

The *absolute* conductivity of wrought iron was investigated with great care by Principal Forbes, by a method which avoided some of the questionable assumptions above enumerated. The end of the bar was heated by a bath of melted lead kept at a uniform temperature, screens being interposed to protect the rest of the bar from the heat radiated by the bath. The temperatures at other points were observed by means of thermometers inserted in small holes drilled in the bar, and kept in metallic contact by fluid metal. In order to determine the loss of heat by radiation at different temperatures, a precisely similar bar, with a thermometer inserted in it, was raised to about the temperature of the bath, and the times of cooling down through different ranges were noted.

The conductivity of one of the two bars experimented on, varied from .01337 at 0° C. to .00801 at 275° C., and the corresponding numbers for the other bar were .00992 and .00724, the units being the foot, the minute, the degree (of any scale), and the foot-degree<sup>1</sup> (of the same scale). In both instances, the conductivity decreased regularly with increase of temperature.

To reduce these results to the C.G.S. scale, we must (as directed in § 168) multiply them by  $\frac{x^2}{t}$ , where  $x$  denotes the number of centimetres in a foot, or 30.48, and  $t$  the number of seconds in a minute;  $\frac{x^2}{t}$  will therefore be

$$\frac{(30.48)^2}{60}, \text{ or } 15.48.$$

The reduced values will therefore be as follows:—

	At 0°.	At 275°
1st bar,.....	.207 .....	.1240
2d bar,.....	.1536 .....	.1121

175. **Experimental Determination of Diffusivity.**—Absolute determinations of the diffusivity  $\kappa$  or  $\frac{k}{c}$  for the soil or rock at three localities in or near Edinburgh were made by Principal Forbes and Lord Kelvin. They were derived from observations on the temperature of the soil as indicated by thermometers having their bulbs buried at depths of 3, 6, 12 and 24 French feet. The annual range of temperature diminished rapidly as the depth increased, and this diminution of range was accompanied by a retardation of the times of maximum and minimum. The greater the diffusivity the more slowly will the range diminish and the less will be the retardation

<sup>1</sup> See § 60.

of phase. By a process described in note C at the end of this chapter the value of  $\kappa$  was deduced; and by combining this with the value of  $c$  (the product of specific heat and density), which was determined by Regnault, from laboratory experiments, the value of  $k$  or  $c\kappa$  was found. The following are the results, expressed in the C.G.S. scale:—

	$\frac{k}{c}$ or Diffusivity.	$\kappa$ or Conductivity.
Trap rock of Calton Hill, .....	.00786 .....	.00415
Sand of Experimental Garden,.....	.03872 .....	.00262
Sandstone of Craighleith Quarry,.....	.02311 .....	.01068

Similar observations made at Greenwich Observatory, and reduced by the editor of the present work, gave .01249 as the diffusivity of the gravel of Greenwich Observatory Hill.

A method based upon similar principles has since been employed by Ångström and also by Neumann for laboratory experiments; a bar of the substance under examination being subjected to regular periodical variations of temperature at one end, and the resulting periodic variations at other points in its length being observed. These gave the means of calculating the diffusivity, and then observations of the specific heat and density gave the conductivity. The following conductivities were thus obtained by Neumann:—

	Conductivity in C.G.S. units.
Copper,.....	1.108
Brass,.....	.302
Zinc,.....	.307
Iron,.....	.164
German silver,.....	.109

176. **Conductivity of Rocks.**—The following values of thermal and thermometric conductivity in C.G.S. units are averages based on the experiments of Professor Alexander Herschel.

	$\frac{k}{c}$	$\frac{k}{c}$
Granite,.....	.0053 .....	.015
Limestone,.....	.005 .....	.009
Sandstone, dry,.....	.0056 .....	.012
Sandstone, thoroughly wet,.....	.0060 .....	.010
Slate, along cleavage,.....	.0060 .....	.010
Slate, across cleavage,.....	.0034 .....	.006
Clay, sun-dried,.....	.0022 .....	.0048
Red brick, .....	.0015 .....	.0044
Plate-glass,.....	.0023 .....	.0040

177. **Conducting Powers of Liquids.**—With the exception of mercury and other melted metals, liquids are exceedingly bad conduc-



tors of heat. This can be shown by heating the upper part of a column of liquid, and observing the variations of temperature below. These will be found to be scarcely perceptible, and to be very slowly produced. If the heat were applied below (Fig. 110), we should have the process called *convection of heat*; the lower layers of liquid would rise to the surface, and be replaced by others which would rise in their turn, thus producing a circulation and a general heating of the liquid. On the other hand, when heat is applied above, the expanded

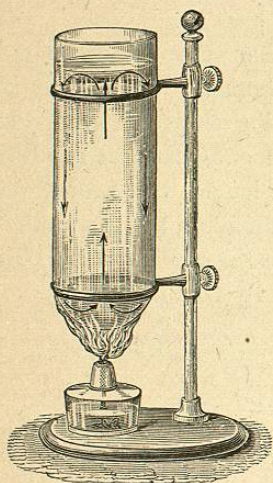


Fig. 110.—Liquid heated from below.

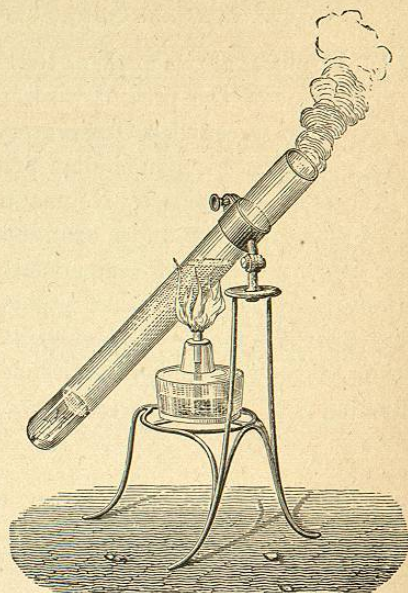


Fig. 111.—Boiling of Water over Ice.

layers remain in their place, and the rest of the liquid can be heated by conduction and radiation only.

The following experiment is one instance of the very feeble conducting power of water. A piece of ice is placed at the bottom of a glass tube (Fig. 111), which is then partly filled with water; heat is applied to the middle of the tube, and the upper portion of the water is readily raised to ebullition, without melting the ice below.

**178. Conducting Power of Water.**—The power of conducting heat possessed by water, though very small, is yet quite appreciable. This was established by Despretz by the following experiment. He took a cylinder of wood (Fig. 112) about a yard in height and eight inches in diameter, which was filled with water. In the side of this

cylinder were arranged twelve thermometers one above another, their bulbs being all in the same vertical through the middle of the liquid column. On the top of the liquid rested a metal box, which was filled with water at 100°, frequently renewed during the course of the experiment. Under these circumstances Despretz observed that the temperature of the thermometers rose gradually, and that a long time—about 30 hours—was required before the permanent state was

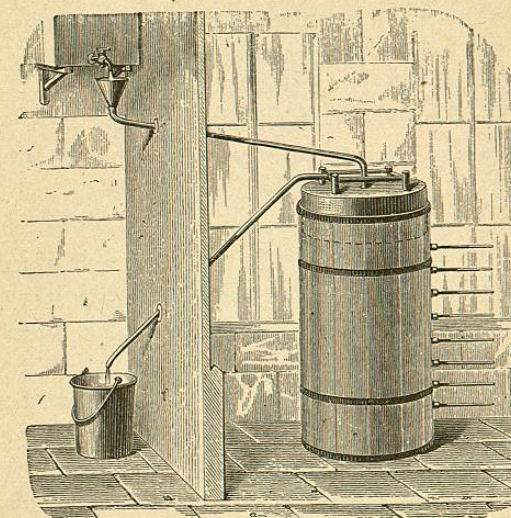


Fig. 112.—Despretz's Experiment.

assumed. Their permanent differences, which formed a decreasing geometric series, were very small, and were inappreciable after the sixth thermometer.

The increase of temperature indicated by the thermometers might be attributed to the heat received from the sides of the cylinder, though the feeble conducting power of wood renders this idea somewhat improbable. But Despretz observed that the temperature was higher in the axis of the cylinder than near the sides, which proves that the elevation of temperature was due to the passage of heat downwards through the liquid.

From experiments by Professor Guthrie,<sup>1</sup> it appears that water conducts better than any other liquid except mercury.

**179. Absolute Measurement of Conductivity of Water.**—The abso-

<sup>1</sup> *B. A. Report*, 1868, and *Trans. R. S.* 1869.



lute value of  $k$  for water has been determined by Mr. J. T. Bottomley. Hot water was gently placed on the top of a mass of water nearly filling a cylindrical wooden vessel. Readings were taken from time to time of two horizontal thermometers, one of them a little lower than the other, which gave the difference of temperature between the two sides of the intervening stratum. The quantity of heat conducted in a given time through this stratum was known from the rise of temperature of the whole mass of water below, as indicated by an upright thermometer with an exceedingly long cylindrical bulb extending downwards from the centre of the stratum in question nearly to the bottom of the vessel. A fourth thermometer, at the level of the bottom of the long bulb, showed when the increase of temperature had extended to this depth, and as soon as this occurred (which was not till an hour had elapsed) the experiment was stopped.

The result of these experiments is that the value of  $k$  for water is from .0020 to .0023, which is nearly identical with its value for ice, this latter element, as determined by Professor George Forbes, being .00223.

The conductivity of water seems to be much greater than that of wood.

180. **Conducting Power of Gases.**—Of the conducting powers of gases it is almost impossible to obtain any direct proofs, since it is exceedingly difficult to prevent the interference of convection and direct radiation. However, we know at least that they are exceedingly bad conductors. In fact, in all cases where gases are inclosed in small cavities where their movement is difficult, the system thus formed is a very bad conductor of heat. This is the cause of the feeble conducting powers of many kinds of cloth, of fur, eider-down, felt, straw, saw-dust, &c. Materials of this kind, when used as articles of clothing, are commonly said to be *warm*, because they hinder the heat of the body from escaping. If a garment of eider-down or fur were compressed so as to expel the greater part of the air, and to reduce the substance to a thin sheet, it would be found to be a much less warm covering than before, having become a better conductor. We thus see that it is the presence of air which gives these substances their feeble conducting power, and we are accordingly justified in assuming that air is a bad conductor of heat.

181. **Conductivity of Hydrogen.**—The conducting power of hydrogen is much superior to that of the other gases—a fact which agrees

with the view entertained by chemists, that this gas is the vapour of a metal. The good conductivity of hydrogen is shown by the following experiments:—

1. Within a glass tube (Fig. 113) is stretched a thin platinum wire, which is raised to incandescence by the passage of an electric current. When air, or any gas other than hydrogen, is passed through the tube, the incandescence continues, though with less vividness than in vacuo; but it disappears as soon as hydrogen is employed.

2. A thermometer is placed at the bottom of a vertical tube, and heated by a vessel containing boiling water which is placed at the top of the tube. The tube is exhausted of air, and different gases are successively admitted. In each case the indication of the thermometer is found to be lower than for vacuum, except when the gas is hydrogen. With this gas, the difference is in the opposite direction, showing that the diminution of radiation has been more than compensated by the conducting power of the hydrogen.

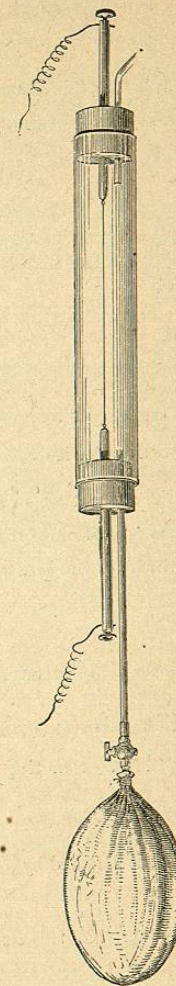


Fig. 113.—Cooling by Contact of Hydrogen

NOTE A. DIFFERENTIAL EQUATION FOR LINEAR FLOW OF HEAT.—The mode of obtaining differential equations for the variation of temperature at each point of a body during the variable stage, may be illustrated by considering the simplest case, that in which the isothermal surfaces (surfaces of equal temperature) are parallel planes, and therefore the lines of flow (which must always be normal to the isothermal surfaces) parallel straight lines.

Let  $x$  denote distance measured in the direction in which heat is flowing,  $v$  the temperature at the time  $t$  at a point specified by  $x$ ,  $k$  the conductivity, and  $c$  the thermal capacity per unit volume (both at the temperature  $v$ ). Then the flow of heat per unit time past a cross section of area  $A$  is  $-kA \frac{dv}{dx}$ , and the flow past an equal and parallel section further on by the small distance  $\delta x$  is greater by the amount

$$A \frac{d}{dx} \left( -k \frac{dv}{dx} \right) \delta x.$$



This latter expression therefore represents the loss of heat from the intervening prism  $A \delta x$ , and the resulting fall of temperature is the quotient of the loss by the thermal capacity  $cA \delta x$ , which quotient is

$$\frac{1}{c} \frac{d}{dx} \left( -k \frac{dv}{dx} \right).$$

This, then, is the fall of temperature per unit time, or is  $-\frac{dv}{dt}$ . If the variation of  $k$  is insensible, so that  $\frac{dk}{dx}$  can be neglected, the equation becomes

$$\frac{dv}{dt} = \frac{k}{c} \frac{d^2v}{dx^2},$$

which applies approximately to the variations of temperature in the soil near the surface of the earth,  $x$  being in this case measured vertically. For the integral of this equation, see Note C.

NOTE B. FLOW OF HEAT IN A BAR (§ 174).—If  $p$  and  $s$  denote the perimeter and section of the bar,  $k$  the conductivity, and  $h$  the coefficient of emission of the surface at the temperature  $v$ , the heat emitted in unit time from the length  $\delta x$  is  $hvp \delta x$ , if we assume as our zero of temperature the temperature of the surrounding air. But the heat which passes a section is  $-sk \frac{dv}{dx}$ , and that which passes a section further on by the amount  $\delta x$  is less by the amount  $sk \frac{d^2v}{dx^2} \delta x$ ; and this difference must equal the amount emitted from the intervening portion of the surface. Hence we have the equation  $\frac{d^2v}{dx^2} = \frac{hp}{ks} v$ , the integral of which for the case supposed is

$$v = V e^{-x} \sqrt{\frac{hp}{ks}}$$

$V$  denoting the temperature at the heated end.

NOTE C. DEDUCTION OF DIFFUSIVITY FROM OBSERVATIONS OF UNDERGROUND TEMPERATURE (§ 175).—Denoting the diffusivity  $\frac{k}{c}$  by  $\kappa$ , the equation of Note A is

$$\frac{dv}{dt} = \kappa \frac{d^2v}{dx^2}. \quad (4)$$

This equation is satisfied by

$$v = e^{-\alpha x} \sin(\beta t - \alpha x), \quad (5)$$

where  $\alpha$  and  $\beta$  are any two constants connected by the relation

$$\frac{\beta}{2\alpha^2} = \kappa; \quad (6)$$

for we find, by actual differentiation,

$$\begin{aligned} \frac{dv}{dx} &= e^{-\alpha x} \{ -\alpha \sin(\beta t - \alpha x) - \alpha \cos(\beta t - \alpha x) \}; \\ \frac{d^2v}{dx^2} &= e^{-\alpha x} \{ \alpha^2 \sin(\beta t - \alpha x) + \alpha^2 \cos(\beta t - \alpha x) + \alpha^2 \cos(\beta t - \alpha x) - \alpha^2 \sin(\beta t - \alpha x) \} \\ &= e^{-\alpha x} 2\alpha^2 \cos(\beta t - \alpha x); \\ \frac{dv}{dt} &= e^{-\alpha x} \beta \cos(\beta t - \alpha x) = \frac{\beta}{2\alpha^2} \frac{d^2v}{dx^2}. \end{aligned}$$

More generally, equation (4) will be satisfied by making  $v$  equal to the sum of any

number of terms similar to the right-hand member of (5), each multiplied by any constant, and a constant term may be added. In fact we may have

$$v = A_0 + A_1 e^{-\alpha_1 x} \sin(\beta_1 t - \alpha_1 x + E_1) + A_2 e^{-\alpha_2 x} \sin(\beta_2 t - \alpha_2 x + E_2) + A_3 e^{-\alpha_3 x} \sin(\beta_3 t - \alpha_3 x + E_3) + \&c., \quad (7)$$

where  $A_0, A_1, E_1, \&c.$ , are any constants.

Let  $x$  be measured vertically downwards from the surface of the ground (supposed horizontal); then at the surface the above expression becomes

$$v = A_0 + A_1 \sin(\beta_1 t + E_1) + A_2 \sin(\beta_2 t + E_2) + A_3 \sin(\beta_3 t + E_3) + \&c. \quad (8)$$

Now, if  $T$  denote a year, it is known that the average temperature of the surface at any time of year can be expressed, in terms of  $t$  the time reckoned from 1st of January or any stated day, by the following series:—

$$v = A_0 + A_1 \sin\left(\frac{2\pi t}{T} + E_1\right) + A_2 \sin\left(\frac{4\pi t}{T} + E_2\right) + A_3 \sin\left(\frac{6\pi t}{T} + E_3\right) + \&c., \quad (9)$$

where  $A_0$  is the mean temperature of the whole year, and  $A_1, A_2, A_3, \&c.$ , which are called the *amplitudes* of the successive terms, diminish rapidly. The term which contains  $A_1$  and  $E_1$  (called the annual term), completes its cycle of values in a year, the next term in half a year, the next in a third of a year, and so on. The annual term is much larger, and more regular in its values from year to year than any of those which follow it. Each term affords two separate determinations of the diffusivity. Thus, for the annual term, we have, by comparing (8) and (9)—

$$\beta_1 = \frac{2\pi}{T}, \text{ whence, by (6),}$$

$$\alpha_1 = \sqrt{\frac{\beta_1}{2\kappa}} = \sqrt{\frac{\pi}{T\kappa}}.$$

At the depth  $x$ , the amplitude of this term will be

$$A_1 e^{-\alpha_1 x},$$

the logarithm of which is

$$\log A_1 - \alpha_1 x.$$

Hence  $\alpha_1$  can be deduced from a comparison of the annual term at two different depths, by dividing the difference of the Napierian logarithms of the amplitudes by the difference of depth.

But  $\alpha_1$  can also be determined by comparing the values of  $\beta_1 t - \alpha_1 x + E_1$  at two depths for the same value of  $t$ , and taking their difference (which is called the *retardation of phase*, since it expresses how much later the maximum, minimum, and other phases, occur at the lower depth than at the upper). This difference, divided by the difference of depth, will be equal to  $\alpha_1$ .

These two determinations of  $\alpha_1$  ought to agree closely, and  $\kappa$  will then be found by the equation

$$\alpha_1 = \sqrt{\frac{\pi}{T\kappa}}.$$