or 18,543 lbs. As in § 68, Fig. 25, this beam action of the post must be neutralized, before the diagram can be drawn, as these diagrams take no account of bending moments, for which see Chap. IX.

We therefore apply at B C the imaginary horizontal force $bc = 5900$ lbs., opposed to the direction of the reaction, and leaving only ac, the vertical component, which is balanced by the post; at C D we apply $cd = 18,543$ lbs.; and at E F, we add $ef = 12,643$ lbs. The sum of these three imaginary horizontal forces being zero, the stresses in the truss are not disturbed. The same steps must be taken at P, the horizontal forces mn, no, and op being obtained by the same process from the horizontal component po of the reaction pa.

The load line therefore finally becomes $bcdefghiklmnop$, the force D E being shifted laterally as shown, and ik being the resultant of ij and jk. The stress in D Q is readily obtained by drawing deq. Then the point D of the post gives the figure $acdqra$, determining the stresses in the upper part of the post and the brace R A. The remainder of the diagram presents no difficulty.

The column must be designed to resist the large bending moment to which it is liable, as well as the thrust qr. For bending moments, etc., see the next chapter, and also Part II. As this structure is supposed to be open below, the lower member should be adapted to resist such compression as may come upon it from the tendency of a gust of wind, entering beneath, to raise the roof.

CHAPTER IX.

BENDING MOMENT AND MOMENT OF RESISTANCE.

78. **Load between Joints.**—Having treated of the action of external forces upon a great variety of trusses, we propose now to investigate the graphical determination of the bending moments which arise from the load on certain pieces, and of the stresses due to the moments of resistance by which the bending moments must be met.

To recapitulate some statements of earlier chapters:—In case the transverse components of the load upon a portion of a rafter, or other piece of a truss, are not immediately resisted by the supporting power of some adjacent parts, or, in other words, unless the load on a structure is actually concentrated at the several joints, such transverse components will exert a bending action on the portion in question, and the additional stress thus caused in the piece may be too great to be safely neglected. Further, in case the piece makes any other than a right angle with the line of action of the load, or has an oblique force acting upon it, the stress along it, given by the diagram, will be less than the maximum, and will generally be the mean stress. Lastly, in case a piece is curved, a bending moment will be exerted upon it by the force acting along the straight line joining its two ends, this bending moment being a maximum at the point where the axis or centre line of the piece is farthest removed from the line drawn between its ends.

79. **Example.**—To illustrate the former statements by a simple example:—Suppose the rafters A C and B C, Fig. 31, to be loaded uniformly over their whole extent. Let us assume, in the first place, that the tie A B is not used, but

that the thrust of the rafters is resisted by the walls which carry the roof. Consider the piece A C. Since the roof is symmetrically loaded, the thrust at C must be horizontal, and therefore the reaction which supports this end of A C will lie in the line C E. The centre of gravity of the load on A C is at D, its middle point, and the resultant of the load will, if prolonged upwards, intersect C E at E. Since the rafter is in equilibrium under the load and the reactions at C and A, the direction of the reaction of the wall at A must also pass through E (compare Figs. 3 and 4). Draw A E and prolong E D to G. Let E G be measured by such a scale as to represent the load on A C. The three forces meeting in the common point E will then be equal to the respective sides of the triangle A E G, drawn parallel to them; and, since A G equals E C, the reactions at A and C will be A E and C E.

We now decompose A E and C E into components along and transverse to the rafter, and have A F, direct compression on the rafter at A, and C F, direct compression at C. The compression on successive sections of the rafter increases from C to A by the successive longitudinal components of the load. The two components A L and C Q, which, combined with A F and C F, give the original forces A E and C E, are analogous to the supporting forces of a beam or truss, and through them we obtain the bending action of the load on this rafter. If, now, the rafters simply rest on the wall, being secured against spreading by the tie A B, the reaction A E will be replaced by the two components, A I, the upward supporting force of the wall, and A G, the stress exerted by the tie; these two forces give the same stress and bending moments on the rafter as before.

80. **Comparison with Diagram.**—Consider, next, the method by diagram. The load is now to be concentrated at the joints, and in place of E G, we shall have A N and C P, each one-half of the load on one rafter. Lay off 1–2 to represent the total load on the roof, make 1–3 equal to A N and 1–4 to A I, and draw 3–5 and 4–5 parallel to the rafter and tie.

A G will equal 4–5, and therefore the stress in the tie is given correctly; but, since A I − A N = A K = 3–4, 3–5 equals A D, and this is the stress given by the diagram as existing from A to C, a supposition which is true when the load is actually concentrated at the joints, but is not true for a distributed load. But A D, or 3–5, is equal to one-half of A F + F C, and is manifestly the value of the direct compression at the middle point D of the rafter; all of the load from A to D was, when we drew the diagram, considered to be concentrated at the joint A. To 3–5, or A D, we should add D F, to obtain the correct compression A F at the lower end; therefore a piece which supports a distributed load should have a compression, equal to the longitudinal component of so much of the load as is transferred to its lower end, added to its stress obtained from the stress diagram. The amount to be added, however, is generally insignificant as compared with the truss stress.

The load on the principal rafters of a roof-truss is usually concentrated at series of equidistant points, by means of the *purlins*, or short cross-beams which extend from one truss to another, and which are themselves weighted at a series of points by the pressure of the secondary rafters. These secondary rafters, when employed, carry the boards, etc., and thus have a uniformly distributed load. It is only in cases where purlins rest at other points than the so-called joints that bending action occurs in the principal rafters, or in very light trusses where the boards are nailed directly to the main rafters. We need to determine the maximum bending moments on such main rafters, on the purlins and secondary rafters, in order to intelligently provide sections sufficiently strong to resist them.

81. **Bending Moment.**—It will first be well to explain what *bending moment* and *moment of resistance* are. A horizontal beam A B, Fig. 32, supported at its two ends, when loaded with a series of weights, distributed in any manner, is in equilibrium under the action of vertical forces, the weights acting downwards and the two supporting forces acting up-

wards. These supporting forces are easily calculated by the principle of the lever, or by taking moments as explained in §§ 26 and 36. They will be found graphically presently. As the beam is at rest, there must be no tendency to rotate, and therefore, if we assume any point for an axis, the sum of the moments, that is of the products of each force by its distance from the axis, must equal zero. A moment which tends to produce rotation in one direction being called plus, one which acts in the other direction is called minus. If then we pass an imaginary vertical plane of section through any point in the beam, such as E, the sum of the moments on one side of the plane of section must balance or equal that on the other. The sum of these moments on one side or the other is called the *bending moment:* the reason for the name will soon be evident.

82. **Moment of Resistance.**—These bending moments on opposite sides of the section in question can balance one another only through the resistance of the material of the beam at the section where stresses between the particles are set in action to resist the tendency to bend. The beam becomes slightly convex, and the particles or fibres on the convex side are extended, while those on the concave side are compressed. Experiment shows that, for flexure within such moderate limits as occur in practice, the horizontal forces exerted between contiguous particles vary uniformly as we go from the top of the beam to the bottom, the compressive stress being most intense on the concave side, diminishing regularly to zero at some point or horizontal plane, called the *neutral axis,* then changing to tension and increasing as we approach the convex side. The two sets of stresses reacting against each other may be represented to the eye by the arrows in the vertical section marked E′.

Since all of the external forces are vertical, these internal stresses, being horizontal, must balance in themselves, or the total tension must equal the total compression, whence it follows that the neutral axis must pass through the centre of

gravity of the section. To make this fact clear, let one consider that the distance of the centre of gravity from any assumed axis or the position of the resultant of parallel forces is found by multiplying each force or weight by its distance from that axis and dividing by the sum of the forces. Now if we attempt to find the centre of gravity of a thin cross-section of this beam, and take our axis through the point where the centre of gravity happens to lie, the sum of the moments of the particles on each side will balance or be equal, and we can see that the distance of each particle from the axis will vary exactly as these given stresses; hence the neutral axis must lie in the centre of gravity of each cross-section.

As these stresses are caused by and resist the external bending moment on each side of the section, the moment in the interior of the beam, made up of the sum of the products of the stress on each particle multiplied by its distance from the neutral axis, or indeed from any axis, and known as the *moment of resistance,* must equal the bending moment at the given section. As the tensions and compressions on one side of the plane of section tend to produce rotation about the neutral axis in the same direction, their moments are added together.

83. **Formula for Bending Moment.**—The bending moment, then, in the beam A B of the figure, at any section E, will be, if P_2 is the supporting force on the right, W_1, W_2, etc., the weights,

$$P_2 \, . \, BE - W_1 \, . \, CE - W_2 \, . \, DE \, ;$$

or, in general, if L equal the arm of any weight, and Σ be the sign of summation,

$$M \text{ (the bending moment)} = P_2 \, . \, BE - \Sigma \, W \, . \, L,$$

it being remembered always to take only the weights between one end and the plane of section.

The moment of resistance, being numerically equal to the bending moment, is therefore equal to the above expression, and the maximum stress at any section can thence be determined, or the required cross-section to conform to the proper working stress for the material. The weights on one side of the section may all be considered to be concentrated at their common centre of gravity, or point of application of their resultant, so far as the bending moment at that section is concerned; the load when continuous is always so taken.

If the reader will take a special case, and, having a beam of known length with weights in given positions, will first find the supporting forces, and then calculate the bending moment on either side of a plane of section, he will obtain the same result with opposite signs, showing that the two moments balance one another. The numerical result, being the product of two quantities, is read as so many foot-pounds or inch-pounds, according to the units employed. As the stress in any material is usually expressed in pounds on the square inch, the latter units are the better.

84. **Equilibrium Polygon.**—Let us suppose that the weights which, in Fig. 32, rest upon the beam are transferred to a cord at the several points c, d, f, and g, vertically below their former positions C, D, F, and G, the cord itself being attached to two fixed points a and b, at equal distances vertically from A and B. Let us further suppose that the amount of the weight at G alone is at present known. This cord can be treated as if it were a frame. Taking the joint g into consideration, draw 5–4 vertically, equal to the weight, then 5–0 parallel to ag and 4–0 parallel to gf. The two lines just drawn must be the tensions in ag and gf. For the joint f, fg is now known; therefore 4–3 parallel to the weight and 3–0 parallel to fd will determine the other forces at f. The side 4–3 must equal the weight at F, and must lie in the same straight line with 5–4; for this triangle was constructed on the side 4–0 previously found. Continuing the construction for the successive angles of the cord, we find that a vertical line 5–1 will represent by its several portions the successive weights, and that the tensions in the different parts of the cord will be given by the lines parallel to these parts, drawn

from the points of division of the load line, and all converging to the common point 0. Draw 0–6 horizontally, and hence parallel to ab; this line will be the horizontal component of the tension at any point of the cord, and is here denoted by H. The form assumed by the cord for a given distribution of weights is called the *Equilibrium Polygon*, as the system will be in equilibrium or at rest; and it is also called in mechanics a funicular polygon. Students of mechanics will recall the fact, so easily shown here, that the horizontal component H is a constant quantity at every point.

85. **Reactions.**—If now the cord, instead of being fastened to fixed points at a and b, is attached to the two ends of a rigid bar ab, and the whole system is then suspended from A and B by two short cords, its equilibrium will not be disturbed. The pull 5–0 at a will be decomposed into 0–6, compression in ba, and 6–5, tension along aA. Similarly at b, 0–1 will be decomposed into 1–6 along bB and 6–0 along ab. 6–0 balances 0–6, while 1–6 and 6–5 must be the supporting forces at b and a. As the supporting forces do not depend upon the form of the frame or truss, the reactions which carry the beam at B and A must be these same quantities.

86. **Equilibrium Polygon, General Construction.**—We may make the construction more general by drawing an equilibrium polygon from any point a', vertically below A, and finding the outline of a cord which will sustain in equilibrium the given weights at the given horizontal distances from A. Lay off the weights in succession from 5 to 1; assume any point 0′ arbitrarily and connect it with all the points of division of the load line. Begin at a', and draw $a'g'$ parallel to 5–0′, stopping at the vertical dropped from G; then draw $g'f'$ parallel to 4–0′, etc., and finally $c'b'$ parallel to 1–0′. That this will be the figure of a cord suspended from a' and b' follows from the preceding demonstration. Connect b' with a'; a line, parallel to $b'a'$, from 0′ must strike the same point 6 which the line from 0, parallel to ba, touched. The sup-