that several insects *walk* on water (Fig. 74), and that a great number of bodies of various natures, provided they be *very minute*, can, if we may so say, be placed on the surface of a liquid without penetrating into its interior. These curious facts depend on the circumstance that the small bodies in question are not wetted by the liquid, and hence, in virtue of principles which will be explained in connection with capillarity (Chap. xvi.), depressions are formed around them on the liquid surface, as represented in Fig. 75. The curvature of the liquid surface in the neighbourhood of the body is very distinctly shown by observing the shadow cast by the floating body, when it is illumined by the sun; it is seen to be bordered by luminous bands, which are owing to the refraction of the rays of light in the portion of the liquid bounded by a curved surface.

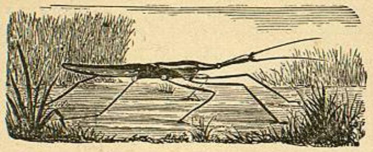

Fig. 74.—Insect Walking on Water.

The existence of the depression about the floating body enables us to bring the condition of equilibrium in this special case under the general enunciation of the principle of Archimedes. Let M (Fig. 75) be the body, CD the region of the depression, and AB the corresponding portion of any horizontal layer; since the pressure at each point of AB must be the same as in other parts of the same horizontal layer, the total weight above AB is the same as if M did not exist and the cavity were filled with the liquid itself.

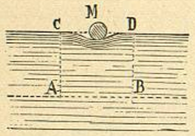

Fig. 75.

We may thus say in this case also that the weight of the floating body is equal to the weight of the *liquid displaced*, understanding by these words the liquid which would occupy the whole of the depression due to the presence of the body.

CHAPTER XIV.

DENSITY AND ITS DETERMINATION.

160. Definitions.—By the *absolute density* of a substance is meant the mass of unit volume of it. By the *relative density* is meant the ratio of its absolute density to that of some standard substance, or, what amounts to the same thing, the ratio of the mass of any volume of the substance in question to the mass of an equal volume of the standard substance. Since equal masses gravitate equally, the comparison of masses can be effected by weighing, and the relative density of a substance is the ratio of its weight to that of an equal volume of the standard substance. Water at a specified temperature and under atmospheric pressure is usually taken as the standard substance, and the density of a substance relative to water is usually called the *specific gravity* of the substance.

Let V denote the volume of a substance, M its mass, and D its absolute density; then by definition, we have $M=VD$.

If s denote the specific gravity of a substance, and d the absolute density of water in the standard condition, then $D=sd$ and $M=Vsd$.

When masses are expressed in lbs. and volumes in cubic feet, the value of d is about 62·4, since a cubic foot of cold water weighs about 62·4 lbs.[1]

In the C.G.S. system, the value of d is sensibly unity, since a cubic centimetre of water, at a temperature which is nearly that of the maximum density of water, weighs exactly a gramme.[2]

The gramme is defined, not by reference to water, but by a standard kilogramme of platinum, which is preserved in Paris, and

[1] In round numbers, a cubic foot of water weighs 1000 oz., which is 62·5 lbs.

[2] According to the best determination yet published, the mass of a cubic centimetre of pure water at 4° is 1·000013, at 3° is 1·000004, and at 2° is ·999982.

of which several very carefully made copies are preserved in other places. In the above statements (as in all very accurate statements of weights), the weighings are supposed to be made in vacuo; for the masses of two bodies are not accurately proportional to their apparent gravitations in air, unless the two bodies happen to have the same density.

161. Ambiguity of the word "Weight."—Properly speaking, "the weight of a body" means the force with which the body gravitates towards the earth. This force, as we have seen, differs slightly according to the place of observation. If m denote the mass of the body, and g the intensity of gravity at the place, the weight of the body is mg. When the body is carried from one place to another without gain or loss of material, m will remain constant and g will vary; hence the weight mg will vary, and in the same ratio as g.

But the employment of gravitation units of force instead of absolute units, obscures this fact. The unit of measurement varies in the same ratio as the thing to be measured, and hence the numerical value remains unaltered. A body weighs the same number of pounds or grammes at one place as at another, because the weights of the pound and gramme are themselves proportional to g. Expressed in absolute units, the weight of unit mass is g, and the weight of a mass m is mg. The latter is m times the former; hence when the weight of unit mass is employed as the unit of weight, the same number m which denotes the mass of a body also denotes its weight. What are usually called standard weights—that is, standard pieces of metal used for weighing—are really standards of mass; and when the result of a weighing is stated in terms of these standards, (as it usually is,) the "weight," as thus stated, is really the *mass* of the body weighed. The standard "weights" which we use in our balances are really standard masses. In discussions relating to density, weights are most conveniently expressed in gravitation measure, and hence the words mass and weight can be used almost indiscriminately.

162. Determination of Density from Weight and Volume.—The absolute density of a substance can be directly determined by weighing a measured volume of it. Thus if v cubic centimetres of it weigh m grammes, its density (in grammes per cubic centimetre) is $\frac{m}{v}$. This method can be easily applied to solids of regular geometrical forms; since their volumes can be computed from their

linear measurements. It can also be applied to liquids, by employing a vessel of known content. The bottle usually employed for this purpose is a bottle of thin glass fitted with a perforated stopper, so that it can be filled and stoppered without leaving a space for air. The difference between its weights when full and empty is the weight of the liquid which fills it; and the quotient of this by the volume occupied (which can be determined once for all by weighing the bottle when filled with water) is the density of the liquid.

The advantage of employing a perforated stopper is that it enables us to ensure constancy of volume. If a wide-mouthed flask were employed, without a stopper, it would be difficult to pronounce when the flask was exactly full. This source of inaccuracy would be diminished by making the mouth narrower: but when it is very narrow, the filling and emptying of the flask are difficult, and there is danger of forcing in bubbles of air with the liquid. When a perforated stopper is employed, the flask is first filled, then the stopper is inserted and some of the liquid is thus forced up through the perforation, overflowing at the top. When the stopper has been pushed home, all the liquid outside is carefully wiped off, and the liquid which remains is as much as just fills the stoppered flask including the perforation in the stopper.

In accurate work, the temperature must be observed, and due allowance made for its effect upon volume.

163. Specific Gravity Flask for Solids.—The volume and density of a solid body of irregular shape, or consisting of a quantity of small pieces, can be determined by putting it into such a bottle (Fig. 76), and weighing the water which it displaces. The most convenient way of doing this is to observe (1) the weight of the solid; (2) the weight of the bottle full of water; (3) the weight of the bottle when it contains the solid, together with as much water as will fill it up. If the

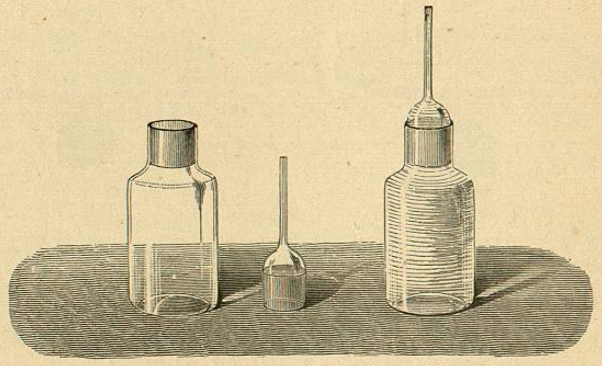

Fig. 76.—Specific Gravity Flask for Solids.

third of these results be subtracted from the sum of the first two, the remainder will be the weight of the water displaced; which, when expressed in grammes, is the same as the volume of the body expressed in cubic centimetres. The weight of the body, divided by this remainder, is the density of the body.

164. Method by Weighing in Water.—The methods of determining density which we are now about to describe depend upon the principle of Archimedes.

One of the commonest ways of determining the density of a solid body is to weigh it first in air and then in water (Fig. 77) the

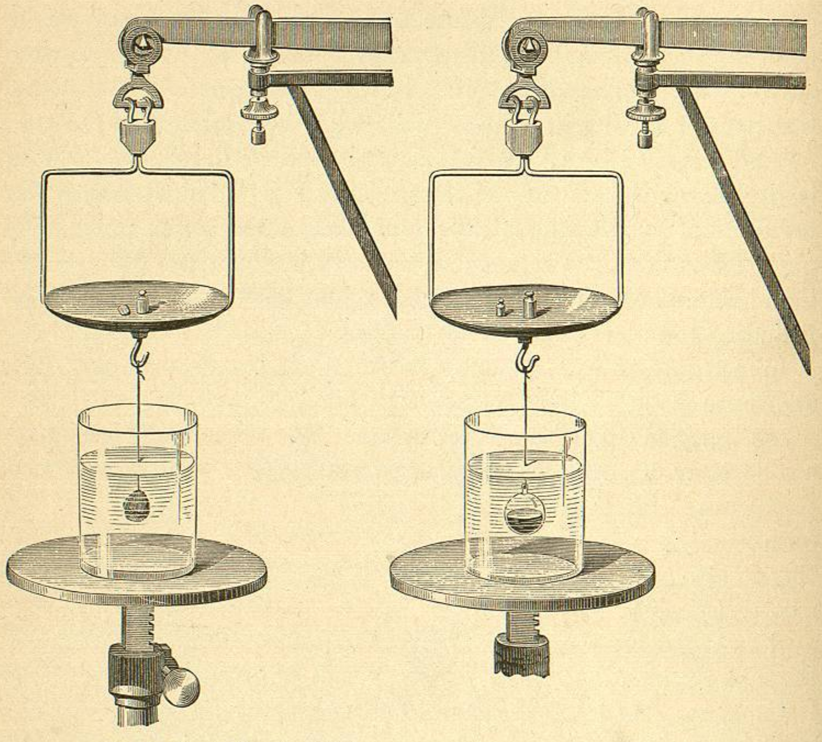

Fig. 77.—Specific Gravity of Solids. Fig. 78.—Specific Gravity of Liquids.

counterpoising weights being in air. Since the loss of weight due to its immersion in water is equal to the weight of the same volume of water, we have only to *divide the weight in air by this loss of weight.* We shall thus obtain the relative density of the body as compared with water—in other words, the specific gravity of the body.

Thus, from the observations

Weight in air,	125 *gm.*
Weight in water,	100 „
Loss of weight,	25 „

we deduce

$$\frac{125}{25} = 5 = \text{density.}$$

A very fine and strong thread or fibre should be employed for suspending the body, so that the volume of liquid displaced by this thread may be as small as possible.

165. Weighing in Water, with a Sinker.—If the body is lighter than water, we may employ a sinker—that is, a piece of some heavy material attached to it, and heavy enough to make it sink. It is not necessary to know the weight of the sinker in air, but we must observe its weight in water. Call this s. Let w be the weight of the body in air, and w' the weight of the body and sinker together in water. Then w' will be less than s. The body has an apparent upward gravitation in water equal to $s-w'$, showing that the resultant pressure upon it exceeds its weight by this amount. Hence the weight of the liquid displaced is $w+s-w'$, and the specific gravity of the body is $\frac{w}{w+s-w'}$.

If any other liquid than water be employed in the methods described in this and the preceding section, the result obtained will be the relative density as compared with that liquid. The result must therefore be multiplied by the density of the liquid, in order to obtain the absolute density.

166. Density of Liquid Inferred from Loss of Weight.—The densities of liquids are often determined by observing the loss of weight of a solid immersed in them, and dividing by the known volume of the solid or by its loss of weight in water.

Thus, from the observations

Weight in air,	200 *gm*
Weight in liquid,	120 „
Weight in water,	110 „

we deduce

Loss in liquid, 80. Loss in water, 90.

$$\text{Density of liquid, } \frac{80}{90} = \frac{8}{9}.$$

A glass ball (sometimes weighted with mercury, as in Fig. 78) is the solid most frequently employed for such observations.

167. Measurement of Volumes of Solids by Loss of Weight.—The volume of a solid body, especially if of irregular shape, can usually be determined with more accuracy by weighing it in a liquid than by any other method. If it weigh w grammes in air, and w' grammes in water, its volume is $w-w'$ cubic centimetres, since it displaces $w-w'$ grammes of water. The mean diameter of a wire can be very accurately determined by an observation of this kind for volume, combined with a direct measurement of length. The volume divided by the length will be the mean sectional area, which is equal to πr^2, where r is the radius.

168. Hydrometers.—The name hydrometer is given to a class of instruments used for determining the densities of liquids by observing either the depths to which they sink in the liquids or the

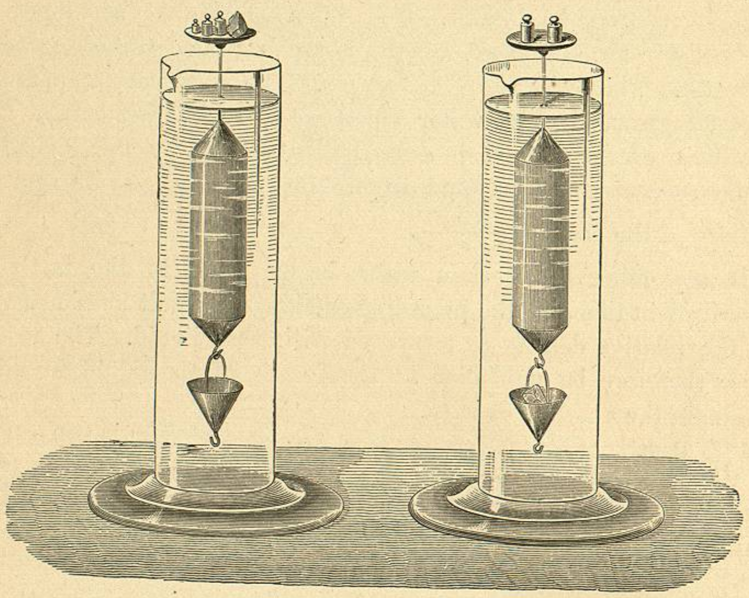

Fig. 79.—Nicholson's Hydrometer.

weights required to be attached to them to make them sink to a given depth. According as they are to be used in the latter or the former of these two ways, they are called hydrometers of constant or of variable immersion. The name areometer (from ἀραιός, rare) is used as synonymous with hydrometer, being probably borrowed from the French name of these instruments, *aréomètre*. The hydrometers of constant immersion most generally known are those of Nicholson and Fahrenheit.

169. Nicholson's Hydrometer.—This instrument, which is represented in Fig. 79, consists of a hollow cylinder of metal with conical ends, terminated above by a very thin rod bearing a small dish, and carrying at its lower end a kind of basket. This latter is of such weight that when the instrument is immersed in water a weight of 100 grammes must be placed in the dish above in order to sink the apparatus as far as a certain mark on the rod. By the principle of Archimedes, the weight of the instrument, together with the 100 grammes which it carries, is equal to the weight of the water displaced. Now, let the instrument be placed in another liquid, and the weights in the dish above be altered until they are just sufficient to make the instrument sink to the mark on the rod. If the weights in the dish be called w, and the weight of the instrument itself W, the weight of liquid displaced is now $W + w$, whereas the weight of the same volume of water was $W + 100$; hence the specific gravity of the liquid is $\frac{W + w}{W + 100}$.

This instrument can also be used either for weighing small solid bodies or for finding their specific gravities. To find the weight of a body (which we shall suppose to weigh less than 100 grammes), it must be placed in the dish at the top, together with weights just sufficient to make the instrument sink in water as far as the mark. Obviously these weights are the difference between the weight of the body and 100 grammes.

To find the specific gravity of a solid, we first ascertain its weight by the method just described; we then transfer it from the dish above to the basket below, so that it shall be under water during the observation, and observe what additional weights must now be placed in the dish. These additional weights represent the weight of the water displaced by the solid; and the weight of the solid itself divided by this weight is the specific gravity required.

170. Fahrenheit's Hydrometer.—This instrument, which is represented in Fig. 80, is generally constructed of glass, and differs from Nicholson's in having at its lower extremity a ball weighted with mercury instead of the basket. It resembles it in having a dish at the top, in which weights are to be placed sufficient to sink the instrument to a definite mark on the stem.

Hydrometers of constant immersion, though still described in text-books, have quite gone out of use for practical work.

171. Hydrometers of Variable Immersion.—These instruments are usually of the forms represented at A, B, C, Fig. 81. The lower end is weighted with mercury in order to make the instrument sink to a convenient depth and preserve an upright position. The stem is cylindrical, and is graduated, the divisions being frequently marked

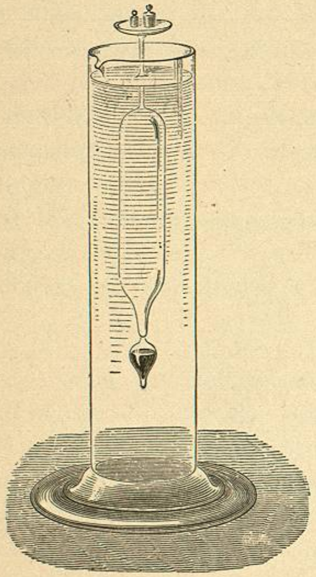

Fig. 80.—Fahrenheit's Hydrometer.

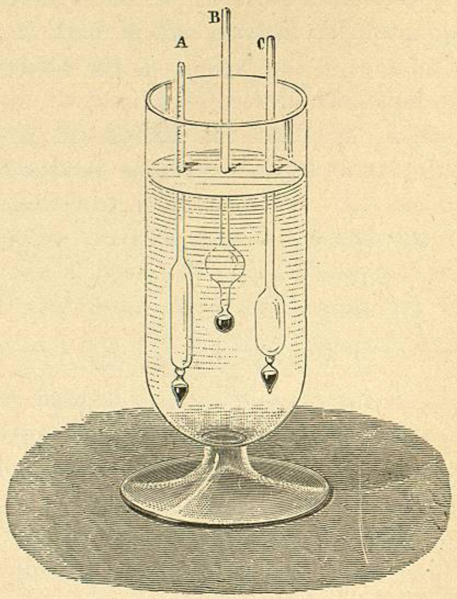

Fig. 81.—Forms of Hydrometers.

upon a piece of paper inclosed within the stem, which must in this case be of glass. It is evident that the instrument will sink the deeper the less is the specific gravity of the liquid, since the weight of the liquid displaced must be equal to that of the instrument. Hence if any uniform system of graduation be adopted, so that all the instruments give the same readings in liquids of the same densities, the density of a liquid can be obtained by a mere immersion of the hydrometer—an operation not indeed very precise, but very easy of execution. These instruments have thus come into general use for commercial purposes and in the excise.

172. General Theory of Hydrometers of Variable Immersion.—Let V be the volume of a hydrometer which is immersed when the instrument floats freely in a liquid whose density is d, then Vd represents the weight of liquid displaced, which by the principle of Archimedes is the same as the weight of the hydrometer itself. If V′, d' be the corresponding values for another liquid, we have therefore

$$Vd = V'd', \text{ or } d : d' :: V' : V,$$

that is, the density varies inversely as the volume immersed. Let $d_1, d_2, d_3 \ldots$ be a series of densities, and $V_1, V_2, V_3 \ldots$ the corresponding volumes immersed, then we have

$$d_1, d_2, d_3 \ldots \text{ proportional to } \frac{1}{V_1}, \frac{1}{V_2}, \frac{1}{V_3} \ldots$$

$$\text{and } V_1, V_2, V_3 \ldots \text{ proportional to } \frac{1}{d_1}, \frac{1}{d_2}, \frac{1}{d_3} \ldots$$

Hence, if we wish the divisions to indicate equal differences of density, we must place them so that the corresponding volumes immersed form a harmonical progression. This implies that the distances between the divisions must diminish as the densities increase.

The following investigation shows how the density of a liquid may be computed from observations made with a hydrometer graduated with equal divisions. It is necessary first to know the divisions to which the instrument sinks in two liquids of known densities. Let these divisions be numbered n_1, n_2, reckoning from the top downwards, and let the corresponding densities be d_1, d_2. Now if we take for our unit of volume one of the equal parts on the stem, and if we take c to denote the volume which is immersed when the instrument sinks to the division marked zero, it is obvious that when the instrument sinks to the nth division (reckoned downwards on the stem from zero) the volume immersed is $c-n$, and if the corresponding density be called d, then $(c-n)\,d$ is the weight of the hydrometer. We have therefore

$$(c-n_1)\,d_1 = (c-n_2)\,d_2, \text{ whence } c = \frac{n_1 d_1 - n_2 d_2}{d_1 - d_2}.$$

This value of c can be computed once for all.

Then the density D corresponding to any other division N can be found from the equation

$$(c-N)\,D = (c-n_1)\,d_1 \text{ which gives } D = \frac{c-n_1}{c-N} d_1.$$

173. Beaumé's Hydrometers.—In these instruments the divisions are equidistant. There are two distinct modes of graduation, according as the instrument is to be used for determining densities greater or less than that of water. In the former case the instrument is called a salimeter, and is so constructed that when immersed in pure water of the temperature 12° Cent. it sinks nearly to the top of the stem, and the point thus determined is the zero of the scale. It is then immersed in a solution of 15 parts of salt to 85 of water, the density of which is about 1·116, and the point to which it sinks is marked 15. The interval is divided into 15 equal parts, and the graduation is continued to the bottom of the stem, the length of which varies according to circumstances; it generally terminates at the degree 66, which corresponds to sulphuric acid, whose density is commonly the greatest that it is required to determine. Referring to the formulæ of last section, we have here

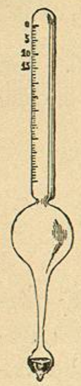

Fig. 82. Baumé's Salimeter.

$$n_1=0,\ d_1=1,\ n_2=15,\ d_2=1{\cdot}116;$$

whence

$$c=\frac{15\times 1{\cdot}116}{{\cdot}116}=144,\ \mathrm{D}=\frac{144}{144-\mathrm{N}}.$$

When the instrument is intended for liquids lighter than water, it is called an alcoholimeter. In this case the point to which it sinks in water is near the bottom of the stem, and is marked 10; the zero of the scale is the point to which it sinks in a solution of 10 parts of salt to 90 of water, the density of which is about 1·085, the divisions in this case being numbered upward from zero.

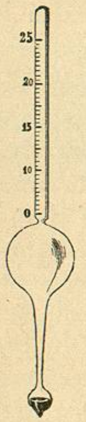

Fig. 83. Fig. 84. Baumé's Alcoholimeters.

In order to adapt the formulæ of last section to the case of graduations numbered upwards, it is merely necessary to reverse the signs of n_1, n_2, and N; that is we must put

$$c=\frac{n_2d_2-n_1d_1}{d_1-d_2},\ \mathrm{D}=\frac{c+n_1}{c+\mathrm{N}}d_1;$$

and as we have now $n_1=10$, $d_1=1$, $n_2=0$, $d_2=1{\cdot}085$ the formulæ give[1]

$$c=\frac{10}{{\cdot}085}=118,\ \mathrm{D}=\frac{128}{118+\mathrm{N}}.$$

174. Twaddell's Hydrometer.—In this instrument the divisions are placed not as in Beaumé's, at equal distances, but at distances corresponding to equal differences of density. In fact the specific gravity of a liquid is found by multiplying the reading by 5, cutting off three decimal places, and prefixing unity. Thus the degree 1 indicates specific gravity 1·005, 2 indicates 1·010, &c.

[1] On comparing the two formulæ for D in this section with the tables in the Appendix to Miller's *Chemical Physics*, I find that as regards the salimeter they agree to two places of decimals and very nearly to three. As regards the alcoholimeter, the table in *Miller* implies that c is about 136, which would make the density corresponding to the zero of the scale about 1·074.

175. Gay-Lussac's Centesimal Alcoholimeter.—When a hydrometer is to be used for a special purpose, it may be convenient to adopt a mode of graduation different in principle from any that we have described above, and adapted to give a direct indication of the proportion in which two ingredients are mixed in the fluid to be examined. It may indicate, for example, the quantity of salt in sea-water, or the quantity of alcohol in a spirit consisting of alcohol and water. Where there are three or more ingredients of different specific gravities the method fails. Gay-Lussac's alcoholimeter is graduated to indicate, at the temperature of 15° Cent., the percentage of pure alcohol in a specimen of spirit. At the top of the stem is 100, the point to which the instrument sinks in pure alcohol, and at the bottom is 0, to which it sinks in water. The position of the intermediate degrees must be determined empirically, by placing the instrument in mixtures of alcohol and water in known proportions, at the temperature of 15°. The law of density, as depending on the proportion of alcohol present, is complicated by the fact that, when alcohol is mixed with water, a diminution of volume (accompanied by rise of temperature) takes place.

Fig. 85. Centesimal Alcoholimeter.

176. Specific Gravity of Mixtures.—When two or more substances are mixed without either shrinkage or expansion (that is, when the volume of the mixture is equal to the sum of the volumes of the components), the density of the mixture can easily be expressed in terms of the quantities and densities of the components.

First, let the volumes $v_1, v_2, v_3 \ldots$ of the components be given, together with their densities $d_1, d_2, d_3 \ldots$

Then their masses (or weights) are $v_1d_1, v_2d_2, v_3d_3 \ldots$

The mass of the mixture is the sum of these masses, and its volume is the sum of the volumes $v_1, v_2, v_3 \ldots$; hence its density is

$$\frac{v_1d_1+v_2d_2+\ldots}{v_1+v_2+\ldots}.$$

Secondly, let the weights or masses $m_1, m_2, m_3 \ldots$ of the components be given, together with their densities $d_1, d_2, d_3 \ldots$

Then their volumes are $\frac{m_1}{d_1}, \frac{m_2}{d_2}, \frac{m_3}{d_3} \ldots$

The volume of the mixture is the sum of these volumes, and its mass is $m_1 + m_2 + m_3 + \ldots$; hence its density is

$$\frac{m_1 + m_2 + \ldots}{\frac{m_1}{d_1} + \frac{m_2}{d_2} + \ldots}$$

177. Graphical Method of Graduation.—When the points on the stem which correspond to some five or six known densities, nearly equidifferent, have been determined, the intermediate graduations can be inserted with tolerable accuracy by the graphical method of interpolation, a method which has many applications in physics besides that which we are now considering. Suppose A and B (Fig. 86) to represent the extreme points, and I, K, L, R intermediate points, all of which correspond to known densities. Erect ordinates (that is to say, perpendiculars) at these points, proportional to the respective densities, or (which will serve our purpose equally well) erect ordinates II′, KK′, LL′, RR′, BC proportional to the excesses of the densities at I, K, L, R, B above the density at A. Any scale of equal parts can be employed for laying off the ordinates, but it is convenient to adopt a scale which will make the greatest ordinate BC not much greater nor much less than the base-line AB. In the figure, the density at B is supposed to be 1·80, the density at A being 1. The difference of density is therefore ·80, as indicated by the figures 80 on the scale of equal parts. Having erected the ordinates, we must draw through their extremities the curve AI′K′L′R′C, making it as free from sudden bends as possible, as it is upon the regularity of this curve that the accuracy of the interpolation depends. Then to find the point on the stem AB at which any other density is to be marked—say 1·60, we must draw through the 60th division, on the line of equal parts, a horizontal line to meet the curve, and, through the point thus found on the curve,

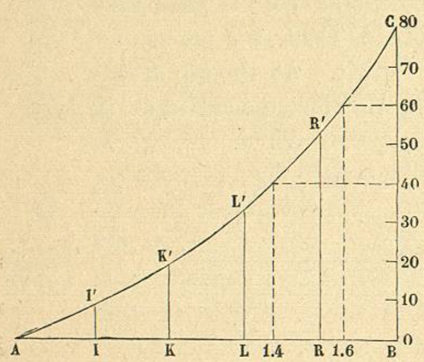

Fig. 86.—Graphical Method of Graduation.

draw an ordinate. This ordinate will meet the base-line AB in the required point, which is accordingly marked 1·6 in the figure. The curve also affords the means of solving the converse problem, that is, of finding the density corresponding to any given point on the stem. At the given point in AB, which represents the stem, we must draw an ordinate, and through the point where this meets the curve we must draw a horizontal line to meet the scale of equal parts. The point thus determined on the scale of equal parts indicates the density required, or rather the excess of this density above the density of A.