carrying at its lower end a frame, in which are fixed one or two glass cylinders containing mercury. When the temperature rises, the lengthening of the rod lowers the centre of gravity and centre of oscillation of the whole; but the expansion of the mercury produces the contrary effect; and if there is exactly the right quantity of mercury the compensation will be nearly perfect.

30. Force of Expansion of Solids.—The *force* of expansion is often very considerable, being equal to the force necessary to compress the body to its original dimensions. Thus, for instance, iron when heated from 0° to 100° increases by ·0012 of its original length. In order to produce a corresponding change of length in a rod an inch square by mechanical means, a force of about 15 tons would be required. This is accordingly the force necessary to prevent such a rod from expanding or contracting when heated or cooled through 100°.

This force has frequently been utilized for bringing in the walls of a building when they have settled outwards. For this purpose the walls are first tied together by iron rods, which pass through the walls, and are furnished at the ends with screws and nuts. All the nuts having been tightened against the wall, alternate bars are heated; and while they are hot, the nuts upon them, which have been thrust away from the wall by the expansion, are screwed home. As these bars cool, they draw the walls in and allow the nuts on the other bars to be tightened. The same operation is then repeated as often as may be necessary.

Iron cannot with safety be used in structures, unless opportunity is given it to expand and contract without doing damage. In laying a railway, small spaces must be left between the ends of the rails to leave room for expansion; and when sheets of lead or zinc are employed for roofing, room must be left for them to overlap.

CHAPTER IV.

EXPANSION OF LIQUIDS.

31. Method of Equilibrating Columns.—Most of the methods employed for measuring the expansion of liquids depend upon a previous knowledge of the expansion of glass, the observation itself consisting in a determination of the apparent expansion of the liquid relative to glass. There is, however, one method which is not liable to this objection, and it has been employed by Dulong and Petit, and afterwards by Regnault, for measuring the expansion of mercury—an element of great importance for many physical applications. It depends upon the hydrostatic principle that the heights of two liquid columns which produce equal pressures are inversely as their densities.
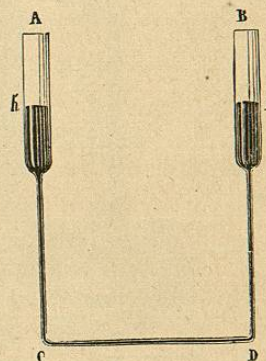
Let A and B (Fig. 30) be two tubes containing mercury, and communicating with each other by a very narrow horizontal tube CD at the bottom. If the temperature of the liquid be uniform, the mercury will stand at the same height in both branches; but if one column be kept at 0° and the other be heated, their densities will be unequal. Let $d \, d'$ be their densities, and $h \, h'$ their heights. Then since their pressures at the bottom are equal, we must have

$$h \, d = h' \, d'.$$

But if v and v' denote the volumes of one and the same mass of liquid at the two temperatures, we have

$$v \, d = v' \, d'.$$

From these two equations, we have

Fig. 30.
Principle of Dulong's Method.

$$v : v' :: h : h',$$

so that the expansion of volume is directly given by a comparison of the heights. Denoting this expansion by m, we shall have

$$m = \frac{h' - h}{h}.$$

Strictly speaking, the mercury in this experiment is not in equilibrium. There will be two very slow currents through the horizontal tube, the current from hot to cold being above, and the current from cold to hot below. Equilibrium of pressure will exist only at the intermediate level—that of the axis of the tube, and it is from this level that h and h' should be measured.

32. The apparatus employed by Dulong and Petit for carrying out this method is represented in Fig. 31. The two upright tubes
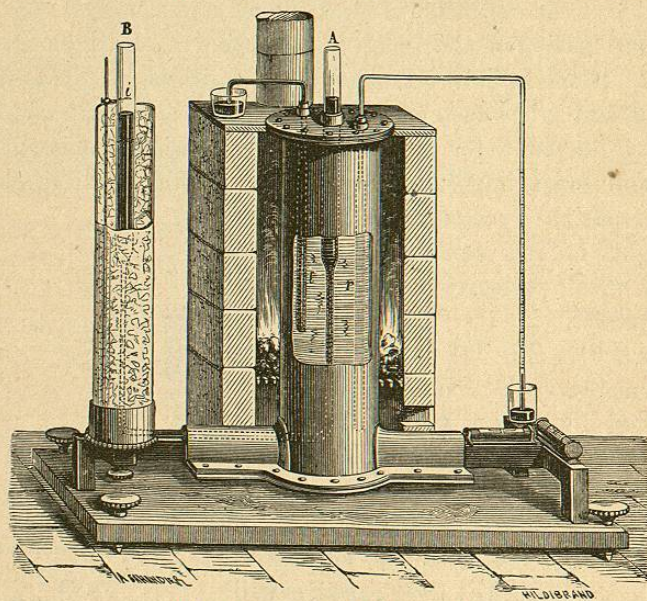

Fig. 31.—Apparatus of Dulong and Petit.

A, B, and the connecting tube at their base, rest upon a massive support furnished with levelling screws, and with two spirit-levels at right angles to each other, for insuring horizontality. The tube B is surrounded by a cylinder containing melted ice. The other tube A is surrounded by a copper cylinder filled with oil, which is heated by a furnace connected with the apparatus. In making an observation, the first step is to arrange the apparatus so that, when

the oil is heated to the temperature required, the mercury in the tube A may just be seen above the top of the cylinder, so as to be sighted with the telescope of a cathetometer; this may be effected by adding or taking away a small quantity of mercury. The extremity of the column B is next sighted, which gives the difference of the heights h' and h. The absolute height h is determined by means of a fixed reference mark i near the top of the column of mercury in the tube B. This reference mark is carried by an iron rod surrounded by the ice, and its distance from the axis of the horizontal connecting tube has been very accurately measured once for all. The temperature of the oil is given by the weight thermometer t, and by the air thermometer r, which latter we shall explain hereafter.

By means of this method Dulong and Petit ascertained that the expansion of mercury is nearly uniform between 0° and 100° C., as compared with the indications of an air-thermometer, and that though its expansion at higher temperatures is more rapid, the difference is less marked than in the case of other liquids. They found the mean coefficient of expansion from 0° to 100° to be $\frac{1}{5550}$; from 0° to 200°, $\frac{1}{5425}$; and from 0° to 300°, $\frac{1}{5300}$.

Regnault, without altering the principle of the apparatus of Dulong and Petit, introduced several improvements in detail, and added greatly to the length of the tubes A and B, thereby rendering the apparatus more sensitive. His results are not very different from those of Dulong and Petit. For example, he makes the mean coefficient from 0° to 100° to be $\frac{1}{5509}$; from 0° to 200°, $\frac{1}{5479}$; and from 0° to 300°, $\frac{1}{5360}$. His experiments show that the mean coefficient from 0° to 50° is $\frac{1}{5547}$, a value almost identical with $\frac{1}{5550}$.

33.. Expansion of Glass.—The expansion of mercury being known, we can find the expansion of any kind of glass by observing the apparent expansion of mercury in a weight thermometer (§ 25) constructed of this glass, and subtracting this apparent expansion from the real expansion of the liquid; or more rigorously, by dividing the factor of real expansion of the liquid by the factor of apparent expansion (§ 20), we shall obtain the factor of expansion of the glass.

Dulong and Petit found $\frac{1}{6480}$ as the mean value of the coefficient

of apparent expansion of mercury in glass, and $\frac{1}{5550}$ as the coefficient of real expansion of mercury. The difference of these two fractions is approximately $\frac{1}{38700}$, which may therefore be taken as the coefficient of expansion of glass. It is about one-seventh of the coefficient of expansion of mercury.

34. Expansion of any Liquid.—The expansion of the glass of which a thermometer is made being known, we may use the instrument to measure the expansion of any liquid. For this purpose we must measure the capacity of the bulb and find how many divisions of the stem it is equal to. We can thus determine how many divisions the liquid occupies at two different temperatures, that is, we can determine the apparent expansion of the liquid; and by adding to this the expansion of the glass, we shall obtain the real expansion of the liquid. Or more rigorously, we shall obtain the factor of real expansion of the liquid by multiplying together the factor of apparent expansion and the factor of expansion of the glass.
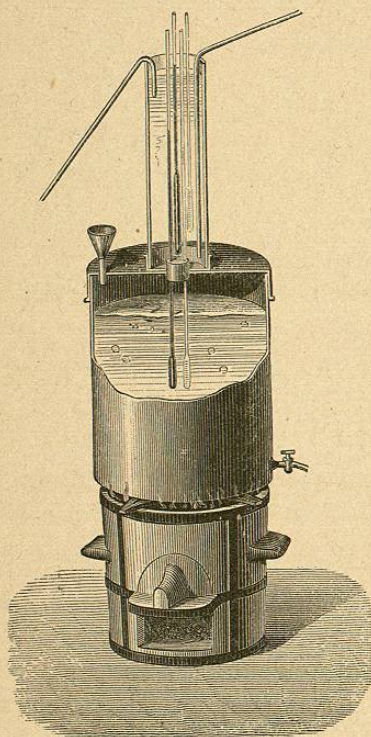
M. Pierre has performed an extensive series of experiments by this method upon a great number of liquids. The apparatus employed by him is shown in Fig. 32. The thermometer containing the given liquid is fixed beside a mercurial thermometer, which marks the temperature. The reservoir and a small part of the tube are immersed in the bath contained in the cylinder below. The upper parts of the stems are inclosed in a second and smaller cylinder, the water in which is maintained at a sensibly constant temperature indicated by a very delicate thermometer.

Fig. 32.—Pierre's Apparatus.

From these experiments it appears that the expansions of liquids are in general much greater than those of solids; also that their ex-

pansion does not proceed uniformly, as compared with the indications of a mercurial thermometer, but increases very perceptibly as the temperature rises. This is shown by the following table:—

	Volume at 0°.	Volume at 10°.	Volume at 40°.
Water....................	1	1·000146	1·007492
Alcohol..................	1	1·010661	1·044882
Ether	1	1·015408	1·066863
Bisulphide of carbon...	1	1·011554	1·049006
Wood-spirit..............	1	1·012020	1·050509

35. Other Methods.—Another method of determining the apparent expansion of a liquid, with a view to deducing its real expansion, consists in weighing a glass bottle full of the liquid at different temperatures. This is virtually employing a weight thermometer.

A third method consists in observing the loss of weight of a piece of glass when weighed in the liquid at different temperatures. Time must be given in each case for the glass to take the temperature of the liquid; and when this condition is fulfilled, the factor of expansion will be equal to the loss of weight at the lower temperature, divided by the loss of weight at the higher.

For if the volume of the glass at the lower temperature be called unity, and its volume at the higher temperature $1+g$, the mass of liquid displaced at the lower temperature will be equal to its density d, and the mass displaced at the higher temperature will be the product of $1+g$ by the density $\frac{d}{1+l}$, where l denotes the expansion of the liquid. The losses of weight, expressed in gravitation measure, are therefore

$$d \text{ and } \frac{(1+g)\,d}{1+l},$$

and the former of these divided by the latter gives $\frac{1+l}{1+g}$, which (§ 20) is the factor of apparent expansion.

36. Formulæ for the Expansion of Liquids.—As we have mentioned above, the expansion of liquids does not advance uniformly with the temperature; whence it follows that the mean coefficient of expansion will vary according to the limiting temperatures between which it is taken.

For a great number of liquids, the mean coefficient of expansion may be taken as increasing uniformly with the temperature. If, therefore, Δ be the expansion from 0 to t, we have

$$\frac{\Delta}{t} = a + bt, \text{ whence } \Delta = at + bt^2,$$

a and *b* being two constants specifying the expansibility of the given liquid.

For some very expansible liquids two constants are not sufficient, and the expansion is represented by the formula

$$\Delta = at + bt^2 + ct^3.$$

We subjoin a few instances of this class taken from the work of M. Pierre:—

Alcohol.....................$\Delta = 0.0010486\ t + 0.0000017510\ t^2 + 0.00000000134518\ t^3$
Ether.......................$\Delta = 0.0015132\ t + 0.0000023592\ t^2 + 0.000000040051\ t^3$
Bisulphide of carbon....$\Delta = 0.0011398\ t + 0.0000013707\ t^2 + 0.00000019123\ t^3$
Bromine....................$\Delta = 0.0010382\ t + 0.0000017114\ t^2 + 0.0000000054471\ t^3$

37. Maximum Density of Water.—Water, unlike other liquids, contracts as its temperature rises from 0° to 4°, at which point its volume is a minimum, and therefore its density a maximum.

The following experiment, which furnishes a means of determining the temperature of maximum density, is due to Hope.

A glass jar is employed, having two lateral openings, one near the top and the other near the bottom, which admit two thermometers placed horizontally. The jar is filled with water at a temperature higher than 4°, and its middle is surrounded with a freezing-mixture. The following phenomena will then be observed.

The lower thermometer descends steadily to 4°, and there remains stationary. The upper thermometer at first undergoes very little change, but when the lower one has reached the fixed temperature, the upper one begins to fall, reaches the temperature of zero, and, finally, the water at the surface freezes, if the action of the freezing-mixture continues for a sufficiently long time. These facts admit of a very simple explanation.
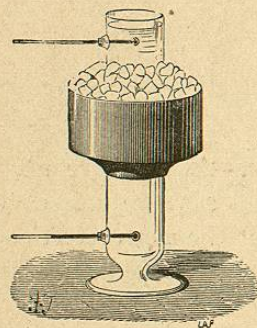
As the water in the middle portion of the jar grows colder, its density increases, and it sinks to the bottom. This process goes on till all the water in the lower part has attained the temperature of 4°. But when all the water from the centre to the bottom has attained this temperature, any further cooling of the water in the centre will produce no circulation in the lower portion, and very little in the upper, until needles of ice are formed. These, being lighter than water, rise to the surface, and thus produce a circulation

Fig. 33.
Hope's Experiment.

which causes the water near the surface to freeze, while that near the bottom remains at the temperature of 4°.

This experiment illustrates what takes place during winter in pools of fresh water. The fall of temperature at the surface does not extend to the bottom of the pool, where the water, whatever be the external temperature, seldom falls below 4°. This is a fact of great interest, as exemplifying the close connection of natural phenomena, and the manner in which they contribute to a common end. It is in virtue of this anomaly exhibited by water in its expansion, taken in conjunction with the specific lightness of ice and the low conducting power of liquids generally, that the temperature at the bottom of deep pools remains moderate even during the severest cold, and that the lives of aquatic animals are preserved.

38. Saline Solutions.—These remarks are not applicable to sea-water, which contracts as its temperature falls till its freezing-point is attained; this latter being considerably lower than the freezing-point of fresh water.

In the case of saline solutions of different strengths, the temperature of maximum density falls along with the freezing-point, and falls more rapidly than this latter, so that for solutions containing more than a certain proportion of salt the temperature of maximum density is below the freezing-point. In order to show this experimentally, the solution must be placed in such circumstances as to remain liquid at a temperature below its ordinary freezing-point.

39. Apparent Expansion of Water.—Fig. 34 represents an apparatus for showing the changes of apparent volume of water in a glass vessel. In the centre are two thermometers, one containing alcohol and the other water. The reservoir of the latter is a long spiral, surrounding the reservoir of the alcohol thermometer and having much greater capacity. Both reservoirs are contained in a metal box, which is at first filled with melting ice. The two instruments are so placed that at zero the extremities of the two liquid columns are on the same horizontal line. This being the case, if the
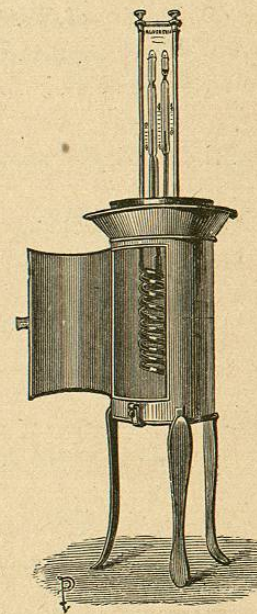

Fig. 34.
Maximum Density of Water.

ice be now removed, and the apparatus left to itself, or if the process be accelerated by placing a spirit-lamp below the box, the alcohol will immediately be seen to rise, while the water will descend; and the two liquids will thus continue to move in opposite directions until a temperature of 5° or 6° is attained. From this moment the water ceases to descend, and begins to move in the same direction as the alcohol. The temperature at which the water thermometer becomes stationary is that at which the coefficient of expansion of water is the same as that of glass. The coefficient of expansion of water is zero at 4°, and at temperatures near 4° is approximately

$$\cdot000016\,(t-4).$$

The average value of the coefficient of expansion of glass is about ·000027, and by equating these two expressions, we have

$$t-4=\frac{27}{16}=1\cdot7 \text{ nearly};$$

hence the water thermometer will be stationary at the temperature 5°·7.

40. Density of Water at Various Temperatures.—The volume, at temperatures near 4°, of a quantity of water which would occupy unit volume at 4°, is approximately

$$1+\cdot000008\,(t-4)^2,$$

and the density of water at these temperatures is therefore

$$1-\cdot000008\,(t-4)^2,$$

the density at 4° being taken as unity.

The density of water at some other temperatures is given in the following table:—

Temperature.		Density.
0°	············	·999871
4°	············	1·000000
8°	············	·999886
12°	············	·999549
16°	············	·999002
20°	············	·998259
50°	············	·9882
100°	············	·9586

41. Expansion of Iron and Platinum.—The coefficient of absolute expansion of mercury being known, that of glass is deduced from it in the manner already indicated (§ 33). Du-
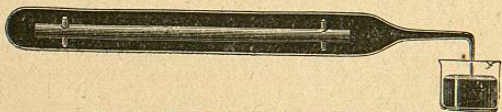

Fig. 35.—Expansion of Iron and Platinum.

long and Petit have deduced from it also the coefficients of expan-

sion of iron and platinum, these metals not being attacked by mercury. The method employed is the following.

The metal in question is introduced, in the shape of a cylindrical bar, into the reservoir of a weight thermometer. Let W be the weight of the metal introduced, and D its density at zero. The process is the same as in using the weight thermometer; that is, after having filled the reservoir with mercury at 0° C., we observe the weight w of the metal which issues at a given temperature t. The volume at 0° C. of the mercury which has issued, is $\frac{w}{d}$, d being the density of mercury at zero; the volume at $t°$ is therefore $\frac{w}{d}(1+mt)$, m being the coefficient of expansion of mercury. This volume evidently represents the expansion of the metal, *plus* that of the mercury, *minus* that of the glass. If then M denote the weight of mercury that fills the apparatus at 0° C., and if K be the coefficient of cubical expansion of glass, and x the expansion of unit volume of the given metal, we have the equation

$$\frac{w}{d}(1+mt)=\frac{W}{D}x+\frac{M}{d}mt-\left(\frac{W}{D}+\frac{M}{d}\right)Kt,$$

whence we can find x.

42. Convection of Heat in Liquids.—When different parts of a liquid or gas are heated to different temperatures, corresponding differences of density arise, leading usually to the formation of currents. This phenomenon is called *convection*.

Thus, for instance, if we apply heat to the bottom of a vessel containing water, the parts immediately subjected to the action of the heat expand and rise to the surface; they are replaced by colder portions, which in their turn are heated and ascend; and thus a continual circulation is maintained. The ascending and descending currents can be rendered visible by putting oak sawdust into the water.

43. Heating of Buildings by Hot Water.—This is a simple application of the principle just stated. One of the most common arrangements for this purpose is shown in Fig. 36. The boiler C is heated by a fire below it, and the products of combustion escape through the chimney A B. At the top of the house is a reservoir D, communicating with the boiler by a tube. From this reservoir the liquid flows into another reservoir E in the story immediately below, thence into another reservoir F, and so on. Finally, the last of these

reservoirs communicates with the bottom of the boiler. The boiler, tubes, and reservoirs are all completely filled with water, with the
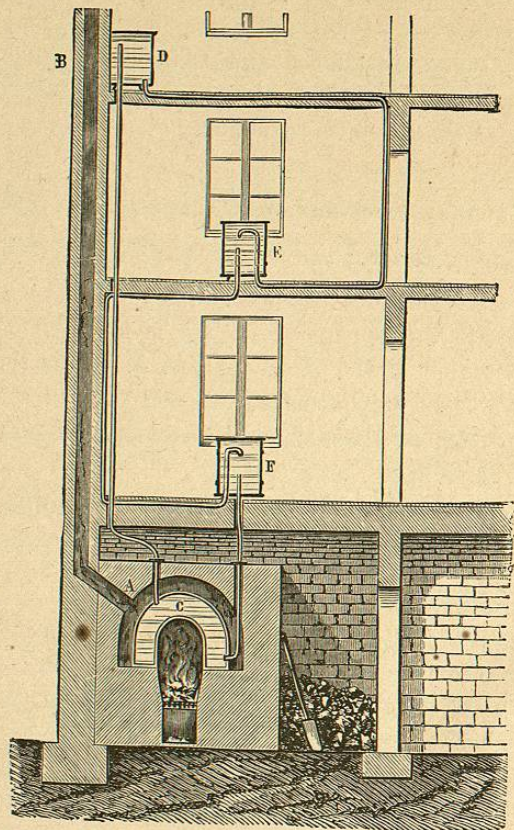

Fig. 36.—Heating by Hot Water.

exception of a small space left above in order to give room for the expansion of the liquid. An ascending current flows through the left-hand tube, and the circulation continues with great regularity, so long as the temperature of the water in the boiler remains constant.

CHAPTER V.

EXPANSION OF GASES.

44. Experiments of Gay-Lussac.—Gay-Lussac conducted a series of researches on the expansion of gases, the results of which were long regarded as classical. He employed a thermometer with a large reservoir A, containing the gas to be operated on; an index of mercury mn separated the gas from the external air, while leaving it full liberty to expand. The gas had previously been dried by pass-
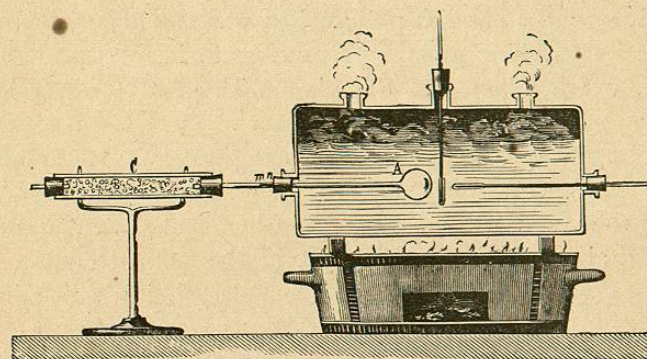

Fig. 37.—Gay-Lussac's Apparatus.

ing it through a tube containing chloride of calcium, or some other desiccating substance. The thermometer was first placed in a vessel filled with melting ice, and when the gas had thus been brought to 0° C., the tube was so adjusted that the index coincided with the opening through which the thermometer passed.

The tube and reservoir having been previously gauged, and the former divided into parts of equal capacity, the apparent volume of the gas (expressed in terms of these divisions) is indicated by the position of the index; let the apparent volume observed at 0° C. be called n, and let H denote the external pressure as indicated by a