CHAPTER VI.

TRUSSES WITH HORIZONTAL THRUST.

59. **Scissor Truss.**—When it is desired to strengthen the rafters in a roof of moderate span by supporting them at their middle points, a simple means, often employed, is to spike on a piece from the lower end of one rafter to the middle of the other, as shown in Fig. 22. The two pieces may or may not be fastened together where they cross. At the first glance we should say that, to draw the diagram, we must lay off the load line *k e*, divide it as usual, and then, beginning at the joint E, draw *a′b′* and *b′f*, parallel to A B and B F. Next, for the joint F G, we should get the lines *b′c′* and *c′g*. For the apex we should have three lines, viz., *h g*, *g c′*, and a line from *c′* parallel to C H to strike *h*. There is evidently something wrong here. If we start from the other point of support K, we obtain the remainder of the diagram in dotted lines, and find that we have two points marked *c′*, some distance apart, which ought to come together; we also have two conspiring forces, *g c′* and *h c′*, whose vertical components ought to balance *h g*.

Abandoning this diagram for the present, let us start at the apex of the roof, where we may feel sure that there are but two unknown forces. Taking the load *h g* at that point, draw the full lines *g c* and *c h*. Next for the joint G F, starting with *c g*, pass down *g f* and draw *f b* and *b c*. The joint H I will similarly give the figure *i h c d i*. Lastly, the joint A C will add *b a* and *a d* to the stresses *d c* and *c b*. To close the polygon for the joint E we must now supply to *a b f e* the line *e a*, which must be the inclined reaction at E, required to keep this truss

from sliding outwards on the wall-plates, on the supposition that the points of meeting of two or more pieces are true joints (ones about which the parts are free to turn). As *e a* may be decomposed into *e a′* and *a′a*, the force *a′a* is called the horizontal thrust of the truss, which may be resisted by the wall or by a tie-rod from E to K. The pieces of this truss are all in compression.

60. **Horizontal Thrust or an Additional Member Necessary.**—That the truss is not in equilibrium without this inclined or horizontal reaction at the walls is seen, if we suppose that E and K are not prevented from sliding laterally; the joint A C will drop, the joints F G and H I will approach one another, and the angle at the apex will become sharper. This change will take place unless the above or some other restraining force is applied. The trouble arises from the four-sided space C, which is here free to change its form. A member added in either diagonal of this space will cure the evil. One from the apex to the joint C A will plainly act as a tie, and will be found to supply the missing line *c′c′* in the dotted diagram first drawn. From this diagram we see that the stresses in most of the pieces will then be greater than when the resistance comes from the wall. A strut between the joints F G and H I will also make the truss secure; the reader can try such a diagram, and see what pieces have their stresses reversed by the change. Either of the above modifications puts the truss into the class having vertical reactions.

61. **Remarks.**—As these trusses are usually made, reliance against change of form, where little or no horizontal thrust is supplied by the walls, is placed upon the stiffness of the rafters, which are of one piece from ridge to eaves, and on that of the two braces; but a failure to get a good horizontal resistance from the walls has sometimes resulted in an unsightly sagging or springing of rafters and braces. The bending moments on these pieces are due to the horizontal

thrust. Bending moments on a rafter or other piece will be considered later.

It is worthy of notice that *c d* equals *b a*, or that the thrust is constant throughout the brace. Two members crossing as at A must naturally give a parallelogram in the stress diagram; the component of the load at H I which starts down the brace will pass to E without being affected by crossing the other brace; yet, to resist the tendency to sag spoken of above, and for the reason that the braces are better able to resist thrust by mutually staying one another, it is advisable to spike them together at their intersection.

62. **Hammer-beam Truss; Curved Members.**—Another example where the horizontal thrust of the truss against the walls must be ascertained is shown in Fig. 23. This frame is called a hammer-beam truss, and is a handsome type often employed, in this country and abroad, for the support of church roofs, the bracing being visible from below, and the spaces containing more or less ornamental work. When the church has a clear-story, the windows come between the trusses at B, the truss is supported on columns, and the roof of the side aisle takes up the horizontal thrust. If there are no side roofs, the main walls are properly strengthened by buttresses.

It will be well to note in advance that a curved piece in a truss, so far as the transmission of the force from one joint to another is concerned, acts as if it lay in the straight line between the two joints. The curved members in the present example are the quadrants of a circle. They may have any other desired curve, depending somewhat upon the pitch of the roof. If, now, we consider the point of support B P of the truss, and remember that the curved brace A O transmits the force between its two extremities as if it were straight, it will be evident that the thrust of the inclined piece, if any thrust exists in it, must have a horizontal component which cannot be neutralized by a vertical supporting force alone. Therefore, in addition to the reaction of half the weight of the roof

and truss, there must be supplied by the wall, assisted perhaps by a buttress or a side roof, a certain horizontal thrust.

63. **Amount of the Horizontal Thrust.**—To determine the value of this thrust:—Let W equal the weight of truss and load. We have nine loaded joints, and there is, therefore, $\frac{1}{8}$ W at each joint except the two extreme ones. The portion 2 1 3 may be considered a small truss, like Fig. 7, superimposed on the lower or main truss 4 6 2 3 7 5, and thus bringing additional loads on the points 2 and 3. If then we regard the main truss as a trapezoidal truss, and consider that the pieces L A and Q A are unnecessary because the load is the same on the two halves of the frame, the trapezoidal truss will be 4 2 3 5, the brace 4–2 being made up of an assemblage of pieces. L A and Q A will be required when wind acts upon the roof. Considering the trapezoidal truss 4 2 3 5 alone, the joint 2 will carry a load equal to that on D M, E K, and F I, or $\frac{3}{8}$ W, the joint 3 will carry the same amount, while 4 will support $\frac{1}{8}$ W from C N, and 5 the remainder. If then we lay off on a vertical line $\frac{3}{8}$ W, for the load on 2, and draw lines parallel to 2–4 and 2–3 from its extremities, the line parallel to 2–3 will be the stress in the same, and will also, since the load is vertical, be the horizontal thrust of the foot of the compound brace 2–4. This force is marked H in the dotted triangle drawn below the truss. A reference to § 25, Fig. 14, may aid one in understanding the above.

64. **Stress Diagram.**—We now have the data for the stress diagram, of which one-half is shown. For the point 4, or B P, we have the upward supporting force $b p = \frac{1}{2}$ W, next $p a =$ H, the horizontal thrust just determined of the wall, etc., against the joint, *a o* parallel to the line of action of AO, and finally *o b*, the pressure of the post O B on 4. The resultant of *b p* and *p a*, or *b a*, may of course be used for the reaction of the wall. Taking next the joint 6, we have *c b* the load, *b o* the thrust of B O, and we then draw *o n* and *n c*. The joint C D gives *d c n m d*. The joint M A already has the lines *m n*, *n o* and *o a*; since the line which is to close on *m* must be

parallel to L M, and a is already vertically over m, $a\,l$ can have no length, and there is no stress in A L, as before assumed. Upon taking the joint D E we find also that no stress exists in L K. The reader must not think this fact at variance with the value H which was said to exist in 2–3 when we considered the trapezoid alone; the triangular truss 1 2 3 will plainly cause a tension in 2–3, and, with this distribution of load, such tension will exactly neutralize the compression caused in the same piece by 4–2. If one will consider the truss as loaded at 6, 2, 1, 3, and 7 only, thus doing away with N M, K I, I G, etc., he will find that a diagram will then give some compression in K L.

Another method of treatment will be applied to this truss later, § 75.

65. **Different Horizontal Thrusts Consistent with Equilibrium.**—In studying Fig. 22 we saw that the stresses in G C and C H were determined by the load G H, and that the space C would become distorted unless a horizontal thrust of a definite amount, here $a'a$, was supplied by the walls. In Fig. 23 also the same things are true; the trapezoidal truss 4 2 3 5 requires a certain horizontal thrust at the points 4 and 5 to balance its load; a greater or less thrust will cause the truss to rise or fall, so long as L A and Q A are neglected, for in that case motion can freely occur at joints 2 and 3. If, however, these pieces are under stress, a greater or less horizontal thrust may be applied, the truss will still be in equilibrium, and the diagram will close. Indeed a vertical reaction is a supposable one, in which case O A must be without stress. The same statement applies to Fig. 22, if one of the diagonals of the space C is put in. As all roof-trusses of small depth in their middle section, as compared with their total rise, have a tendency to spread under a load, and hence to thrust against their supports, their diagrams should be drawn for a moderate amount of thrust at least, if it is desired to have them maintain their shape; and the supports should be able to offer this resistance, or a tie should be carried across

below. Otherwise, in addition to the sagging, a large increase of stress is likely to be found in some of the parts as a result of a vertical reaction. The determination of the horizontal thrust in a braced frame of this kind is not very simple, but may be worked out by a method given in Part III, "Arches," Chap. XII.

66. **Proof.**—That such trusses are in equilibrium under a greater or less amount of horizontal thrust, or even when the reactions are vertical, provided the pieces are able to withstand the resulting stresses, is illustrated by Fig. 24. Here the load C B is taken as twice D C. The vertical reactions $b\,a$ and $a\,d$ are calculated by the method of § 26. The diagram with unaccented letters is then drawn and closed as usual. Next, any horizontal thrust $a\,a'$ at the points of support is assumed and the diagram with accented letters is drawn. This diagram also closes. The reduction of all of the stresses except that in $f\,g$ is most marked. We see from these cases that only when the truss admits of deformation by the distortion of some interior space such as C of Fig. 22, or R of Fig. 27, is the horizontal thrust determinate by the method of these chapters; and that moderately inclined reactions or the tension of a horizontal tie between points of support are favorable to a reduction of the stresses.

Arched ribs of a nearly constant depth, not infrequently employed in railroad stations and public halls, will be treated in Part III.

CHAPTER VII.

FORCES NOT APPLIED AT JOINTS.

67. **First Diagram.**—In the trusses heretofore treated the loads have been concentrated at those points only which were directly supported. It sometimes happens that the cross-beams or purlins, which connect the trusses and convey the weight from the secondary rafters to the main rafters, rest upon the latter at points between the joints. Let us, in Fig. 25, assume that a load rests upon the middle of each of the upper rafters. If we neglect the bending action of the load E G upon the rafter and proceed as usual, we consider that one-half of the load E G will be supported at each of the joints C E and G K, and similarly for the load K M. Therefore, having laid off the weights and the two equal reactions of the walls on the load-line of the first diagram, we may increase the loads on the joints C E, G K, and M O by the new points of division, and complete this diagram, taking first B, then the next joint on the inside, and then the outside one. It will be noticed that all of the pieces except the rafters are ties.

68. **Supplying Imaginary Forces.**—This diagram gives but one stress along the whole of the upper rafter; but it is plain that the vertical force E G must have a component along the rafter and cause a different stress to exist in E T from what exists in G T. If, however, we suppose a joint to be at E G, the transverse component of E G will cause it to yield, as there is no brace beneath to hold it in place. To secure equilibrium here we may supply an imaginary force E F, shown by the dotted line, equal and directly opposed to this

transverse component. This imaginary force will take the place of a perpendicular strut, will steady the joint, and will leave the longitudinal component to affect the rafter. But the transverse component of F G actually gives a pressure at the joints C E and G K, while the imaginary force E F, just added, will lift the ends of this rafter by the same amount; therefore we must restore the pressure, and the equilibrium of the rafter F T as a whole, by adding imaginary forces, each one-half of E F, at C D and G H. This added system of forces cannot interfere with the stresses in any other pieces, for they balance by themselves. Treat the similar load K M in the same way.

69. **Second Diagram.**—In the second diagram the two supporting forces, pa and ab, are each equal to one-half the total load. Lay off bc as before; draw the dotted line cd, equal and parallel to the first imaginary force C D; then de vertical, as before; then ef, equal to, and in the direction of E F; then fg, and so on, arriving finally at p, as usual.

The construction of the rest of the diagram presents no difficulty; the joints are taken in the same order as before, and, when we have more than one external force on a joint, we take them in succession, in the order first observed for the external forces. When we reach the upper rafters, we find that g falls on the line et; et is greater and gt is less than the line for the same piece in the first diagram.

70. **Comparison of Results.**—Thus it appears that the first diagram gives the stress which would exist in the whole length of the rafter E T G, if the load F G were actually at its extremities; but, being at its middle point, one-half of the longitudinal component of F G goes to diminish the compression otherwise existing in G T, and the other half to increase the compression in E T. A comparison of the two diagrams will also show the truth of the former statement, that the system of imaginary forces does not affect any of the truss outside of the particular pieces to which it may be applied. It is still necessary to provide for the bending action of the

transverse portion of F G, or a force equal and opposite to E F upon the rafter, considered as a beam extending from hip to apex, a joint of course not being made at E G. This subject will be treated in Chapter IX.

71. **Remarks.**—If the action of the wind upon this truss is considered, it will be seen at once that no special treatment is needed; for the wind pressure is normal, and the addition of the opposite force E F at once balances the force on this joint, and transfers it to the ends D and H as the first analysis did. The bending action on the rafter must, however, be provided for.

The treatment of loads or forces not directly resisted, as above, is given by Mr. Bow in his "Economics of Construction," and may be applied to frames where one or more of the internal spaces are not triangles, but quadrilaterals. If such spaces are not surrounded by triangular spaces on at least all sides but one, the truss is liable to distortion, unless the resistance of some of the pieces to bending or the stiffness of the *theoretical* joints is called into play. A use of this treatment at many points in the same diagram will, however, be apt to make confusion.

Another application of imaginary forces, where a bending moment exists, will be made at the close of the next chapter.

CHAPTER VIII.

SPECIAL SOLUTIONS.

72. **Reversal of Diagonal.**—Difficulty is sometimes experienced in completing the diagram for a truss because, after passing a certain point, no joint can be found where but two stresses are unknown; while yet, judging from the arrangement of the pieces, the stresses ought apparently to be determinate. Such a case was found in Fig. 11, and was solved in § 20 by what might be called the law of symmetry. A method of more general application to these cases is what may be styled *Reversal of a Diagonal.*

It has been pointed out already that, if any quadrangular figure in a truss is crossed by one diagonal, the other diagonal of the quadrangle may be substituted for the former without affecting the stresses in any pieces except those which make up the quadrangle. See §§ 26 and 53. It will be found that such a change often reduces the stress in one or more pieces of the quadrangle to zero, and thus makes the truss solvable graphically. It will be well, if the reader fails to distinguish readily the altered truss from the original one, to temporarily erase from a pencil sketch the pieces thus rendered superfluous, or to draw the truss anew with the proper changes as has been done in Figs. 26 and 27. The modified truss will then be easily analyzed, and, when the old members are restored, enough stresses will be known to make the final solution practicable.

73. **Example.**—This method will first be applied to the roof-truss, Fig. 26, of a railroad station at Worcester, Mass. The span of this roof is 125 feet; entire height, wall to apex,