transverse portion of F G, or a force equal and opposite to E F upon the rafter, considered as a beam extending from hip to apex, a joint of course not being made at E G. This subject will be treated in Chapter IX.

71. **Remarks.**—If the action of the wind upon this truss is considered, it will be seen at once that no special treatment is needed; for the wind pressure is normal, and the addition of the opposite force E F at once balances the force on this joint, and transfers it to the ends D and H as the first analysis did. The bending action on the rafter must, however, be provided for.

The treatment of loads or forces not directly resisted, as above, is given by Mr. Bow in his "Economics of Construction," and may be applied to frames where one or more of the internal spaces are not triangles, but quadrilaterals. If such spaces are not surrounded by triangular spaces on at least all sides but one, the truss is liable to distortion, unless the resistance of some of the pieces to bending or the stiffness of the *theoretical* joints is called into play. A use of this treatment at many points in the same diagram will, however, be apt to make confusion.

Another application of imaginary forces, where a bending moment exists, will be made at the close of the next chapter.

CHAPTER VIII.

SPECIAL SOLUTIONS.

72. **Reversal of Diagonal.**—Difficulty is sometimes experienced in completing the diagram for a truss because, after passing a certain point, no joint can be found where but two stresses are unknown; while yet, judging from the arrangement of the pieces, the stresses ought apparently to be determinate. Such a case was found in Fig. 11, and was solved in § 20 by what might be called the law of symmetry. A method of more general application to these cases is what may be styled *Reversal of a Diagonal.*

It has been pointed out already that, if any quadrangular figure in a truss is crossed by one diagonal, the other diagonal of the quadrangle may be substituted for the former without affecting the stresses in any pieces except those which make up the quadrangle. See §§ 26 and 53. It will be found that such a change often reduces the stress in one or more pieces of the quadrangle to zero, and thus makes the truss solvable graphically. It will be well, if the reader fails to distinguish readily the altered truss from the original one, to temporarily erase from a pencil sketch the pieces thus rendered superfluous, or to draw the truss anew with the proper changes as has been done in Figs. 26 and 27. The modified truss will then be easily analyzed, and, when the old members are restored, enough stresses will be known to make the final solution practicable.

73. **Example.**—This method will first be applied to the roof-truss, Fig. 26, of a railroad station at Worcester, Mass. The span of this roof is 125 feet; entire height, wall to apex,

45 feet; camber of main tie 8 feet; rafter divided into six equal panels; trusses 50 feet apart.

Under steady load the tie bars S T. T U, U W, WS, which cross the centre line of the truss, will be without stress, as in Fig. 14, § 25. Indeed, as these two centre ties are independent of one another, but one can be in action at a time, as, for instance, S W and T U when the wind is on the left side. If we begin our diagram from B with *c b a k c*, we meet with no difficulty until we have passed the joint E F, for which we drew *f e n o p f*. At either of the next joints are three unknown stresses. As all stresses are determined up to the piece P Q, change the diagonal Q R in the adjoining quadrilateral from the position of the full line to the dotted one. Then the joint F G, as seen in the sketch below, will give us *g f p q′ g*. As the full-lined diagonal has been removed, the joint R W has disappeared; for, if three supposed forces are in equilibrium at one point, § 17, and two of them act in one line, the third force must be zero, and R S therefore can have no stress. The stress in S W will also be zero unless it resists wind on the left, and the stress in S T is then zero. In either case we can draw *h g q′ r′ h* for the upper joint, and then find *a w* and *w r′*, if it exists, at the lower joint. The dotted peak is not in the main truss, but in the jack-rafters which transfer their load to G H and H I; if one prefers, he may put a load at the peak and draw the triangle of forces for that point.

After using the above expedient on the other half of the truss also, if the load is unsymmetrical, we replace the *reversed diagonal* and find the true stresses in the pieces affected by the change,—the diagonal and the four sides of the containing quadrilateral. Hence we may draw *p o a w q p* for the lower joint or *h g r s h* for the upper joint, and finally *g f p q r g* for the left-hand joint of the quadrilateral.

74. **Polonceau Truss.**—The left half of Fig. 29 is the same as Fig. 11. It will be remembered that we were stopped at the piece D E of Fig. 29 by having three unknown stresses at

either end. Change the full line E F to the dotted one. The stress in F G at once becomes zero, as did R S in Fig. 26. We may now find the stresses in D E and E L at the joint K L; in dotted E F and G M at joint L M, and in A H and H F at the lower joint. Then the diagonal may be replaced and the stresses in D E, E F, F G, E H, and F L rectified. The right half of Fig. 29 may be similarly solved by reversing the diagonal P Q, which change makes the stress in O P zero.

75. **Hammer-Beam Truss, by Reversal of Diagonals.**—The hammer-beam truss of Fig. 27 differs from that of Fig. 23 by the omission of the vertical in the space R. As pointed out in § 66, this omission renders the horizontal thrust of this truss definite. In attempting to draw a diagram, however, we cannot apparently begin at the wall until we know the horizontal thrust, and, if we begin at F G, we soon meet with joints where three unknown forces are found. The method of the preceding sections will first be applied to the right half. Draw *g f r* for the upper joint, *h g r s h* for joint G H, and *f e q r f* for E F. As joints H I and R A are now insoluble, draw dotted T W for the full-lined diagonal T W, and do the same with X Y. The truss will thus be changed to the form of the sketch below. For, since T A and Y A act in the same straight line (shown dotted on left half of truss), the stress in W X is now zero, and T A and Y A have the same stress. Further, at joint K L there remain K Y, Y L, and the exterior force or load K L, which latter acts in the vertical line Y L; hence the stress in K Y is now zero, and Y L carries K L only. We can therefore draw *i h s t′ i* for joint H I, *k i t′ w′ k* for joint I K, *w′ t′ s r q . . . a w′* for joint A R, and *l k w′ a l* for the abutment. The reaction *a l*, being thus determined, can be used to draw the diagram, as in Fig. 23. The diagram for the left half of the truss is given in full lines, and it may be seen that A P and A T are now useful.

76. **Method of Trial and Error.**—Where the unknown stress in but one piece appears to stand in the way of a

solution, the diagram may sometimes be drawn with comparative ease by trial. Thus, in the left half of Fig. 27, we may assume the value of the horizontal thrust or of the stress in P Q and proceed with the diagram. Upon its failing to close, we can change the assumed quantity and try again. Thus, beginning at the apex, draw $gfrg$, $feqrf$, and $hgrsh$; then assume qp' and its equal st'. The middle joint will give $t'srqp'a't'$; the joint D E, $p'qedo'p'$, etc.; and finally the horizontal line from n' will fail to meet a line parallel to A M on the load line, to give mb in the post. It is evident, upon a slight inspection, that qp' is too long. The reader will find that he can soon bring the diagram to a closure by diminishing qp'.

By the use of such approximations one of necessity loses that check on the accuracy of the diagram, of having it close with reasonable exactness.

Fig. 30, in case one or the other of the dotted diagonals is used, will serve as an example for the practice of the preceding suggestions. Which diagonal tie, if either, will be needed for wind, and which for steady load?

77. **Example.**—We will close this branch of the subject with an example which will introduce one or two new points in addition to a combination of principles heretofore illustrated separately. The example shows the capabilities of this method in handling complex problems. The structure drawn in Fig. 28 is to be treated as a whole in its resistance to wind pressure.

The steady-load diagram would present no difficulty. The truss is carried upon columns which are hinged at their lower ends B and P, each being connected by a pin to its pedestal. The brace at R is therefore necessary to prevent overturning. The proportions of the frame are as follows: Distance between columns, 76 ft.; A C = 15 ft.; Q R = 7 ft.; camber of lower tie, 3 ft.; 1–A = 19 ft.; height of space 1 = 16 ft.; of Y = 7 ft.; extreme height, ground to peak, 48 ft.

Distance between trusses, 12 ft. Scale 40 ft. = 1 in. Scale of diagram, 8000 lbs. = 1 inch. No wind on C.

Wind pressure on main roof, 12,000 lbs. = bj; therefore fg, gh, etc., = 3000 lbs.; wind pressure on K X = 3360 lbs. = j–10; on L Y = 3500 lbs. = 10–m. The dotted arrows are resultants of wind pressure on the sloping surfaces. By moments about P, or by proportion of segments of span B P, as in § 48, we find

that 8368 lbs. of bj	is carried	at B,	and	3632 lbs.	at P.
that 940 " " 10–m	"	" "	"	2460 "	" "
9308 lbs. = b–9	"	" "	"	6092 "	" "

The horizontal force, j–10, at K, may be supposed to be resisted equally at each point of support, since the two posts will be alike. Hence $jk = 9\text{–}a' = \frac{1}{2}(j\text{–}10) = 1680$ lbs. is carried at B. The moment of this horizontal force K about B or P, tending to overturn the frame, or the couple formed by K and the equal reaction in the line P B, will cause an increased upward vertical force at P and an equal downward force or diminished pressure at B. Its value, § 42, will be

$\dfrac{3360 \times 39\frac{3}{4}}{76} = 1760$ pounds $= a'a$. The reaction at B must balance the components, b–9, 9–a', and a'–a, and hence will be ab. The reaction at P will then be m (or p) a, which may be checked in detail, if desired.

The reaction ab, at B, will now be decomposed into its vertical and horizontal components ac and cb. The piece A C can resist ac as a strut or post, but must carry cb 5900 lbs. by acting like a beam. Were there a real joint at D the structure would fall. It is therefore necessary to make the post of one piece, or as one member from B to R. The magnitude of the horizontal force at F caused by the 5900 lbs. of horizontal force at B will be in the ratio of the two segments of the column (beam) or as 15 to 7, or 12,643 lbs. These two forces must be balanced at D by a force equal to their sum, or 18,543 lbs. As in § 68, Fig. 25, this beam action of the post must be neutralized, before the diagram can be drawn, as these diagrams take no account of bending moments, for which see Chap. IX.

We therefore apply at B C the imaginary horizontal force $bc = 5900$ lbs., opposed to the direction of the reaction, and leaving only ac, the vertical component, which is balanced by the post; at C D we apply $cd = 18,543$ lbs.; and at E F, we add $ef = 12,643$ lbs. The sum of these three imaginary horizontal forces being zero, the stresses in the truss are not disturbed. The same steps must be taken at P, the horizontal forces mn, no, and op being obtained by the same process from the horizontal component po of the reaction pa.

The load line therefore finally becomes $bcdefghiklmnop$, the force D E being shifted laterally as shown, and ik being the resultant of ij and jk. The stress in D Q is readily obtained by drawing deq. Then the point D of the post gives the figure $acdqra$, determining the stresses in the upper part of the post and the brace R A. The remainder of the diagram presents no difficulty.

The column must be designed to resist the large bending moment to which it is liable, as well as the thrust qr. For bending moments, etc., see the next chapter, and also Part II. As this structure is supposed to be open below, the lower member should be adapted to resist such compression as may come upon it from the tendency of a gust of wind, entering beneath, to raise the roof.

CHAPTER IX.

BENDING MOMENT AND MOMENT OF RESISTANCE.

78. **Load between Joints.**—Having treated of the action of external forces upon a great variety of trusses, we propose now to investigate the graphical determination of the bending moments which arise from the load on certain pieces, and of the stresses due to the moments of resistance by which the bending moments must be met.

To recapitulate some statements of earlier chapters:—In case the transverse components of the load upon a portion of a rafter, or other piece of a truss, are not immediately resisted by the supporting power of some adjacent parts, or, in other words, unless the load on a structure is actually concentrated at the several joints, such transverse components will exert a bending action on the portion in question, and the additional stress thus caused in the piece may be too great to be safely neglected. Further, in case the piece makes any other than a right angle with the line of action of the load, or has an oblique force acting upon it, the stress along it, given by the diagram, will be less than the maximum, and will generally be the mean stress. Lastly, in case a piece is curved, a bending moment will be exerted upon it by the force acting along the straight line joining its two ends, this bending moment being a maximum at the point where the axis or centre line of the piece is farthest removed from the line drawn between its ends.

79. **Example.**—To illustrate the former statements by a simple example:—Suppose the rafters A C and B C, Fig. 31, to be loaded uniformly over their whole extent. Let us assume, in the first place, that the tie A B is not used, but