extremities of the beam are two knife-edges, parallel to the axis of rotation, and facing upwards. On these knife-edges rests, by a hard plane surface of agate or steel, a stirrup, the front of which has been taken away in the figure. On the lower part of the stirrup rests another knife-edge, at right angles to the former, the two being together equivalent to a universal joint supporting the scale-pan and its contents. By this arrangement, whatever may be the position of the weights, their action is always reduced to a vertical force acting on the upper knife-edge.

Fig. 34 represents a balance of great delicacy, with the glass case that contains it. At the bottom is seen the extremity of a lever, which enables us to raise the beam, and thus avoid wearing the knife-edge when not in use. At the top may be remarked an arrangement employed by some makers, consisting of a horizontal graduated circle, on which a small metallic index can be made to travel; its different displacements, whose value can be determined once for all, are used for the final adjustment to produce exact equilibrium.

73. Steelyard.—The steelyard (Fig. 35) is an instrument for weighing bodies by means of a single weight, P, which can be hung at any point of a graduated arm O B. As P is moved further from the fulcrum O, its moment round O increases, and therefore the weight which must be hung from the fixed point A to counterbalance it increases. Moreover, equal movements of P along the arm produce equal additions to its moment, and equal additions to the weight at A produce equal additions to the opposing moment. Hence the divisions on the arm (which indicate the weight in the pan at A) must be equidistant.

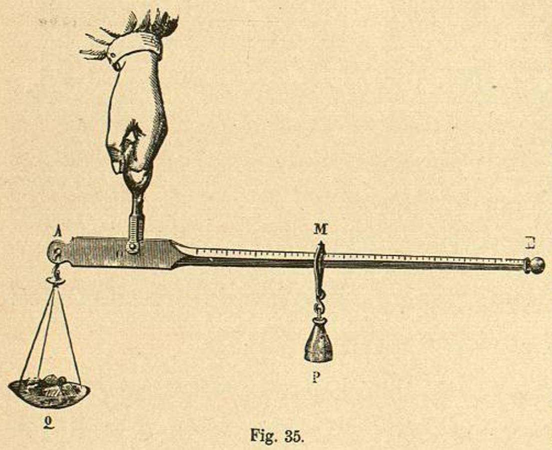

Fig. 35.

CHAPTER VI.

FIRST PRINCIPLES OF KINETICS.

77. Principle of Inertia.—A body not acted on by any forces, or only acted on by forces which are in equilibrium, will not commence to move; and if it be already in motion with a movement of pure translation, it will continue its velocity of translation unchanged, so that each of its points will move in a straight line with uniform velocity. This is Newton's first law of motion, and is stated by him in the following terms:—

"Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state."

The tendency to continue in a state of rest is manifest to the most superficial observation. The tendency to continue in a state of uniform motion can be clearly understood from an attentive study of facts. If, for example, we make a pendulum oscillate, the amplitude of the oscillations slowly decreases and at last vanishes altogether. This is because the pendulum experiences resistance from the air which it continually displaces; and because the axis of suspension rubs on its supports. These two circumstances combine to produce a diminution in the velocity of the apparatus until it is completely annihilated. If the friction at the point of suspension is diminished by suitable means, and the apparatus is made to oscillate *in vacuo*, the duration of the motion will be immensely increased.

Analogy evidently indicates that if it were possible to suppress entirely these two causes of the destruction of the pendulum's velocity, its motion would continue for an indefinite time unchanged.

This tendency to continue in motion is the cause of the effects which are produced when a carriage or railway train is suddenly stopped. The passengers are thrown in the direction of the motion, in virtue of the velocity which they possessed at the moment when the stoppage occurred. If it were possible to find a brake sufficiently powerful to stop a train suddenly at full speed, the effects of such a stoppage would be similar to the effects of a collision.

Inertia is also the cause of the severe falls which are often received in alighting incautiously from a carriage in motion; all the particles of the body have a forward motion, and the feet alone being reduced to rest, the upper portion of the body continues to move, and is thus thrown forward.

When we fix the head of a hammer on the handle by striking the end of the handle on the ground, we utilize the inertia of matter. The handle is suddenly stopped by the collision, and the head continues to move for a short distance in spite of the powerful resistances which oppose it.

78. Second Law of Motion.—Newton's second law of motion is that "Change of motion is proportional to the impressed force and is in the direction of that force."

Change of motion is here spoken of as a quantity, and as a directed quantity. In order to understand how to estimate change of motion, we must in the first place understand how to compound motions.

When a boat is sailing on a river, the motion of the boat relative to the shore is compounded of its motion relative to the water and the motion of the water relative to the shore. If a person is walking along the deck of the boat in any direction, his motion relative to the shore is compounded of three motions, namely the two above mentioned and his motion relative to the boat.

Let X, Y and Z be any three bodies or systems. The motion of X relative to Y, compounded with the motion of Y relative to Z, is the motion of X relative to Z. This is to be taken as the definition of what is meant by compounding two motions; and it leads very directly to the result that two rectilinear motions are compounded by the parallelogram law. For if a body moves along the deck of a ship from O to A (Fig. 36), and the ship in the meantime advances through the distance OB, it is obvious that, if we complete the parallelogram OBCA, the point A of the ship will be brought to C, and the movement of the body in space will be from O to C. If the motion along OA is uniform.

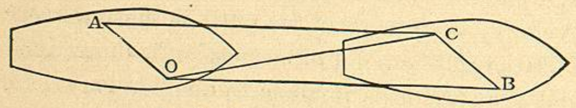

Fig. 36.—Composition of Motions.

and the motion of the ship is also uniform, the motion of the body through space will be a uniform motion along the diagonal OC. Hence, *if two component velocities be represented by two lines drawn from a point, and a parallelogram be constructed on these lines, its diagonal will represent the resultant velocity.*

It is obvious that if OA in the figure represented the velocity of the ship and OB the velocity of the body relative to the ship, we should obtain the same resultant velocity OC. This is a general law; the interchanging of velocities which are to be compounded does not affect their resultant.

Now suppose the velocity OB to be changed into the velocity OC, what are we to regard as the change of velocity? The change of velocity is that velocity which compounded with OB would give OC It is therefore OA. The same force which, in a given time, acting always parallel to itself, changes the velocity of a body from OB to OC, would give the body the velocity OA if applied to it for the same time commencing from rest. Change of motion, estimated in this way, depends only on the acting force and the body acted on by the force; it is entirely independent of any previous motion which the body may possess. No experiments on forces and motions inside a carriage or steamboat which is travelling with perfect smoothness in a straight course, will enable us to detect that it is travelling at all. We cannot even assert that there is any such thing as absolute rest, or that there is any difference between absolute rest and uniform straight movement of translation.

As change of motion is independent of the initial condition of rest or motion, so also is the change of motion produced by one force acting on a body independent of the change produced by any other force acting on the body, provided that each force remains constant in magnitude and direction. The actual motion will be that which is compounded of the initial motion and the motions due to the two forces considered separately. If AB represent one of these motions, BC another, and CD the third, the actual or resultant motion will be AD.

The change produced in the motion of a body by two forces acting jointly can therefore be found by compounding the changes which would be produced by each force separately. This leads at once to the "parallelogram of forces," since the changes of motion produced in one and the same body are proportional to the forces which produce them, and are in the directions of these forces.

In case any student should be troubled by doubt as to whether the "changes of motion" which are proportional to the forces, are to be understood as distances, or as velocities, we may remark that the law is equally true for both, and its truth for one implies its truth for the other, as will appear hereafter from comparing the formula for the distance $s = \frac{1}{2}ft^2$, with the formula for the velocity $v = ft$, since both of these expressions are proportional to f.

79. Explanation of Second Law continued.—It is convenient to distinguish between the *intensity* of a force and the *magnitude* or *amount* of a force. The intensity of a force is measured by the change of velocity which the force produces during the unit of time; and can be computed from knowing the motion of the body acted on, without knowing anything as to its mass. Two bodies are said to be of equal *mass* when the same change of motion (whether as regards velocity or distance) which is produced by applying a given force to one of them for a given time, would also be produced by applying this force to the other for an equal time. If we join two such bodies, we obtain a body of double the mass of either; and if we apply the same force as before for the same time to this double mass, we shall obtain only half the change of velocity or distance that we obtained before. Masses can therefore be compared by taking the inverse ratio of the changes produced in their velocities by equal forces.

The velocity of a body multiplied by its mass is called the *momentum* of the body, and is to be regarded as a directed magnitude having the same direction as the velocity. The change of velocity, when multiplied by the mass of the body, gives the change of momentum; and the second law of motion may be thus stated:—

The change of momentum produced in a given time is proportional to the force which produces it, and is in the direction of this force. It is independent of the mass; the change of velocity in a given time being inversely as the mass.

80. Proper Selection of Unit of Force.—If we make a proper selection of units, the change of momentum produced *in unit time* will be not only proportional but numerically *equal* to the force which produces it; and the change of momentum produced in any time will be the product of the force by the time. Suppose any units of length, time, and mass respectively to have been selected. Then the unit velocity will naturally be defined as the velocity with which unit length would be passed over in unit time; the unit momentum will be the momentum of the unit mass moving with this velocity:

and the unit force will be that force which produces this momentum in unit time. We define the unit force, then, as *that force which acting for unit time upon unit mass produces unit velocity.*

81. Relation between Mass and Weight.—The *weight* of a body, strictly speaking, is the force with which the body tends towards the earth. This force depends partly on the body and partly on the earth. It is not exactly the same for one and the same body at all parts of the earth's surface, but is decidedly greater in the polar than in the equatorial regions. Bodies which, when weighed in a balance *in vacuo*, counterbalance each other, or counterbalance one and the same third body, have equal *weights* at that place, and will also be found to have equal weights at any other place. Experiments which we shall hereafter describe (§ 89) show that such bodies have equal masses; and this fact having been established, the most convenient mode of comparing masses is by weighing them. A pound of iron has the same mass as a pound of brass or of any other substance. A pound of any kind of matter tends to the earth with different forces at different places. The weight of a pound of matter is therefore not a definite standard of force. But the pound of matter itself is a perfectly definite standard of mass. If we weigh one and the same portion of matter in different states; for instance water in the states of ice, snow, liquid water, or steam; or compare the weight of a chemical compound with the weights of its components; we find an exact equality; hence it has been stated that the mass of a body is a measure of the quantity of matter which it contains; but though this statement expresses a simple fact when applied to the comparison of different quantities of one and the same substance, it expresses no known fact of nature when applied to the comparison of different substances. A pound of iron and a pound of lead tend to the earth with equal forces; and if equal forces are applied to them both their velocities are equally affected. We may if we please agree to measure "quantity of matter" by these tests; but we must beware of assuming that two things which are essentially different in kind can be equal in themselves.

82. Third Law of Motion. Action and Reaction.—Forces always occur in pairs, every exertion of force being a mutual action between two bodies. Whenever a body is acted on by a force, the body from which this force proceeds is acted on by an equal and opposite force. The earth attracts the moon, and the moon attracts the earth. A magnet attracts iron and is attracted by iron. When two boats are floating freely, a rope attached to one and hauled in by a person in the other, makes each boat move towards the other. Every exertion of force generates equal and opposite momenta in the two bodies affected by it, since these two bodies are acted on by equal forces for equal times.

If the forces exerted by one body upon the other are equivalent to a single force, the forces of reaction will also be equivalent to a single force, and these two equal and opposite resultants will have the same line of action. We have seen in § 29 that the general resultant of any set of forces applied to a body is a *wrench;* that is to say it consists of a force with a definite line of action (called the *axis*), accompanied by a couple in a perpendicular plane. The reaction upon the body which exerts these forces will always be an equal and opposite wrench; the two wrenches having the same axis, equal and opposite forces along this axis, and equal and opposite couples in the perpendicular plane.

83. Motion of Centre of Gravity Unaffected.—A consequence of the equality of the mutual forces between two bodies is, that these forces produce no movement of the common centre of gravity of the two bodies. For if A be the centre of gravity of a mass m_1, and B the centre of gravity of a mass m_2, their common centre of gravity C will divide AB inversely as the masses. Let the masses be originally at rest, and let them be acted on only by their mutual attraction or replusion. The distances through which they are moved by these equal forces will be inversely as the masses, that is, will be directly as AC and BC; hence if A′ B′ are their new positions after any time, we have

$$\frac{AC}{BC} = \frac{AA'}{BB'} = \frac{AC \pm AA'}{BC \pm BB'} = \frac{A'C}{B'C}.$$

The line A′B′ is therefore divided at C in the same ratio in which the line AB was divided; hence C is still the centre of gravity.

84. Velocity of Centre of Gravity.—If any number of masses are moving with any velocities, and in any directions, but so that each of them moves uniformly in a straight line, their common centre of gravity will move uniformly in a straight line.

To prove this, we shall consider their component velocities in any one direction,

let these component velocities be $u_1 \quad u_2 \quad u_3$ &c.,
the masses being $m_1 \quad m_2 \quad m_3$ &c.,
and the distances of the bodies (strictly speaking the distances of their respective centres of gravity) from a fixed plane to which the given direction is normal, be $x_1 \quad x_2 \quad x_3$ &c.

The formula for the distance of their common centre of gravity from this plane is

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \&c.}{m_1 + m_2 + \&c.} \qquad (1)$$

In the time t, x_1 is increased by the amount $u_1 t$, x_2 by $u_2 t$, and so on; hence the numerator of the above expression is increased by

$$m_1 u_1 t + m_2 u_2 t + \&c.,$$

and the value of $\bar{x}$ is increased in each unit of time by

$$\frac{m_1 u_1 + m_2 u_2 + \&c.}{m_1 + m_2 + \&c.}, \qquad (2)$$

which is therefore the component velocity of the centre of gravity in the given direction. As this expression contains only given constant quantities, its value is constant; and as this reasoning applies to all directions, the velocity of the centre of gravity must itself be constant both in magnitude and direction.

We may remark that the above formula (2) correctly expresses the component velocity of the centre of gravity at the instant considered, even when u_1, u_2, &c., are not constant.

85. Centre of Mass.—The point which we have thus far been speaking of under the name of "centre of gravity," is more appropriately called the "centre of mass," a name which is at once suggested by formula (1) § 84. When gravity acts in parallel lines upon all the particles of a body, the resultant force of gravity upon the body is a single force passing through this point; but this is no longer the case when the forces of gravity upon the different parts of the body (or system of bodies) are not parallel.

86. Units of Measurement.—It is a matter of importance, in scientific calculations, to express the various magnitudes with which we have to deal in terms of units which have a simple relation to each other. The British weights and measures are completely at fault in this respect, for the following reasons:—

1. They are not a decimal system; and the reduction of a measurement (say) from inches and decimals of an inch to feet and decimals of a foot, cannot be effected by inspection.

2. It is still more troublesome to reduce gallons to cubic feet or inches.

3. The weight (properly the mass) of a cubic foot of a substance in lbs., cannot be written down by inspection, when the specific gravity of the substance (as compared with water) is given.

87. The C.G.S. System.—A committee of the British Association, specially appointed to recommend a system of units for general adoption in scientific calculation, have recommended that the *centimetre* be adopted as the unit of length, the *gramme* as the unit of mass, and the *second* as the unit of time. We shall first give the rough and afterwards the more exact definitions of these quantities.

The centimetre is approximately $\frac{1}{10^9}$ of the distance of either pole of the earth from the equator; that is to say 1 followed by 9 zeros expresses this distance in centimetres.

The gramme is approximately the mass of a cubic centimetre of cold water. Hence the same number which expresses the specific gravity of a substance referred to water, expresses also the mass of a cubic centimetre of the substance, in grammes.

The second is $\frac{1}{24 \times 60 \times 60}$ of a mean solar day.

More accurately, the centimetre is defined as one hundredth part of the length, at the temperature 0° Centigrade, of a certain standard bar, preserved in Paris, carefully executed copies of which are preserved in several other places; and the gramme is defined as one thousandth part of the mass of a certain standard which is preserved at Paris, and of which also there are numerous copies preserved elsewhere.

For brevity of reference, the committee have recommended that the system of units based on the Centimetre, Gramme, and Second, be called the C.G.S. system.

The unit of area in this system is the square centimetre.

The unit of volume is the cubic centimetre.

The unit of velocity is a velocity of a centimetre per second.

The unit of momentum is the momentum of a gramme moving with a velocity of a centimetre per second.

The unit force is that force which generates this momentum in one second. It is therefore that force which, acting on a gramme for one second, generates a velocity of a centimetre per second. This force is called the *dyne*, an abbreviated derivative from the Greek δύναμις (force).

The unit of work is the work done by a force of a dyne working through a distance of a centimetre. It might be called the dyne-centimetre, but a shorter name has been provided and it is called the *erg*, from the Greek ἔργον (work).

CHAPTER VII.

LAWS OF FALLING BODIES.

88. Effect of the Resistance of the Air.—In air, bodies fall with unequal velocities; a sovereign or a ball of lead falls rapidly, a piece of down or thin paper slowly. It was formerly thought that this difference was inherent in the nature of the materials; but it is easy to show that this is not the case, for if we compress a mass of down or a piece of paper by rolling it into a ball, and compare it with a piece of gold-leaf, we shall find that the latter body falls more slowly than the former. The inequality of the velocities which we observe is due to the resistance of the air, which increases with the extent of surface exposed by the body.

It was Galileo who first discovered the cause of the unequal rapidity of fall of different bodies. To put the matter to the test, he prepared small balls of different substances, and let them fall at the same time from the top of the tower of Pisa; they struck the ground almost at the same instant. On changing their forms, so as to give them very different extents of surface, he observed that they fell with very unequal velocities. He was thus led to the conclusion that gravity acts on all substances with the same intensity, and that in a vacuum all bodies would fall with the same velocity.

This last proposition could not be put to the test of experiment in the time of Galileo, the air-pump not having yet been invented. The experiment was performed by Newton, and is now well known as the "guinea and feather" experiment. For this purpose a tube from a yard and a half to two yards long is used, which can be exhausted of air, and which contains bodies of various densities, such as a coin, pieces of paper, and feathers. When the tube is full of air and is inverted, these different bodies are seen to fall with very unequal velocities; but if the experiment is repeated after the tube