the stiffness of the rafters, which are of one piece from ridge to eaves, and on that of the two braces; but a failure to get a good horizontal resistance from the walls has sometimes resulted in an unsightly sagging or springing of rafters and braces. The bending moments on these pieces are due to the horizontal

thrust. Bending moments on a rafter or other piece will be considered later.

It is worthy of notice that cd equals ba, or that the thrust is constant throughout the brace. Two members crossing as at A must naturally give a parallelogram in the stress diagram; the component of the load at H I which starts down the brace will pass to E without being affected by crossing the other brace; yet, to resist the tendency to sag spoken of above, and for the reason that the braces are better able to resist thrust by mutually staying one another, it is advisable to spike them together at their intersection.

62. **Hammer-beam Truss; Curved Members.**—Another example where the horizontal thrust of the truss against the walls must be ascertained is shown in Fig. 23. This frame is called a hammer-beam truss, and is a handsome type often employed, in this country and abroad, for the support of church roofs, the bracing being visible from below, and the spaces containing more or less ornamental work. When the church has a clear-story, the windows come between the trusses at B, the truss is supported on columns, and the roof of the side aisle takes up the horizontal thrust. If there are no side roofs, the main walls are properly strengthened by buttresses.

It will be well to note in advance that a curved piece in a truss, so far as the transmission of the force from one joint to another is concerned, acts as if it lay in the straight line between the two joints. The curved members in the present example are the quadrants of a circle. They may have any other desired curve, depending somewhat upon the pitch of the roof. If, now, we consider the point of support B P of the truss, and remember that the curved brace A O transmits the force between its two extremities as if it were straight, it will be evident that the thrust of the inclined piece, if any thrust exists in it, must have a horizontal component which cannot be neutralized by a vertical supporting force alone. Therefore, in addition to the reaction of half the weight of the roof

and truss, there must be supplied by the wall, assisted perhaps by a buttress or a side roof, a certain horizontal thrust.

63. **Amount of the Horizontal Thrust.**—To determine the value of this thrust:—Let W equal the weight of truss and load. We have nine loaded joints, and there is, therefore, $\frac{1}{8}$ W at each joint except the two extreme ones. The portion 2 1 3 may be considered a small truss, like Fig. 7, superimposed on the lower or main truss 4 6 2 3 7 5, and thus bringing additional loads on the points 2 and 3. If then we regard the main truss as a trapezoidal truss, and consider that the pieces L A and Q A are unnecessary because the load is the same on the two halves of the frame, the trapezoidal truss will be 4 2 3 5, the brace 4–2 being made up of an assemblage of pieces. L A and Q A will be required when wind acts upon the roof. Considering the trapezoidal truss 4 2 3 5 alone, the joint 2 will carry a load equal to that on D M, E K, and F I, or $\frac{3}{8}$ W, the joint 3 will carry the same amount, while 4 will support $\frac{1}{8}$ W from C N, and 5 the remainder. If then we lay off on a vertical line $\frac{3}{8}$ W, for the load on 2, and draw lines parallel to 2–4 and 2–3 from its extremities, the line parallel to 2–3 will be the stress in the same, and will also, since the load is vertical, be the horizontal thrust of the foot of the compound brace 2–4. This force is marked H in the dotted triangle drawn below the truss. A reference to § 25, Fig. 14, may aid one in understanding the above.

64. **Stress Diagram.**—We now have the data for the stress diagram, of which one-half is shown. For the point 4, or B P, we have the upward supporting force $bp = \frac{1}{2}$ W, next $pa =$ H, the horizontal thrust just determined of the wall, etc., against the joint, ao parallel to the line of action of AO, and finally ob, the pressure of the post O B on 4. The resultant of bp and pa, or ba, may of course be used for the reaction of the wall. Taking next the joint 6, we have cb the load, bo the thrust of B O, and we then draw on and nc. The joint C D gives $dcnmd$. The joint M A already has the lines mn, no and oa; since the line which is to close on m must be

parallel to L M, and a is already vertically over m, al can have no length, and there is no stress in A L, as before assumed. Upon taking the joint D E we find also that no stress exists in L K. The reader must not think this fact at variance with the value H which was said to exist in 2–3 when we considered the trapezoid alone; the triangular truss 1 2 3 will plainly cause a tension in 2–3, and, with this distribution of load, such tension will exactly neutralize the compression caused in the same piece by 4–2. If one will consider the truss as loaded at 6, 2, 1, 3, and 7 only, thus doing away with N M, K I, I G, etc., he will find that a diagram will then give some compression in K L.

Another method of treatment will be applied to this truss later, § 75.

65. **Different Horizontal Thrusts Consistent with Equilibrium.**—In studying Fig. 22 we saw that the stresses in G C and C H were determined by the load G H, and that the space C would become distorted unless a horizontal thrust of a definite amount, here $a'a$, was supplied by the walls. In Fig. 23 also the same things are true; the trapezoidal truss 4 2 3 5 requires a certain horizontal thrust at the points 4 and 5 to balance its load; a greater or less thrust will cause the truss to rise or fall, so long as L A and Q A are neglected, for in that case motion can freely occur at joints 2 and 3. If, however, these pieces are under stress, a greater or less horizontal thrust may be applied, the truss will still be in equilibrium, and the diagram will close. Indeed a vertical reaction is a supposable one, in which case O A must be without stress. The same statement applies to Fig. 22, if one of the diagonals of the space C is put in. As all roof-trusses of small depth in their middle section, as compared with their total rise, have a tendency to spread under a load, and hence to thrust against their supports, their diagrams should be drawn for a moderate amount of thrust at least, if it is desired to have them maintain their shape; and the supports should be able to offer this resistance, or a tie should be carried across

below. Otherwise, in addition to the sagging, a large increase of stress is likely to be found in some of the parts as a result of a vertical reaction. The determination of the horizontal thrust in a braced frame of this kind is not very simple, but may be worked out by a method given in Part III, "Arches," Chap. XII.

66. **Proof.**—That such trusses are in equilibrium under a greater or less amount of horizontal thrust, or even when the reactions are vertical, provided the pieces are able to withstand the resulting stresses, is illustrated by Fig. 24. Here the load C B is taken as twice D C. The vertical reactions ba and ad are calculated by the method of § 26. The diagram with unaccented letters is then drawn and closed as usual. Next, any horizontal thrust aa' at the points of support is assumed and the diagram with accented letters is drawn. This diagram also closes. The reduction of all of the stresses except that in fg is most marked. We see from these cases that only when the truss admits of deformation by the distortion of some interior space such as C of Fig. 22, or R of Fig. 27, is the horizontal thrust determinate by the method of these chapters; and that moderately inclined reactions or the tension of a horizontal tie between points of support are favorable to a reduction of the stresses.

Arched ribs of a nearly constant depth, not infrequently employed in railroad stations and public halls, will be treated in Part III.

CHAPTER VII.

FORCES NOT APPLIED AT JOINTS.

67. **First Diagram.**—In the trusses heretofore treated the loads have been concentrated at those points only which were directly supported. It sometimes happens that the cross-beams or purlins, which connect the trusses and convey the weight from the secondary rafters to the main rafters, rest upon the latter at points between the joints. Let us, in Fig. 25, assume that a load rests upon the middle of each of the upper rafters. If we neglect the bending action of the load E G upon the rafter and proceed as usual, we consider that one-half of the load E G will be supported at each of the joints C E and G K, and similarly for the load K M. Therefore, having laid off the weights and the two equal reactions of the walls on the load-line of the first diagram, we may increase the loads on the joints C E, G K, and M O by the new points of division, and complete this diagram, taking first B, then the next joint on the inside, and then the outside one. It will be noticed that all of the pieces except the rafters are ties.

68. **Supplying Imaginary Forces.**—This diagram gives but one stress along the whole of the upper rafter; but it is plain that the vertical force E G must have a component along the rafter and cause a different stress to exist in E T from what exists in G T. If, however, we suppose a joint to be at E G, the transverse component of E G will cause it to yield, as there is no brace beneath to hold it in place. To secure equilibrium here we may supply an imaginary force E F, shown by the dotted line, equal and directly opposed to this

transverse component. This imaginary force will take the place of a perpendicular strut, will steady the joint, and will leave the longitudinal component to affect the rafter. But the transverse component of F G actually gives a pressure at the joints C E and G K, while the imaginary force E F, just added, will lift the ends of this rafter by the same amount; therefore we must restore the pressure, and the equilibrium of the rafter F T as a whole, by adding imaginary forces, each one-half of E F, at C D and G H. This added system of forces cannot interfere with the stresses in any other pieces, for they balance by themselves. Treat the similar load K M in the same way.

69. **Second Diagram.**—In the second diagram the two supporting forces, pa and ab, are each equal to one-half the total load. Lay off bc as before; draw the dotted line cd, equal and parallel to the first imaginary force C D; then de vertical, as before; then ef, equal to, and in the direction of E F; then fg, and so on, arriving finally at p, as usual.

The construction of the rest of the diagram presents no difficulty; the joints are taken in the same order as before, and, when we have more than one external force on a joint, we take them in succession, in the order first observed for the external forces. When we reach the upper rafters, we find that g falls on the line et; et is greater and gt is less than the line for the same piece in the first diagram.

70. **Comparison of Results.**—Thus it appears that the first diagram gives the stress which would exist in the whole length of the rafter E T G, if the load F G were actually at its extremities; but, being at its middle point, one-half of the longitudinal component of F G goes to diminish the compression otherwise existing in G T, and the other half to increase the compression in E T. A comparison of the two diagrams will also show the truth of the former statement, that the system of imaginary forces does not affect any of the truss outside of the particular pieces to which it may be applied. It is still necessary to provide for the bending action of the

transverse portion of F G, or a force equal and opposite to E F upon the rafter, considered as a beam extending from hip to apex, a joint of course not being made at E G. This subject will be treated in Chapter IX.

71. **Remarks.**—If the action of the wind upon this truss is considered, it will be seen at once that no special treatment is needed; for the wind pressure is normal, and the addition of the opposite force E F at once balances the force on this joint, and transfers it to the ends D and H as the first analysis did. The bending action on the rafter must, however, be provided for.

The treatment of loads or forces not directly resisted, as above, is given by Mr. Bow in his "Economics of Construction," and may be applied to frames where one or more of the internal spaces are not triangles, but quadrilaterals. If such spaces are not surrounded by triangular spaces on at least all sides but one, the truss is liable to distortion, unless the resistance of some of the pieces to bending or the stiffness of the *theoretical* joints is called into play. A use of this treatment at many points in the same diagram will, however, be apt to make confusion.

Another application of imaginary forces, where a bending moment exists, will be made at the close of the next chapter.

CHAPTER VIII.

SPECIAL SOLUTIONS.

72. **Reversal of Diagonal.**—Difficulty is sometimes experienced in completing the diagram for a truss because, after passing a certain point, no joint can be found where but two stresses are unknown; while yet, judging from the arrangement of the pieces, the stresses ought apparently to be determinate. Such a case was found in Fig. 11, and was solved in § 20 by what might be called the law of symmetry. A method of more general application to these cases is what may be styled *Reversal of a Diagonal.*

It has been pointed out already that, if any quadrangular figure in a truss is crossed by one diagonal, the other diagonal of the quadrangle may be substituted for the former without affecting the stresses in any pieces except those which make up the quadrangle. See §§ 26 and 53. It will be found that such a change often reduces the stress in one or more pieces of the quadrangle to zero, and thus makes the truss solvable graphically. It will be well, if the reader fails to distinguish readily the altered truss from the original one, to temporarily erase from a pencil sketch the pieces thus rendered superfluous, or to draw the truss anew with the proper changes as has been done in Figs. 26 and 27. The modified truss will then be easily analyzed, and, when the old members are restored, enough stresses will be known to make the final solution practicable.

73. **Example.**—This method will first be applied to the roof-truss, Fig. 26, of a railroad station at Worcester, Mass. The span of this roof is 125 feet; entire height, wall to apex,

transverse portion of F G, or a force equal and opposite to E F upon the rafter, considered as a beam extending from hip to apex, a joint of course not being made at E G. This subject will be treated in Chapter IX.

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73. **Example.**—This method will first be applied to the roof-truss, Fig. 26, of a railroad station at Worcester, Mass. The span of this roof is 125 feet; entire height, wall to apex,

45 feet; camber of main tie 8 feet; rafter divided into six equal panels; trusses 50 feet apart.

Under steady load the tie bars S T, T U, U W, W S, which cross the centre line of the truss, will be without stress, as in Fig. 14, § 25. Indeed, as these two centre ties are independent of one another, but one can be in action at a time, as, for instance, S W and T U when the wind is on the left side. If we begin our diagram from B with *c b a k c*, we meet with no difficulty until we have passed the joint E F, for which we drew *f e n o p f*. At either of the next joints are three unknown stresses. As all stresses are determined up to the piece P Q, change the diagonal Q R in the adjoining quadrilateral from the position of the full line to the dotted one. Then the joint F G, as seen in the sketch below, will give us *g f p q' g*. As the full-lined diagonal has been removed, the joint R W has disappeared; for, if three supposed forces are in equilibrium at one point, § 17, and two of them act in one line, the third force must be zero, and R S therefore can have no stress. The stress in S W will also be zero unless it resists wind on the left, and the stress in S T is then zero. In either case we can draw *h g q' r' h* for the upper joint, and then find *a w* and *w r'*, if it exists, at the lower joint. The dotted peak is not in the main truss, but in the jack-rafters which transfer their load to G H and H I; if one prefers, he may put a load at the peak and draw the triangle of forces for that point.

After using the above expedient on the other half of the truss also, if the load is unsymmetrical, we replace the *reversed diagonal* and find the true stresses in the pieces affected by the change,—the diagonal and the four sides of the containing quadrilateral. Hence we may draw *p o a w q p* for the lower joint or *h g r s h* for the upper joint, and finally *g f p q r g* for the left-hand joint of the quadrilateral.

74. **Polonceau Truss.**—The left half of Fig. 29 is the same as Fig. 11. It will be remembered that we were stopped at the piece D E of Fig. 29 by having three unknown stresses at either end. Change the full line E F to the dotted one. The stress in F G at once becomes zero, as did R S in Fig. 26. We may now find the stresses in D E and E L at the joint K L; in dotted E F and G M at joint L M, and in A H and H F at the lower joint. Then the diagonal may be replaced and the stresses in D E, E F, F G, E H, and F L rectified. The right half of Fig. 29 may be similarly solved by reversing the diagonal P Q, which change makes the stress in O P zero.

75. **Hammer-Beam Truss, by Reversal of Diagonals.**—The hammer-beam truss of Fig. 27 differs from that of Fig. 23 by the omission of the vertical in the space R. As pointed out in § 66, this omission renders the horizontal thrust of this truss definite. In attempting to draw a diagram, however, we cannot apparently begin at the wall until we know the horizontal thrust, and, if we begin at F G, we soon meet with joints where three unknown forces are found. The method of the preceding sections will first be applied to the right half. Draw *g f r* for the upper joint, *h g r s h* for joint G H, and *f e q r f* for E F. As joints H I and R A are now insoluble, draw dotted T W for the full-lined diagonal T W, and do the same with X Y. The truss will thus be changed to the form of the sketch below. For, since T A and Y A act in the same straight line (shown dotted on left half of truss), the stress in W X is now zero, and T A and Y A have the same stress. Further, at joint K L there remain K Y, Y L, and the exterior force or load K L, which latter acts in the vertical line Y L; hence the stress in K Y is now zero, and Y L carries K L only. We can therefore draw *i h s t' i* for joint H I, *k i t' w' k* for joint I K, *w' t' s r q . . . a w'* for joint A R, and *l k w' a l* for the abutment. The reaction *a l*, being thus determined, can be used to draw the diagram, as in Fig. 23. The diagram for the left half of the truss is given in full lines, and it may be seen that A P and A T are now useful.

76. **Method of Trial and Error.**—Where the unknown stress in but one piece appears to stand in the way of a

solution, the diagram may sometimes be drawn with comparative ease by trial. Thus, in the left half of Fig. 27, we may assume the value of the horizontal thrust or of the stress in P Q and proceed with the diagram. Upon its failing to close, we can change the assumed quantity and try again. Thus, beginning at the apex, draw *g f r g*, *f e q r f*, and *h g r s h*; then assume *q p′* and its equal *s t′*. The middle joint will give *t′ s r q p′ a′ t′*; the joint D E, *p′ q e d o′ p′*, etc.; and finally the horizontal line from *n′* will fail to meet a line parallel to A M on the load line, to give *m b* in the post. It is evident, upon a slight inspection, that *q p′* is too long. The reader will find that he can soon bring the diagram to a closure by diminishing *q p′*.

By the use of such approximations one of necessity loses that check on the accuracy of the diagram, of having it close with reasonable exactness.

Fig. 30, in case one or the other of the dotted diagonals is used, will serve as an example for the practice of the preceding suggestions. Which diagonal tie, if either, will be needed for wind, and which for steady load?

77. **Example.**—We will close this branch of the subject with an example which will introduce one or two new points in addition to a combination of principles heretofore illustrated separately. The example shows the capabilities of this method in handling complex problems. The structure drawn in Fig. 28 is to be treated as a whole in its resistance to wind pressure.

The steady-load diagram would present no difficulty. The truss is carried upon columns which are hinged at their lower ends B and P, each being connected by a pin to its pedestal. The brace at R is therefore necessary to prevent overturning. The proportions of the frame are as follows: Distance between columns, 76 ft.; A C = 15 ft.; Q R = 7 ft.; camber of lower tie, 3 ft.; 1–A = 19 ft.; height of space 1 = 16 ft.; of Y = 7 ft.; extreme height, ground to peak, 48 ft.

Distance between trusses, 12 ft. Scale 40 ft. = 1 in. Scale of diagram, 8000 lbs. = 1 inch. No wind on C.

Wind pressure on main roof, 12,000 lbs. = *b j*; therefore *f g*, *g h*, etc., = 3000 lbs.; wind pressure on K X = 3360 lbs. = *j*–10; on L Y = 3500 lbs. = 10–*m*. The dotted arrows are resultants of wind pressure on the sloping surfaces. By moments about P, or by proportion of segments of span B P, as in § 48, we find

that 8368 lbs. of *b j*	is carried	at B,	and	3632 lbs.	at P.
that 940 " " 10–*m*	"	" "	"	2460 "	" "
9308 lbs. = *b*–9	"	" "	"	6092 "	" "

The horizontal force, *j*–10, at K, may be supposed to be resisted equally at each point of support, since the two posts will be alike. Hence $j k = 9 - a' = \frac{1}{2}(j-10) = 1680$ lbs. is carried at B. The moment of this horizontal force K about B or P, tending to overturn the frame, or the couple formed by K and the equal reaction in the line P B, will cause an increased upward vertical force at P and an equal downward force or diminished pressure at B. Its value, § 42, will be

$$\frac{3360 \times 39\frac{3}{4}}{76} = 1760 \text{ pounds} = a'a.$$

The reaction at B must balance the components, *b*–9, 9–*a′*, and *a′*–*a*, and hence will be *a b*. The reaction at P will then be *m* (or *p*) *a*, which may be checked in detail, if desired.

The reaction *a b*, at B, will now be decomposed into its vertical and horizontal components *a c* and *c b*. The piece A C can resist *a c* as a strut or post, but must carry *c b* 5900 lbs. by acting like a beam. Were there a real joint at D the structure would fall. It is therefore necessary to make the post of one piece, or as one member from B to R. The magnitude of the horizontal force at F caused by the 5900 lbs. of horizontal force at B will be in the ratio of the two segments of the column (beam) or as 15 to 7, or 12,643 lbs. These two forces must be balanced at D by a force equal to their sum, or 18,543 lbs. As in § 68, Fig. 25, this beam action of the post must be neutralized, before the diagram can be drawn, as these diagrams take no account of bending moments, for which see Chap. IX.

We therefore apply at B C the imaginary horizontal force $bc = 5900$ lbs., opposed to the direction of the reaction, and leaving only ac, the vertical component, which is balanced by the post; at C D we apply $cd = 18{,}543$ lbs.; and at E F, we add $ef = 12{,}643$ lbs. The sum of these three imaginary horizontal forces being zero, the stresses in the truss are not disturbed. The same steps must be taken at P, the horizontal forces mn, no, and op being obtained by the same process from the horizontal component po of the reaction pa.

The load line therefore finally becomes $bcdefghiklmnop$, the force D E being shifted laterally as shown, and ik being the resultant of ij and jk. The stress in D Q is readily obtained by drawing deq. Then the point D of the post gives the figure $acdqra$, determining the stresses in the upper part of the post and the brace R A. The remainder of the diagram presents no difficulty.

The column must be designed to resist the large bending moment to which it is liable, as well as the thrust qr. For bending moments, etc., see the next chapter, and also Part II. As this structure is supposed to be open below, the lower member should be adapted to resist such compression as may come upon it from the tendency of a gust of wind, entering beneath, to raise the roof.

CHAPTER IX.

BENDING MOMENT AND MOMENT OF RESISTANCE.

78. **Load between Joints.**—Having treated of the action of external forces upon a great variety of trusses, we propose now to investigate the graphical determination of the bending moments which arise from the load on certain pieces, and of the stresses due to the moments of resistance by which the bending moments must be met.

To recapitulate some statements of earlier chapters:—In case the transverse components of the load upon a portion of a rafter, or other piece of a truss, are not immediately resisted by the supporting power of some adjacent parts, or, in other words, unless the load on a structure is actually concentrated at the several joints, such transverse components will exert a bending action on the portion in question, and the additional stress thus caused in the piece may be too great to be safely neglected. Further, in case the piece makes any other than a right angle with the line of action of the load, or has an oblique force acting upon it, the stress along it, given by the diagram, will be less than the maximum, and will generally be the mean stress. Lastly, in case a piece is curved, a bending moment will be exerted upon it by the force acting along the straight line joining its two ends, this bending moment being a maximum at the point where the axis or centre line of the piece is farthest removed from the line drawn between its ends.

79. **Example.**—To illustrate the former statements by a simple example:—Suppose the rafters A C and B C, Fig. 31, to be loaded uniformly over their whole extent. Let us assume, in the first place, that the tie A B is not used, but

or 18,543 lbs. As in § 68, Fig. 25, this beam action of the post must be neutralized, before the diagram can be drawn, as these diagrams take no account of bending moments, for which see Chap. IX.

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BENDING MOMENT AND MOMENT OF RESISTANCE.

78. **Load between Joints.**—Having treated of the action of external forces upon a great variety of trusses, we propose now to investigate the graphical determination of the bending moments which arise from the load on certain pieces, and of the stresses due to the moments of resistance by which the bending moments must be met.

To recapitulate some statements of earlier chapters:—In case the transverse components of the load upon a portion of a rafter, or other piece of a truss, are not immediately resisted by the supporting power of some adjacent parts, or, in other words, unless the load on a structure is actually concentrated at the several joints, such transverse components will exert a bending action on the portion in question, and the additional stress thus caused in the piece may be too great to be safely neglected. Further, in case the piece makes any other than a right angle with the line of action of the load, or has an oblique force acting upon it, the stress along it, given by the diagram, will be less than the maximum, and will generally be the mean stress. Lastly, in case a piece is curved, a bending moment will be exerted upon it by the force acting along the straight line joining its two ends, this bending moment being a maximum at the point where the axis or centre line of the piece is farthest removed from the line drawn between its ends.

79. **Example.**—To illustrate the former statements by a simple example:—Suppose the rafters A C and B C, Fig. 31, to be loaded uniformly over their whole extent. Let us assume, in the first place, that the tie A B is not used, but

that the thrust of the rafters is resisted by the walls which carry the roof. Consider the piece A C. Since the roof is symmetrically loaded, the thrust at C must be horizontal, and therefore the reaction which supports this end of A C will lie in the line C E. The centre of gravity of the load on A C is at D, its middle point, and the resultant of the load will, if prolonged upwards, intersect C E at E. Since the rafter is in equilibrium under the load and the reactions at C and A, the direction of the reaction of the wall at A must also pass through E (compare Figs. 3 and 4). Draw A E and prolong E D to G. Let E G be measured by such a scale as to represent the load on A C. The three forces meeting in the common point E will then be equal to the respective sides of the triangle A E G, drawn parallel to them; and, since A G equals E C, the reactions at A and C will be A E and C E.

We now decompose A E and C E into components along and transverse to the rafter, and have A F, direct compression on the rafter at A, and C F, direct compression at C. The compression on successive sections of the rafter increases from C to A by the successive longitudinal components of the load. The two components A L and C Q, which, combined with A F and C F, give the original forces A E and C E, are analogous to the supporting forces of a beam or truss, and through them we obtain the bending action of the load on this rafter. If, now, the rafters simply rest on the wall, being secured against spreading by the tie A B, the reaction A E will be replaced by the two components, A I, the upward supporting force of the wall, and A G, the stress exerted by the tie; these two forces give the same stress and bending moments on the rafter as before.

80. **Comparison with Diagram.**—Consider, next, the method by diagram. The load is now to be concentrated at the joints, and in place of E G, we shall have A N and C P, each one-half of the load on one rafter. Lay off 1–2 to represent the total load on the roof, make 1–3 equal to A N and 1–4 to A I, and draw 3–5 and 4–5 parallel to the rafter and tie.

A G will equal 4–5, and therefore the stress in the tie is given correctly; but, since A I − A N = A K = 3–4, 3–5 equals A D, and this is the stress given by the diagram as existing from A to C, a supposition which is true when the load is actually concentrated at the joints, but is not true for a distributed load. But A D, or 3–5, is equal to one-half of A F + F C, and is manifestly the value of the direct compression at the middle point D of the rafter; all of the load from A to D was, when we drew the diagram, considered to be concentrated at the joint A. To 3–5, or A D, we should add D F, to obtain the correct compression A F at the lower end; therefore a piece which supports a distributed load should have a compression, equal to the longitudinal component of so much of the load as is transferred to its lower end, added to its stress obtained from the stress diagram. The amount to be added, however, is generally insignificant as compared with the truss stress.

The load on the principal rafters of a roof-truss is usually concentrated at series of equidistant points, by means of the *purlins*, or short cross-beams which extend from one truss to another, and which are themselves weighted at a series of points by the pressure of the secondary rafters. These secondary rafters, when employed, carry the boards, etc., and thus have a uniformly distributed load. It is only in cases where purlins rest at other points than the so-called joints that bending action occurs in the principal rafters, or in very light trusses where the boards are nailed directly to the main rafters. We need to determine the maximum bending moments on such main rafters, on the purlins and secondary rafters, in order to intelligently provide sections sufficiently strong to resist them.

81. **Bending Moment.**—It will first be well to explain what *bending moment* and *moment of resistance* are. A horizontal beam A B, Fig. 32, supported at its two ends, when loaded with a series of weights, distributed in any manner, is in equilibrium under the action of vertical forces, the weights acting downwards and the two supporting forces acting up-

wards. These supporting forces are easily calculated by the principle of the lever, or by taking moments as explained in §§ 26 and 36. They will be found graphically presently. As the beam is at rest, there must be no tendency to rotate, and therefore, if we assume any point for an axis, the sum of the moments, that is of the products of each force by its distance from the axis, must equal zero. A moment which tends to produce rotation in one direction being called plus, one which acts in the other direction is called minus. If then we pass an imaginary vertical plane of section through any point in the beam, such as E, the sum of the moments on one side of the plane of section must balance or equal that on the other. The sum of these moments on one side or the other is called the *bending moment*: the reason for the name will soon be evident.

82. **Moment of Resistance.**—These bending moments on opposite sides of the section in question can balance one another only through the resistance of the material of the beam at the section where stresses between the particles are set in action to resist the tendency to bend. The beam becomes slightly convex, and the particles or fibres on the convex side are extended, while those on the concave side are compressed. Experiment shows that, for flexure within such moderate limits as occur in practice, the horizontal forces exerted between contiguous particles vary uniformly as we go from the top of the beam to the bottom, the compressive stress being most intense on the concave side, diminishing regularly to zero at some point or horizontal plane, called the *neutral axis*, then changing to tension and increasing as we approach the convex side. The two sets of stresses reacting against each other may be represented to the eye by the arrows in the vertical section marked E′.

Since all of the external forces are vertical, these internal stresses, being horizontal, must balance in themselves, or the total tension must equal the total compression, whence it follows that the neutral axis must pass through the centre of

gravity of the section. To make this fact clear, let one consider that the distance of the centre of gravity from any assumed axis or the position of the resultant of parallel forces is found by multiplying each force or weight by its distance from that axis and dividing by the sum of the forces. Now if we attempt to find the centre of gravity of a thin cross-section of this beam, and take our axis through the point where the centre of gravity happens to lie, the sum of the moments of the particles on each side will balance or be equal, and we can see that the distance of each particle from the axis will vary exactly as these given stresses; hence the neutral axis must lie in the centre of gravity of each cross-section.

As these stresses are caused by and resist the external bending moment on each side of the section, the moment in the interior of the beam, made up of the sum of the products of the stress on each particle multiplied by its distance from the neutral axis, or indeed from any axis, and known as the *moment of resistance*, must equal the bending moment at the given section. As the tensions and compressions on one side of the plane of section tend to produce rotation about the neutral axis in the same direction, their moments are added together.

83. **Formula for Bending Moment.**—The bending moment, then, in the beam A B of the figure, at any section E, will be, if P_2 is the supporting force on the right, W_1, W_2, etc., the weights,

$$P_2 . BE - W_1 . CE - W_2 . DE;$$

or, in general, if L equal the arm of any weight, and Σ be the sign of summation,

$$M \text{ (the bending moment)} = P_2 . BE - \Sigma W . L,$$

it being remembered always to take only the weights between one end and the plane of section.

The moment of resistance, being numerically equal to the bending moment, is therefore equal to the above expression, and the maximum stress at any section can thence be determined, or the required cross-section to conform to the proper working stress for the material. The weights on one side of the section may all be considered to be concentrated at their common centre of gravity, or point of application of their resultant, so far as the bending moment at that section is concerned; the load when continuous is always so taken.

If the reader will take a special case, and, having a beam of known length with weights in given positions, will first find the supporting forces, and then calculate the bending moment on either side of a plane of section, he will obtain the same result with opposite signs, showing that the two moments balance one another. The numerical result, being the product of two quantities, is read as so many foot-pounds or inch-pounds, according to the units employed. As the stress in any material is usually expressed in pounds on the square inch, the latter units are the better.

84. **Equilibrium Polygon.**—Let us suppose that the weights which, in Fig. 32, rest upon the beam are transferred to a cord at the several points *c*, *d*, *f*, and *g*, vertically below their former positions C, D, F, and G, the cord itself being attached to two fixed points *a* and *b*, at equal distances vertically from A and B. Let us further suppose that the amount of the weight at G alone is at present known. This cord can be treated as if it were a frame. Taking the joint *g* into consideration, draw 5–4 vertically, equal to the weight, then 5–0 parallel to *a g* and 4–0 parallel to *g f*. The two lines just drawn must be the tensions in *a g* and *g f*. For the joint *f*, *f g* is now known; therefore 4–3 parallel to the weight and 3–0 parallel to *f d* will determine the other forces at *f*. The side 4–3 must equal the weight at F, and must lie in the same straight line with 5–4; for this triangle was constructed on the side 4–0 previously found. Continuing the construction for the successive angles of the cord, we find that a vertical line 5–1 will represent by its several portions the successive weights, and that the tensions in the different parts of the cord will be given by the lines parallel to these parts, drawn

from the points of division of the load line, and all converging to the common point 0. Draw 0–6 horizontally, and hence parallel to *a b*; this line will be the horizontal component of the tension at any point of the cord, and is here denoted by H. The form assumed by the cord for a given distribution of weights is called the *Equilibrium Polygon*, as the system will be in equilibrium or at rest; and it is also called in mechanics a funicular polygon. Students of mechanics will recall the fact, so easily shown here, that the horizontal component H is a constant quantity at every point.

85. **Reactions.**—If now the cord, instead of being fastened to fixed points at *a* and *b*, is attached to the two ends of a rigid bar *a b*, and the whole system is then suspended from A and B by two short cords, its equilibrium will not be disturbed. The pull 5–0 at *a* will be decomposed into 0–6, compression in *b a*, and 6–5, tension along *a* A. Similarly at *b*, 0–1 will be decomposed into 1–6 along *b* B and 6–0 along *a b*. 6–0 balances 0–6, while 1–6 and 6–5 must be the supporting forces at *b* and *a*. As the supporting forces do not depend upon the form of the frame or truss, the reactions which carry the beam at B and A must be these same quantities.

86. **Equilibrium Polygon, General Construction.**—We may make the construction more general by drawing an equilibrium polygon from any point *a*′, vertically below A, and finding the outline of a cord which will sustain in equilibrium the given weights at the given horizontal distances from A. Lay off the weights in succession from 5 to 1; assume any point 0′ arbitrarily and connect it with all the points of division of the load line. Begin at *a*′, and draw *a*′*g*′ parallel to 5–0′, stopping at the vertical dropped from G; then draw *g*′*f*′ parallel to 4–0′, etc., and finally *c*′*b*′ parallel to 1–0′. That this will be the figure of a cord suspended from *a*′ and *b*′ follows from the preceding demonstration. Connect *b*′ with *a*′; a line, parallel to *b*′*a*′, from 0′ must strike the same point 6 which the line from 0, parallel to *b a*, touched. The sup-

porting forces, if $b'a'$ exists, will be 1–6 and 6–5 as before; but 0′–6′ will be the horizontal component H′ for this cord.

87. **The Equilibrium Polygon Gives Bending Moments.**—If we turn again to the first cord, attached at a and b, the piece ab being dispensed with, the moment of all the forces on one side of any point, such as e, must be the bending moment there; but as the cord is perfectly flexible and at rest, this bending moment will equal zero. Using, instead of 1–0, its two components 1–6 = P_2 and 6–0 = H, multiplying each force by the perpendicular distance of its line of action from e, calling the combined moments of the weights on one side of e Σ W . L as before, and denoting the tendency to produce rotation in opposite ways by opposite signs, we shall have, for moments of forces on the right of, and around e,

$$P_2 \, . \, bk - \Sigma W . L - H \, . \, ek = 0,$$

or

$$H \, . \, ek = P_2 \, . \, bk - \Sigma W \, . \, L.$$

But $P_2 . bk = P_2 . BE$, and $P_2 . BE - \Sigma W . L = M$, the bending moment at the section E of the beam, as shown in § 83; therefore

$$M = H \, . \, ek.$$

By a similar analysis of the lower cord we have

$$P_2 \, . \, ik' - \Sigma W \, . \, L = (6\text{–}0') \, . \, e'l = M.$$

From similarity of triangles $le'k'$ and 6′ 0′ 6, we have

$$e'l \, : \, e'k' = 6'\text{–}0' \, : \, 6\text{–}0',$$

or

$$(6\text{–}0') \, . \, e'l = (6'\text{–}0') \, . \, e'k';$$

therefore

$$M = (6'\text{–}0') \, . \, e'k' = H' \, . \, e'k',$$

as in the other case. The solution is therefore general, and the bending moment at any section of the beam equals the product of H from the stress diagram 0 1 5 by the vertical ordinate, below the section, from the cord to the line connecting its two extremities.

88. **Remarks.**—The relative situations of a' and b' will depend upon the choice of the position of 0′, and this point may be taken wherever convenient. H′ is measured by the same scale used in plotting 5–1, while $e'k'$ must be measured by the scale to which A B is laid off. The two scales, one representing pounds, the other inches, need not be numerically the same; their product will be inch-pounds.

A single load on the beam will have for its equilibrium polygon two straight lines from a' and b', meeting at a point vertically under the weight. A uniformly distributed load will give a parabola with the maximum ordinate at the middle of the span. This load may be treated as if concentrated at any convenient number of points along the beam, as we have done in getting the loads at the several divisions of a rafter, and the angles of the polygon will lie in the desired parabola. When the beam is inclined the transverse components alone of the load produce any bending, as explained for a uniform load in § 79. Wind pressure will act as a uniform normal or transverse load on the piece which directly resists it.

The equilibrium polygon has much more extended applications in Parts II. and III.

89. **Moment of Resistance of Rectangular Cross-Section.**—Next, to determine the moment of resistance for a particular form of cross-section:—Consider a beam of rectangular cross-section, represented by A B C D of Fig. 33. The intensity of stress, as shown at E′, Fig. 32, varies uniformly each way from the neutral axis which, lying through the centre of gravity G of the cross-section, will be at E F, the middle of the depth. The stress on a square inch will be most intense on the fibres at the edge A B or C D, and less intense on any intermediate layer, such as I K, in the proportion of E I to E A. If then we draw from G the lines G A and G B, and imagine that the layer I K is replaced by I′ K′, which has its breadth diminished in the same proportion, the total stress on I′ K′, if of the intensity found at A B, will be equal to the total stress of less intensity actually existing on I K. The

former stress will also have the same leverage about E F as does the actual stress on I K. By the same reasoning for all layers of the cross-section, we obtain two triangular, shaded areas, A B G and G D C, which may be termed *equivalent areas of uniform stress* of intensity equal to the actual maximum; one of them, usually the upper one, when multiplied by this maximum intensity of stress, represents the total compression, and the other the total tension at the section. The moments of this tension and compression about the neutral axis will be most readily obtained by considering the stress, which is now uniformly distributed over the triangle, as concentrated at its centre of action, the centre of gravity G′ of the triangle, distant two-thirds of its height from the apex G.

Let b represent the breadth and h the height of the cross-section in inches; the area of one triangle will be $\frac{1}{2}b \cdot \frac{1}{2}h$; and the lever arm about E F will be $\frac{2}{3} \cdot \frac{1}{2}h$. Let f represent the maximum stress on the square inch at A B. Since the tension and compression tend to produce rotation in the same direction, we add the moments of the two forces together and have

$$2\left(\frac{bh}{4} \cdot f \cdot \frac{1}{3}h\right) = \text{moment of resistance} = \tfrac{1}{6}fbh^2.$$

Putting this value equal to the bending moment M, we obtain

$$H' \cdot e'k' = \tfrac{1}{6}fbh^2.$$

If we select the maximum value of $e'k'$, introduce the safe working stress f for the extreme fibres, and assume either b or h, we can compute the other required dimension, and thus determine the beam when of uniform section throughout. If the cross-section is to vary, its moment of resistance at different points must at least be equal to the bending moments. As the stiffness of the beam depends principally upon h, the depth must not be made too small. If the beam has too little breadth the compressed edge will yield sideways.

90. **Moment of Resistance of T Section.**—It is easy to compute the size of a beam of rectangular cross-section by the above formula, but for less regular sections the determination of the moment of resistance by this graphical method may prove of service. In applying it to a beam of the section shown in Fig. 34 we must begin by finding the centre of gravity of the section. By multiplying each rectangular area by the distance of its centre of gravity from either the top or the bottom, adding these products, and dividing by the whole area, we find the distance of the neutral axis from that edge. If $GI = b$, $AB = b'$, $GE = h$, and $CA = h'$, we have

$$\frac{bh \cdot \frac{1}{2}h + b'h'(h + \frac{1}{2}h')}{bh + b'h'} = \text{distance of neutral axis from GI.}$$

The construction of the shaded area A P B needs no explanation, as it follows the previous example. The stress on the fibres at the edge G I will not be so great as at the edge A B, because they are not so far from the neutral axis. If the fibres at G I were removed to K L, so as to be equally remote with A B, they would be equally strained. Then to reduce the layer G I to one which, if it had the same intensity of stress with A B, would give the same total stress which now exists on G I, project G I to K L, draw K P and L P, and G′ I′ will be the desired reduced length. The remainder of the shaded area for the lower rectangle follows the usual rule. In the same way, the fibres at C D will be projected at Q R, and, by drawing Q P and R P, we determine C′D′, and thus complete the shaded portion. These triangles, etc., can be readily scaled, or computed from the known proportions of the beam, their centres of gravity found and the moment of resistance calculated.

91. **Moment of Resistance of an Irregular Section.**—A good example of a section whose moment of resistance is not readily determined by computation alone is afforded by a deck-beam, Fig. 35, often employed in floors and roofs. It is here drawn to one-quarter scale, showing height of section 6 inches, breadth of flange A B $3\frac{1}{2}$ inches, thickness of web $\frac{3}{8}$ inch, weight per yard 44 lbs.; therefore the area of cross-section is about 4.4 square inches.

The readiest way to determine the moment of resistance of such a cross-section is as follows :—Transfer its outlines from the book of shapes or by such data as you have to a sheet of heavy paper, and make a tracing for construction purposes. Cut the section from the heavy paper, balance on a knife-edge and thus determine the neutral axis C D. Then on the tracing draw K L horizontally at the same distance from C D that S T is. A B will be projected at K L, and lines from K and L to P, the middle point of C D, or the centre of gravity of this section, will cut A B at A′ and B′, making A′ B′ the reduced length of A B, and now considered to have the same stress per square inch as exists at I G. In the same way the end M of M N will be projected at O, the point U at V, and the lines from O and V to P will cut the horizontal lines through M and U at new points in the desired curve. Thus enough points are soon obtained to locate the boundary of the shaded portion from B′ to P. The part of the web with straight sides gives of course a triangle, found at once by drawing a line from W to P. The curve A′ P corresponds with B′ P. For the lower portion, project E F on T S, draw lines to P, and get in a similar way enough points for this curve. Cut out the two shaded figures from the heavy paper, balance each one over a knife-edge and thus determine their respective centres of gravity Q and R. Calculate the area of one; the area of the other should exactly equal it, for the total tension equals the total compression. Calling this area A and the safe working stress on the square inch f, we shall then have for the moment of resistance

$$f \cdot A \cdot PQ + f \cdot A \cdot PR = f \cdot A \cdot QR.$$

In this example $A = 1.29$ sq. inches, $PQ = 2.12$ inches, and $PR = 2.66$ inches. If therefore for a static load $f = 12{,}000$ lbs., the moment of resistance equals

$$12{,}000 \times 1.29 \times 4.78 = 74{,}000 \text{ inch-pounds.}$$

92. **Moment of Resistance of I Beam.**—In simpler cases the required size of beam to sustain a given load is more readily found by formula. If I beams are used, the web being thin, and the top and bottom flanges alike, an approximate formula may be used. If F represents the area in square inches of the cross-section of either flange, W the area of the web, h the depth from centre to centre of flanges or the entire depth minus thickness of one flange (that is, between centres of gravity approximately), and f the safe stress on the square inch, the moment of resistance is nearly equal to

$$f h (F + \tfrac{1}{6} W).$$

CHAPTER X.

LOAD AND DETAILS.

93. **Lateral Bracing.**—The principal trusses, if large, should be braced together in the planes of the rafters to prevent wind, in a direction perpendicular to the gable ends, from producing any lateral movement. The roof boards, if laid close, and well nailed, will stiffen trusses of moderate span. It is often customary also to fasten the trusses down to the walls, especially in those buildings where wind may get below the roof. In such cases it is proper to consider and provide for the tendency of the wind to reverse the stresses in a roof which has a light covering.

94. **Weight of Materials.**—The weight of the roof covering can be ascertained in advance. The bending moments on the jack-rafters and the purlins can then be found, their sizes computed and their weights added in. The weight of the truss must then be assumed from such data as may be at hand. After the diagrams have been drawn and the truss has been roughly designed, its weight should be calculated to see how well it agrees with the assumed weight. If this agreement is not sufficiently exact, the proper allowance is then to be made.

Trautwine says that, for spans not exceeding about 75 feet, and trusses 7 feet apart, of the type shown in Figs. 11 and 29, the total load per square foot, including the truss itself, purlins, etc., complete, may be taken as follows:

Roof covered with corrugated iron, unboarded, . .	8 lbs.
Same if plastered below the rafters, ...	18 "
Roof covered with corrugated iron, on boards, . .	11 "

Same if plastered below the rafters, ...	21 lbs.
Roof covered with slate, unboarded or on laths, . .	13 "
Same on boards $1\frac{1}{4}$ inches thick, ...	16 "
Same if plastered below the rafters, ...	26 "
Roof covered with shingles on laths, ...	10 "

For spans from 75 feet to 150 feet it will suffice to add 4 lbs. to each of these totals.

The weight of an ordinary lathed and plastered ceiling is about 10 lbs. per square foot; and that of an ordinary floor of 1-inch boards, together with the usual 2×12 inch joists, 12 inches apart from centre to centre, is from 9 to 12 lbs. per square foot. White pine timber, if dry, may be considered to weigh about 25 lbs., northern yellow pine 35 lbs., and southern yellow pine 45 lbs. per cubic foot; if wet, add from 20 to 50 per cent. Oak may be reckoned at from 40 to 50 lbs. per cubic foot; cast iron at 450 lbs. per cubic foot; wrought iron at 480 lbs. per cubic foot.

The allowance to be made for the weight of snow will depend upon the latitude; from 12 to 15 lbs. per square foot of roof will suffice for most places. In some situations snow may accumulate in considerable quantities, becoming saturated with water and turning to ice; but snow saturated with water will generally slide off from roofs of ordinary pitch. The weight of a cubic foot varies much; freshly fallen snow may weigh from 5 to 12 lbs.; snow and hail, sleet or ice may weigh from 30 to 50 lbs. per cubic foot, but the quantity on a roof will usually be small.

95. **Action of Materials under Stress.**—After the stresses in the frame are determined, the several parts must be designed to withstand them. While it is not the purpose of this work to explain minutely the method of proportioning the members of a truss and of working out the details, a few suggestions as to safe stresses and some points which should be borne in mind in designing may not be amiss.

As materials, if repeatedly strained to an amount at all approaching the breaking strain, will fail sooner or later, the severe action weakening them, and as we must provide for unforeseen and unknown defects of material and workmanship, as well as for more or less of shock and vibration, it is customary to so proportion the several parts of a structure that they will be able to resist without failure much larger forces than those obtained from the stress diagrams. The smaller the load or stress on a piece the greater number of applications and removals before the piece is injured or broken. If the stress is reduced so much by increase of cross-section of the member that the piece will safely sustain an indefinitely great number of repetitions of it, such cross-section will be the proper one for a piece in a bridge or machine.

The stress arising from a stationary load, such as the weight of the structure, which is constant, is not so trying as repeated application and release of the same stress. The heavy wind-stresses determined in the previous chapters are not likely to occur more than once or twice, if at all, in the life of the structure. Hence good practice will authorize the employment of stresses some fifty per cent. in excess of those considered allowable in first-class bridge structures and those subjected to frequent change of load, to shock and vibration.

96. **Allowable Stresses.**—In accordance with this view, the following values may be used, where the wind-pressure of Chapter IV. has been allowed for.

Material.	Bending Stress.	Tension.	Compression with grain.	Compression across grain.	Shear with grain.
White Oak....	1,600	1,500	1,400	400	180
Long-leaf Southern Pine....	1,600	1,400	1,400	300	150
Oregon Pine or Fir........	1,600	1,800	1,300	250	200
White Pine (Eastern)......	1,400	800	1,200	200	100
Spruce	1,200	1,200	1,200	200	100
			Compression		Shear
Wrought Iron...........	10,000	12,000	10,000		8,000
" " best quality..	12,000	15,000	12,000		10,000

The quality of the iron employed materially affects the force which it may safely resist

The above values must not be applied to parts subjected to moving loads, such as floor-beams and suspending rods for same, unless the load is moderate in total amount and very gradually applied and removed. For bridge work they must be reduced from 20 to 33 per cent.

97. **Tension Members.**—Pieces in tension will be liable to break at the smallest cross-section. It is therefore economical to enlarge the screw-ends of long iron rods and bolts so that the cross-section at the bottom of the threads shall be at least as large as at any other point. It is desirable that the centre of resistance of the cross-section of struts and ties shall coincide with the centre of figure, as a deviation from that position greatly weakens the piece. To calculate the net or smallest cross-section of a tension member where the pull is axial or central it is sufficient to divide the force by the safe working tensile stress. Allowance must be made for diminution of cross-section by any cutting away, bolt or rivet holes.

98. **Compression Members.**—For very short pieces or blocks in compression, whose lengths do not exceed six times the least dimension, the same process may be followed. But as the length increases the strut has a tendency to yield sideways when compressed, and the cross-section must be increased. The most comprehensive formula for such pieces is that known as Gordon's Formula. Letting l denote the length of the piece in inches, h its least external diameter in inches, S the cross-section in square inches, P the given force to be resisted in pounds, f the safe working compressive stress, and a a certain constant, this formula, for pieces with flat, securely bedded ends, or ends fixed in direction by bolting or riveting, may be written

$$P = \frac{fS}{1 + a\frac{l^2}{h^2}},$$

where $a = \frac{1}{250}$ for rectangular timber struts, and $\frac{1}{3000}$ for rectangular wrought-iron struts.

If the struts are jointed at their ends by pin connections, or are so narrow as to readily yield sideways at these points, use $2a$ in place of a; if one end is firmly fixed in direction while the other end is jointed use $\frac{4}{3}a$ in place of a.

It is convenient to assume h and compute S. If the other dimension then comes smaller than h, a less value must be taken for h than before, and the calculation made anew. For cross-sections in wrought iron not rectangles, such as L, T, and H sections, $1\frac{1}{2}l$ may be used in place of l.

Pieces subjected alternately to tension and compression should have a materially larger section than would be required for either stress alone.

Cast iron is not in favor with the best designers for any but short compression pieces, packing blocks and pedestals, although it is still employed for columns. The values in Gordon's Formula for cast iron may be $f = 15{,}000$; $a = \frac{1}{800}$.

99. **Beams.**—The values of f to be used in the moment of resistance, for pieces subjected to bending, are marked bending stress in the preceding table. In determining the moment of resistance of a piece exposed to bending, or in calculating the cross-section required at the point of maximum bending moment, allowance must be made for portions cut away on the tension side in attaching fastenings, bolting or riveting together parts, and also on the compression side unless the holes, etc., are so tightly filled that the compression can be fairly considered as resisted by those portions also.

Those pieces which resist both a bending moment and a direct stress may first be designed to safely carry the bending moment, and then the dimension transverse to that in which the piece will bend may be so much increased that the added slice will resist the direct pull or thrust. If that force is thrust, it will be well to test the size of the piece by Gordon's Formula also.

100. **Pins and Eyes.**—A reasonable rule for proportioning pins and eyes of tension bars is as follows:—Make the diameter of the pin from three-fourths to four-fifths of the width of the bar in flats, and one and one-fourth times the diameter of the bar in rounds, giving the eye a sectional area of fifty per cent. in excess of that of the bar. The thickness of flat bars should be at least one-fourth of the width in order to secure a good bearing surface on the pin, and the metal at the eyes should be as thick as the bars. As the bending moment on a pin generally determines its diameter, pieces assembled on a pin should be packed closely, and those having opposing stresses should be brought into juxtaposition if possible.

101. **Details.**—Very close attention must be given to all minor details; to so proportion all the parts of a joint that it will be no more likely to yield in one way than another; to weaken as little as possible the pieces connected at a splice; to give sufficient bearing surface so as to bring the intensity of the compression on the surface within proper limits; to distribute rivets and bolts so as to give the greatest resistance with the least cutting away of other parts; to keep the action line of every piece as near its axis as possible; and to examine all sections and parts for tension, compression, and shear. The failure of a joint or connection is as fatal to a frame as to have a member too small for the stress upon it.

The pocket and hand-books issued by the different iron companies, for the use of their patrons, give the sections and weights of the various shapes of rolled iron, the safe loads for beams of different spans, data for columns, details of construction, and much miscellaneous useful information.

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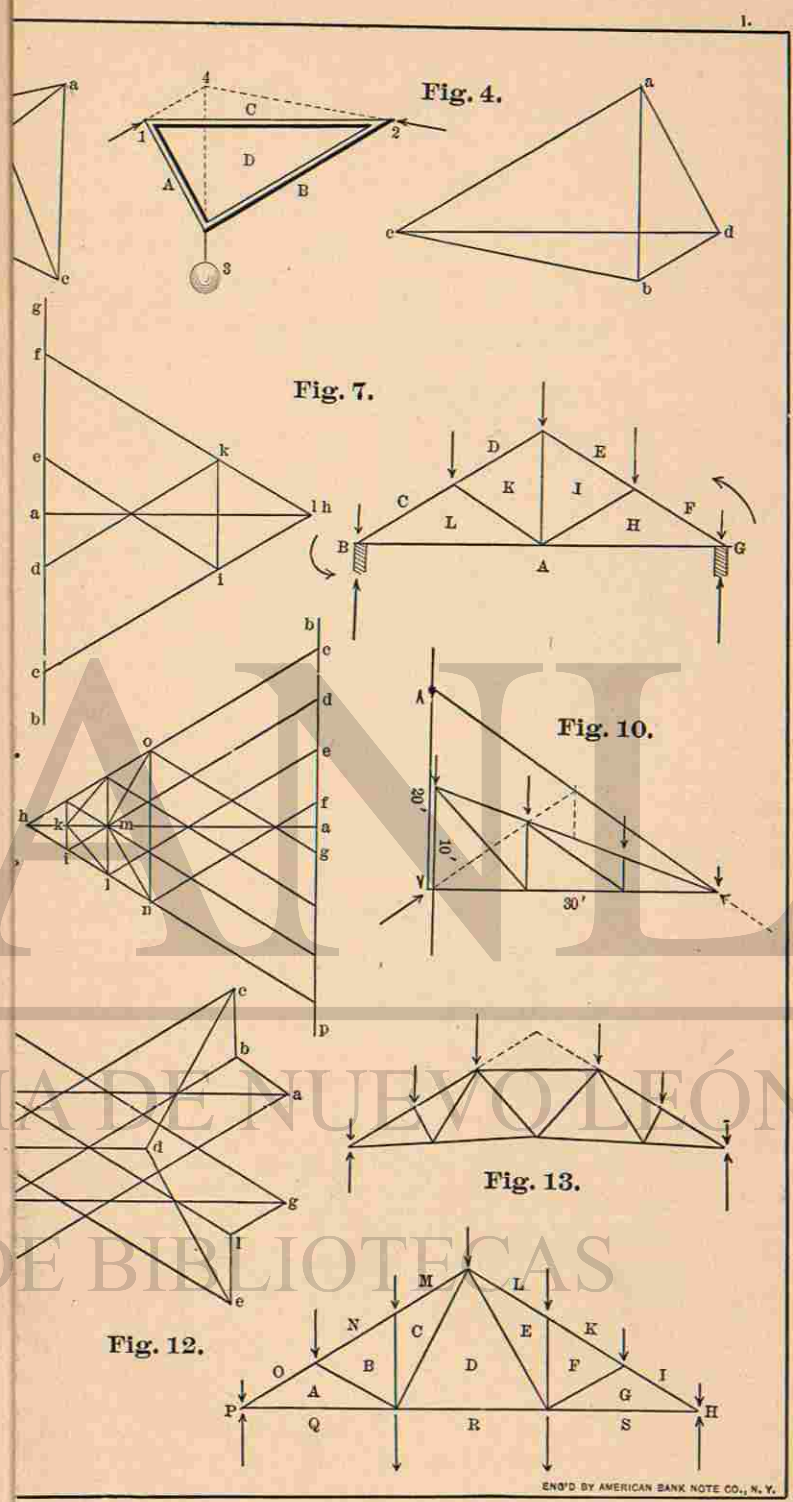
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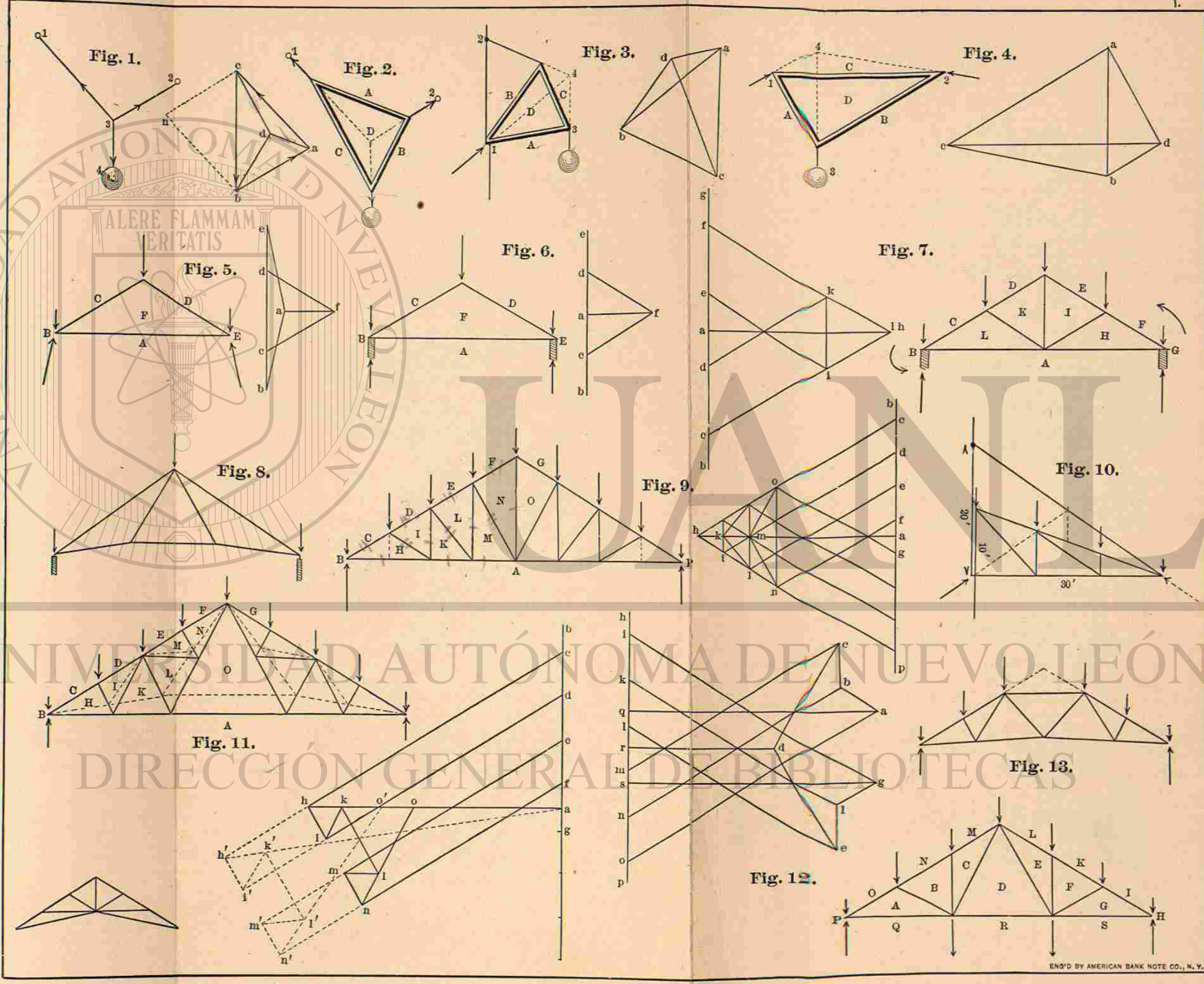

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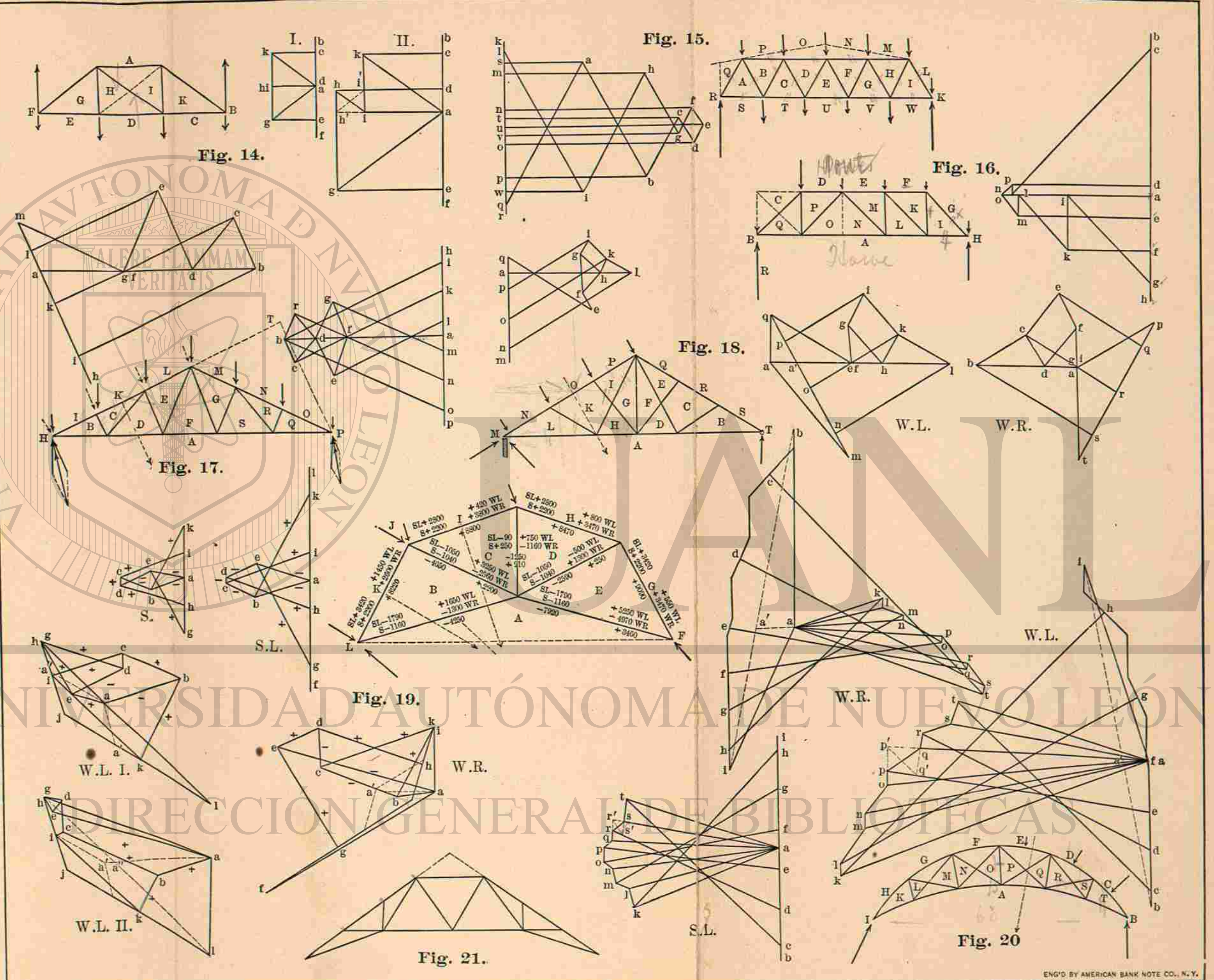

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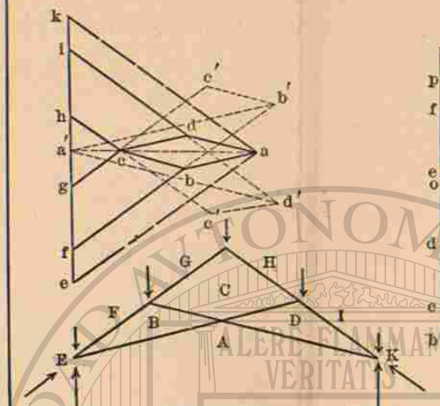

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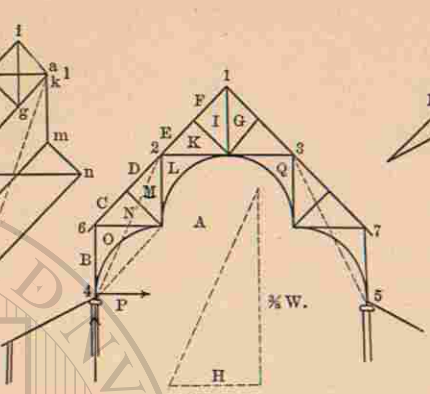

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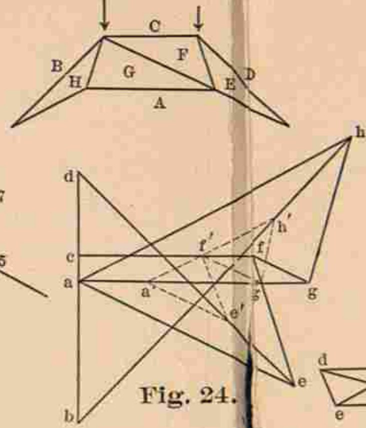

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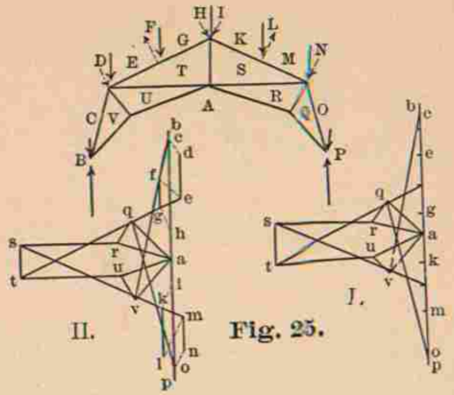

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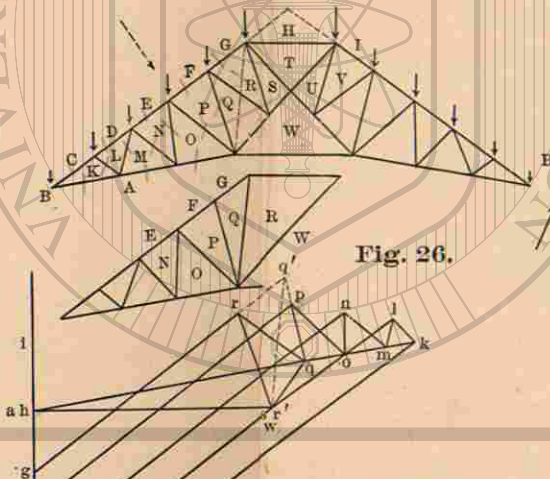

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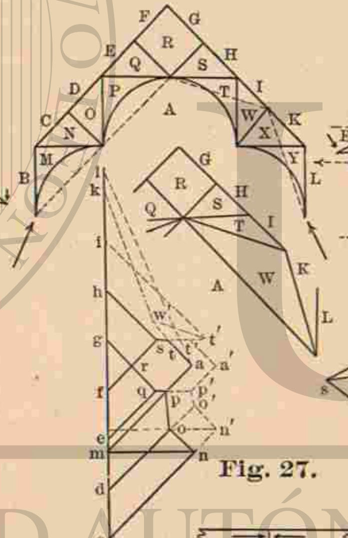

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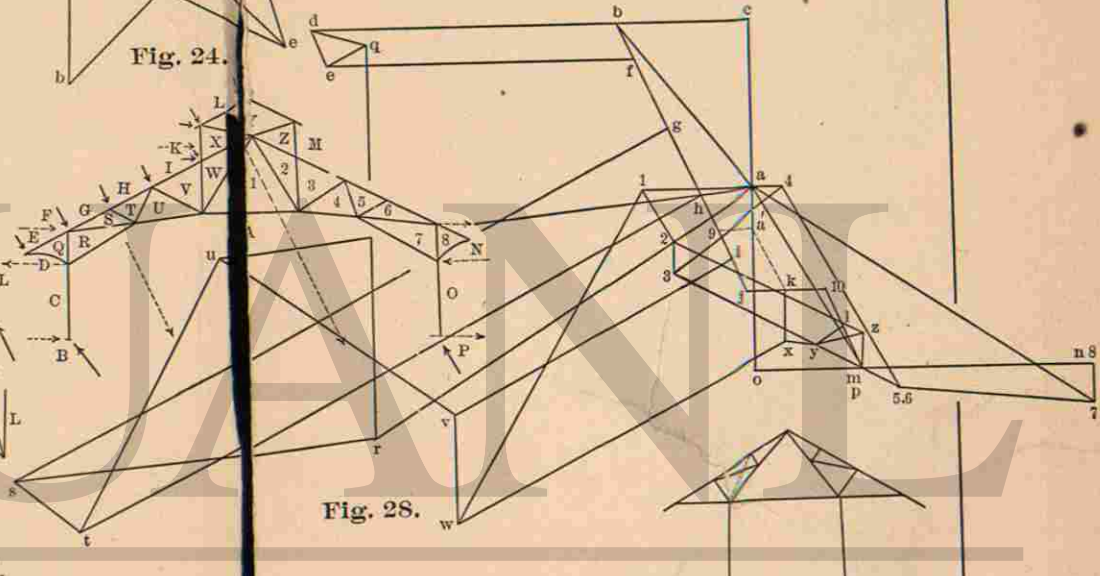

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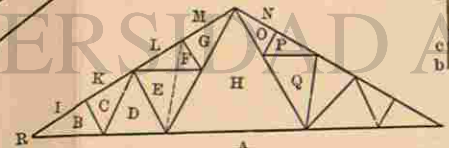

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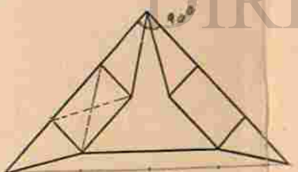

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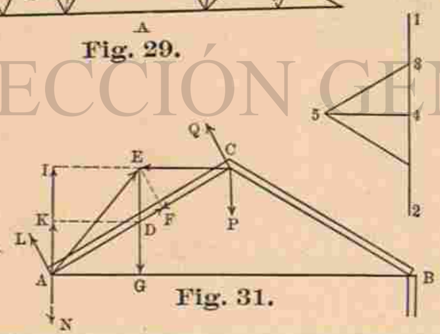

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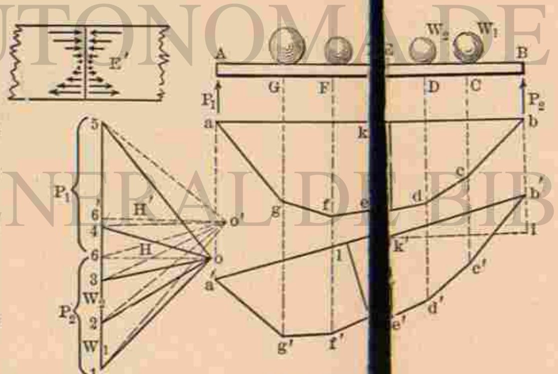

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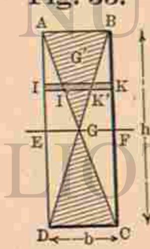

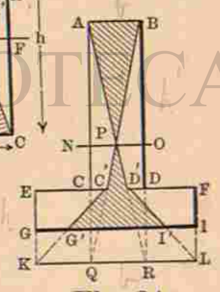

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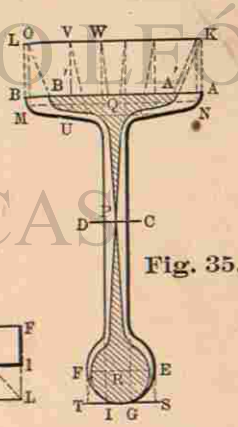

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