3. Those phenomena which specially belong to the domain of Natural Philosophy are called *physical;* and Natural Philosophy itself is called *Physics.* It may be divided into the following branches.

I. Dynamics, or the general laws of force and of the relations which exist between force, mass, and velocity. These laws may be applied to solids, liquids, or gases. Thus we have the three divisions, *Mechanics, Hydrostatics,* and *Pneumatics.*

II. Thermics; the science of Heat.

III. The science of Electricity, with the closely related subject of Magnetism.

IV. Acoustics; the science of Sound.

V. Optics; the science of Light.

The branches here numbered I. II. III. are treated in Parts I. II. III. respectively, of the present Work. The two branches numbered IV. V. are treated in Part IV.

CHAPTER II.

FIRST PRINCIPLES OF DYNAMICS. STATICS.

4. Force.—Force may be defined as that which tends to produce motion in a body at rest, or to produce change of motion in a body which is moving. A particle is said to have uniform or unchanged motion when it moves in a straight line with constant velocity; and every deviation of material particles from uniform motion is due to forces acting upon them.

5. Translation and Rotation.—When a body moves so that all lines in it remain constantly parallel to their original positions (or, to use the ordinary technical phrase, *move parallel to themselves*), its movement is called a *pure translation.* Since the lines joining the extremities of equal and parallel straight lines are themselves equal and parallel, it can easily be shown that, in such motion, all points of the body have equal and parallel velocities, so that the movement of the whole body is completely represented by the movement of any one of its points.

On the other hand, if one point of a rigid body be fixed, the only movement possible for the body is *pure rotation,* the axis of the rotation at any moment being some straight line passing through this point.

Every movement of a rigid body can be specified by specifying the movement of one of its points (any point will do) together with the rotation of the body about this point.

6. Force which acts uniformly on all the particles of a body, as gravity does sensibly in the case of bodies of moderate size on the earth's surface (equal particles being urged with equal forces and in parallel directions), tends to give the body a movement of pure translation.

In elementary statements of the laws of force, it is necessary, for the sake of simplicity, to confine attention to forces tending to produce pure translation.

7. Instruments for Measuring Force.—We obtain the idea of force through our own conscious exercise of muscular force, and we can approximately estimate the amount of a force (if not too great or too small) by the effort which we have to make to resist it; as when we try the weight of a body by lifting it.

Dynamometers are instruments in which force is measured by means of its effect in bending or otherwise distorting elastic springs, and the spring-balance is a dynamometer applied to the measurement of weights, the spring in this case being either a flat spiral (like the mainspring of a watch), or a helix (resembling a cork-screw).

A force may also be measured by causing it to act vertically downwards upon one of the scale-pans of a balance and counter-poising it by weights in the other pan.

8. Gravitation Units of Force.—In whatever way the measurement of a force is effected, the result, that is, the magnitude of the force, is usually stated in terms of weight; for example, in pounds or in kilogrammes. Such units of force (called gravitation units) are to a certain extent indefinite, inasmuch as gravity is not exactly the same over the whole surface of the earth; but they are sufficiently definite for ordinary commercial purposes.

9. Equilibrium, Statics, Kinetics.—When a body free to move is acted on by forces which do not move it, these forces are said to be *in equilibrium*, or to *equilibrate* each other. They may equally well be described as *balancing* each other. Dynamics is usually divided into two branches. The first branch, called *Statics*, treats of the conditions of equilibrium. The second branch, called *Kinetics*, treats of the movements produced by forces not in equilibrium.

10. Action and Reaction.—Experiment shows that force is always a mutual action between two portions of matter. When a body is urged by a force, this force is exerted by some other body, which is itself urged in the opposite direction with an equal force. When I press the table downwards with my hand, the table presses my hand upwards; when a weight hangs by a cord attached to a beam, the cord serves to transmit force between the beam and the weight, so that, by the instrumentality of the cord, the beam pulls the weight upwards and the weight pulls the beam downwards. Electricity and magnetism furnish no exception to this universal law. When a magnet attracts a piece of iron, the piece of iron attracts the magnet with a precisely equal force.

11. Specification of a Force acting at a Point.—Force may be applied over a finite area, as when I press the table with my hand; or may be applied through the whole substance of a body, as in the case of gravity; but it is usual to begin by discussing the action of forces applied to a *single particle*, in which case each force is supposed to act along a mathematical straight line, and the particle or material point to which it is applied is called its *point* of application. A force is completely specified when its *magnitude*, its *point of application*, and its *line of action* are all given.

12. Rigid Body. Fundamental Problem of Statics.—A force of finite magnitude applied to a mathematical point of any actual solid body would inevitably fracture the body. To avoid this complication and other complications which would arise from the bending and yielding of bodies under the action of forces, the fiction of a perfectly rigid body is introduced, a body which cannot bend or break under the action of any force however intense, but always retains its size and shape unchanged.

The fundamental problem of Statics consists in determining the conditions which forces must fulfil in order that they may be in equilibrium when applied to a rigid body.

13. Conditions of Equilibrium for Two Forces.—In order that two forces applied to a rigid body should be in equilibrium, it is necessary and sufficient that they fulfil the following conditions:—

1st. Their lines of action must be one and the same.

2nd. The forces must act in opposite directions along this common line.

3rd. They must be equal in magnitude.

It will be observed that nothing is said here about the points of application of the forces. They may in fact be anywhere upon the common line of action. *The effect of a force upon a rigid body is not altered by changing its point of application to any other point in its line of action.* This is called the principle of the *transmissibility of force.*

It follows from this principle that the condition of equilibrium for any number of forces with the same line of action is simply that the sum of those which act in one direction shall be equal to the sum of those which act in the opposite direction.

14. Three Forces Meeting in a Point. Triangle of Forces.—If three forces, not having one and the same line of action, are in equilibrium, their lines of action must lie in one plane, and must either meet in a point or be parallel. We shall first discuss the case in which they meet in a point.

From any point A (Fig. 1) draw a line AB parallel to one of the two given forces, and so that in travelling from A to B we should be travelling in the same direction in which the force acts (not in the opposite direction). Also let it be understood that the length of AB represents the magnitude of the force.

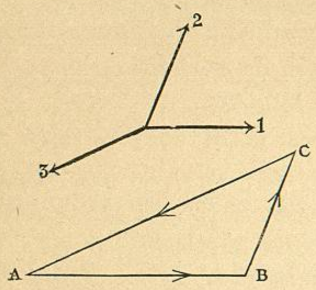

Fig. 1.—Triangle of Forces.

From the point B draw a line BC representing the second force in direction, and on the same scale of magnitude on which AB represents the first.

Then the line CA will represent both in direction and magnitude the third force which would equilibrate the first two.

The principle embodied in this construction is called the *triangle of forces*. It may be briefly stated as follows:—*The condition of equilibrium for three forces acting at a point is, that they be represented in magnitude and direction by the three sides of a triangle, taken one way round.* The meaning of the words "taken one way round" will be understood from an inspection of the arrows with which the sides of the triangle in Fig. 1 are marked. If the directions of all three arrows are reversed the forces represented will still be in equilibrium. The arrows must be so directed that it would be possible to travel completely round the triangle by moving along the sides in the directions indicated.

When a line is used to represent a force, it is always necessary to employ an arrow or some other mark of direction, in order to avoid ambiguity between the direction intended and its opposite. In naming such a line by means of two letters, one at each end of it, the order of the letters should indicate the direction intended. The direction of AB is from A to B; the direction of BA is from B to A.

15. Resultant and Components.—Since two forces acting at a point can be balanced by a single force, it is obvious that they are equivalent to a single force, namely, to a force equal and opposite to that which would balance them. This force to which they are equivalent

is called their *resultant*. Whenever one force acting on a rigid body is equivalent to two or more forces, it is called their resultant, and they are called its *components*. When any number of forces are in equilibrium, a force equal and opposite to any one of them is the resultant of all the rest.

The "triangle of forces" gives us the resultant of any two forces acting at a point. For example, in Fig. 1, AC (with the arrow in the figure reversed) represents the resultant of the forces represented by AB and BC.

16. Parallelogram of Forces.—The proposition called the "parallelogram of forces" is not essentially distinct from the "triangle of forces," but merely expresses the same fact from a slightly different point of view. It is as follows:—*If two forces acting upon the same rigid body in lines which meet in a point, be represented by two lines drawn from the point, and a parallelogram be constructed on these lines, the diagonal drawn from this point to the opposite corner of the parallelogram represents the resultant.* For example, if AB, AC, Fig. 2, represent the two forces, AD will represent their resultant.

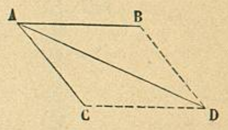

Fig. 2.—Parallelogram of Forces.

To show the identity of this proposition with the triangle of forces, we have only to substitute BD for AC (which is equal and parallel to it). We have then two forces represented by AB, BD (two sides of a triangle) giving as their resultant a force represented by the third side AD. We might equally well have employed the triangle ACD, by substituting CD for AB.

17. Gravesande's Apparatus.—An apparatus for verifying the parallelogram of forces is represented in Fig. 3. ACDB is a light frame in the form of a parallelogram. A weight P″ can be hung at A, and weights P, P′ can be attached, by means of cords passing over pulleys, to the points B, C. When the weights P, P′, P″ are proportional to AB, AC and AD respectively, the strings attached at B and C will be observed to form prolongations of the sides, and the diagonal AD will be vertical.

18. Resultant of any Number of Forces at a Point.—To find the resultant of any number of forces whose lines of action meet in a point, it is only necessary to draw a crooked line composed of straight lines which represent the several forces. The resultant will be represented by a straight line drawn from the beginning to the end of this crooked line. For by what precedes, if ABCDE be a crooked line such that the straight lines AB, BC, CD, DE represent four forces acting at a point, we may substitute for AB and BC

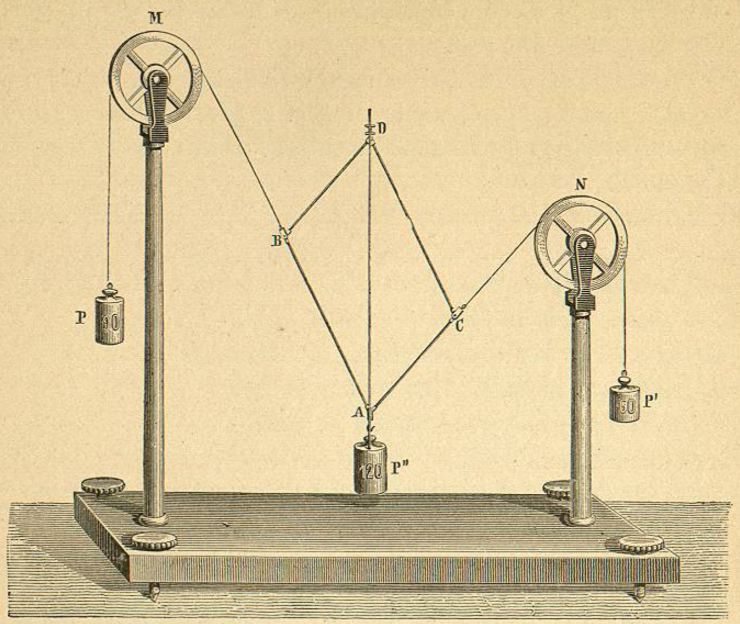

Fig. 3.—Gravesande's Apparatus.

the straight line AC, since this represents their resultant. We may then substitute AD for AC and CD, and finally AE for AD and DE.

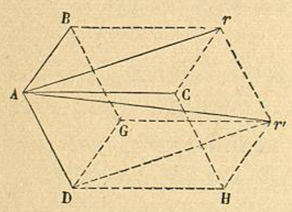

Fig. 4.—Parallelepiped of Forces.

One of the most important applications of this construction is to three forces not lying in one plane. In this case the crooked line will consist of three edges of a parallelepiped, and the line which represents the resultant will be the diagonal. This is evident from Fig. 4, in which AB, AC, AD represent three forces acting at A. The resultant of AB and AC is Ar, and the resultant of Ar and AD is Ar'. The crooked line whose parts represent the forces, may be either ABrr', or ABGr', or ADGr', &c., the total number of alternatives being six, since three things can be taken in six different orders. We have here an excellent illustration of the fact that the same final resultant is obtained in whatever order the forces are combined

19. Equilibrium of Three Parallel Forces.—If three parallel forces, P, Q, R, applied to a rigid body, balance each other, the following conditions must be fulfilled:—

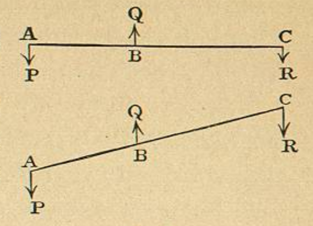

Fig. 5.

1. The three lines of action AP, BQ, CR, Fig. 5, must be in one plane.

2. The two outside forces P, R, must act in the opposite direction to the middle force Q, and their sum must be equal to Q.

3. Each force must be proportional to the distance between the lines of action of the other two; that is, we must have

$$\frac{P}{BC} = \frac{Q}{AC} = \frac{R}{AB}. \qquad (1)$$

The figure shows that AC is the sum of AB and BC; hence it follows from these equations, that Q is equal to the sum of P and R, as above stated.

20. Resultant of Two Parallel Forces.—Any two parallel forces being given, a third parallel force which will balance them can be found from the above equations; and a force equal and opposite to this will be their resultant. We may distinguish two cases.

1. Let the two given forces be in the same direction. Then their resultant is equal to their sum, and acts in the same direction, along a line which cuts the line joining their points of application into two parts which are inversely as the forces.

2. Let the two given forces be in opposite directions. Then their resultant will be equal to their difference, and will act in the direction of the greater of the two forces, along a line which cuts the production of the line joining their points of application on the side of the greater force; and the distances of this point of section from the two given points of application are inversely as the forces.

21. Centre of Two Parallel Forces.—In both cases, if the points of application are not given, but only the magnitudes of the forces and their lines of action, the magnitude and line of action of the resultant are still completely determined; for all straight lines which are drawn across three parallel straight lines are cut by them in the same ratio; and we shall obtain the same result whatever points of application we assume.

If the points of application are given, the resultant cuts the line