than the coefficient of kinetical friction; in other words, friction offers more resistance to the commencement of sliding than to the continuance of it.

A body which has small coefficients of friction with other bodies is called slippery.

133. Coefficient = tan θ. Inclined Plane.—If θ be the inclination of the mutual force P to the common normal, the tangential component will be P sin θ, the normal component P cos θ, and the ratio of the former to the latter will be tan θ. Hence *the coefficient of statical friction is equal to the tangent of the limiting angle of friction.*

When a heavy body rests on an inclined plane, the mutual pressure is vertical, and the angle θ is the same as the inclination of the plane. Hence if an inclined plane is gradually tilted till a body lying on it slides under the action of gravity, the inclination of the plane at which sliding begins is the limiting angle of friction between the body and the plane, and the tangent of this angle is the coefficient of statical friction.

Again, if the inclination of a plane be such that the motion of a body sliding down it under the action of gravity is neither accelerated nor retarded, the tangent of this inclination will be the coefficient of kinetical friction.

CHAPTER XII.

HYDROSTATICS.

134. Hydrodynamics.—We shall now treat of the laws of force as applied to fluids. This branch of the general science of dynamics is called *hydrodynamics* (ὕδωρ, water), and is divided into *hydrostatics* and *hydrokinetics*. Our discussions will be almost entirely confined to hydrostatics.

FLUIDS.—TRANSMISSION OF PRESSURE.

The name *fluid* comprehends both liquids and gases.

135. No Statical Friction in Fluids.—A fluid at rest cannot exert any tangential force against a surface in contact with it; its pressure at every point of such a surface is entirely normal. A slight tangential force is exerted by fluids in motion; and this fact is expressed by saying that all fluids are more or less *viscous*. An imaginary perfect fluid would be perfectly free from viscosity; its pressure against any surface would be entirely normal, whether the fluid were in motion or at rest.

136. Intensity of Pressure.—When pressure is uniform over an area, the total amount of the pressure, divided by the area, is called the *intensity of the pressure*. The C.G.S. unit of intensity of pressure is a pressure of a *dyne on each square centimetre* of surface. A rough unit of intensity frequently used is the pressure of a pound per square inch. This unit varies with the intensity of gravity, and has an average value of about 69,000 C.G.S. units. Another rough unit of intensity of pressure frequently employed is "an atmosphere"—that is to say, the average intensity of pressure of the atmosphere at the surface of the earth. This is about 1,000,000 C.G.S. units.

The single word "pressure" is used sometimes to denote "amount of pressure" (which can be expressed in dynes) and sometimes "intensity of pressure" (which can be expressed in dynes per square centimetre). The context usually serves to show which of these two meanings is intended.

137. Pressure the Same in all Directions.—The intensity of pressure at any point of a fluid is the same in all directions; it is the same whether the surface which receives the pressure faces upwards, downwards, horizontally, or obliquely.

This equality is a direct consequence of the absence of tangential force between two contiguous portions of a fluid.

For in order that a small triangular prism of the fluid (its ends being right sections) may be in equilibrium, the pressures on its three faces must balance each other. But when three forces balance each other, they are proportional to the sides of a triangle to which they are perpendicular;[1] hence the *amounts* of pressure on the three faces are proportional to the faces, in other words the *intensities* of these three pressures are equal. As we can take two of the faces perpendicular to any two given directions, this proves that the pressures in all directions at a point are of equal intensity.

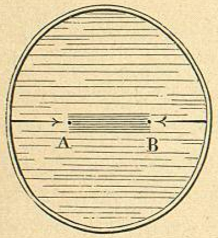

Fig. 50.

138. Pressure the Same at the Same Level.—In a fluid at rest, the pressure is the same at all points in the same horizontal plane. This appears from considering the equilibrium of a horizontal cylinder AB (Fig. 50), of small sectional area, its ends being right sections. The pressures on the sides are normal, and therefore give no component in the direction of the length; hence the pressures on the ends must be equal in amount; but they act on equal areas; therefore their intensities are equal.

A horizontal surface in a liquid at rest may therefore be called a "surface of equal pressure."

139. Difference of Pressure at Different Levels.—The increase of pressure with depth, in a fluid of uniform density, can be investigated as follows:—Consider the equilibrium of a vertical cylinder mm' (Fig. 51), its ends being right sections. The pressures on its

[1] This is an obvious consequence of the triangle of forces (art. 14); for if the sides of a triangle are parallel to three forces, we have only to turn the triangle through a right angle, and its sides will then be perpendicular to the forces.

sides are normal, and therefore horizontal. The only vertical forces acting upon it are its own weight and the pressures on its ends, of which it is to be observed that the pressure on the upper end acts downwards and that on the lower end upwards. The pressure on the lower end therefore exceeds that on the upper end by an amount equal to the weight of the cylinder. If a be the sectional area, w the weight of unit volume of the liquid, and h the length of the cylinder, the volume of the cylinder is ha, and its weight wha, which must be equal to $(p-p')\,a$ if p, p' are the intensities of pressure on the lower and upper ends respectively. We have therefore

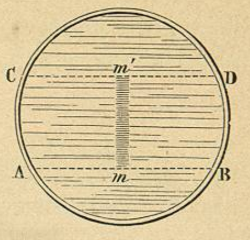

Fig. 51.

$$p-p'=wh,$$

that is, *the increase of pressure in descending through a depth h is wh.*

The principles of this and the preceding section remain applicable whatever be the shape of the containing vessel, even if it be such as to render a circuitous route necessary in passing from one of two points compared to the other; for this route can always be made to consist of a succession of vertical and horizontal lines, and the preceding principles when applied to each of these lines separately, will give as the final result a difference of pressure wh for a difference of heights h.

If d denote the density of the liquid, in grammes per cub. cm., the weight of a cubic cm. will be gd dynes. The increase of pressure for an increase of depth h cm. is therefore ghd dynes per sq. cm. If there be no pressure at the surface of the liquid, this will be the actual pressure at the depth h.

140. Free Surface.—It follows from these principles that the free surface of a liquid at rest—that is, the surface in contact with the atmosphere—must be horizontal; since all points in this surface are at the same pressure. If the surface were not horizontal, but were higher at n than at n' (Fig. 52), the pressures at the two points m, m' vertically beneath them in any horizontal plane AB would be unequal, for they would be due to the weights

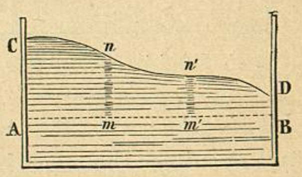

Fig. 52.

of unequal columns nm, $n'm'$, and motion would ensue from m towards m'.

The same conclusion can be deduced from considering the equilibrium of a particle at the surface, as M (Fig. 53). If the tangent plane at M were not horizontal there would be a component of gravity tending to make the particle slide down; and this tendency would produce motion, since there is no friction to oppose it.

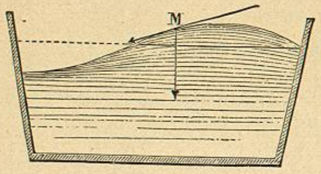

Fig. 53.

141. Transmissibility of Pressure in Fluids.—Since the difference of the pressures at two points in a fluid can be determined by the foregoing principles, independently of any knowledge of the absolute intensity of either, it follows that when increase or diminution of pressure occurs at one point, an equal increase or diminution must occur throughout the whole fluid. *A fluid in a closed vessel perfectly transmits through its whole substance whatever pressure we apply to any part.* The changes in *amount* of pressure will be equal for all equal areas. For unequal areas they will be proportional to the areas.

Thus if the two vertical tubes in Fig. 54 have sectional areas which are as 1 to 16, a weight of 1 kilogram acting on the surface of the liquid in the smaller tube will be balanced by 16 kilograms acting on the surface of the liquid in the larger.

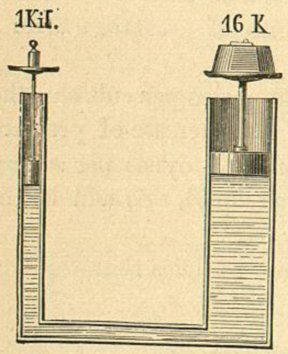

Fig. 54.—Principle of the Hydraulic Press.

This principle of the perfect transmission of pressure by fluids appears to have been first discovered and published by Stevinus; but it was rediscovered by Pascal a few years later, and having been made generally known by his writings is often called "Pascal's principle." In his celebrated treatise on the *Equilibrium of Liquids*, he says, "If a vessel full of water, closed on all sides, has two openings, the one a hundred times as large as the other, and if each be supplied with a piston which fits exactly, a man pushing the small piston will exert a force which will equilibrate that of a hundred men pushing the piston which is a hundred times as large, and will overcome that of ninety-nine. And whatever may be the proportion of these openings, if the forces applied to the pistons are to each other as the openings, they will be in equilibrium."

142. Hydraulic Press.—This mode of multiplying force remained for a long time practically unavailable on account of the difficulty of making the pistons water-tight. The hydraulic press was first successfully made by Bramah, who invented the *cupped leather collar* illustrated in Fig. 166, § 264. Fig. 165 shows the arrangements of the press as a whole. Instead of pistons, *plungers* are employed; that is to say, solid cylinders of metal which can be pushed down into the liquid, or can be pushed up by the pressure of the liquid against their bases. The volume of liquid displaced by the advance of a plunger is evidently equal to that displaced by a piston of the same sectional area, and the above calculations for pistons apply to plungers as well. The plungers work through openings which are kept practically water-tight by means of the cup-leather arrangement. The cup-leather, which is shown both in plan and section in Fig. 166, consists of a leather ring bent so as to have a semicircular section. It is fitted into a hollow in the interior of the sides of the opening, so that water leaking up along the circumference of the plunger will fill the concavity of the leather, and, by pressing on it, will produce a packing which fits more tightly as the pressure on the plunger increases.

143. Principle of Work Applicable.—In Fig. 54, when the smaller piston advances and forces the other back, the volume of liquid driven out of the smaller tube is equal to the sectional area multiplied by the distance through which the piston advances. In like manner, the volume of liquid driven into the larger tube is equal to its sectional area multiplied by the distance that its piston is forced back. But these two volumes are equal, since the same volume of liquid that leaves one tube enters the other. The distances through which the two pistons move are therefore inversely as their sectional areas, and hence are inversely as the amounts of pressure applied to them. The *work done* in pushing forward the smaller piston is therefore equal to the work done by the liquid in pushing back the larger. This was remarked by Pascal, who says—

"It is, besides, worthy of admiration that in this new machine we find that constant rule which is met with in all the old ones such as the lever, wheel and axle, screw, &c., which is that the distance is increased in proportion to the force; for it is evident that as one of these openings is a hundred times as large as the other, if the man who pushes the small piston drives it forward one inch, he will drive the large piston backward only one-hundredth part of that length."

144. Experiment on Upward Pressure.—The upward pressure exerted by a liquid against a horizontal surface facing downwards can be exhibited by the following experiment. Take a tube open at both ends (Fig. 55), and keeping the lower end covered with a piece of card, plunge it into water. The liquid will press the card against the bottom of the tube with a force which increases as it is plunged deeper. If water be now poured into the tube, the card will remain in its place as long as the level of the liquid is lower within the tube than without; but at the moment when equality of levels is attained it will become detached.

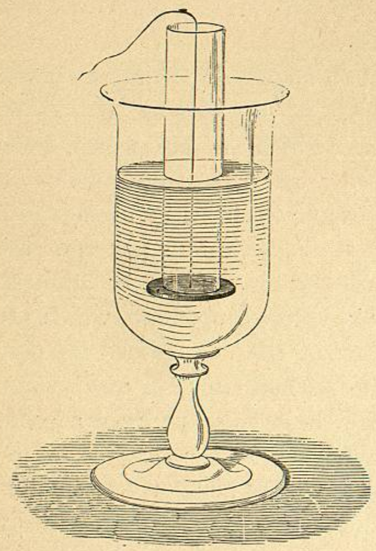

Fig. 55.—Upward Pressure.

145. Liquids in Superposition.—When one liquid rests on the top of another of different density, the foregoing principles lead to the result that the surface of demarcation must be horizontal. For the free surface of the upper liquid must, as we have seen, be horizontal. If now we take two small equal areas n and n' (Fig. 56) in a horizontal layer of the lower liquid, they must be subjected to equal pressures. But these pressures are measured by the weights of the liquid cylinders nrs, $n'tl$; and these latter cannot be equal unless the points r and t at the junction of the two liquids are at the same level. All points in the surface of demarcation are therefore in the same horizontal plane.

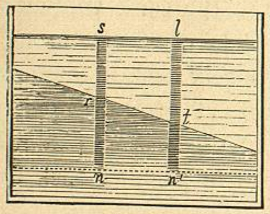

Fig. 56.

The same reasoning can be extended downwards to any number of liquids of unequal densities, which rest one upon another, and shows that all the surfaces of demarcation between them must be horizontal.

An experiment in illustration of this result is represented in Fig. 57. Mercury, water, and oil are poured into a glass jar. The mercury, being the heaviest, goes to the bottom; the oil, being the lightest, floats at the top; and the surfaces of contact of the liquids are seen to be horizontal.

Even when liquids are employed which gradually mix with one another, as water and alcohol, or fresh water and salt water, so that there is no definite surface of demarcation, but a gradual increase of density with depth, it still remains true that the density at all points in a horizontal plane is the same.

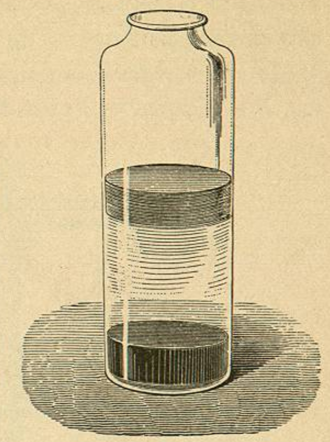

Fig. 57.
Phial of the Four Elements.

146. Two Liquids in Bent Tube.—If we pour mercury into a bent tube open at both ends (Fig. 58), and then pour water into one of the arms, the heights of the two liquids above the surface of junction will be very unequal, as shown in the figure. The general rule for the equilibrium of any two liquids in these circumstances is that *their heights above the surface of junction must be inversely as their densities*, since they correspond to equal pressures.

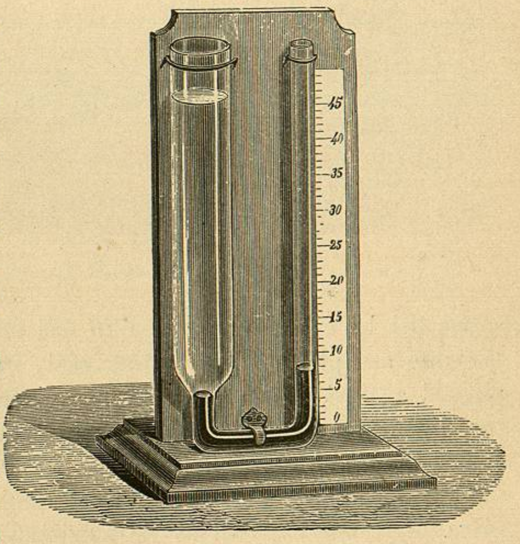

Fig. 58.—Equilibrium of Two Fluids in Communicating Vessels.

147. Experiment of Pascal's Vases.—Since the amount of pressure on a horizontal area A at the depth h in a liquid is whA, where w denotes the weight of unit volume of the liquid, it follows that the pressure on the bottom of a vessel containing liquid is not affected by the breadth or narrowness of the upper part of the