Another mode of discussing the case, is to treat it as one of statical equilibrium under the joint action of gravity and a fictitious force called centrifugal force, the latter force being, for each particle, equal and opposite to that which would produce the actual acceleration of the particle. This so-called centrifugal force is therefore to be regarded as a force directed radially outwards from the axis; and by compounding the centrifugal force of each particle with its weight we shall obtain what we are to treat as the resultant force on that particle. The form of the surface will then be determined by the condition that *at every point of the surface the normal must coincide with this resultant force;* just as in a liquid at rest, the normals must coincide with the direction of gravity.

The plan here adopted of introducing fictitious forces equal and opposite to those which if directly applied to each particle of a system would produce the actual accelerations, and then applying the conditions of statical equilibrium, is one of very frequent application, and will always lead to correct results. This principle was first introduced, or at least systematically expounded, by D'Alembert, and is known as D'Alembert's Principle.

# CHAPTER XIII.

## PRINCIPLE OF ARCHIMEDES.

**153. Pressure of Liquids on Bodies Immersed.**—When a body is immersed in a liquid, the different points of its surface are subjected to pressures which obey the rules laid down in the preceding chapter. As these pressures increase with the depth, those which tend to raise the body exceed those which tend to sink it, so that the resultant effect is a force in the direction opposite to that of gravity.

By resolving the pressure on each element into horizontal and vertical components, it can be shown that this resultant upward force is exactly equal to the weight of the liquid displaced by the body.

The reasoning is particularly simple in the case of a right cylinder (Fig. 66) plunged vertically in a liquid. It is evident, in the first place, that if we consider any point on the sides of the cylinder, the normal pressure on that point is horizontal and is destroyed by the equal and contrary pressure at the point diametrically opposite; hence, the horizontal pressures destroy each other. As regards the vertical pressures on the ends, one of them, that on the upper end AB, is in a downward direction, and equal to the weight of the liquid column ABNN; the other, that on the lower end CD, is in an upward direction, and equal to the weight of the liquid column CNND; this latter pressure exceeds the former by the weight of the liquid cylinder ABDC, so that the resultant effect of the pressure is to raise the body with a force equal to the weight of the liquid displaced.

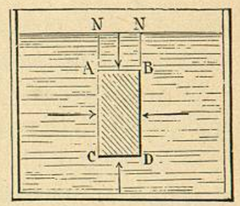


Fig. 66.—Principle of Archimedes.

By a synthetic process of reasoning, we may, without having recourse to the analysis of the different pressures, show that this conclusion is perfectly general. Suppose we have a liquid mass in equilibrium, and that we consider specially the portion M (Fig. 67); this portion is likewise in equilibrium. If we suppose it to become solid, without any change in its weight or volume, equilibrium will still subsist. Now this is a heavy mass, and as it does not fall, we must conclude that the effect of the pressures on its surface is to produce a resultant upward pressure exactly equal to its weight, and acting in a line which passes through its centre of gravity. If we now suppose M replaced by a body exactly occupying its place, the exterior pressures will remain the same, and their resultant effect will therefore be the same.

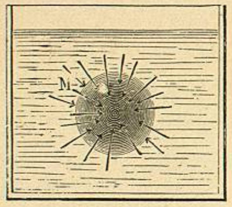


Fig. 67.—Principle of Archimedes.

The name *centre of buoyancy* is given to the centre of gravity of the liquid displaced,—that is, if the liquid be uniform, to the centre of gravity of the space occupied by the immersed body; and the above reasoning shows that the resultant pressure acts vertically upwards in a line which passes through this point. The results of the above explanations may thus be included in the following proposition: *Every body immersed in a liquid is subjected to a resultant pressure equal to the weight of the liquid displaced, and acting vertically upwards through the centre of buoyancy.*

This proposition constitutes the celebrated principle of Archimedes. The first part of it is often enunciated in the following form: *Every body immersed in a liquid loses a portion of its weight equal to the weight of the liquid displaced;* for when a body is immersed in a liquid, the force required to sustain it will evidently be diminished by a quantity equal to the upward pressure.

**154. Experimental Demonstration of the Principle of Archimedes.**—The following experimental demonstration of the principle of Archimedes is commonly exhibited in courses of physics:—

From one of the scales of a hydrostatic balance (Fig. 68) is suspended a hollow cylinder of brass, and below this a solid cylinder, whose volume is equal to the interior volume of the hollow cylinder; these are balanced by weights in the other scale. A vessel of water is then placed below the cylinders, in such a position that the lower cylinder shall be immersed in it. The equilibrium is immediately



destroyed, and the upward pressure of the water causes the scale with the weights to descend. If we now pour water into the hollow cylinder, equilibrium will gradually be re-established; and the beam

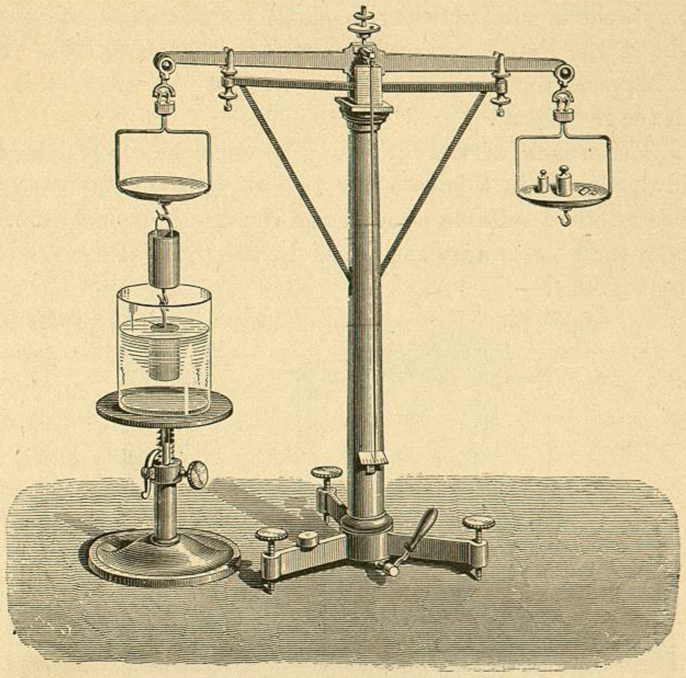


Fig. 68.—Experimental Verification of Principle of Archimedes.

will be observed to resume its horizontal position when the hollow cylinder is full of water, the other cylinder being at the same time completely immersed. The upward pressure upon this latter is thus equal to the weight of the water added, that is, to the weight of the liquid displaced.

**155. Body Immersed in a Liquid.**—It follows from the principle of Archimedes that when a body is immersed in a liquid, it is subjected to two forces: one equal to its weight and applied at its centre of gravity, tending to make the body descend; the other equal to the weight of the displaced liquid, applied at the centre of buoyancy, and tending to make it rise. There are thus three different cases to be considered:

(1.) The weight of the body may exceed the weight of the liquid displaced, or, in other words, the mean density of the body may be greater than that of the liquid; in this case, the body sinks in the liquid, as, for instance, a piece of lead dropped into water.

(2.) The weight of the body may be less than that of the liquid displaced; in this case the body will not remain submerged unless forcibly held down, but will rise partly out of the liquid, until the weight of the liquid displaced is equal to its own weight. This is what happens, for instance, if we immerse a piece of cork in water and leave it to itself.

(3.) The weight of the body may be equal to the weight of the liquid displaced; in this case, the two opposite forces being equal, the body takes a suitable position and remains in equilibrium.

These three cases are exemplified in the three following experiments (Fig. 69):—

(1.) An egg is placed in a vessel of water; it sinks to the bottom

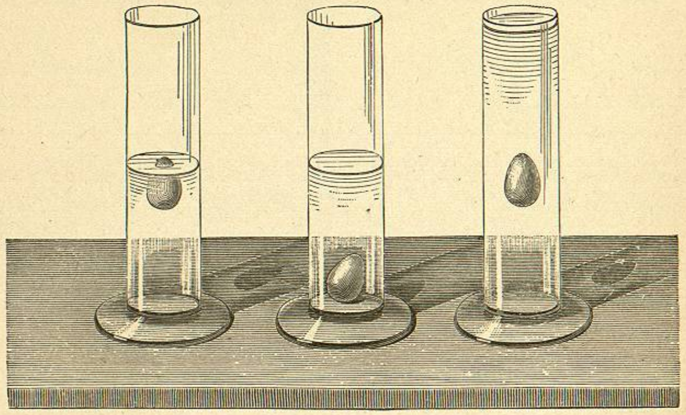


Fig. 69.—Egg Plunged in Fresh and Salt Water.

of the vessel, its mean density being a little greater than that of the liquid.

(2.) Instead of fresh water, salt water is employed; the egg floats at the surface of the liquid, which is a little denser than it.

(3.) Fresh water is carefully poured on the salt water; a mixture of the two liquids takes place where they are in contact; and if the egg is put in the upper part, it will be seen to descend, and, after a few oscillations, remain at rest at such a depth that it displaces its own weight of the liquid. In speaking of the liquid displaced in this case, we must imagine each horizontal layer of liquid surrounding the egg to be produced through the space which the egg occupies; and by the centre of buoyancy we must understand the centre of



gravity of the portion of liquid which would thus take the place of the egg. We may remark that, in this position the egg is in stable equilibrium; for, if it rises, the upward pressure diminishing, its weight tends to make it descend again; if, on the contrary, it sinks, the pressure increases and tends to make it reascend.

**156. Cartesian Diver.**—The experiment of the *Cartesian diver*, which is described in old treatises on physics, shows each of the different cases that can present themselves when a body is immersed. The diver (Fig. 70) consists of a hollow ball, at the bottom of which is a small opening O; a little porcelain figure is attached to the ball, and the whole floats upon water contained in a glass vessel, the mouth of which is closed by a strip of caoutchouc or a bladder. If we press with the hand on the bladder, the air is compressed, and the pressure, transmitted through the different horizontal layers, condenses the air in the ball, and causes the entrance of a portion of the liquid by the opening O; the floating body becomes heavier, and in consequence of this increase of weight the diver descends. When we cease to press upon the bladder, the pressure becomes what it was before, some water flows out and the diver ascends. It must be observed, however, that as the diver continues to descend, more and more water enters the ball, in consequence of the increase of pressure, so that if the depth of the water exceeded a certain limit, the diver would not be able to rise again from the bottom.

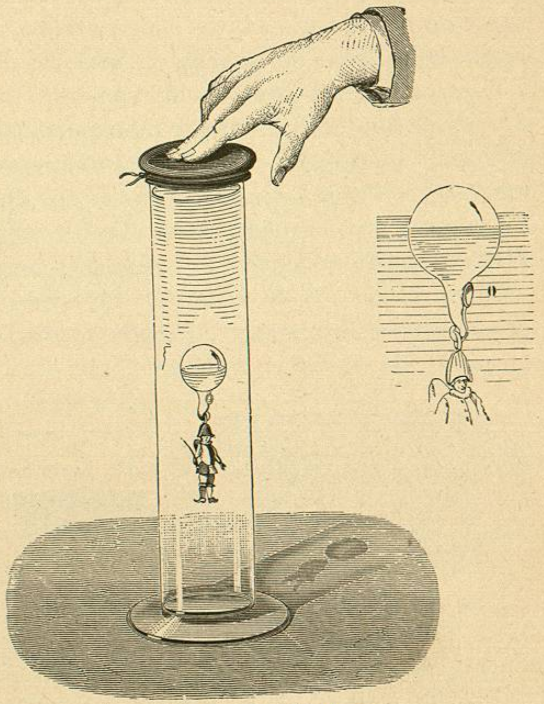




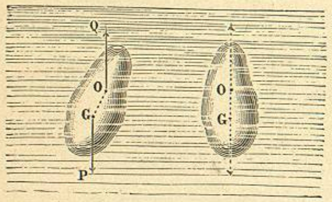
Fig. 70.—Cartesian Diver.

If we suppose that at a certain moment the weight of the diver becomes exactly equal to the weight of an equal volume of the liquid, there will be equilibrium; but, unlike the equilibrium in the experiment (3) of last section, this will evidently be *unstable*, for a slight movement either upwards or downwards will alter the resultant force so as to produce further movement in the same direction. As a consequence of this instability, if the diver is sent down below a certain depth he will not be able to rise again.

**157. Relative Positions of the Centre of Gravity and Centre of Buoyancy.**—In order that a floating body either wholly or partially immersed in a liquid, may be in equilibrium, it is necessary that its weight be equal to the weight of the liquid displaced.

This condition is however not sufficient; we require, in addition, that the action of the upward pressure should be exactly opposite to that of the weight; that is, that the centre of gravity and the centre of buoyancy be in the same vertical line; for if this were not the case, the two contrary forces would compose a couple, the effect of which would evidently be to cause the body to turn.

In the case of a body completely immersed, it is further necessary for stable equilibrium that *the centre of gravity should be below* the centre of buoyancy; in fact we see, by Fig. 71, that in any other



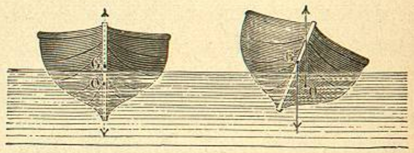



Fig. 71. Relative Positions of Centre of Gravity and Centre of Pressure. Fig. 72.

position than that of equilibrium, the effect of the two forces applied at the two points G and O would be to turn the body, so as to bring the centre of gravity lower, relatively to the centre of buoyancy. But this is not the case when the body is only partially immersed, as most frequently happens. In this case it may indeed happen that, with stable equilibrium, the centre of gravity is below the centre of pressure; but this is not necessary, and in the majority of instances is not the case. Let Fig. 72 represent the lower part of a floating body—a boat, for instance. The centre of pressure is at O, the centre of gravity at G, considerably above; if the body



is displaced, and takes the position shown in the figure, it will be seen that the effect of the two forces acting at O and at G is to restore the body to its former position. This difference from what takes place when the body is completely immersed, depends upon the fact that, in the case of the floating body, the figure of the liquid displaced changes with the position of the body, and the centre of buoyancy moves towards the side on which the body is more deeply immersed. It will depend upon the form of the body whether this lateral movement of the centre of buoyancy is sufficient to carry it beyond the vertical through the centre of gravity. The two equal forces which act on the body will evidently turn it to or from the original position of equilibrium, according as the new centre of buoyancy lies beyond or falls short of this vertical.[1]

**158. Advantage of Lowering the Centre of Gravity.**—Although stable equilibrium may subsist with the centre of gravity above the centre of buoyancy, yet for a body of given external form the stability is always increased by lowering the centre of gravity; as we thus lengthen the arm of the couple which tends to right the body when displaced. It is on this principle that the use of ballast depends.

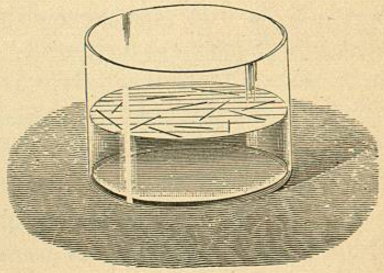


Fig. 73.—Steel Needles Floating on Water.

**159. Phenomena in Apparent Contradiction to the Principle of Archimedes.**—The principle of Archimedes seems at first sight to be contradicted by some well-known facts. Thus, for instance, if small needles are placed carefully on the surface of water, they will remain there in equilibrium (Fig. 73). It is on a similar principle

[1] If a vertical through the new centre of buoyancy be drawn upwards to meet that line in the body which in the position of equilibrium was a vertical through the centre of gravity, the point of intersection is called the *metacentre*. Evidently when the forces tend to restore the body to the position of equilibrium, the metacentre is above the centre of gravity; when they tend to increase the displacement, it is below. In ships the distance between these two points is usually nearly the same for all amounts of heeling, and this distance is a measure of the stability of the ship.

We have defined the metacentre as the intersection of two lines. When these lines lie in different planes, and do not intersect each other, there is no metacentre. This indeed is the case for most of the displacements to which a floating body of irregular shape can be subjected. There are in general only two directions of heeling to which metacentres correspond, and these two directions are at right angles to each other.

that several insects *walk* on water (Fig. 74), and that a great number of bodies of various natures, provided they be *very minute*, can, if we may so say, be placed on the surface of a liquid without penetrating into its interior. These curious facts depend on the circumstance that the small bodies in question are not wetted by the liquid, and hence, in virtue of principles which will be explained in connection with capillarity (Chap. xvi.), depressions are formed around them on the liquid surface, as represented in Fig. 75. The curvature of the liquid surface in the neighbourhood of the body is very distinctly shown by observing the shadow cast by the floating body, when it is illumined by the sun; it is seen to be bordered by luminous bands, which are owing to the refraction of the rays of light in the portion of the liquid bounded by a curved surface.

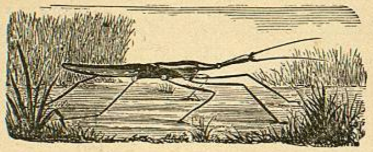


Fig. 74.—Insect Walking on Water.

The existence of the depression about the floating body enables us to bring the condition of equilibrium in this special case under the general enunciation of the principle of Archimedes. Let M (Fig. 75) be the body, CD the region of the depression, and AB the corresponding portion of any horizontal layer; since the pressure at each point of AB must be the same as in other parts of the same horizontal layer, the total weight above AB is the same as if M did not exist and the cavity were filled with the liquid itself.

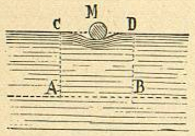



Fig. 75.

We may thus say in this case also that the weight of the floating body is equal to the weight of the *liquid displaced*, understanding by these words the liquid which would occupy the whole of the depression due to the presence of the body.

# CHAPTER XIV.

## DENSITY AND ITS DETERMINATION.

**160. Definitions.**—By the *absolute density* of a substance is meant the mass of unit volume of it. By the *relative density* is meant the ratio of its absolute density to that of some standard substance, or, what amounts to the same thing, the ratio of the mass of any volume of the substance in question to the mass of an equal volume of the standard substance. Since equal masses gravitate equally, the comparison of masses can be effected by weighing, and the relative density of a substance is the ratio of its weight to that of an equal volume of the standard substance. Water at a specified temperature and under atmospheric pressure is usually taken as the standard substance, and the density of a substance relative to water is usually called the *specific gravity* of the substance.

Let V denote the volume of a substance, M its mass, and D its absolute density; then by definition, we have $M=VD$.

If $s$ denote the specific gravity of a substance, and $d$ the absolute density of water in the standard condition, then $D=sd$ and $M=Vsd$.

When masses are expressed in lbs. and volumes in cubic feet, the value of $d$ is about 62·4, since a cubic foot of cold water weighs about 62·4 lbs.[1]

In the C.G.S. system, the value of $d$ is sensibly unity, since a cubic centimetre of water, at a temperature which is nearly that of the maximum density of water, weighs exactly a gramme.[2]

The gramme is defined, not by reference to water, but by a standard kilogramme of platinum, which is preserved in Paris, and

[1] In round numbers, a cubic foot of water weighs 1000 oz., which is 62·5 lbs.

[2] According to the best determination yet published, the mass of a cubic centimetre of pure water at 4° is 1·000013, at 3° is 1·000004, and at 2° is ·999982.