The differential thermometer is an instrument of great sensibility, and enabled Leslie to conduct some important investigations on the subject of the radiation of heat. It is now, however, superseded by the thermo-pile invented by Melloni. This latter instrument will be

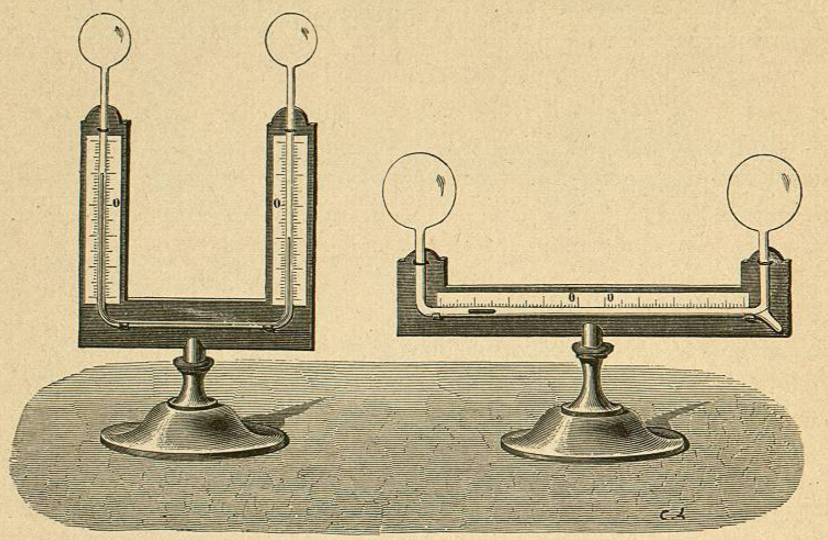

Fig. 21.—Leslie's Differential Thermometer. Fig. 22.—Rumford's Thermoscope.

described in another portion of this work. Rumford's thermoscope (Fig. 22) is analogous to Leslie's differential thermometer. It differs from it in having the horizontal part much longer, and the vertical branches shorter. In the horizontal tube is an alcohol index, which, when the two globes are at the same temperature, occupies exactly the middle.

CHAPTER II.

MATHEMATICS OF EXPANSION.

17. **Expansion. Factor of Expansion.**—When a body expands from volume V to volume $V+v$, the ratio $\frac{v}{V}$ is called the *expansion of volume* or the *cubical expansion* of the body.

In like manner if the length, breadth, or thickness of a body increases from L to $L+l$, the ratio $\frac{l}{L}$ is called the *linear expansion.*

The ratio $\frac{V+v}{V}$ will be called, in this treatise, the *factor of cubical expansion*, and the ratio $\frac{L+l}{L}$ the *factor of linear expansion.* In each case the factor of expansion is *unity plus the expansion.*

Similar definitions apply to expansion of area or superficial expansion; but it is seldom necessary to consider this element in thermal discussions.

18. **Relation between Linear and Cubical Expansion.**—If a cube, whose edge is the unit length, expands equally in all directions, the length of each edge will become $1+l$, where l is the linear expansion; and the volume of the cube will become $(1+l)^3$ or $1+3l+3l^2+l^3$.

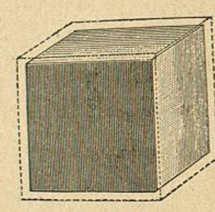

Fig. 23.

In the case of the thermal expansion of solid bodies l is always very small, so that l^2 and l^3 can be neglected, and the expansion of volume is therefore $3l$; that is to say, the *cubical expansion is three times the linear expansion.* This is illustrated geometrically by Fig. 23, which represents a unit cube with a plate of thickness l and therefore of volume l applied to each of three faces; the total volume added being therefore $3l$.

Similar reasoning shows that *the superficial expansion* is *double the linear expansion.*

These results have been deduced from the supposition of equal expansion in all directions. If the expansions of the cube in the directions of three conterminous edges be denoted by a, b, c, the angles being supposed to remain right angles, the volume will become $(1+a)(1+b)(1+c)$ or $1+a+b+c+ab+ac+bc+abc$, which, when a, b and c are so small that their products can be neglected, becomes $1+a+b+c$; so that the expansion of volume is the sum of the expansions of length, breadth, and thickness.

19. Variation of Density.—Since the density of a body varies inversely as its volume, the density after expansion will be obtained by *dividing* the original density by the factor of expansion. In fact, if V, D denote the volume and density before, and V′, D′ after expansion, the mass of the body, which remains unchanged, is equal to V D, and also to V′ D′. We have therefore $\frac{D'}{D}=\frac{V}{V'}=\frac{1}{1+e}$, where e denotes the expansion of volume, and therefore $1+e$ the factor of expansion.

Since $\frac{1}{1+e}$ is $1-e+e^2-e^3+$&c., it is sensibly equal to $1-e$ when e is small. We have therefore $D'=D(1-e)$.

20. Real and Apparent Expansion.—When the volume of a liquid is specified by the number of divisions which it occupies in a graduated vessel, it is necessary to take into account the expansion of the vessel, if we wish to determine the true expansion of the liquid.

Let a denote the apparent expansion computed by disregarding the expansion of the vessel and attending only to the number of divisions occupied. Then if n be the number of divisions occupied before, and n' after expansion, we have

$$n'=n\,(1+a).$$

Let g denote the real expansion of the containing vessel; then if d be the volume of each division before, and d' after expansion, we have

$$d'=d\,(1+g).$$

Let m denote the real expansion of the liquid. Then if v denote the real volume of the liquid before, and v' after expansion, we have

$$v'=v\,(1+m).$$

But since the volume v consists of n parts each having the volume d, we have

$$v=nd,$$

and in like manner

$$v'=n'd'.$$

Substituting for n' and d' in this last equation, we have

$$v'=n\,(1+a)\,d\,(1+g)=v\,(1+a)\,(1+g).$$

But $v'=v\,(1+m)$.

Hence we have

$$(1+a)\,(1+g)=1+m;$$

that is, *the factor of real expansion of the liquid is the product of the factor of real expansion of the vessel and the factor of apparent expansion.* Multiplying out, we have

$$1+a+g+ag=1+m,$$

and as the term ag, being the product of two small quantities, is usually negligible, we have sensibly

$$a+g=m;$$

that is, the expansion of the liquid is the sum of the expansion of the glass and the apparent expansion.

This investigation is applicable to the mercurial thermometer when the capacity of the bulb has been expressed in degrees of the stem.

Similar reasoning applies to the apparent expansion of a bar of one metal as measured by means of a graduated bar of a less expansible metal. The real expansion of the bar to be measured will be sensibly equal to the sum of the expansion of the measuring bar and the apparent expansion.

In adopting the mercurial thermometer as the standard of temperature (the tube being graduated into equal parts), we virtually adopt the apparent expansion of mercury in glass as our standard of *uniform* expansion.

21. Physical Meaning of the Degrees of the Mercurial Thermometer.—Since the stem of a mercurial thermometer is divided into degrees of equal capacity, we can express the capacity of the bulb in degrees. Let the capacity of the bulb together with as much of the stem as is below the freezing-point be N degrees, and let the interval from freezing to boiling point be n degrees; then $\frac{n}{N}$ is the apparent expansion of the mercury from freezing to boiling point. When the Centigrade scale is employed, this apparent expansion is $\frac{100}{N}$, and the apparent expansion from zero to $t°$ is $\frac{t}{N}$. Hence the apparent expansion from zero to $t°$ is $\frac{t}{100}$ of the apparent expansion from zero to 100°. This last statement constitutes the definition of the temperature $t°$ when the mercurial thermometer is regarded as the standard.

22. Comparability of Mercurial Thermometers.—If two mercurial thermometers, each of them constructed so as to have its degrees rigorously equal in capacity, agree in their indications at all temperatures, the above investigation shows that the apparent expansions of the mercury in the two instruments must be exactly proportional. But we have shown in § 20 that the apparent expansion a is equal to $m-g$, m denoting the real expansion of the mercury, and g that of the glass. Mercury, being a liquid and an elementary substance, can always be obtained in the same condition, so that m will have the same value in the two thermometers; but it is difficult to ensure that two specimens of glass shall be exactly alike; hence g has different values in different thermometers. The agreement of the two thermometers does not, however, require identity in the values of $m-g$, but only proportionality; in other words it requires that the fraction

$$\frac{m-g_1}{m-g_2}$$

(where g_1 and g_2 are the values of g for the two instruments) shall have the same value at all temperatures.

The average value of g is about $\frac{1}{7}$ of that of m. In other words mercury expands about 7 times as much as glass.

23. Steadiness of Zero in Spirit Thermometers.—It is obvious from § 21 that the volume of a degree can be computed by multiplying the capacity of the bulb by the number which denotes the apparent expansion for one degree. Alcohol expands about 6 times as much as mercury, and its apparent expansion in glass is about 7 times that of mercury. Hence with the same size of bulb, the degrees of an alcohol thermometer will be about 7 times as large as those of a mercurial thermometer, and a contraction of the bulb which produces a change of one degree in the reading of a mercurial thermometer, would only produce a change of one-seventh of a degree in the reading of an alcohol thermometer. This is the reason, or at all events one reason, why displacement of the zero point (§ 9) is insignificant in spirit thermometers.

24. Length of a Degree on the Stem.—Since the length of a degree upon the stem of a thermometer is equal to the volume of a degree divided by the sectional area of the tube, the formula for this length is $\frac{aC}{s}$, where a denotes the apparent expansion for one degree, C the capacity of the bulb with as much of the stem as is below zero, and

s the sectional area of the stem. The value of a for the mercurial Centigrade thermometer is about $\frac{1}{6480}$.

25. Weight Thermometer.—In the weight thermometer (Fig. 24) the apparent expansion of mercury is observed by comparing the weight of the mercury which passes the zero point with that of the mercury which remains below it. The tube is open, and its mouth is the zero point. The instrument is first filled with mercury at zero, and is then exposed to the temperature which it is required to measure. The mercury which overflows is caught and weighed, and the weight of the mercury which remains in the instrument is also determined—usually by subtracting the weight of the overflow from that of the original contents. The weight of the overflow, divided by the weight of what remains, is equal to the apparent expansion; for it is the same as the ratio of the volume of mercury above the zero point to the volume below it in an ordinary thermometer.

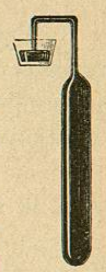

Fig. 24. Weight Thermometer.

In order to measure temperatures in degrees, with this thermometer, the apparent expansion from 0° to 100° C. must be determined once for all and put on record. One hundredth part of this must be divided into the apparent expansion observed at the unknown temperature t°, and the quotient will be t.

26. Expansion of Gases.—In the case of solids and liquids the expansions produced by heat are usually very small, so that it is not important to distinguish between the value of $\frac{v}{V}$ and the value of $\frac{v}{V+v}$ (§ 17). But in the case of gases much larger expansions occur, and it is essential to attend to the above distinction. By general agreement, the volume of a gas at zero (Centigrade) is taken as the standard with which the volume at any other temperature is to be compared. We shall denote the volume at zero by V_0, and the volume at temperature t° by V_t. Then, if the pressure be the same at both temperatures, we shall write

$$V_t = V_0 (1 + at)$$

where a is called the mean coefficient of expansion between the temperatures 0° and t°. Experiment has shown that when temperatures are measured by the mercurial thermometer, graduated in the manner which we have already described, a is practically the same at all temperatures which lie within the range of the mercurial thermometer. In other words, the expansions of gases are sensibly proportional to the apparent expansion of mercury in glass. Moreover, the coefficient α is not only the same for different temperatures, but it is also the same for different gases; its value being always very approximately

$$\cdot 00366 \text{ or } \frac{1}{273}.$$

By Boyle's law, the product of the volume and pressure of a gas remains constant when the temperature is constant. We have been supposing the pressure to remain constant, so that the product in question is proportional to the volume only. If the volume is kept constant the pressure will vary in proportion to $1+\alpha t$, so that we shall have

$$P_t = P_0 (1+\alpha t),$$

P_0 and P_t denoting the pressures at 0° and t° respectively. If we remove all restriction, we have

$$(V P)_t = (V P)_0 (1+\alpha t),$$

where $(V P)_0$, $(V P)_t$ denote the products of volume and pressure at 0° and t° respectively. Hence the value of the expression

$$\frac{V P}{1+\alpha t}$$

will be the same for all values of V, P and t. Since the mass is unchanged, the density D varies inversely as the volume, and therefore

$$\frac{P}{D(1+\alpha t)}$$

is also constant.

27. General Definition of Coefficient of Expansion.—If V_0 denote the volume of any substance at temperature 0° (Centigrade), V_t its volume under the same pressure at temperature t°, and $V_{t'}$ its volume at a higher temperature t'°, the *mean coefficient of expansion* α between the temperatures t and t' is defined by the equation

$$V_{t'} - V_t = V_0 \, \alpha \, (t'-t),$$

and the *coefficient of expansion at the temperature* t° is the limit to which α approaches as t' approaches t; that is, in the language of the differential calculus, it is

$$\frac{1}{V_0}\frac{dV}{dt}.$$

If we make V_0 unity, the coefficient of expansion at temperature t will be simply

$$\frac{dV}{dt}.$$

CHAPTER III.

EXPANSION OF SOLIDS.

28. Observations of Linear Expansion.—Laplace and Lavoisier determined the linear expansion of a great number of solids by the following method.

The bar AB (Fig. 25) whose expansion is to be determined, has one end fixed at A, while the other can move freely, pushing before it the lever OB, which is movable about the point O, and carries a telescope whose line of sight is directed to a scale at some distance. A displacement BB′ corresponds to a considerably greater length CC′ on the scale, the ratio of the former to the latter being the same as that of OB to OC.

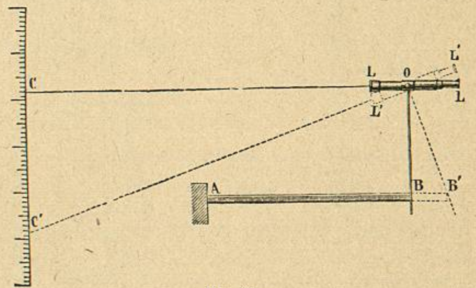

Fig. 25.
Principle of the Method of Laplace and Lavoisier.

The apparatus employed by Laplace and Lavoisier is shown in Fig. 26. The trough C, in which is laid the bar whose expansion is to be determined, is placed between four massive uprights of hewn stone N. One of the extremities of the bar rests against a fixed bar B′, firmly joined to two of the uprights; the other extremity, which rests upon a roller to give it greater freedom of movement, pushes the bar B, which produces the rotation of the axis aa'. This axis carries with it in its rotation the telescope LL′, which is directed to the scale. The first step is to surround the bar with melting ice, and take a reading through the telescope when the bar is at the temperature zero. The temperature of the trough is then raised, and readings are taken, which, by comparison with the first, give the increase of length.

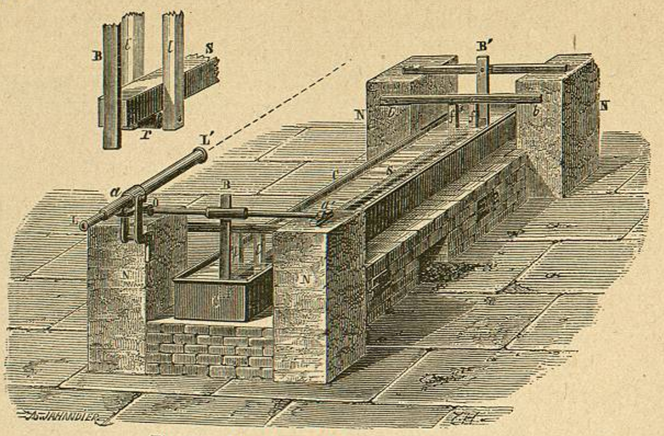

Fig. 26.—Apparatus of Laplace and Lavoisier.

The following table contains the most important results thus obtained:—

COEFFICIENTS OF LINEAR EXPANSION.

Gold, Paris standard, annealed,	0·000015153	Soft wrought iron, ...	0·000012204
„ „ unannealed,	0·000015515	Round iron, wire drawn, . .	0·000012350
Steel not tempered, . . .	0·000010792	English flint-glass, ...	0·000008116
Tempered steel reheated to 65°,	0·000012395	Gold, procured by parting, .	0·000014660
Silver obtained by cupellation,	0·000019075	Platina, ...	0·000009913
Silver, Paris standard, . .	0·000019086	Lead, ...	0·000088483
Copper, ...	0·000017173	French glass with lead, . .	0·000008715
Brass, ...	0·000018782	Sheet zinc, ...	0·000029416
Malacca tin, ...	0·000019376	Forged zinc, ...	0·000031083
Falmouth tin, ...	0·000021729		

The coefficient of expansion of a metal is not precisely the same at all temperatures, but it is sensibly constant from 0° to 100° C.

A simpler and probably more accurate method of observing expansions was employed by Ramsden and Roy. It consists in the direct observation of the distances moved by the ends of the bar, by means of two microscopes furnished with micrometers, the microscopes themselves being attached to an apparatus which is kept at a constant temperature by means of ice.

29. Compensated Pendulum.—The rate of a clock is regulated by the motion of its pendulum. Suppose the clock to keep correct time at a certain temperature. Then at higher temperatures the pendulum will be too long and will therefore vibrate too slowly, so that

the clock will lose. At lower temperatures, on the other hand, the clock will gain. To obviate or, at least, diminish this source of irregularity, the following methods of compensation are employed.

1. *Harrison's Gridiron Pendulum.*—This consists of four oblong frames, the uprights of which are alternately of steel F and of brass

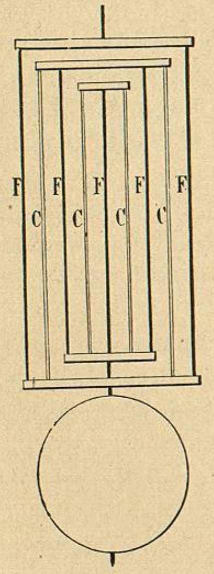

Fig. 27
Plan of Gridiron Pendulum.

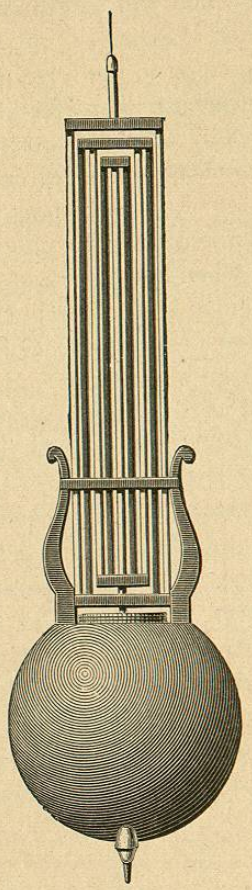

Fig. 28.
Gridiron Pendulum.

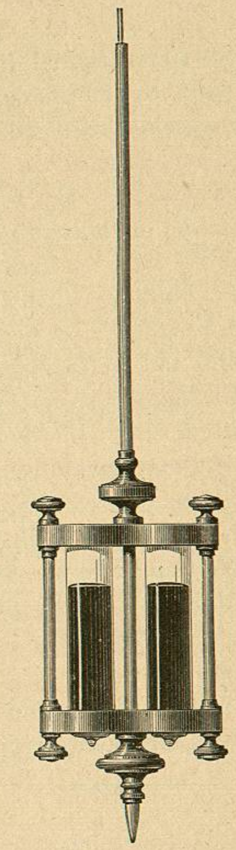

Fig. 29.
Graham's Mercurial Pendulum.

C (Fig. 27), so arranged that the bob will rise or fall through a distance equal to the difference between the total expansion of 3 steel rods and that of 2 brass rods. As the coefficients of expansion of these metals are nearly as 2 to 3, it is possible to make the compensation nearly exact.

2. *Graham's Mercurial Pendulum.*—This consists of an iron rod

carrying at its lower end a frame, in which are fixed one or two glass cylinders containing mercury. When the temperature rises, the lengthening of the rod lowers the centre of gravity and centre of oscillation of the whole; but the expansion of the mercury produces the contrary effect; and if there is exactly the right quantity of mercury the compensation will be nearly perfect.

30. **Force of Expansion of Solids.**—The *force* of expansion is often very considerable, being equal to the force necessary to compress the body to its original dimensions. Thus, for instance, iron when heated from 0° to 100° increases by ·0012 of its original length. In order to produce a corresponding change of length in a rod an inch square by mechanical means, a force of about 15 tons would be required. This is accordingly the force necessary to prevent such a rod from expanding or contracting when heated or cooled through 100°.

This force has frequently been utilized for bringing in the walls of a building when they have settled outwards. For this purpose the walls are first tied together by iron rods, which pass through the walls, and are furnished at the ends with screws and nuts. All the nuts having been tightened against the wall, alternate bars are heated; and while they are hot, the nuts upon them, which have been thrust away from the wall by the expansion, are screwed home. As these bars cool, they draw the walls in and allow the nuts on the other bars to be tightened. The same operation is then repeated as often as may be necessary.

Iron cannot with safety be used in structures, unless opportunity is given it to expand and contract without doing damage. In laying a railway, small spaces must be left between the ends of the rails to leave room for expansion; and when sheets of lead or zinc are employed for roofing, room must be left for them to overlap.

CHAPTER IV.

EXPANSION OF LIQUIDS.

31. **Method of Equilibrating Columns.**—Most of the methods employed for measuring the expansion of liquids depend upon a previous knowledge of the expansion of glass, the observation itself consisting in a determination of the apparent expansion of the liquid relative to glass. There is, however, one method which is not liable to this objection, and it has been employed by Dulong and Petit, and afterwards by Regnault, for measuring the expansion of mercury—an element of great importance for many physical applications. It depends upon the hydrostatic principle that the heights of two liquid columns which produce equal pressures are inversely as their densities.

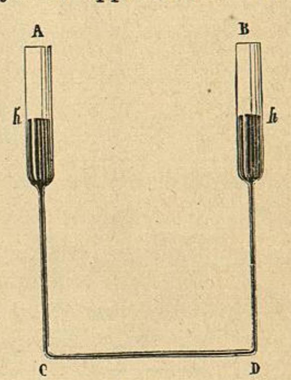

Fig. 30.
Principle of Dulong's Method.

Let A and B (Fig. 30) be two tubes containing mercury, and communicating with each other by a very narrow horizontal tube CD at the bottom. If the temperature of the liquid be uniform, the mercury will stand at the same height in both branches; but if one column be kept at 0° and the other be heated, their densities will be unequal. Let $d\,d'$ be their densities, and $h\,h'$ their heights. Then since their pressures at the bottom are equal, we must have

$$hd=h'd'.$$

But if v and v' denote the volumes of one and the same mass of liquid at the two temperatures, we have

$$vd=v'd'.$$

From these two equations, we have