from the other. The pendulum is swung first upon one of these edges and then upon the other, and, if any difference is detected in the times of vibration, it is corrected by moving the adjustable edge. When the difference has been completely destroyed, the distance between the two edges is the length of the equivalent simple pendulum. It is necessary, in any arrangement of this kind, that the two knife-edges should be in a plane passing through the centre of gravity; also that they should be on opposite sides of the centre of gravity, and at unequal distances from it.

120. Determination of the Value of *g*.—Returning to the formula for the simple pendulum $T=\pi\sqrt{\frac{l}{g}}$, we easily deduce from it $g=\frac{\pi^2 l}{T^2}$, whence it follows that the value of g can be determined by making a pendulum vibrate and measuring T and l. T is determined by counting the number of vibrations that take place in a given time; l can be calculated, when the pendulum is of regular form, by the aid of formulæ which are given in treatises on rigid dynamics, but its value is more easily obtained by Kater's method, described above, founded on the principle of the convertibility of the centres of suspension and oscillation.

It is from pendulum observations, taken in great numbers at different parts of the earth, that the approximate formula for the intensity of gravity which we have given at § 91 has been deduced. Local peculiarities prevent the possibility of laying down any general formula with precision; and the exact value of g for any place can only be ascertained by observations on the spot.

CHAPTER IX.

CONSERVATION OF ENERGY.

121. Definition of Kinetic Energy.—We have seen in § 93 that the work which must be done upon a mass of m grammes to give it a velocity of v centimetres per second is $\frac{1}{2}mv^2$ ergs. Though we have proved this only for the case of falling bodies, with gravity as the working force, the result is true universally, as is shown in advanced treatises on mathematical physics. It is true whether the motion be rectilinear or curvilinear, and whether the working force act in the line of motion or at an angle with it.

If the velocity of a mass increases from v_1 to v_2, the work done upon it in the interval is $\frac{1}{2}m\ (v_2^2-v_1^2)$; in other words, is the increase of $\frac{1}{2}mv^2$.

On the other hand, if a force acts in such a manner as to oppose the motion of a moving mass, the force will do negative work, the amount of which will be equal to the decrease in the value of $\frac{1}{2}mv^2$.

For example, during any portion of the ascent of a projectile, the diminution in the value of $\frac{1}{2}mv^2$ is equal to gm multiplied by the increase of height; and during any portion of its descent the increase in $\frac{1}{2}mv^2$ is equal to gm multiplied by the decrease of height.

The work which must have been done upon a body to give it its actual motion, supposing it to have been initially at rest, is called the *energy of motion* or the *kinetic energy* of the body. It can be computed by multiplying *half the mass by the square of the velocity.*

122. Definition of Static or Potential Energy.—When a body of mass m is at a height s above the ground, which we will suppose level, gravity is ready to do the amount of work gms upon it by making it fall to the ground. A body in an elevated position may therefore be regarded as a reservoir of work. In like manner a wound-up clock, whether driven by weights or by a spring, has work stored up in it. In all these cases there is force between parts of a system tending to produce relative motion, and there is room for such relative motion to take place. There is force ready to act, and space for it to act through. Also the force is always the same in the same relative position of the parts. Such a system possesses energy, which is usually called *potential*. We prefer to call it *statical*, inasmuch as its amount is computed on statical principles alone.[1] Statical energy depends jointly on mutual force and relative position. Its amount in any given position is the amount of work which would be done by the forces of the system in passing from this position to the standard position. When we are speaking of the energy of a heavy body in an elevated position above level ground, we naturally adopt as the standard position that in which the body is lying on the ground. When we speak of the energy of a wound-up clock, we adopt as the standard position that in which the clock has completely run down. Even when the standard position is not indicated, we can still speak definitely of the difference between the energies of two given positions of a system; just as we can speak definitely of the difference of level of two given points without any agreement as to the datum from which levels are to be reckoned.

123. Conservation of Mechanical Energy.—When a frictionless system is so constituted that its forces are always the same in the same positions of the system, the amount of work done by these forces during the passage from one position A to another position B will be independent of the path pursued, and will be equal to *minus* the work done by them in the passage from B to A. The earth and any heavy body at its surface constitute such a system; the force of the system is the mutual gravitation of these two bodies; and the work done by this mutual gravitation, when the body is moved by any path from a point A to a point B, is equal to the weight of the body multiplied by the height of A above B. When the system passes through any series of movements beginning with a given position and ending with the same position again, the algebraic total of work done by the forces of the system in this series of movements is zero. For instance, if a heavy body be carried by a roundabout path back to the point from whence it started, no work is done upon it by gravity upon the whole.

Every position of such a system has therefore a definite amount

[1] That is to say, the computation involves no reference to the laws of motion.

of statical energy, reckoned with respect to an arbitrary standard position. The work done by the forces of the system in passing from one position to another is (by definition) equal to the loss of static energy; but this loss is made up by an equal gain of kinetic energy. Conversely if kinetic energy is lost in passing from one position to another, the forces do negative work equal to this loss, and an equal amount of static energy is gained. The total energy of the system (including both static and kinetic) therefore remains unaltered.

An approximation to such a state of things is exhibited by a pendulum. In the two extreme positions it is at rest, and has therefore no kinetic energy; but its statical energy is then a maximum. In the lowest position its motion is most rapid; its kinetic energy is therefore a maximum, but its statical energy is zero. The difference of the statical energies of any two positions, will be the weight of the pendulum multiplied by the difference of levels of its centre of gravity, and this will also be the difference (in inverse order) between the kinetic energies of the pendulum in these two positions.

As the pendulum is continually setting the air in motion and thus doing external work, it gradually loses energy and at last comes to rest, unless it be supplied with energy from a clock or some other source. If a pendulum could be swung in a perfect vacuum, with an entire absence of friction, it would lose no energy, and would vibrate for an indefinite time without decrease of amplitude.

124. Illustration from Pile-driving.—An excellent illustration of transformations of energy is furnished by pile-driving. A large mass of iron called a *ram* is slowly hauled up to a height of several yards above the pile, and is then allowed to fall upon it. During the ascent, work must be supplied to overcome the force of gravity; and this work is represented by the statical energy of the ram in its highest position. While falling, it continually loses statical and gains kinetic energy; the amount of the latter which it possesses immediately before the blow being equal to the work which has been done in raising it. The effect of the blow is to drive the pile through a small distance against a resistance very much greater than the weight of the ram; the work thus done being nearly equal to the total energy which the ram possessed at any point of its descent. We say *nearly* equal, because a portion of the energy of the blow is spent in producing vibrations.

125. Hindrances to Availability of Energy.—There is almost always some waste in utilizing energy. When water turns a mill-wheel, it runs away from the wheel with a velocity, the square of which multiplied by half the mass of the water represents energy which has run to waste.

Friction again often consumes a large amount of energy; and in this case we cannot (as in the preceding one) point to any palpable motion of a mass as representing the loss. Heat, however, is produced, and the energy which has disappeared as regarded from a gross mechanical point of view, has taken a molecular form. Heat is a form of molecular energy; and we know, from modern researches, what quantity of heat is equivalent to a given amount of mechanical work. In the steam-engine we have the converse process; mechanical work is done by means of heat, and heat is destroyed in the doing of it, so that the amount of heat given out by the engine is less than the amount supplied to it.

The sciences of electricity and magnetism reveal the existence of other forms of molecular energy; and it is possible in many ways to produce one form of energy at the expense of another; but in every case there is an exact equivalence between the quantity of one kind which comes into existence and the quantity of another kind which simultaneously disappears. Hence the problem of constructing a self-driven engine, which we have seen to be impossible in mechanics, is equally impossible when molecular forms of energy are called to the inventor's aid.

Energy may be transformed, and may be communicated from one system to another; but it cannot be increased or diminished in total amount. This great natural law is called the *principle of the conservation of energy*.

CHAPTER X.

ELASTICITY.

126. Elasticity and its Limits.—There is no such thing in nature as an absolutely rigid body. All bodies yield more or less to the action of force; and the property in virtue of which they tend to recover their original form and dimensions when these are forcibly changed, is called *elasticity*. Most solid bodies possess almost perfect elasticity for small deformations; that is to say, when distorted, extended, or compressed, within certain small limits, they will, on the removal of the constraint to which they have been subjected, instantly regain almost completely their original form and dimensions. These limits (which are called the limits of elasticity) are different for different substances; and when a body is distorted beyond these limits, it takes a *set*, the form to which it returns being intermediate between its original form and that into which it was distorted.

When a body is distorted within the limits of its elasticity, the force with which it reacts is directly proportional to the amount of distortion. For example, the force required to make the prongs of a tuning-fork approach each other by a tenth of an inch, is double of that required to produce an approach of a twentieth of an inch; and if a chain is lengthened a twentieth of an inch by a weight of 1 cwt., it will be lengthened a tenth of an inch by a weight of 2 cwt., the chain being supposed to be strong enough to experience no permanent set from this greater weight. Also, within the limits of elasticity, equal and opposite distortions, if small, are resisted by equal reactions. For example, the same force which suffices to make the prongs of a tuning-fork approach by a twentieth of an inch, will, if applied in the opposite direction, make them separate by the same amount.

127. Isochronism of Small Vibrations.—An important consequence of these laws is, that when a body receives a slight distortion within the limits of its elasticity, the vibrations which ensue when the constraint is removed are isochronous. This follows from § 111, inasmuch as the accelerations are proportional to the forces, and are therefore proportional at each instant to the deformation at that instant.

128. Stress, Strain, and Coefficients of Elasticity.—A body which, like indian-rubber, can be subjected to large deformations without receiving a permanent set, is said to have wide limits of elasticity.

A body which, like steel, opposes great resistance to deformation, is said to have large coefficients of elasticity.

Any change in the shape or size of a body produced by the application of force to the body is called a *strain;* and an action of force tending to produce a strain is called a *stress.*

When a wire of cross-section A is stretched with a force F, the longitudinal stress is $\frac{F}{A}$; this being the intensity of force per unit area with which the two portions of the wire separated by any cross-section are pulling each other. If the length of the wire when unstressed is L and when stressed $L+l$, the longitudinal strain is $\frac{l}{L}$. A stress is always expressed in units of force per unit of area. A strain is always expressed as the ratio of two magnitudes of the same kind (in the above example, two lengths), and is therefore independent of the units employed.

The quotient of a stress by the strain (of a given kind) which it produces, is called a *coefficient* or *modulus of elasticity.* In the above example, the quotient $\frac{FL}{Al}$ is called *Young's modulus* of elasticity.

As the wire, while it extends lengthwise, contracts laterally, there will be another coefficient of elasticity obtained by dividing the longitudinal stress by the lateral strain.

It is shown, in special treatises, that a solid substance may have 21 independent coefficients of elasticity; but that when the substance is *isotropic,* that is, has the same properties in all directions, the number reduces to 2.

129. Volume-elasticity.—The only coefficient of elasticity possessed by liquids and gases is elasticity of volume. When a body of volume V is reduced by the application of uniform normal pressure over its whole surface to volume $V-v$, the volume-strain is $\frac{v}{V}$, and if this

effect is produced by a pressure of p units of force per unit of area, the elasticity of volume is the quotient of the stress p by the strain $\frac{v}{V}$, or is $\frac{pV}{v}$. This is also called the *resistance to compression;* and its reciprocal $\frac{v}{pV}$ is called the *compressibility* of the substance. In dealing with gases, p must be understood as a pressure superadded to the original pressure of the gas.

Since a strain is a mere numerical quantity, independent of units, a coefficient of elasticity must be expressed, like a stress, in units of force per unit of area. In the C.G.S. system, stresses and coefficients of elasticity are expressed in dynes per square centimetre. The following are approximate values (thus expressed) of the two coefficients of elasticity above defined:—

	Young's Modulus.	Elasticity of Volume.
Glass (flint),	60×10^{10}	40×10^{10}
Steel,	210×10^{10}	180×10^{10}
Iron (wrought),	190×10^{10}	140×10^{10}
Iron (cast),	130×10^{10}	96×10^{10}
Copper,	120×10^{10}	160×10^{10}
Mercury,		54×10^{10}
Water,		2×10^{10}
Alcohol,		$1{\cdot}2 \times 10^{10}$

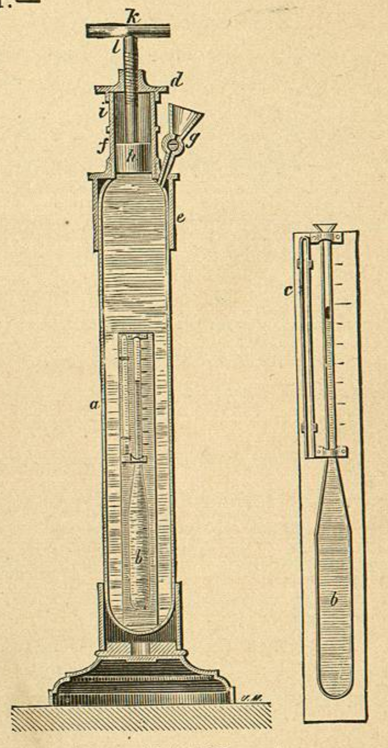
Fig. 49.—Œrsted's Piezometer.

130. Œrsted's Piezometer.—The compression of liquids has been observed by means of Œrsted's piezometer, which is represented in Fig. 49. The liquid whose compression is to be observed is contained in a glass vessel b, resembling a thermometer with a very large bulb and short tube. The tube is open above, and a globule of mercury at the top of the liquid column serves as an index. This apparatus is placed in a very strong glass vessel a full of water. When pressure is exerted by means of the piston klh, the index of mercury is seen to descend, showing a diminution of volume of the liquid, and showing moreover that this diminution of volume exceeds that of the containing vessel b. It might at first sight appear that since this vessel is subjected to equal pressure within and without, its volume is unchanged; but in fact, its volume is altered to the same extent as that of a solid vessel of the same material; for the interior shells would react with a force precisely equivalent to that which is exerted by the contained liquid.

CHAPTER XI.

FRICTION.

131. Friction, Kinetical and Statical.—When two bodies are pressed together in such a manner that the direction of their mutual pressure is not normal to the surface of contact, the pressure can be resolved into two parts, one normal and the other tangential. The tangential component is called the *force of friction* between the two bodies. The friction is called *kinetical* or *statical* according as the bodies are or are not sliding one upon the other.

As regards kinetical friction, experiment shows that if the normal pressure between two given surfaces be changed, the tangential force changes almost exactly in the same proportion; in other words, the ratio of the force of friction to the normal pressure is nearly constant for two given surfaces. This ratio is called the *coefficient of kinetical friction* between the two surfaces, and is nearly independent of the velocity.

132. Statical Friction. Limiting Angle.—It is obvious that the statical friction between two given surfaces is zero when their mutual pressure is normal, and increases with the obliquity of the pressure if the normal component be preserved constant. The obliquity, however, cannot increase beyond a certain limit, depending on the nature of the bodies, and seldom amounting to so much as 45°. Beyond this limit sliding takes place. The limiting obliquity, that is, the greatest angle that the mutual force can make with the normal, is called the *limiting angle of friction* for the two surfaces; and the ratio of the tangential to the normal component when the mutual force acts at the limiting angle, is called the *coefficient of statical friction* for the two surfaces. The coefficient and limiting angle remain nearly constant when the normal force is varied.

The coefficient of statical friction is in almost every case greater

6

than the coefficient of kinetical friction; in other words, friction offers more resistance to the commencement of sliding than to the continuance of it.

A body which has small coefficients of friction with other bodies is called slippery.

133. Coefficient = tan θ. Inclined Plane.—If θ be the inclination of the mutual force P to the common normal, the tangential component will be P sin θ, the normal component P cos θ, and the ratio of the former to the latter will be tan θ. Hence *the coefficient of statical friction is equal to the tangent of the limiting angle of friction.*

When a heavy body rests on an inclined plane, the mutual pressure is vertical, and the angle θ is the same as the inclination of the plane. Hence if an inclined plane is gradually tilted till a body lying on it slides under the action of gravity, the inclination of the plane at which sliding begins is the limiting angle of friction between the body and the plane, and the tangent of this angle is the coefficient of statical friction.

Again, if the inclination of a plane be such that the motion of a body sliding down it under the action of gravity is neither accelerated nor retarded, the tangent of this inclination will be the coefficient of kinetical friction.

CHAPTER XII.

HYDROSTATICS.

134. Hydrodynamics.—We shall now treat of the laws of force as applied to fluids. This branch of the general science of dynamics is called *hydrodynamics* (ὕδωρ, water), and is divided into *hydrostatics* and *hydrokinetics*. Our discussions will be almost entirely confined to hydrostatics.

FLUIDS.—TRANSMISSION OF PRESSURE.

The name *fluid* comprehends both liquids and gases.

135. No Statical Friction in Fluids.—A fluid at rest cannot exert any tangential force against a surface in contact with it; its pressure at every point of such a surface is entirely normal. A slight tangential force is exerted by fluids in motion; and this fact is expressed by saying that all fluids are more or less *viscous*. An imaginary perfect fluid would be perfectly free from viscosity; its pressure against any surface would be entirely normal, whether the fluid were in motion or at rest.

136. Intensity of Pressure.—When pressure is uniform over an area, the total amount of the pressure, divided by the area, is called the *intensity of the pressure*. The C.G.S. unit of intensity of pressure is a pressure of a *dyne on each square centimetre* of surface. A rough unit of intensity frequently used is the pressure of a pound per square inch. This unit varies with the intensity of gravity, and has an average value of about 69,000 C.G.S. units. Another rough unit of intensity of pressure frequently employed is "an atmosphere"—that is to say, the average intensity of pressure of the atmosphere at the surface of the earth. This is about 1,000,000 C.G.S. units.