PREFACE.

At a meeting of the American Association for the Advancement of Science, held in August, 1876, at Buffalo, the writer read two papers, entitled respectively, "Certain New Constructions in Graphical Statics," and "A New Fundamental Method in Graphical Statics." These papers, with considerable additions and amplifications, are presented on the following pages; and to them is added a third on *The Theory of Internal Stress.*

The paper, entitled *New Constructions in Graphical Statics*, is largely occupied with the various forms of the elastic arch. The possibility of obtaining a complete graphical solution of the elastic arch in all cases depends upon a theorem not hitherto recognized as to the relative position of the equilibrium curve due to the loading and the curve of the arch itself. The demonstration of this theorem, which may be properly named the Theorem Respecting the Coincidence of Closing Lines, as given on page 12, is somewhat obscure. However, a second demonstration is given on page 98, and this latter, stated at somewhat greater length, may also be found in the *American Journal of Pure and Applied Mathematics*, Vol. I, No. 3. Prof. Wm. Cain, A.M., C.E., has also published a third demonstration in *Van Nostrand's Magazine*, Vol. XVIII. The solution of the elastic arch is further simplified so that it depends upon that of the straight girder of the same cross section. Moreover, it is shown that the processes employed not only serve to obtain the moment, thrust and shear due the loading, but also to obtain those due to changes of temperature, or to any cause which alters the span of the arch. It is not known that a graphical solution of temperature stresses has been heretofore attempted.

A new general theorem is also enunciated which affords the basis for a direct solution of the flexible arch rib, or suspension cable, and its stiffening truss.

These discussions have led to a new graphical solution of the continuous girder in the most general case of variable moment of inertia. This is accompanied by an analytic investigation of the Theorem of Three Moments, in which the general equation of three moments appears for the first time in simple form. This investigation, slightly extended and amplified, may be also found in the *American Journal of Pure and Applied Mathematics*, Vol. I, No. 1.

Intermediate between the elastic and flexible arch is the arch with blockwork joints, such as are found in stone or brick arches. A graphical solution of this problem was given by Poncelet, which may be found in Woodbury's treatise on the *Stability of the Arch*, page 404. Woodbury states that this solution is correct in case of an unsymmetrical arch, but in this he is mistaken. The solution proposed in the following pages is simpler, susceptible of greater accuracy, and is not restricted to the case when either the arch or loading is symmetrical about the crown.

The graphical construction for determining the stability of retaining walls is the first one proposed, so far as known, which employs the true thrust in its real direction, as shown by Rankine in his investigation of the stress of homogeneous solids. It is in fact an adaptation of that most useful conception, Coulomb's *Wedge of Maximum Thrust*, to Rankine's investigation.

It has also been found possible to obtain a complete solution of the dome of metal and of masonry by employing constructions analogous to those employed for the arch; and in particular, it is believed that the dome of masonry is here investigated correctly for the first time, and the proper distinctions pointed out between it and the dome of metal.

In the paper entitled, *A New General Method in Graphical Statics*, a fundamental process or method is established of the same generality as the well-known method of the Equilibrium Polygon The new method is designated as that of the Frame Pencil, and both the methods are discussed side by side in order that their reciprocal relationship may be made the more apparent. The reader who is not familiar with the properties of the equilibrium polygon will find it advantageous to first read this paper, or, at least, defer the others until he has read it as far as page 83.

As an example affording a comparison of the two methods, the moments of inertia and resistance have been discussed in a novel manner, and this is accompanied by a new graphical discussion of the distribution of shearing stress.

In the paper entitled, *The Theory of Internal Stress in Graphical Statics*, there is considerable new matter, especially in those problems which relate to the combination of states of stress, a subject which has not been, heretofore, sufficiently treated.

It is hoped that these graphical investigations which afford a pictorial representation, so to speak, of the quantities involved and their relations may not present the same difficulties to the reader as do the intricate formulae arising from the analytic solutions of the same problems. Indeed, analysis almost always requires some kind of uniformity in the loading and in the structure sustaining the load, while a graphical construction treats all cases with the same ease; and especially are cases of discontinuity, either in the load or structure, difficult by analysis but easy by graphics.

H. T. E.

CONTENTS.

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