with the first known force, so that the two unknown forces will be the last two sides of that particular polygon. 3°. The direction in which any line is passed over, in going round the polygon as above directed, shows whether the stress in the piece to which that line was drawn parallel acts towards or from the joint to which the polygon belongs, and hence is compression or tension. The reader must understand this principle in order to correctly interpret his diagrams.

10. **Reciprocal Figures.**—Prof. Clerk-Maxwell called the frame and stress diagrams *reciprocal* figures; for, referring to the figures already drawn, we see that the forces which meet at one point in the frame diagram give us a triangle or closed polygon in the stress diagram, and the pieces which make the triangular frame have their stresses represented by the lines which meet at one point in the stress diagram. The same reciprocity will exist in more complex figures, and it is one of the checks which we have upon the correctness of our diagrams.

The convenience of the notation explained in § 3 depends upon the above property.

CHAPTER II.

TRUSSES WITH STRAIGHT RAFTERS; VERTICAL FORCES.

11. **Triangular Truss; Inclined Reactions.**—Suppose that the roof represented in Fig. 5 has a certain load per foot over each rafter, and let the whole weight be denoted by W. It is evident that one-half of the load on the rafter C F will be supported by the joint B and one-half by the upper joint; the same will be true for the rafter D F; therefore the joint B will carry $\frac{1}{4}$ W, the upper joint $\frac{1}{2}$ W, and the joint at E $\frac{1}{4}$ W. The additional stress produced in C F by the bending action of the load which it carries is not considered at this time, but must be noticed and allowed for separately. (See Chap. IX.) Taking the external forces in order from right to left over the roof, lay off *ed*, or $\frac{1}{4}$ W, vertically, to represent the weight E D acting downward at the joint E, next *dc* equal to $\frac{1}{2}$ W, for the weight D C, and lastly *cb* for the weight at B. Call *eb* the *load line.*

Let the two reactions or supporting forces for the present be considered as a little inclined from the vertical, as shown by the arrows B A and A E. Since the truss is symmetrical and symmetrically loaded, the resultant of the load must pass through the apex of the roof, and, as the two supporting forces must meet this resultant at one point, the two reactions must be equally inclined. Then, to complete the polygon of external forces:—as we have drawn *ed*, *dc*, and *cb* in order, passing over the frame to the left,—draw next *ba*, up from the extremity *b* of the load line, and parallel to the upward reaction B A; and lastly a line *ae*, parallel to the other reaction A E, to close on *e*, the point of beginning.

12. **Triangular Truss: Stresses.**—While in this truss we might find the stresses at any joint, let us begin at B. Here

we have equilibrium under the action of four forces, of which the two external ones are known. Taking the latter in the same order as above, and beginning at *c* (§ 9, 2°), pass over *cb* downwards and *ba* upwards; then draw *af* parallel to A F, in such a direction that *fc*, drawn from *f* parallel to F C, will strike *c*, the point of beginning. Because we passed from *a* to *f*, A F will pull on the joint B, and as we then passed from *f* to *c*, F C will exert a thrust on B. (It is usual to draw *af* from *a* and *fc* from *c* till they meet at *f*; but to determine the *kind* of stress, one must pass over the lines in the directions noted.)

Passing next to the apex of the roof, and again taking the forces in the same order, pass down the line *dc* for the external force, thence up to *f* for the thrust *cf*, and finally draw *fd* parallel to F D, thus determining the thrust of that rafter against the top joint. If this line does not close on *d*, the drawing has not been made with care. As all the stresses are now found we need not examine the remaining joint. It may again be noted that we pass over a stress line in one direction when we analyze the stresses at the joint at one end of the piece to which the line is parallel, and in the reverse direction when we consider the joint at the other end of the same piece.

13. **Effect of Inclined Reactions.**—If the supporting forces had been more inclined from the vertical, the point *a*, of their meeting in the stress diagram, would have been nearer *f*, thus diminishing the tension in A F, but not affecting the compression in the rafters. The inclination might be so much increased that *a* would fall on *f*, when the piece A F would have no stress, the thrust of the rafters being balanced without it. If *a* fell to the right of *f*, *af* would be a thrust.

14. **Triangular Truss: Vertical Reactions.**—If the two reactions are vertical, as will be the case when the roof truss is simply placed upon the wall, B A and A E, Fig. 6, will each be $\frac{1}{2}$ W, and the point *a* will therefore be found at the middle of *eb*. The polygon of external forces has closed up and become a straight line, but in the analysis it must still be used. Thus we pass down $ed + dc + cb$ for the weights at the joints and back over $ba + ae$ for the reactions. The rest of the diagram follows from § 12.

The diagrams which the reader draws may be inked in black and red, one denoting compression, the other tension, or the two kinds of stress may be indicated by the signs + and —.

15. **King-post Truss.**—In the truss of Fig. 7 the rafters are supported at points midway between their extremities. Each point of junction of two or more pieces is considered a joint around which the pieces would be free to turn were they not restrained by their connections with other points. Whatever stiffness the joint may possess from friction between its parts, or from the continuity of a piece, such as a rafter, through the joint, is not taken into account, and may add somewhat to the strength of the truss.

In this example, therefore, half of the uniform load on C L will be carried at B, and be represented by the arrow B C; the other half together with half of the uniform load on D K will make the force C D, and so on, three of the joints carrying each one-quarter of the whole load, and the two extreme ones one-eighth each.

On a vertical line lay off $gf = \frac{1}{8}$ W, $fe = ed = dc = \frac{1}{4}$ W and $cb = \frac{1}{8}$ W; then $ba = ag = \frac{1}{2}$ W, the two supporting forces. In the order shown by the arrow, for the joint B we have *cb* external load, *ba* reaction; then draw *al*, tension, § 9, 3°, parallel to A L and *lc*, compression, parallel to L C. At the joint C D the unknown forces now are those in L K and K D. Begin with the load *dc*, following with *cl*, the stress just found in C L; then draw *lk*, compression, parallel to L K, and *kd*, compression, parallel to K D, to close on *d*. Passing next to the joint D E, *ed* is the load, *dk* the thrust of D K on this joint, *ki* the tension in K I,* and *ie*, to close on *e*, is the compression in I E. Take next the joint in the middle of the

* It will be seen that K I is a tension member or tie, and not a post as would be inferred from the name given to this truss by old builders.

lower tie; we know ik, kl, and la; the next stress lies in A H; as we have just arrived at a from l, we must pass back horizontally until a line from h parallel to H I will close on i, the point from which we started. The remaining line hf is easily determined by taking either the joint E F or the one at G.

It will be noticed that, since the truss is symmetrically made and loaded, the stress diagram is symmetrical; ki must be bisected by al; dk and ei must intersect on al. Attention to such points ensures the accuracy of the drawing.

A truss, Fig. 8, is now submitted, which the reader is advised to analyze for himself, as a test whether the principles thus far explained are clearly understood.

16. **Wooden Truss with Frequent Joints.**—The truss represented by Fig. 9, a simple extension of Fig. 7, is one well adapted for construction in timber, the verticals alone being made of iron. It can be used for roofs of large span. In any actual case, before beginning to draw the diagram, assume an approximate value for the weight of the truss, add so much of the weight of the purlins, small rafters, boards and slates, or other covering, as is supported by one truss, and divide this total weight by the number of equal parts, such as D I or E L, in the two rafters. We thus obtain the weight which is supposed to act at each joint where two pieces of the rafter meet. The weight at each abutment joint will be half as much. If the rafter is not supported at equidistant points, divide the total load by the combined length of both rafters, to obtain the load per foot of rafter, and then multiply the load per foot by the distance from the middle of one piece of the rafter to the middle of the next, to obtain the load on the joint which connects them. Numerical values will be introduced in later chapters.

Draw the vertical load line equal to the total weight, and beginning with bc as the load on B from one-half of C H, space off the weights cd, de, etc., in succession, closing at p with a half load as at b. The point of division a, at the middle of pb, marks off the two supporting forces pa and ab, which close the polygon of external forces. Beginning now at B, draw, as heretofore directed, § 9, $abcha$ for this joint. The order of these letters gives the directions of the forces on the joint B. Then for the joint C D we have $hcdih$; for H K we have $ahika$; for D E we have $kidelk$, etc. Observe that, by taking the joints in this order, first the one on the rafter, and then the one below it on the tie, we have in each case only two unknown forces, out of, at some joints, five forces. We repeat, also, the remark that it is expedient, when possible, first to pass over all the known forces at any joint, taking them in the order observed with the external forces when laying off the load line. The rest of the diagram presents no difficulty.

After the stress in N O is obtained, the diagram will begin to repeat itself inversely, the stress in O G being equal to that in F N. It is therefore unnecessary to draw more than one-half of this figure, except for a check on the accuracy of the drawing by the intersections which are seen on inspection of this diagram. Noting the stresses found in the several polygons, we see that all the inclined pieces are in compression, while the horizontal and vertical members are in tension.

17. **Superfluous Pieces.**—Sometimes a vertical rod is introduced in the first and last triangles, where dotted lines are drawn. It is evident that this rod will be of no service if all the load is assumed to be concentrated on the joints of the rafters, and this fact can be determined from the stress diagram as well. Thus, taking the joint below H, Fig. 9, we have three forces in equilibrium; begin at a in the stress diagram and pass to h along the line already found for A H; then we are required to draw a vertical line from h and, from its extremity, a horizontal line to close on the point a from which we started; the vertical line therefore can have no length. All that this vertical rod can do is to keep the horizontal tie from sagging, by sustaining whatever small weight is found at its foot.

Therefore, whenever there are at a joint but three pieces or lines along which forces can act, and two of these pieces lie in one straight line, it follows from the above that the third piece must be without stress, and that the first two pieces or lines will have the same stress. Thus, L K of Fig. 7 and H I of Fig. 9 would have no compression if the external load C D were removed. This fact will often prove of service in analysis.

18. **Problem.**—Draw the stress diagram for the truss illustrated by Fig. 10, which is supported on a shoulder at the wall and by an overhead tie running from the right end. It will be convenient to imagine that tie replaced by the inclined reaction shown by the arrow at the right, as thus the reaction is kept on the right of the load at that joint. The reaction at the wall will cut the tie where the resultant of the load cuts it; if the load is uniform over the rafter, that intersection is at the middle of the tie.

Next, try this problem with the two inclined diagonals reversed, so as to slant up to the right. Notice the upper left-hand joint. Compare the two cases, as to difference in magnitude and kind of stress.

19. **Joints where three Forces are Unknown.**—It appears impracticable to determine the stresses at any joint where more than two forces are unknown. In Fig. 9, we could not start with the joint C D or at D E; for we should know only the external force or load, and have three unknown stresses to find; therefore our quadrilateral, of which one side is known, might have the other sides of various lengths, but still parallel to the original pieces of the frame. When the joints were taken in the order observed this difficulty was not met with.

When, in some cases, we find three or more apparently unknown forces at a joint we may have some knowledge of the proportion which exists between one or more of them and a known force, and can thus determine the proper length of the line in the stress diagram. An example of such a case will be given in Fig. 11. In Chapter VIII. will be found a treatment

that is applicable to certain trusses which otherwise offer difficulties in solution.

20. **Polonceau or Fink Truss.**—Fig. 11 shows a truss which is often built in iron. The loads at the several joints of the rafters are found by the method prescribed in § 16. It will be unnecessary to dwell upon the manner of finding the stresses at the joints B, C D, and H K, for which the stresses will be *ch*, *ha*, *ak*, *ki*, *hi* and *id*. But when we attempt to analyze the joint D E, we find that, with the external load, we have six forces in equilibrium, of which those along E M, M L, and L K are unknown. If we try the joint L A we find four forces, three of which are at present unknown. We are therefore obliged to seek some other way of determining one of the stresses.

It will be seen, upon inspection, that the joint E F is like the joint C D; and it will appear reasonable that N M should have an equal stress with I H. We may then expect that there must be as much and the same kind of stress exerted by M L to keep the foot of the strut N M from moving laterally as is found necessary in K I to restrain the foot of I H.

Returning then to the joint D E, and beginning with *ki*, pass next over *id*, then *de*, then draw *em*, parallel to E M, to such a point *m*, that (having drawn *ml* until its extremity *l* comes in the middle of what will be the space between *em* and *fn*, or until *ml* equals in length *ik*), the line *lk* shall close on *k* whence we started. The ties and struts can be readily selected by the direction of movement over these lines in reference to the joint D E. The remaining joints when taken in the usual order of succession offer no difficulty, and the other half of the diagram need not be added, unless one desires a check on the results.

This truss will be treated again in § 74.

The polygon which we have just traced, *kidemlk*, affords a good illustration of the rule that the forces which meet at a joint make a closed polygon in the stress diagram. The symmetry of the triangles *hik* and *mnl*, and their resemblance to

klo, are worth noting, and will assist one in drawing diagrams for trusses of this type.

21. **Cambering the Lower Tie.**—Sometimes it is thought desirable to raise the tie A O, either to give more height below the truss or to improve its appearance. The effect on the stresses of such an alteration is very readily traced, and one then can judge how much change it is expedient to make. Let it be proposed to raise the portion A O of the tie to the position indicated by the dotted line, and thus to introduce such changes in the other members that they shall coincide with the other dotted lines in Fig. 11, while the load remains unchanged.

The line $c\,h$ for joint B now becomes $c\,h'$, being prolonged until $h'\,a$ can be drawn parallel to H A in its new position. Next come $h'\,i'$ and $i'\,d$; then we easily draw $i'\,k'$, $k'\,l'$, $l'\,m'$, $m'\,n'$, etc. The struts H I, K L, and M N are the only pieces in this half of the truss unaffected by the change; the amount of increase, and the serious increase, of the other stresses for any considerable elevation of the lower member can be readily seen.

22. **Load on all Joints.**—If one prefers to consider that a portion of the weight of the truss, or that a floor, ceiling or other load is supported at the lower joints, the load may be distributed as in Fig. 12. Here the joints Q R and R S carry their share of the weight of the pieces which touch these joints, as well as such other load as may properly be put there. Each supporting force, if the load is symmetrical, will still be one-half the total load, but the two will no longer divide the load line equally, nor can the load line be at once measured off as equal to the total weight.

Begin, if convenient, with the extremity H of the truss, and lay off $h\,i$, $i\,k$, $k\,l$, etc., downwards, ending with $o\,p$. Passing on, around the truss, lay off next the reaction $p\,q$ upwards, equal to one-half the *total* weight, then $q\,r$ and $r\,s$ downwards, and finally $s\,h$ upwards, for the other supporting force, to close on h. The polygon of external forces, therefore, doubles back

on itself as it were, and $h\,p$ is still the load on the exterior of the roof. The diagram can now be drawn, by taking three joints on the rafter in succession before trying the joint Q R; when taking that joint remember that there is a load upon it. The loads on the horizontal tie cause the stresses in its three parts to be drawn as three separate lines, instead of being superimposed as in the figures before given.

A diagram may now be drawn for Fig. 13. The upper part of the roof, dotted in the figure, throws its load, through the small rafters, on the upper joints of the truss.

23. **Stresses by Calculation.**—It is evident, from inspection of the preceding diagrams, that the stresses may be calculated by means of the known inclinations of the parts of the trusses. The degree of accuracy with which they can be scaled equals, however, if it does not exceed the approximation which designing and actual construction make to the theoretical structure.

24. **Distribution of Load on the Joints.**—In Unwin's "Iron Bridges and Roofs" the rafter is treated as a beam continuous over three or more supports, and the distribution of the load on the several joints is there determined by that hypothesis. That such an analysis may be true, it is necessary that all the points at which the rafter is supported shall remain in definite positions, usually a straight line. As slight deformations of the truss and unequal loading of the joints will prevent the realization of that assumption, a division of the load at any point of a rafter or other piece so that the joints at its two ends shall be loaded in the inverse ratio of the two segments into which the point divides the piece will best represent the case. Uniform loads will be distributed easily by § 16. A different distribution of the load, however, if one prefers it, will only require a corresponding division of the load line. (See Part II., Bridge Trusses, Chaps. VIII. and IX.)