ROOF-TRUSSES.

CHAPTER I.

GENERAL PRINCIPLES.

1. **Aim of the Book.**—It is proposed, in this volume, to explain and illustrate a simple method for finding the stresses in all of the pieces of such roof or other trusses, under the action of a steady load, as permit of an exact analysis; to show how the wind or any oblique force alters the amount of the stresses arising from the weight; to add a device for solving some systems of trussing which otherwise appear insoluble by the above method; and to conclude with such an explanation of bending moments and moments of resistance as will make this part reasonably complete for roof designing.

2. **Triangle of Forces.**—Taking it for granted that, if two forces, acting at a common point, are represented in length and direction by the two adjacent sides of a parallelogram $c\,a$ and $c\,n$, Fig. 2, their resultant will be equal to the diagonal $c\,b$ of the figure, drawn from the same point,—it follows that a force equal to this resultant, and acting in the opposite direction, will balance the first two forces. Hence, considering one-half of the parallelogram, we have the well-known proposition that, if three forces in equilibrium act at a single point, and a triangle be drawn with sides parallel to the three forces, these sides will be proportional in length, by a definite scale, to these forces. The forces will also be found to act in order

round the triangle, and must necessarily lie in one plane. If the magnitude of one force is known, the other two can be readily determined.

For example :—Let a known weight be suspended from the points 1 and 2, Fig. 1, by the cords 1–3, 3–2, and 3–4. Draw $c\,b$ vertically to represent the weight by any convenient scale of pounds to the inch. This line will then be parallel to, and will equal the tension in 3–4. Draw $c\,a$ parallel to 1–3, and $b\,a$ parallel to 3–2. Then will the sides of the triangle $c\,b\,a$ represent the forces which act on the point 3, and they will be found to follow one another round the triangle, as shown by the arrows.

3. **Notation.**—A notation will now be introduced which will be found very convenient when applied to trusses and diagrams. In the frame diagram write a capital letter in every *space* which is cut off from the rest of the figure by lines, real or imaginary, along which forces act. See Fig. 2 and following figures. Thus D represents the space within the triangular frame, A the space limited by the external forces acting at 1 and 2, B the space between the line to 2 and the line which carries the weight. Then let that piece of the frame or that force which lies between any two letters be called by those letters; thus, the upper bar of the triangle is A D, the right hand bar is B D, the cord to the point 1 is A C, that to the weight, or the weight itself, is C B, etc. In the diagrams drawn to determine the magnitude and kind of the several forces acting upon or in the frames the corresponding small letters will be used; thus cb will be the vertical line representing the force in C B, $b\,a$ the tension of the cord B A, and $a\,c$ the pull on 1.

4. **External Forces.**—Returning to Fig. 1, let us suppose that a rigid, triangular frame is made fast to those cords, so that, as shown by Fig. 2, the cords are attached to the vertices of the triangle, while their directions are undisturbed. It is evident that the same stresses still exist in those cords, if the frame has no weight, and that the portion of the cords

within the triangle may be cut away without destroying the equilibrium of this combination. Hence we see that the equilibrium of this frame is assured, if the directions of these cords, or forces external to the frame, meet, if prolonged, at a common point.

The external forces C B, B A and A C, taken in the order C B A, or passing around the exterior of the triangle in a direction contrary to the movement of the hands of a watch, give the triangle of forces $c\,b\,a$, in which $c\,b$ acting in a known direction, i.e. downwards, determines the direction of $b\,a$ and $a\,c$ in relation to their points of application to the frame, since for equilibrium, by § 2, they must follow one another in order round the stress triangle.

5. **Stresses in the Frame.**—Consider the left-hand apex of the triangle. This point is in equilibrium under the action of three forces, viz., those in A C, C D, and D A, which we read around the point *in the same order as before;* we found the direction and magnitude of A C in the previous section, and the inclinations of the other two are known. The three forces at this joint must therefore be equal to the three sides of a stress triangle, as before.

Begin with A C, the fully known force, and pass from a to c, because that is the direction of the action of the force A C on the joint under consideration. Next, from c, draw $c\,d$ parallel to C D, prolonging it until a line from its extremity d, parallel to the piece D A, will strike or close on a. The stress $c\,d$ is found in C D, and the stress $d\,a$ exists in D A. The direction in which we passed around $a\,c\,d$, that is, from c to d, and then to a, shows that C D and D A both exert tension on the joint where they meet.

Next take the lowest joint. Remembering again to take the three forces in equilibrium here in the order in which the external forces were taken, and commencing with the first known one, we go, in the stress diagram, from d to c; because, since we have just found that $c\,d$ represents the pull of C D on the left-hand apex of the frame, $d\,c$ must be the equal and opposite pull of D C on the lowest joint. Next comes *c b*, along which we pass *down*, the direction in which the weight acts; and finally we draw from *b*, *b d* parallel to the piece B D. This last line will close on the point *d*, if the construction has been carefully made, and the direction in which we pass over it, from *b* to *d*, shows that the piece B D exerts tension on the lowest joint. If the reader will now run over the triangle *d b a*, which must belong to the right-hand joint, he will see that the directions just given are again complied with.

The reader can invert Fig. 2; then the weight will press down upon the upper apex of the triangle, and he will find, upon drawing the stress diagram, that the three external forces are thrusts, and that compression exists in each piece of the frame.

6. **Second Illustration: External Forces.**—In order to make these first principles more plain let us take another case. Suppose a triangular frame, Fig. 3, to rest against a wall by one angle, to have a weight of known amount suspended from the outer corner, and to be sustained by a cord attached to the third angle and secured to a point 2. Since this frame is at rest under the action of three external forces which are not parallel, their lines of action must, by § 2, meet at one common point; and since the known directions of two of these forces, A C and C B, will meet at 4, if prolonged, the force exerted on the frame by the wall at 1 must have the direction of the line 1–4. The magnitude and kind of the two unknown external forces therefore will be found by the following construction, observing the rules of interpretation already laid down:—

Draw *a c*, vertically down, equal to the known weight and force A C; next, from *c*, a line parallel to the cord and force C B, and prolong it until, from its extremity *b*, a line may be drawn parallel to B A, to strike *a*. As we went from *c* to *b*, and from *b* to *a*, C B must pull on, and B A must thrust against, the frame.

7. **Stresses in the Frame.**—Take whichever joint is most

convenient, for instance the one where the weight is attached; pass down *ac* for the external force and then, observing the order in which the triangle of external forces was drawn, draw *cd* parallel to C D and *d a* parallel to D A. Since *cd*, in the triangle *a c d* (made up of forces *ac*, *cd*, and *d a*), must represent a force acting upwards, C D exerts tension on this joint; and, similarly, *d a* (not *a d*) shows that D A thrusts against the same joint.

Take next the joint at 1. Here the reaction, as before ascertained, is *b a*; next comes *a d*, the thrust of the piece A D against this joint; and lastly *d b*, drawn parallel to D B, to close on *b* the point of beginning, shows that D B also thrusts with this amount at 1.

8. **Third Illustration.**—Once more, suppose that the triangular frame, Fig. 4, has a weight attached to its lowest angle and that the two other points are supported by inclined posts. The forces 1–4 and 2–4 must intersect 3–4 at the same point. Draw *a b* vertically downwards, and equal to the given weight; draw *b c* parallel to 2–4 or B C and *c a* parallel to 1–4 or C A. Hence *b c* and *c a* are thrusts. For the lowest joint, after passing down *a b* for the weight, draw *b d* parallel to B D and *d a* parallel to D A, thus finding that B D and D A both pull on the joint A B, and hence are tension members. As in former cases, find *d c*, which proves to be compression.

9. **General Application.**—Since, in Mechanics, the polygon of forces follows naturally from the triangle of forces, being simply a combination of several triangles, the same rules will apply when we have to deal with several external forces or a number of pieces meeting at one joint. 1°. Draw the polygon of external forces for the whole frame, taking them in order round the truss, either to the left or right, as may seem convenient. 2°. Take any joint where not more than two stresses in the pieces are unknown, and draw the polygon of forces for it. Treat the pieces and external forces which meet at the joint in that order, to the left or right, in which the external forces were taken, and begin, if possible,

with the first known force, so that the two unknown forces will be the last two sides of that particular polygon. 3°. The direction in which any line is passed over, in going round the polygon as above directed, shows whether the stress in the piece to which that line was drawn parallel acts towards or from the joint to which the polygon belongs, and hence is compression or tension. The reader must understand this principle in order to correctly interpret his diagrams.

10. **Reciprocal Figures.**—Prof. Clerk-Maxwell called the frame and stress diagrams *reciprocal* figures; for, referring to the figures already drawn, we see that the forces which meet at one point in the frame diagram give us a triangle or closed polygon in the stress diagram, and the pieces which make the triangular frame have their stresses represented by the lines which meet at one point in the stress diagram. The same reciprocity will exist in more complex figures, and it is one of the checks which we have upon the correctness of our diagrams.

The convenience of the notation explained in § 3 depends upon the above property.

CHAPTER II.

TRUSSES WITH STRAIGHT RAFTERS; VERTICAL FORCES.

11. **Triangular Truss; Inclined Reactions.**—Suppose that the roof represented in Fig. 5 has a certain load per foot over each rafter, and let the whole weight be denoted by W. It is evident that one-half of the load on the rafter C F will be supported by the joint B and one-half by the upper joint; the same will be true for the rafter D F; therefore the joint B will carry $\frac{1}{4}$ W, the upper joint $\frac{1}{2}$ W, and the joint at E $\frac{1}{4}$ W. The additional stress produced in C F by the bending action of the load which it carries is not considered at this time, but must be noticed and allowed for separately. (See Chap. IX.) Taking the external forces in order from right to left over the roof, lay off *ed*, or $\frac{1}{4}$ W, vertically, to represent the weight E D acting downward at the joint E, next *dc* equal to $\frac{1}{2}$ W, for the weight D C, and lastly *cb* for the weight at B. Call *eb* the *load line.*

Let the two reactions or supporting forces for the present be considered as a little inclined from the vertical, as shown by the arrows B A and A E. Since the truss is symmetrical and symmetrically loaded, the resultant of the load must pass through the apex of the roof, and, as the two supporting forces must meet this resultant at one point, the two reactions must be equally inclined. Then, to complete the polygon of external forces:—as we have drawn *ed*, *dc*, and *cb* in order, passing over the frame to the left,—draw next *ba*, up from the extremity *b* of the load line, and parallel to the upward reaction B A; and lastly a line *ae*, parallel to the other reaction A E, to close on *e*, the point of beginning.

12. **Triangular Truss: Stresses.**—While in this truss we might find the stresses at any joint, let us begin at B. Here