The "tumbler" (Fig. 19) consists of a light figure attached to a hemisphere of lead, the centre of gravity of the whole being between the centre of gravity of the hemisphere and the centre of the sphere to which it belongs. When placed upon a level table, the lowest position of the centre of gravity is that in which the figure is upright, and it accordingly returns to this position when displaced.

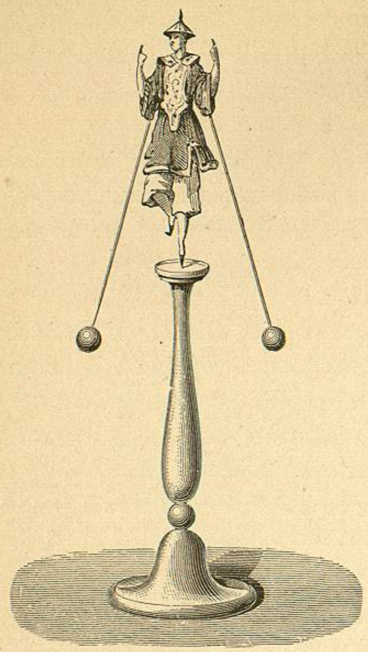

Fig. 18.—Balancer.

54. Limits of Stability.—In the foregoing discussion we have employed the term "stability" in its strict mathematical sense. But there are cases in which, though small displacements would merely produce small oscillations, larger displacements would cause the body, when left to itself, to fall entirely away from the given position of equilibrium. This may be expressed by saying that the equilibrium is stable for displacements lying within certain limits, but unstable for displacements beyond these limits. The equilibrium

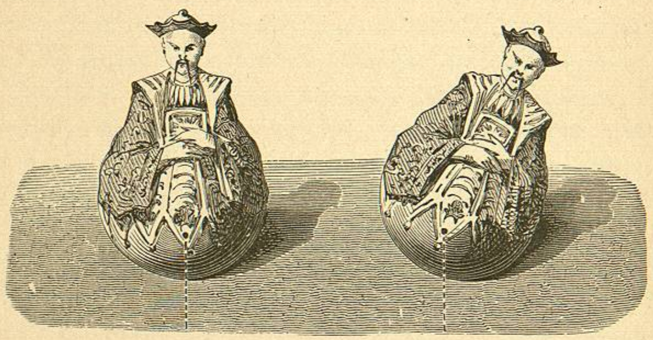

Fig. 19.—Tumblers.

of a system is *practically* unstable when the displacements which it is likely to receive from accidental disturbances lie beyond its limits of stability.

CHAPTER IV.

THE MECHANICAL POWERS.

55. We now proceed to a few practical applications of the foregoing principles; and we shall begin with the so-called "mechanical powers," namely, the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*.

56. Lever.—Problems relating to the lever are usually most conveniently solved by taking moments round the fulcrum. The general condition of equilibrium is, that the moments of the power and the weight about the fulcrum must be in opposite directions, and must be equal. When the power and weight act in parallel directions, the conditions of equilibrium are precisely those of three parallel forces (§ 19), the third force being the reaction of the fulcrum.

It is usual to distinguish three "orders" of lever. In levers of the first order (Fig. 20) the fulcrum is between the power and the

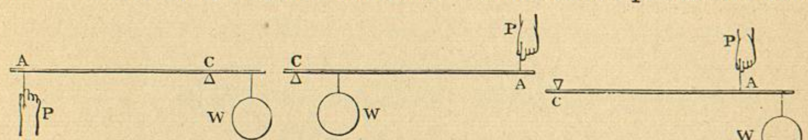

Fig. 20. Fig. 21. Fig. 22.

Three Orders of Lever.

weight. In those of the second order (Fig. 21) the weight is between the power and the fulcrum. In those of the third order (Fig. 22) the power is between the weight and the fulcrum.

In levers of the second order (supposing the forces parallel), the weight is equal to the sum of the power and the pressure on the fulcrum; and in levers of the third order, the power is equal to the sum of the weight and the pressure on the fulcrum; since the middle one of three parallel forces in equilibrium must always be equal to the sum of the other two.

57. Arms.—The *arms of a lever* are the two portions of it intermediate, respectively, between the fulcrum and the power, and between the fulcrum and the weight. If the lever is bent, or if, though straight, it is not at right angles to the lines of action of the power and weight, it is necessary to distinguish between the arms of the lever as above defined (which are parts of the lever), and the *arms of the power and weight* regarded as forces which have moments round the fulcrum. In this latter sense (which is always to be understood unless the contrary is evidently intended), the arms are the perpendiculars dropped from the fulcrum upon the lines of action of the power and weight.

58. Weight of Lever.—In the above statements of the conditions of equilibrium, we have neglected the weight of the lever itself. To take this into account, we have only to suppose the whole weight of the lever collected at its centre of gravity, and then take its moment round the fulcrum. We shall thus have three moments to take account of, and the sum of the two that tend to turn the lever one way, must be equal to the one that tends to turn it the opposite way.

59. Mechanical Advantage.—Every machine when in action serves to transmit *work* without altering its amount; but the *force* which the machine gives out (equal and opposite to what is commonly called the *weight*) may be much greater or much less than that by which it is driven (commonly called the *power*). When it is greater, the machine is said to confer *mechanical advantage*, and the mechanical advantage is measured by the ratio of the weight to the power for equilibrium. In the lever, when the power has a longer arm than the weight, the mechanical advantage is equal to the quotient of the longer arm by the shorter.

60. Wheel and Axle.—The wheel and axle (Fig. 23) may be regarded as an endless lever. The condition of equilibrium is at once given by taking moments round the common axis of the wheel and axle (§ 24). If we neglect the thickness of the ropes, the condition is that the power multiplied by the radius of the wheel must equal the weight multiplied by the radius of the axle; but it is more exact to regard the lines of action of the two forces as coinciding with the axes of the two ropes, so that each of the two radii should be increased by half the thickness of its own rope. If we neglect the thickness of the ropes, the

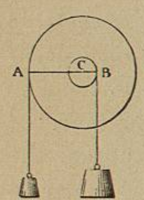

Fig. 23.

mechanical advantage is the quotient of the radius of the wheel by the radius of the axle.

61. Pulley.—A pulley, when fixed in such a way that it can only turn about a fixed axis (Fig. 24), confers no mechanical advantage. It may be regarded as an endless lever of the first order with its two arms equal.

The arrangement represented in Fig. 25 gives a mechanical advantage of 2; for the lower or movable pulley may be regarded as an endless lever of the second order, in which the arm of the power is the diameter of the pulley, and the arm of the weight is

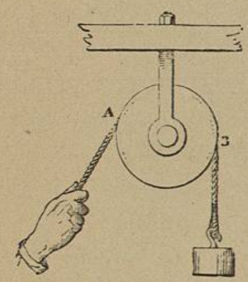

Fig. 24.

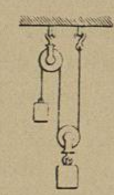

Fig. 25.

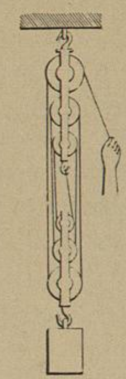

Fig. 26.

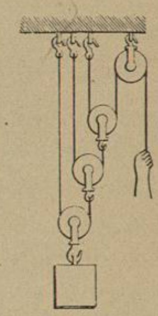

Fig. 27.

half the diameter. The same result is obtained by employing the principle of work; for if the weight rises 1 inch, 2 inches of slack are given over, and therefore the power descends 2 inches.

62. In Fig. 26 there are six pulleys, three at the upper and three at the lower block, and one cord passes round them all. All portions of this cord (neglecting friction) are stretched with the same force, which is equal to the power; and six of these portions, parallel to one another, support the weight. The power is therefore one-sixth of the weight, or the mechanical advantage is 6.

63. In the arrangement represented in Fig. 27, there are three movable pulleys, each hanging by a separate cord. The cord which supports the lowest pulley is stretched with a force equal to half the weight, since its two parallel portions jointly support the weight. The cord which supports the next pulley is stretched with a force half of this, or a quarter of the weight; and the next cord with a force half of this, or an eighth of the weight; but this cord is directly attached to the power. Thus the power is an eighth of the weight, or the mechanical advantage is 8. If the weight and the block[1] to which it is attached rise 1 inch, the next block rises 2 inches, the next 4, and the power moves through 8 inches. Thus, the work done by the power is equal to the work done upon the weight.

In all this reasoning we neglect the weights of the blocks themselves; but it is not difficult to take them into account when necessary.

64. Inclined Plane.—We now come to the inclined plane. Let AB (Fig. 28) be any portion of such a plane, and let AC and BC be drawn vertically and horizontally. Then AB is called the *length*, AC the *height*, and CB the *base* of the inclined plane. The force of gravity upon a heavy body M resting on the plane, may be represented by a vertical line MP, and may be resolved by the parallelogram of forces (§ 16) into two components, MT, MN, the former parallel and the latter perpendicular to the plane. A force equal and opposite to the component MT will suffice to prevent the body from slipping down the plane. Hence, if the power act parallel to the plane, and the weight be that of a heavy body resting on the plane, the power is to the weight as MT to MP; but the two triangles MTP and ACB are similar, since the angles at M and A are equal, and the angles at T and C are right angles; hence MT is to MP as AC to AB, that is, as the height to the length of the plane.

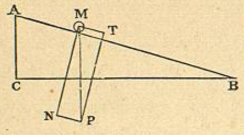

Fig. 28.

65. The investigation is rather easier by the principle of work (§ 49). The work done by the power in drawing the heavy body up the plane, is equal to the power multiplied by the length of the plane. But the work done upon the weight is equal to the weight multiplied by the height through which it is raised, that is, by the height of the plane. Hence we have

$$\text{Power} \times \text{length of plane} = \text{weight} \times \text{height of plane; or}$$
$$\text{power} : \text{weight} :: \text{height of plane} : \text{length of plane}.$$

66. If, instead of acting parallel to the plane, the power acted parallel to the base, the work done by the power would be the product of the power by the base; and this must be equal to the product of the weight by the height; so that in this case the condition of equilibrium would be—

[1] The "pulley" is the revolving wheel. The pulley, together with the frame in which it is inclosed, constitute the "block."

$$\text{Power} : \text{weight} :: \text{height of plane} : \text{base of plane}.$$

67. Wedge.—In these investigations we have neglected friction. The wedge may be regarded as a case of the inclined plane; but its practical action depends to such a large extent upon friction and impact[1] that we cannot profitably discuss it here.

68. Screw.—The screw (Fig. 29) is also a case of the inclined plane. The length of one convolution of the thread is the length of the corresponding inclined plane, the step of the screw, or distance between two successive convolutions (measured parallel to the axis of the screw), is the height of the plane, and the circumference of the screw is the base of the plane. This is easily shown by cutting out a right-angled triangle in paper, and bending it in cylindrical fashion so that its base forms a circle.

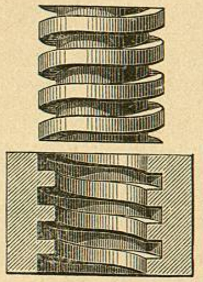
Fig 29.

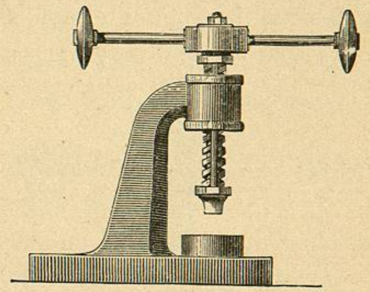
Fig. 30.

69. Screw Press.—In the screw press (Fig. 30) the screw is turned by means of a lever, which gives a great increase of mechanical advantage. In one complete revolution, the pressures applied to the two handles of the lever to turn it, do work equal to their sum multiplied by the circumference of the circle described (approximately) by either handle (we suppose the two handles to be equidistant from the axis of revolution); and the work given out by the machine, supposing the resistance at its lower end to be constant, is equal to this resistance multiplied by the distance between the threads. These two products must be equal, friction being neglected.

[1] An *impact* (for example a blow of a hammer) may be regarded as a very great (and variable) force acting for a very short time. The magnitude of an impact is measured by the momentum which it generates in the body struck.

CHAPTER V.

THE BALANCE.

70. General Description of the Balance.—In the common *balance* (Fig. 31) there is a stiff piece of metal, A B, called the *beam*, which turns about the sharp edge O of a steel wedge forming part of the beam and resting upon two hard and smooth supports. There are two other steel wedges at A and B, with their edges upwards, and upon these edges rest the hooks for supporting the scale pans. The three edges (called knife-edges) are parallel to one another and perpendicular to the length of the beam, and are very nearly in one plane.

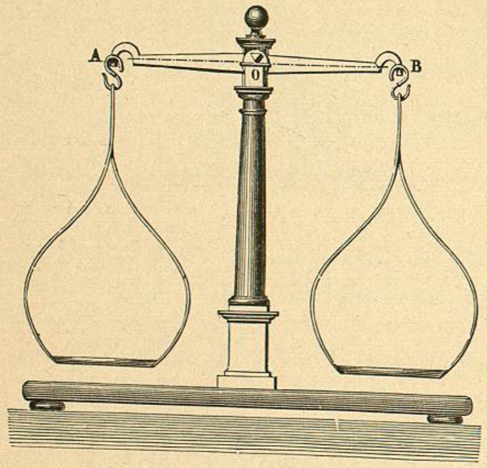

Fig. 31.—Balance.

71. Qualities Requisite.—The qualities requisite in a balance are:

1. That it be consistent with itself; that is, that it shall give the same result in successive weighings of the same body. This depends chiefly on the trueness of the knife-edges.

2. That it be just. This requires that the distances A O, O B, be equal, and also that the beam remain horizontal when the pans are empty. Any inequality in the distances A O, O B, can be detected by putting equal (and tolerably heavy) weights into the two pans. This adds equal moments if the distances are equal, but unequal

moments if they are unequal, and the greater moment will preponderate.

3. Delicacy or sensibility (that is, the power of indicating inequality between two weights even when their difference is very small).

This requires a minimum of friction, and a very near approach to neutral equilibrium (§ 40). In absolutely neutral equilibrium, the smallest conceivable force is sufficient to produce a displacement to the full limit of neutrality; and in barely stable equilibrium a small force produces a large displacement. The condition of stability is that if the weights supported at A and B be supposed collected at these edges, the centre of gravity of the system composed of the beam and these two weights shall be below the middle edge O. The equilibrium would be neutral if this centre of gravity exactly coincided with O; and it is necessary as a condition of delicacy that its distance below O be very small.

4. Facility for weighing quickly is desirable, but must sometimes be sacrificed when extreme accuracy is required.

The delicate balances used in chemical analysis are provided with a long pointer attached to the beam. The end of this pointer moves along a graduated arc as the beam vibrates; and if the weights in the two pans are equal, the excursions of the pointer on opposite sides of the zero point of this arc will also be equal. Much time is consumed in watching these vibrations, as they are very slow; and the more nearly the equilibrium approaches to neutrality, the slower they are. Hence quick weighing and exact weighing are to a certain extent incompatible.

72. Double Weighing.—Even if a balance be not just, yet if it be consistent with itself, a correct weighing can be made with it in the following manner:—Put the body to be weighed in one pan, and counterbalance it with sand or other suitable material in the other. Then remove the body and put in its place such weights as are just sufficient to counterpoise the sand. These weights are evidently equal to the weight of the body. This process is called *double weighing*, and is often employed (even with the best balances) when the greatest possible accuracy is desired.

73. Investigation of Sensibility.—Let A and B (Fig. 32) be the points from which the scale-pans are suspended, O the axis about which the beam turns, and G the centre of gravity of the beam. If when the scale-pans are loaded with equal weights, we put into one of them an excess of weight p, the beam will become inclined, and will take a position such as A′B′, turning through an angle which we will call α, and which is easily calculated.

In fact let the two forces P and $P+p$ act at A′ and B′ respectively, where P denotes the less of the two weights, including the weight of the pan. Then the two forces P destroy each other in consequence of the resistance of the axis O; there is left only the force p applied at B′, and the weight π of the beam applied at G′, the new position of the centre of gravity. These two forces are parallel, and are in equilibrium about the axis O, that is, their resultant passes through the point O. The distances of the points of application of the forces from a vertical through O are therefore inversely proportional to the forces themselves, which gives the relation

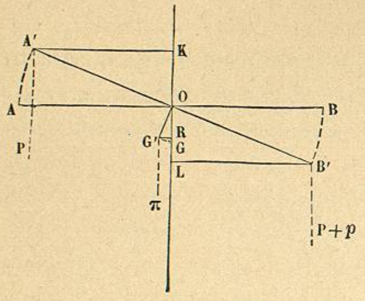

Fig. 32.

$$\pi . \, G'R = p . \, B'L.$$

But if we call half the length of the beam l, and the distance OG r we have

$$G'R = r \sin \alpha, \quad B'L = l \cos \alpha.$$

whence $\pi r \sin \alpha = pl \cos \alpha$, and consequently

$$\tan \alpha = \frac{pl}{\pi r}. \qquad (a)$$

The formula (a) contains the entire theory of the sensibility of the balance when properly constructed. We see, in the first place, that $\tan \alpha$ increases with the excess of weight p, which was evident beforehand. We see also that the sensibility increases as l increases and as π diminishes, or, in other words, as the beam becomes longer and lighter. At the same time it is obviously desirable that, under the action of the weights employed, the beam should be stiff enough to undergo no sensible change of shape. The problem of the balance then consists in constructing a beam of the greatest possible length and lightness, which shall be capable of supporting the action of given forces without bending.

Fortin, whose balances are justly esteemed, employed for his beams bars of steel placed edgewise; he thus obtained great rigidity, but

certainly not all the lightness possible. At present the makers of balances employ in preference beams of copper or steel made in the form of a frame, as shown in Fig 33. They generally give them the shape of a very elongated lozenge, the sides of which are connected by bars variously arranged. The determination of the best shape is, in fact, a special problem, and is an application on a small scale of that principle of applied mechanics which teaches us that hollow pieces have greater resisting power in proportion to their weight than solid pieces, and consequently, for equal resisting power, the former are lighter than the latter. Aluminium, which with a rigidity nearly equal to that of copper, has less than one-fourth of its density, seems naturally marked out as adapted to the construction of beams. It has as yet, however, been little used.

The formula (a) shows us, in the second place, that the sensibility increases as r diminishes; that is, as the centre of gravity approaches the centre of suspension. These two points, however, must not coincide, for in that case for any excess of weight, however small, the beam would deviate from the horizontal as far as the mechanism would permit, and would afford no indication of approach to equality in the weights. With equal weights it would remain in equilibrium in any position. In virtue of possessing this last property, such a balance is called *indifferent*. Practically the distance between the centre of gravity and the point of suspension must not be less than a certain amount depending on the use for which the balance is designed. The proper distance is determined by observing what difference of weights corresponds to a division of the graduated arc along which the needle moves. If, for example, there are 20 divisions on each side of zero, and if 2 milligrammes are necessary for the total displacement of the needle, each division will correspond to an excess of weight of $\frac{2}{20}$ or $\frac{1}{10}$ of a milligramme. That this may be the case we must evidently have a suitable value of r, and the maker is enabled to regulate this value with precision by means of the screw which is shown in the figure above the beam, and which enables him slightly to vary the position of the centre of gravity.

74. Weighing with Constant Load.—In the above analysis we have supposed that the three points of suspension of the beam and of the two scale-pans are in one straight line; in which case the value of $\tan \alpha$ does not include P, that is, the sensibility is independent of the weight in the pans. This follows from the fact that the resultant of the two forces P passes through O, and is thus destroyed, because the axis is fixed. This would not be the case if, for example, the points of suspension of the pans were above that of the beam; in this case the point of application of the common load is above the point O, and, when the beam is inclined, acts in the same direction as the excess of weight; whence the sensibility increases with the load up to a certain limit, beyond which the equilibrium becomes unstable.[1] On the other hand, when the points of suspension of the pans are below that of the beam, the sensibility increases as the load diminishes, and, as the centre of gravity of the beam may in this case be above the axis, equilibrium may become unstable when the load is less than a certain amount. This variation of the sensibility with the load is a serious disadvantage; for, as we have just shown, the displacement of the needle is used as the means of estimating weights, and for this purpose we must have the same displacement corresponding to the same excess of weight. If we wish to employ

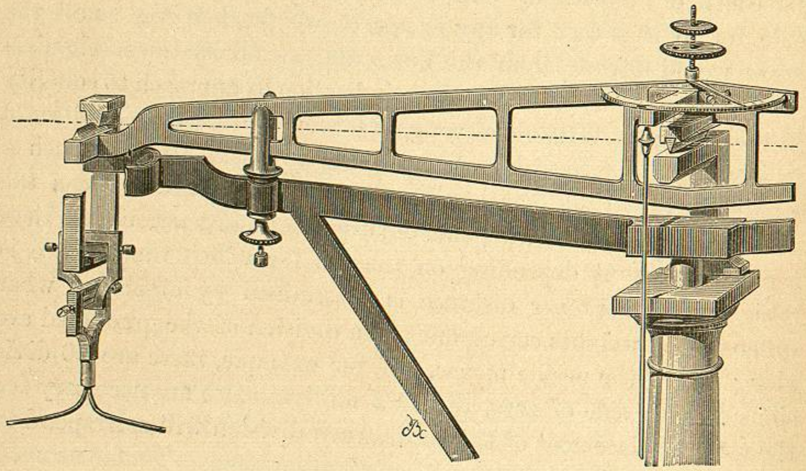

Fig. 33.—Beam of Balance.

either of the two above arrangements, we should weigh with a constant load. The method of doing so, which constitutes a kind of double weighing, consists in retaining in one of the pans a weight equal to this constant load. In the other pan is placed the same load subdivided into a number of marked weights. When the body

[1] This is an illustration of the general principle, applicable to a great variety of philosophical apparatus, that a maximum of sensibility involves a minimum of stability; that is, a very near approach to instability. This near approach is usually indicated by excessive slowness in the oscillations which take place about the position of equilibrium.

to be weighed is placed in this latter pan, we must, in order to maintain equilibrium, remove a certain number of weights, which evidently represent the weight of the body.

We may also remark that, strictly speaking, the sensibility always depends upon the load, which necessarily produces a variation in the friction of the axis of suspension. Besides, it follows from the nature

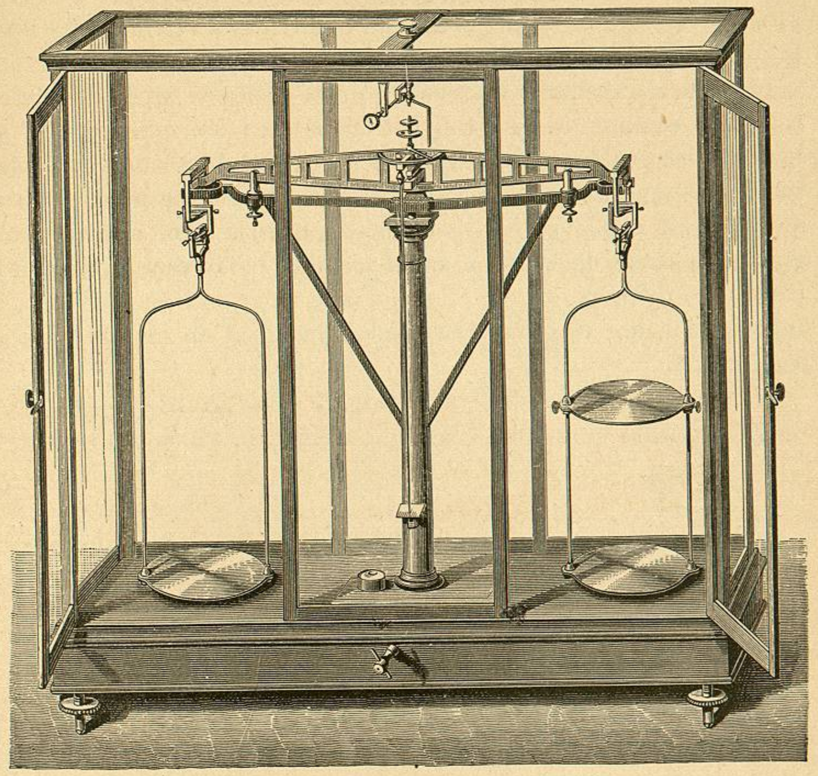

Fig. 34.—Balance for Purposes of Accuracy.

of bodies that there is no system that does not yield somewhat even to the most feeble action. For these reasons, there is a decided advantage in operating with constant load.

75. Details of Construction.—A fundamental condition of the correctness of the balance is, that the weight of each pan and of the load which it contains should always act exactly at the same point, and therefore at the same distance from the axis of suspension. This important result is attained by different methods. The arrangement represented in Fig. 33 is one of the most effectual. At the extremities of the beam are two knife-edges, parallel to the axis of rotation, and facing upwards. On these knife-edges rests, by a hard plane surface of agate or steel, a stirrup, the front of which has been taken away in the figure. On the lower part of the stirrup rests another knife-edge, at right angles to the former, the two being together equivalent to a universal joint supporting the scale-pan and its contents. By this arrangement, whatever may be the position of the weights, their action is always reduced to a vertical force acting on the upper knife-edge.

Fig. 34 represents a balance of great delicacy, with the glass case that contains it. At the bottom is seen the extremity of a lever, which enables us to raise the beam, and thus avoid wearing the knife-edge when not in use. At the top may be remarked an arrangement employed by some makers, consisting of a horizontal graduated circle, on which a small metallic index can be made to travel; its different displacements, whose value can be determined once for all, are used for the final adjustment to produce exact equilibrium.

76. Steelyard.—The steelyard (Fig. 35) is an instrument for weighing bodies by means of a single weight, P, which can be hung at any point of a graduated arm O B. As P is moved further from the fulcrum O, its moment round O increases, and therefore the weight which must be hung from the fixed point A to counterbalance it increases. Moreover, equal movements of P along the arm produce equal additions to its moment, and equal additions to the weight at A produce equal additions to the opposing moment. Hence the divisions on the arm (which indicate the weight in the pan at A) must be equidistant.

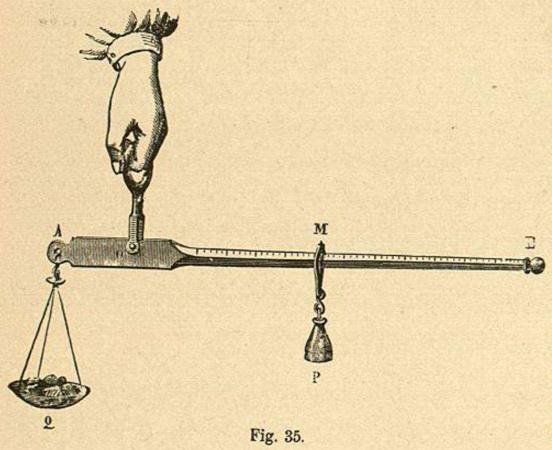

Fig. 35.

CHAPTER VI.

FIRST PRINCIPLES OF KINETICS.

77. Principle of Inertia.—A body not acted on by any forces, or only acted on by forces which are in equilibrium, will not commence to move; and if it be already in motion with a movement of pure translation, it will continue its velocity of translation unchanged, so that each of its points will move in a straight line with uniform velocity. This is Newton's first law of motion, and is stated by him in the following terms:—

"Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state."

The tendency to continue in a state of rest is manifest to the most superficial observation. The tendency to continue in a state of uniform motion can be clearly understood from an attentive study of facts. If, for example, we make a pendulum oscillate, the amplitude of the oscillations slowly decreases and at last vanishes altogether. This is because the pendulum experiences resistance from the air which it continually displaces; and because the axis of suspension rubs on its supports. These two circumstances combine to produce a diminution in the velocity of the apparatus until it is completely annihilated. If the friction at the point of suspension is diminished by suitable means, and the apparatus is made to oscillate *in vacuo*, the duration of the motion will be immensely increased.

Analogy evidently indicates that if it were possible to suppress entirely these two causes of the destruction of the pendulum's velocity, its motion would continue for an indefinite time unchanged.

This tendency to continue in motion is the cause of the effects which are produced when a carriage or railway train is suddenly stopped. The passengers are thrown in the direction of the motion,