vessel, provided the height of the free surface of the liquid be given. Pascal verified this fact by an experiment which is frequently exhibited in courses of physics. The apparatus employed (Fig. 59) is a tripod supporting a ring, into which can be screwed three vessels of different shapes, one widened upwards, another cylindrical, and the third tapering upwards. Beneath the ring is a movable disc

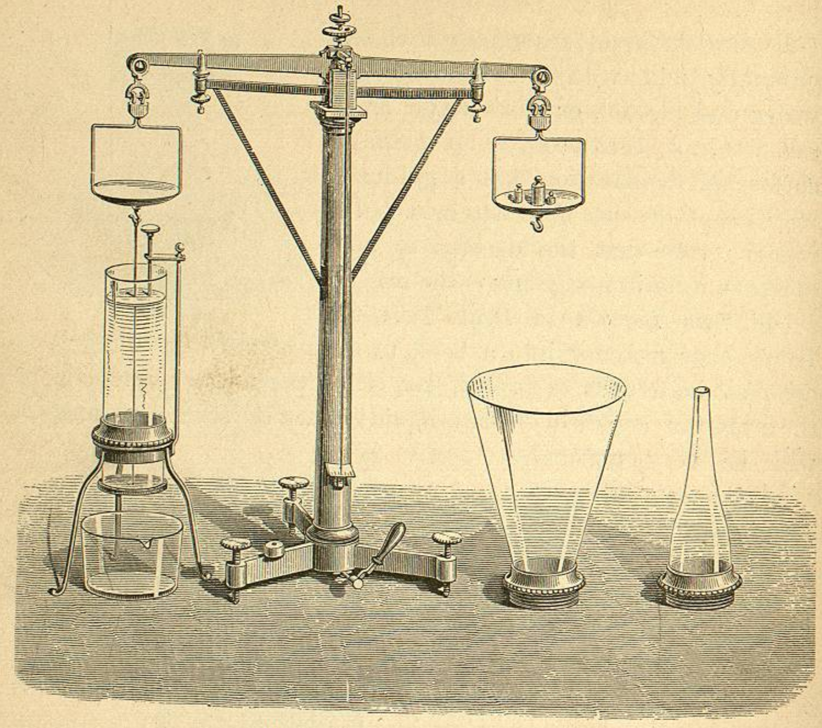

Fig. 59.—Experiment of Pascal's Vases.

supported by a string attached to one of the scales of a balance. Weights are placed in the other scale in order to keep the disc pressed against the ring. Let the cylindrical vase be mounted on the tripod, and filled up with water to such a level that the pressure is just sufficient to detach the disc from the ring. An indicator, shown in the figure, is used to mark the level at which this takes place. Let the experiment be now repeated with the two other vases, and the disc will be detached when the water has reached the same level as before.

In the case of the cylindrical vessel, the pressure on the bottom is evidently equal to the weight of the liquid. Hence in all three

cases the pressure on the bottom of the vessel is equal to the weight of a cylindrical column of the liquid, having the bottom as its base, and having the same height as the liquid in the vessel.

148. Resultant Pressure on Vessel.—The pressure exerted by the bottom of the vessel upon the stand on which it rests, consists of the weight of the vessel itself, together with the resultant pressure of the contained liquid against it. The actual pressure of the liquid against any portion of the vessel is normal to this portion, and if we resolve it into two components, one vertical and the other horizontal, only the vertical component need be attended to, in computing the resultant; for the horizontal components will always destroy one another. At such points as n, n' (Fig. 60) the vertical component is downwards; at s and s' it is upwards; at r and r' there is no vertical component; and at AB the whole pressure is vertical. It can be demonstrated mathematically that the resultant pressure is always equal to the total weight of the contained liquid; a conclusion which can also be deduced from the consideration that the pressure exerted by the vessel upon the stand on which it rests must be equal to its own weight together with that of its contents.

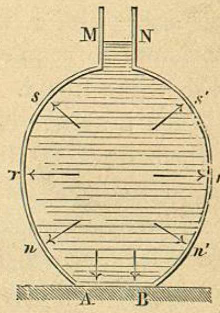

Fig. 60.—Total Pressure.

Some cases in which the proof above indicated becomes especially obvious, are represented in Fig. 61. In the cylindrical vessel ABDC, it is evident that the only pressure transmitted to the stand is that exerted upon the bottom, which is equal to the weight of the liquid. In the case of the vessel which is wider at the top, the stand is subjected to the weight of the liquid column ABSK, which presses on the bottom AB, together with the columns GHKC, RLDS, pressing on GH and RL; the sum of which weights composes the total weight of liquid contained in the vessel. Finally, in the third case, the pressure on the bottom AB, which is equal to the weight of a liquid column ABSK, must be diminished by the

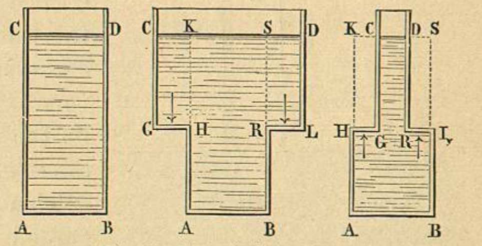

Fig. 61.—Hydrostatic Paradox.

upward pressures on HG and RL. These last being represented by liquid columns HGCK, RLSD, there is only left to be transmitted to the stand a pressure equal to the weight of the water in the vessel.

149. Back Pressure in Discharging Vessel.—The same analysis which shows that the resultant vertical pressure of a liquid against the containing vessel is equal to the weight of the liquid, shows also that the horizontal components of the pressures destroy one another. This conclusion is in accordance with everyday experience. However susceptible a vessel may be of horizontal displacement, it is not found to acquire any tendency to horizontal motion by being filled with a liquid.

When a system of forces are in equilibrium, the removal of one of them destroys the equilibrium, and causes the resultant of the system to be a force equal and opposite to the force removed. Accordingly if we remove an element of one side of the containing vessel, leaving a hole through which the liquid can flow out, the remaining pressure against this side will be insufficient to preserve equilibrium, and there will be an excess of pressure in the opposite direction.

This conclusion can be directly verified by the experiment represented in Fig. 62. A tall floating vessel of water is fitted with a horizontal discharge-pipe on one side near its base. The vessel is to be filled with water, and the discharge-pipe opened while the vessel is at rest. As the water flows out, the vessel will be observed to acquire a velocity, at first very slow, but continually increasing, in the opposite direction to that of the issuing stream.

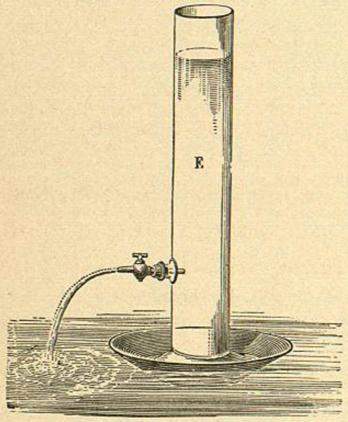

Fig. 62.—Backward Movement of Discharging Vessel.

This experiment may also be regarded as an illustration of the law of action and reaction, which asserts that momentum cannot be imparted to any body without equal and opposite momentum being imparted to some other body. The water in escaping from the vessel acquires horizontal momentum in one direction, and the vessel with its remaining contents acquires horizontal momentum in the opposite direction.

The movements of the vessel in this experiment are slow. More marked effects of the same kind can be obtained by means of the hydraulic tourniquet (Fig. 63), which when made on a larger scale is called Barker's mill. It consists of a vessel of water free to rotate about a vertical axis, and having at its lower end bent arms through which the water is discharged horizontally, the direction of discharge being nearly at right angles to a line joining the discharging orifice to the axis. The unbalanced pressures at the bends of the tube, opposite to the openings, cause the apparatus to revolve in the opposite direction to the issuing liquid.

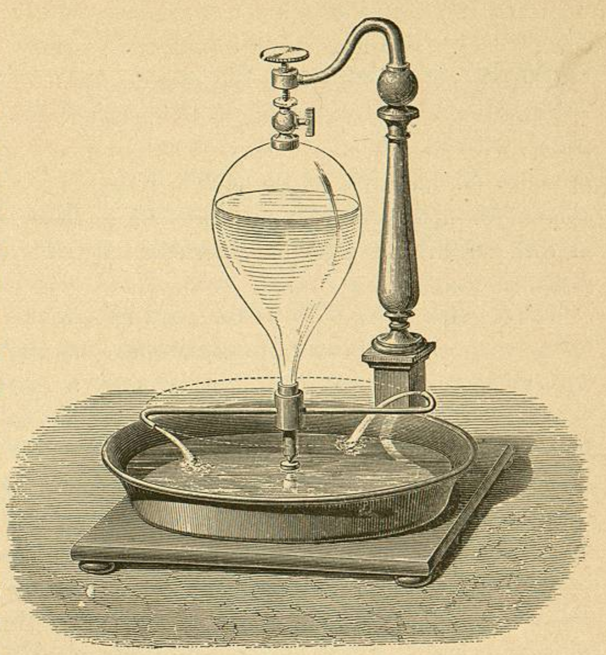

Fig. 63.—Hydraulic Tourniquet.

150. Total and Resultant Pressures. Centre of Pressure.—The intensity of pressure on an area which is not horizontal is greatest on those parts which are deepest, and the average intensity can be shown to be equal to the actual intensity at the centre of gravity of the area. Hence if A denote the area, h the depth of its centre of gravity, and w the weight of unit volume of the liquid, the total pressure will be $w\,Ah$. Strictly speaking, this is the pressure due to the weight of the liquid, the transmitted atmospheric pressure being left out of account.

In attaching numerical values to w, A, and h, the same unit of length must be used throughout. For example, if h be expressed in feet, A must be expressed in square feet, and w must stand for the weight of a cubic foot of the liquid.

When we employ the centimetre as the unit of length, the value of w will be sensibly 1 gramme if the liquid be water, so that the amount of pressure in grammes will be simply the product of the depth of the centre of gravity in centimetres by the area in square centimetres. For any other liquid, the pressure will be found by multiplying this product by the specific gravity of the liquid.

These rules for computing total pressure hold for areas of all forms, whether plane or curved; but the investigation of the total pressure on an area which is not plane is a mere mathematical exercise of no practical importance; for as the elementary pressures in this case are not parallel, their sum (which is the total pressure) is not the same thing as their resultant.

For a plane area, in whatever position, the elementary pressures, being everywhere normal to its plane, are parallel and give a resultant equal to their sum; and it is often a matter of interest to determine that point in the area through which the resultant passes. This point is called the *Centre of Pressure.* It is not coincident with the centre of gravity of the area unless the pressure be of equal intensity over the whole area. When the area is not horizontal, the pressure is most intense at those parts of it which are deepest, and the centre of pressure is accordingly lower down than the centre of gravity. For a horizontal area the two centres are coincident, and they are also sensibly coincident for any plane area whose dimensions are very small in comparison with its depth in the liquid, for the pressure over such an area is sensibly uniform.

151. Construction for Centre of Pressure.—If at every point of a plane area immersed in a liquid, a normal be drawn, equal to the depth of the point, the normals will represent the intensity of pressure at the respective points, and the volume of the solid constituted by all the normals will represent the total pressure. That normal which passes through the centre of gravity of this solid will be the line of action of the resultant, and will therefore pass through the centre of pressure.

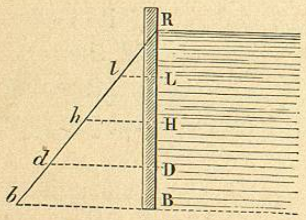

Fig. 64.—Centre of Pressure.

Thus, if RB (Fig. 64) be a rectangular surface (which we may suppose to be the surface of a flood-gate or of the side of a dam), its lower side B being at the bottom of the water and its upper side R at the top, the pressure is zero at R and goes on increasing uniformly to B. The normals Bb, Dd, Hh, Ll, equal to the depths of a

series of points in the line BR will have their extremities b, d, h, l, in one straight line. To find the centre of pressure, we must find the centre of gravity of the triangle RBb and draw a normal through it. As the centre of gravity of a triangle is at one-third of its height, the centre of pressure will be at one-third of the height of BR. It will lie on the line joining the middle points of the upper and lower sides of the rectangle, and will be at one-third of the length of this line from its lower end.

The total pressure will be equal to the weight of a quantity of the liquid whose volume is equal to that of the triangular prism constituted by the aggregate of the normals, of which prism the triangle RBb is a right section. It is not difficult to show that the volume of this prism is equal to the product of the area of the rectangle by the depth of the centre of gravity of the rectangle, in accordance with the rule above given.

152. Whirling Vessel. D'Alembert's Principle.—If an open vessel of liquid is rapidly rotated round a vertical axis, the surface of the liquid assumes a concave form, as represented in Fig. 65, where the dotted line is the axis of rotation. When the rotation has been going on at a uniform rate for a sufficient time, the liquid mass rotates bodily as if its particles were rigidly connected together, and when this state of things has been attained the form of the surface is that of a paraboloid of revolution, so that the section represented in the figure is a parabola.

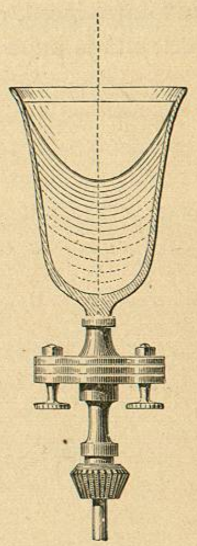
Fig. 65.—Rotating Vessel of Liquid.

We have seen in § 101 that a particle moving uniformly in a circle is acted on by a force directed towards the centre. In the present case, therefore, there must be a force acting upon each particle of the liquid urging it towards the axis. This force is supplied by the pressure of the liquid, which follows the usual law of increase with depth *for all points in the same vertical.* If we draw a horizontal plane in the liquid, the pressure at each point of it is that due to the height of the point of the surface vertically over it. The pressure is therefore least at the point where the plane is cut by the axis, and increases as we recede from this centre. Consequently each particle of liquid receives unequal pressures on two opposite sides, being more strongly pressed towards the axis than from it.

Another mode of discussing the case, is to treat it as one of statical equilibrium under the joint action of gravity and a fictitious force called centrifugal force, the latter force being, for each particle, equal and opposite to that which would produce the actual acceleration of the particle. This so-called centrifugal force is therefore to be regarded as a force directed radially outwards from the axis; and by compounding the centrifugal force of each particle with its weight we shall obtain what we are to treat as the resultant force on that particle. The form of the surface will then be determined by the condition that *at every point of the surface the normal must coincide with this resultant force;* just as in a liquid at rest, the normals must coincide with the direction of gravity.

The plan here adopted of introducing fictitious forces equal and opposite to those which if directly applied to each particle of a system would produce the actual accelerations, and then applying the conditions of statical equilibrium, is one of very frequent application, and will always lead to correct results. This principle was first introduced, or at least systematically expounded, by D'Alembert, and is known as D'Alembert's Principle.

CHAPTER XIII.

PRINCIPLE OF ARCHIMEDES.

153. Pressure of Liquids on Bodies Immersed.—When a body is immersed in a liquid, the different points of its surface are subjected to pressures which obey the rules laid down in the preceding chapter. As these pressures increase with the depth, those which tend to raise the body exceed those which tend to sink it, so that the resultant effect is a force in the direction opposite to that of gravity.

By resolving the pressure on each element into horizontal and vertical components, it can be shown that this resultant upward force is exactly equal to the weight of the liquid displaced by the body.

The reasoning is particularly simple in the case of a right cylinder (Fig. 66) plunged vertically in a liquid. It is evident, in the first place, that if we consider any point on the sides of the cylinder, the normal pressure on that point is horizontal and is destroyed by the equal and contrary pressure at the point diametrically opposite; hence, the horizontal pressures destroy each other. As regards the vertical pressures on the ends, one of them, that on the upper end AB, is in a downward direction, and equal to the weight of the liquid column ABNN; the other, that on the lower end CD, is in an upward direction, and equal to the weight of the liquid column CNND; this latter pressure exceeds the former by the weight of the liquid cylinder ABDC, so that the resultant effect of the pressure is to raise the body with a force equal to the weight of the liquid displaced.

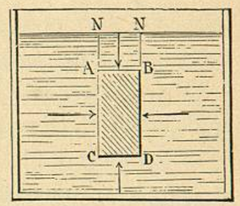

Fig. 66.—Principle of Archimedes.