porting forces, if $b'a'$ exists, will be 1–6 and 6–5 as before; but 0′–6′ will be the horizontal component H′ for this cord.

87. **The Equilibrium Polygon Gives Bending Moments.**—If we turn again to the first cord, attached at a and b, the piece ab being dispensed with, the moment of all the forces on one side of any point, such as e, must be the bending moment there; but as the cord is perfectly flexible and at rest, this bending moment will equal zero. Using, instead of 1–0, its two components $1–6 = P_2$ and $6–0 = H$, multiplying each force by the perpendicular distance of its line of action from e, calling the combined moments of the weights on one side of e $\Sigma W.L$ as before, and denoting the tendency to produce rotation in opposite ways by opposite signs, we shall have, for moments of forces on the right of, and around e,

$$P_2 \,.\, bk - \Sigma W.L - H \,.\, ek = 0,$$

or

$$H \,.\, ek = P_2 \,.\, bk - \Sigma W.L.$$

But $P_2 . bk = P_2 . BE$, and $P_2 . BE - \Sigma W.L = M$, the bending moment at the section E of the beam, as shown in § 83; therefore

$$M = H \,.\, ek.$$

By a similar analysis of the lower cord we have

$$P_2 \,.\, ik' - \Sigma W.L = (6–0') \,.\, e'l = M.$$

From similarity of triangles $le'k'$ and 6′ 0′ 6, we have

$$e'l : e'k' = 6'–0' : 6–0',$$

or

$$(6–0') \,.\, e'l = (6'–0') \,.\, e'k';$$

therefore

$$M = (6'–0') \,.\, e'k' = H' \,.\, e'k',$$

as in the other case. The solution is therefore general, and the bending moment at any section of the beam equals the product of H from the stress diagram 0 1 5 by the vertical ordinate, below the section, from the cord to the line connecting its two extremities.

88. **Remarks.**—The relative situations of a' and b' will depend upon the choice of the position of 0′, and this point may be taken wherever convenient. H′ is measured by the same scale used in plotting 5–1, while $e'k'$ must be measured by the scale to which AB is laid off. The two scales, one representing pounds, the other inches, need not be numerically the same; their product will be inch-pounds.

A single load on the beam will have for its equilibrium polygon two straight lines from a' and b', meeting at a point vertically under the weight. A uniformly distributed load will give a parabola with the maximum ordinate at the middle of the span. This load may be treated as if concentrated at any convenient number of points along the beam, as we have done in getting the loads at the several divisions of a rafter, and the angles of the polygon will lie in the desired parabola. When the beam is inclined the transverse components alone of the load produce any bending, as explained for a uniform load in § 79. Wind pressure will act as a uniform normal or transverse load on the piece which directly resists it.

The equilibrium polygon has much more extended applications in Parts II. and III.

89. **Moment of Resistance of Rectangular Cross-Section.**—Next, to determine the moment of resistance for a particular form of cross-section:—Consider a beam of rectangular cross-section, represented by ABCD of Fig. 33. The intensity of stress, as shown at E′, Fig. 32, varies uniformly each way from the neutral axis which, lying through the centre of gravity G of the cross-section, will be at EF, the middle of the depth. The stress on a square inch will be most intense on the fibres at the edge AB or CD, and less intense on any intermediate layer, such as IK, in the proportion of EI to EA. If then we draw from G the lines GA and GB, and imagine that the layer IK is replaced by I′K′, which has its breadth diminished in the same proportion, the total stress on I′K′, if of the intensity found at AB, will be equal to the total stress of less intensity actually existing on IK. The

former stress will also have the same leverage about E F as does the actual stress on I K. By the same reasoning for all layers of the cross-section, we obtain two triangular, shaded areas, A B G and G D C, which may be termed *equivalent areas of uniform stress* of intensity equal to the actual maximum; one of them, usually the upper one, when multiplied by this maximum intensity of stress, represents the total compression, and the other the total tension at the section. The moments of this tension and compression about the neutral axis will be most readily obtained by considering the stress, which is now uniformly distributed over the triangle, as concentrated at its centre of action, the centre of gravity G′ of the triangle, distant two-thirds of its height from the apex G.

Let b represent the breadth and h the height of the cross-section in inches; the area of one triangle will be $\frac{1}{2}b \, . \, \frac{1}{2}h$; and the lever arm about E F will be $\frac{2}{3} \, . \, \frac{1}{2}h$. Let f represent the maximum stress on the square inch at A B. Since the tension and compression tend to produce rotation in the same direction, we add the moments of the two forces together and have

$$2\left(\frac{bh}{4} \, . f \, . \, \tfrac{1}{3}h\right) = \text{moment of resistance} = \tfrac{1}{6} f b h^2.$$

Putting this value equal to the bending moment M, we obtain

$$\mathrm{H}' \, . \, e'k' = \tfrac{1}{6} f b h^2.$$

If we select the maximum value of $e'k'$, introduce the safe working stress f for the extreme fibres, and assume either b or h, we can compute the other required dimension, and thus determine the beam when of uniform section throughout. If the cross-section is to vary, its moment of resistance at different points must at least be equal to the bending moments. As the stiffness of the beam depends principally upon h, the depth must not be made too small. If the beam has too little breadth the compressed edge will yield sideways.

90. **Moment of Resistance of T Section.**—It is easy to compute the size of a beam of rectangular cross-section by the

above formula, but for less regular sections the determination of the moment of resistance by this graphical method may prove of service. In applying it to a beam of the section shown in Fig. 34 we must begin by finding the centre of gravity of the section. By multiplying each rectangular area by the distance of its centre of gravity from either the top or the bottom, adding these products, and dividing by the whole area, we find the distance of the neutral axis from that edge. If G I $= b$, A B $= b'$, G E $= h$, and C A $= h'$, we have

$$\frac{bh \, . \, \frac{1}{2}h + b'h'\left(h + \frac{1}{2}h'\right)}{bh + b'h'} = \text{distance of neutral axis from G I.}$$

The construction of the shaded area A P B needs no explanation, as it follows the previous example. The stress on the fibres at the edge G I will not be so great as at the edge A B, because they are not so far from the neutral axis. If the fibres at G I were removed to K L, so as to be equally remote with A B, they would be equally strained. Then to reduce the layer G I to one which, if it had the same intensity of stress with A B, would give the same total stress which now exists on G I, project G I to K L, draw K P and L P, and G′ I′ will be the desired reduced length. The remainder of the shaded area for the lower rectangle follows the usual rule. In the same way, the fibres at C D will be projected at Q R, and, by drawing Q P and R P, we determine C′D′, and thus complete the shaded portion. These triangles, etc., can be readily scaled, or computed from the known proportions of the beam, their centres of gravity found and the moment of resistance calculated.

91. **Moment of Resistance of an Irregular Section.**—A good example of a section whose moment of resistance is not readily determined by computation alone is afforded by a deck-beam, Fig. 35, often employed in floors and roofs. It is here drawn to one-quarter scale, showing height of section 6 inches, breadth of flange A B $3\frac{1}{2}$ inches, thickness of web $\frac{3}{8}$ inch, weight per yard 44 lbs.; therefore the area of cross-section is about 4.4 square inches.

The readiest way to determine the moment of resistance of such a cross-section is as follows :—Transfer its outlines from the book of shapes or by such data as you have to a sheet of heavy paper, and make a tracing for construction purposes. Cut the section from the heavy paper, balance on a knife-edge and thus determine the neutral axis C D. Then on the tracing draw K L horizontally at the same distance from C D that S T is. A B will be projected at K L, and lines from K and L to P, the middle point of C D, or the centre of gravity of this section, will cut A B at A′ and B′, making A′ B′ the reduced length of A B, and now considered to have the same stress per square inch as exists at I G. In the same way the end M of M N will be projected at O, the point U at V, and the lines from O and V to P will cut the horizontal lines through M and U at new points in the desired curve. Thus enough points are soon obtained to locate the boundary of the shaded portion from B′ to P. The part of the web with straight sides gives of course a triangle, found at once by drawing a line from W to P. The curve A′ P corresponds with B′ P. For the lower portion, project E F on T S, draw lines to P, and get in a similar way enough points for this curve. Cut out the two shaded figures from the heavy paper, balance each one over a knife-edge and thus determine their respective centres of gravity Q and R. Calculate the area of one; the area of the other should exactly equal it, for the total tension equals the total compression. Calling this area A and the safe working stress on the square inch f, we shall then have for the moment of resistance

$$f \,.\, A \,.\, PQ + f \,.\, A \,.\, PR = f \,.\, A \,.\, QR.$$

In this example $A = 1.29$ sq. inches, $PQ = 2.12$ inches, and $PR = 2.66$ inches. If therefore for a static load $f = 12{,}000$ lbs., the moment of resistance equals

$$12{,}000 \times 1.29 \times 4.78 = 74{,}000 \text{ inch-pounds.}$$

92. **Moment of Resistance of I Beam.**—In simpler cases the required size of beam to sustain a given load is more readily found by formula. If I beams are used, the web being thin, and the top and bottom flanges alike, an approximate formula may be used. If F represents the area in square inches of the cross-section of either flange, W the area of the web, h the depth from centre to centre of flanges or the entire depth minus thickness of one flange (that is, between centres of gravity approximately), and f the safe stress on the square inch, the moment of resistance is nearly equal to

$$fh(F + \tfrac{1}{6}W).$$