it out into a plane). It then presents the appearance shown in Fig. 40.

If we take on the horizontal line equal distances at 1, 2, 3, 4, 5 . . . , and draw perpendiculars at their extremities to meet the curve, it is evident that the points thus found are those which were traced by the pencil when the cylinder had turned through the distances 1, 2, 3, 4, 5. . . . The corresponding verticals represent the spaces traversed in the times 1, 2, 3, 4, 5. . . . Now we find, as the figure shows, that these spaces are represented by the numbers 1, 4, 9, 16, 25 . . . , thus verifying the principle that the spaces described are proportional to the squares of the times employed in their description.

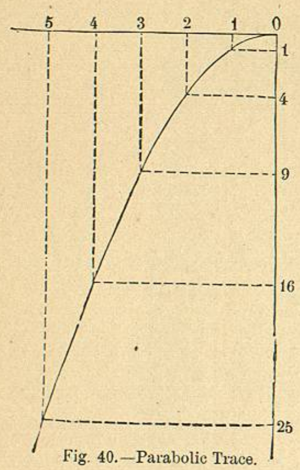

Fig. 40.—Parabolic Trace.

We may remark that the proportionality of the vertical lines to the squares of the horizontal lines shows that the curve is a parabola. The parabolic trace is thus the consequence of the law of fall, and from the fact of the trace being parabolic we can infer the proportionality of the spaces to the squares of the times.

The law of velocities might also be verified separately by Morin's apparatus; we shall not describe the method which it would be necessary to employ, but shall content ourselves with remarking that the law of velocities is a logical consequence of the law of spaces.[1]

99. Atwood's Machine.—Atwood's machine, which affords great facilities for illustrating the effects of force in producing motion, consists essentially of a very freely moving pulley over which a fine cord passes, from the ends of which two equal weights can be suspended. A small additional weight of flat and elongated form is laid upon one of them, which is thus caused to descend with uniform *acceleration*, and means are provided for suddenly removing

[1] Consider, in fact, the space traversed in any time t; this space is given by the formula $s=Kt^2$; during the time $t+\theta$ the space traversed will be $K(t+\theta)^2=Kt^2+2Kt\theta+K\theta^2$, whence it follows that the space traversed during the time θ after the time t is $2Kt\,\theta+K\theta^2$. The average velocity during this time θ is obtained by dividing the space by θ, and is $2Kt+K\theta$, which, by making θ very small, can be made to agree as accurately as we please with the value $2Kt$. This limiting value $2Kt$ must therefore be the velocity at the end of time t.—*D.*

this additional weight at any point of the descent, so as to allow the motion to continue from this point onward with uniform *velocity*.

The machine is represented in Fig. 41. The pulley over which the string passes is the largest of the wheels shown at the top of the apparatus. In order to give it greater freedom of movement, the ends of its axis are made to rest, not on fixed supports, but on the circumferences of four wheels (two at each end of the axis) called friction-wheels, because their office is to diminish friction. Two small equal weights are shown, suspended from this pulley by a string passing over it. One of them P′ is represented as near the bottom of the supporting pillar, and the other P as near the top. The latter is resting upon a small platform, which can be suddenly dropped when it is desired that the motion shall commence. A little lower down and vertically beneath the platform, is seen a ring, large enough to let the weight pass through it without danger of

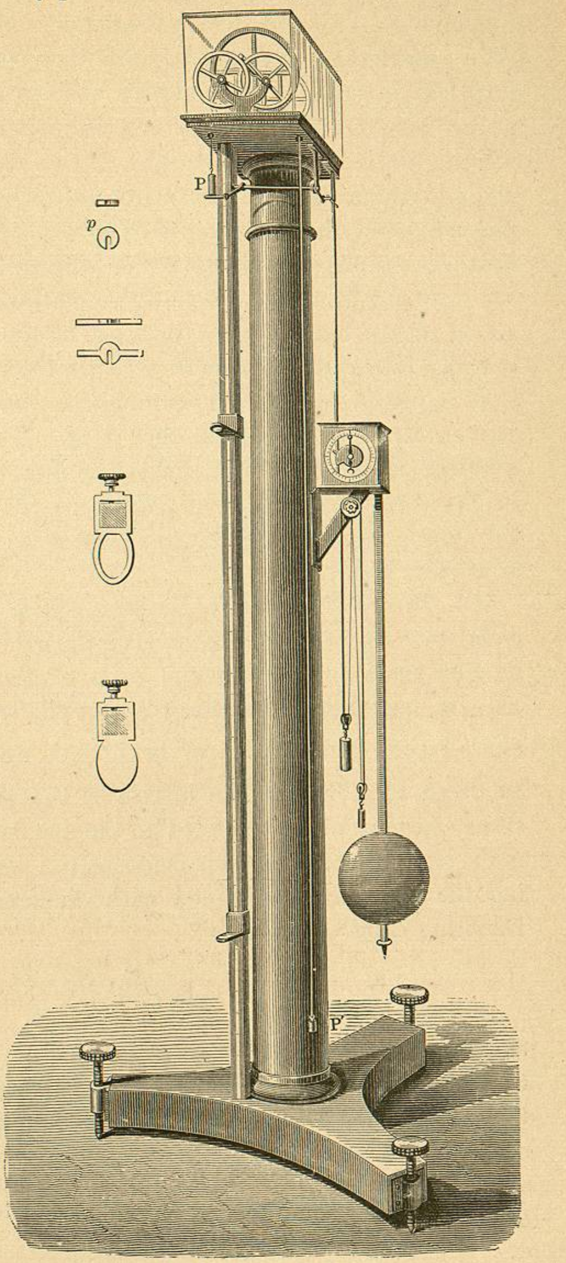

Fig. 41.—Atwood's Machine.

contact. This ring can be shifted up or down, and clamped at any height by a screw; it is represented on a larger scale in the margin. At a considerable distance beneath the ring, is seen the stop, which is also represented in the margin, and can like the ring be clamped at any height. The office of the ring is to intercept the additional weight, and the office of the stop is to arrest the descent. The upright to which they are both clamped is marked with a scale of equal parts, to show the distances moved over. A clock with a pendulum beating seconds, is provided for measuring the time; and there is an arrangement by which the movable platform can be dropped by the action of the clock precisely at one of the ticks. To measure the distance fallen in one or more seconds, the ring is removed, and the stop is placed by trial at such heights that the descending weight strikes it precisely at another tick. To measure the velocity acquired in one or more seconds, the ring must be fixed at such a height as to intercept the additional weight at one of the ticks, and the stop must be placed so as to be struck by the descending weight at another tick.

100. Theory of Atwood's Machine.—If M denote each of the two equal masses, in grammes, and m the additional mass, the whole moving mass (neglecting the mass of the pulley and string) is $2M+m$, but the moving force is only the weight of m. The acceleration produced, instead of being g, is accordingly only $\frac{m}{2M+m}g$. In order to allow for the inertia of the pulley and string, a constant quantity must be added to the denominator in the above formula, and the value of this constant can be determined by observing the movements obtained with different values of M and m. Denoting it by C, we have

$$\frac{m}{m+2M+C}g \qquad (A)$$

as the expression for the acceleration. As m is usually small in comparison with M, the acceleration is very small in comparison with that of a freely falling body, and is brought within the limits of convenient observation. Denoting the acceleration by a, and using v and s, as in § 92, to denote the velocity acquired and space described in time t, we shall have

$$v=at, \qquad (1)$$
$$s=\tfrac{1}{2}at^2, \qquad (2)$$
$$as=\tfrac{1}{2}v^2, \qquad (3)$$

and each of these formulæ can be directly verified by experiments with the machine.

Fig. 42.

101. Uniform Motion in a Circle.—A body cannot move in a curved path unless there be a force urging it towards the concave side of the curve. We shall proceed to investigate the intensity of this force when the path is circular and the velocity uniform. We shall denote the velocity by v, the radius of the circle by r, and the intensity of the force by f. Let AB (Figs. 42, 43) be a small portion of the path, and BD a perpendicular upon AD the tangent at A. Then, since the arc AB is small in comparison with the whole circumference, it is sensibly equal to AD, and the body would have been found at D instead of at B if no force had acted upon it since leaving A. DB is accordingly the distance due to the force; and if t denote the time from A to B, we have

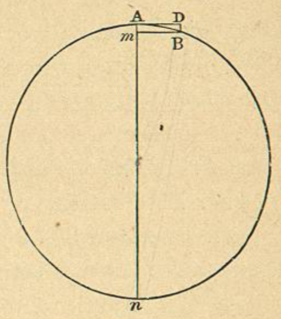

Fig 43.

$$AD = vt \qquad (1)$$
$$DB = \tfrac{1}{2}ft^2. \qquad (2)$$

The second of these equations gives

$$f = \frac{2DB}{t^2}$$

and substituting for t from the first equation, this becomes

$$f = \frac{2DB}{AD^2}v^2, \qquad (3)$$

But if An (Fig. 43) be the diameter at A, and Bm the perpendicular upon it from B, we have, by Euclid, $AD^2 = mB^2 = Am \cdot mn = 2r \cdot Am$ sensibly, $= 2r \cdot DB$.

Therefore $\frac{2DB}{AD^2} = \frac{1}{r}$, and hence by (3)

$$f = \frac{v^2}{r}. \qquad (4)$$

Hence the force necessary for keeping a body in a circular path without change of velocity, is a force of intensity $\frac{v^2}{r}$ directed towards the centre of the circle. If m denote the mass of the body, the amount of the force will be $\frac{mv^2}{r}$. This will be in dynes, if m be in grammes, r in centimetres, and v in centimetres per second.

If the time of revolution be denoted by T, and π as usual denote the ratio of circumference to diameter, the distance moved in time

T is $2\pi r$; hence $v = \frac{2\pi r}{T}$, and another expression for the intensity of the force will be

$$f = \left(\frac{2\pi}{T}\right)^2 r. \qquad (5)$$

102. Deflecting Force in General.—In general, when a body is moving in any path, and with velocity either constant or varying, the force acting upon it at any instant can be resolved into two components, one along the tangent and the other along the normal. The intensity of the tangential component is measured by the rate at which the velocity increases or diminishes, and the intensity of the normal component is given by formula (4) of last article, if we make r denote the radius of curvature.

103. Illustrations of Deflecting Force.—When a stone is swung round by a string in a vertical circle, the tension of the string in the lowest position consists of two parts:—

(1) The weight of the stone, which is mg if m be the mass of the stone.

(2) The force $m\frac{v^2}{r}$ which is necessary for deflecting the stone from a horizontal tangent into its actual path in the neighbourhood of the lowest point.

When the stone is at the highest point of its path, the tension of the string is the difference of these two forces, that is to say it is

$$m\left(\frac{v^2}{r} - g\right),$$

and the motion is not possible unless the velocity at the highest point is sufficient to make $\frac{v^2}{r}$ greater than g.

The tendency of the stone to persevere in rectilinear motion and to resist deflection into a curve, causes it to exert a force upon the string, of amount $m\frac{v^2}{r}$, and this is called *centrifugal force*. It is not a force acting upon the stone, but a force exerted by the stone upon the string. Its direction is *from* the centre of curvature, whereas the deflecting force which acts upon the stone is *towards* the centre of curvature.

104. Centrifugal Force at the Equator.—Bodies on the earth's surface are carried round in circles by the diurnal rotation of the earth upon its axis. The velocity of this motion at the equator is about 46,500 centimetres per second, and the earth's equatorial radius is about $6{\cdot}38 \times 10^8$ centimetres. Hence the value of $\frac{v^2}{r}$ is found to be about 3·39. The case is analogous to that of the stone

at the highest point of its path in the preceding article, if instead of a string which can only exert a pull we suppose a stiff rod which can exert a push upon the stone. The rod will be called upon to exert a pull or a push at the highest point according as $\frac{v^2}{r}$ is greater or less than g. The force of the push in the latter case will be

$$m\left(g - \frac{v^2}{r}\right),$$

and this is accordingly the force with which the surface of the earth at the equator pushes a body lying upon it. The push, of course, is mutual, and this formula therefore gives the apparent weight or apparent gravitating force of a body at the equator, mg denoting its true gravitating force (due to attraction alone). A body falling in vacuo at the equator has an acceleration 978·10 relative to the surface of the earth in its neighbourhood; but this portion of the surface has itself an acceleration of 3·39, directed towards the earth's centre, and therefore in the same direction as the acceleration of the body. The absolute acceleration of the body is therefore the sum of these two, that is 981·49, which is accordingly the intensity of true gravity at the equator.

The apparent weight of bodies at the equator would be *nil* if $\frac{v^2}{r}$ were equal to g. Dividing 3·39 into 981·49, the quotient is approximately 289, which is $(17)^2$. Hence this state of things would exist if the velocity of rotation were about 17 times as fast as at present.

Since the movements and forces which we actually observe depend upon *relative* acceleration, it is usual to understand, by the value of g or the intensity of gravity at a place, the *apparent* values, unless the contrary be expressed. Thus the value of g at the equator is usually stated to be 978·10.

105. Direction of Apparent Gravity.—The total amount of centrifugal force at different places on the earth's surface, varies directly as their distance from the earth's axis; for this is the value of r in the formula (5) of § 101, and the value of T in that formula is the same for the whole earth. The direction of this force, being perpendicular to the earth's axis, is not vertical except at the equator; and hence, when we compound it with the force of true gravity, we obtain a resultant force of apparent gravity differing in direction as well as in magnitude from true gravity. What is always understood by a *vertical*, is the direction of *apparent* gravity; and a plane perpendicular to it is what is meant by a horizontal plane.