CHAPTER XVI.

CAPILLARITY.

182. Capillarity—General Phenomena.—The laws which we have thus far stated respecting the levels of liquid surfaces are subject to remarkable exceptions when the vessels in which the liquids are contained are very narrow, or, as they are called, capillary (*capillus*, a hair); and also in the case of vessels of any size, when we consider the portion of the liquid which is in close proximity to the sides.

1. *Free Surface.*—The surface of a liquid is not horizontal in the neighbourhood of the sides of the vessel, but presents a very decided curvature. When the liquid wets the vessel, as in the case of water in a glass vessel (Fig. 96), the surface is concave; on the contrary

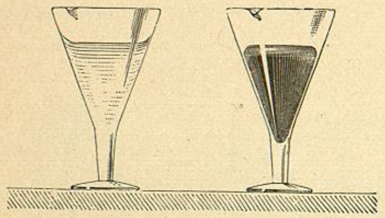

Fig. 96. Fig. 97.

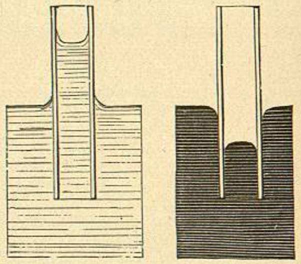

Fig. 98. Fig. 99.

when the liquid does not wet the vessel, as in the case of mercury in a glass vessel (Fig. 97), the surface is, generally speaking, convex.

2. *Capillary Elevation and Depression.*—If a very narrow tube of glass be plunged in water, or any other liquid that will wet it (Fig. 98), it will be observed that the level of the liquid, instead of remaining at the same height inside and outside of the tube, stands perceptibly higher in the tube; a *capillary ascension* takes place, which varies in amount according to the nature of the liquid and

the diameter of the tube. It will also be seen that the liquid column thus raised terminates in a concave surface. If a glass tube be dipped in mercury, which does not wet it, it will be seen, by bringing the tube to the side of the vessel, that the mercury is depressed in its interior, and that it terminates in a convex surface (Fig. 99).

3. *Capillary Vessels in Communication with Others.*—If we take two bent tubes (Fig. 100), each having one branch of a considerable diameter and the other extremely narrow, and pour into one of them a liquid which wets it, and into the other mercury, the liquid will be observed in the former case to stand higher in the capillary than in the principal branch, and in the latter case to stand lower; the free surfaces being at the same time concave in the case of the liquid which wets the tubes, and convex in the case of the mercury.

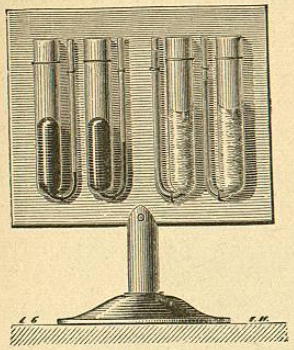

Fig. 100.

183. Circumstances which influence Capillary Elevation and Depression.—In wetted tubes the elevation depends upon the nature of the liquid; thus, at the temperature of 18° Cent., water rises 29·79^{mm} in a tube 1 millimetre in diameter, alcohol rises 12·18^{mm}, nitric acid 22·57^{mm}, essence of lavender 4·28^{mm}, &c. The nature of the tube is almost entirely immaterial, provided the precaution be first taken of wetting it with the liquid to be employed in the experiment, so as to leave a film of the liquid adhering to the sides of the tube.

Capillary depression, on the other hand, depends both on the nature of the liquid and on that of the tube. Both ascension and depression diminish as the temperature increases; for example, the elevation of water, which in a tube of a certain diameter is equal to 132^{mm} at 0° Cent., is only 106^{mm} at 100°.

184. Law of Diameters.—*Capillary elevations and depressions, when all other circumstances are the same, are inversely proportional to the diameters of the tubes.* As this law is a consequence of the mathematical theories which are generally accepted as explaining capillary phenomena, its verification has been regarded as of great importance.

The experiments of Gay-Lussac, which confirmed this law, have been repeated, with slight modifications, by several observers. The method employed consists essentially in measuring the capillary elevation of a liquid by means of a cathetometer (Fig. 101). The telescope ll is directed first to the top n of the column in the tube, and then to the end of a pointer b, which touches the surface of the

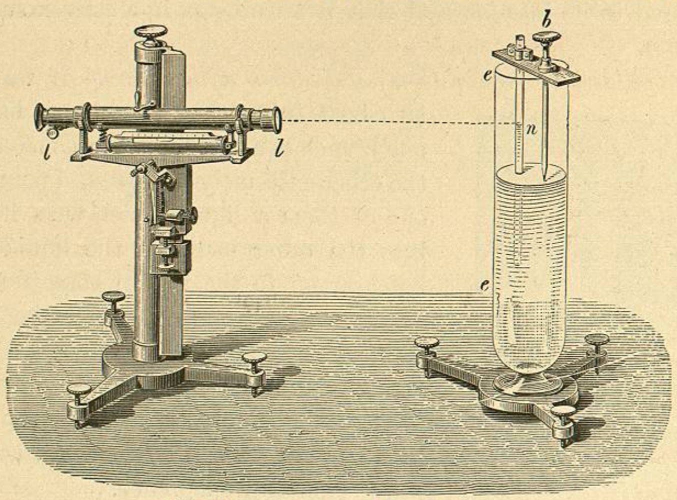

Fig. 101.—Verification of Law of Diameters.

liquid at a point where it is horizontal. In observing the depression of mercury, since the opacity of the metal prevents us from seeing the tube, we must bring the tube close to the side of the vessel e.

The diameter of the tube can be measured directly by observing its section through a microscope, or we may proceed by the method employed by Gay-Lussac. He weighed the quantity of mercury which filled a known length l of the tube; this weight w is that of a cylinder of mercury whose radius x is determined by the equation $13{\cdot}59\ \pi x^2 l = w$, where x and l are in centimetres, and w in grammes.

The result of these different experiments is, that in the case of wetted tubes the law is exactly fulfilled, provided that they be previously washed with the greatest care, so as to remove all foreign matters, and that the liquid on which the experiment is to be performed be first passed through them. When the liquid does not wet the tube, various causes combine to affect the form of the surface in which the liquid column terminates; and we cannot infer the depression from knowing the diameter, unless we also take into consideration some element connected with the form of the terminal surface, such as the length of the sagitta, or the angle made with the sides

of the tube by the extremities of the curved surface, which is called the *angle of contact.*

185. Fundamental Laws of Capillary Phenomena.—Capillary phenomena, as they take place alike in air and in vacuo, cannot be attributed to the action of the atmosphere. They depend upon molecular actions which take place between the particles of the liquid itself, and between the liquid and the solid containing it, the actions in question being purely superficial—that is to say, being confined to an extremely thin layer forming the external boundary of the liquid, and to an extremely thin superficial layer of the solid in contact with the liquid. For example, it is found in the case of glass tubes, that the amount of capillary elevation or depression is not at all affected by the thickness of the sides of the tube. The following are some of the principles which govern capillary phenomena.

1. For a given liquid in contact with a given solid, with a definite intimateness of contact (this last element being dependent upon the cleanness of the surface, upon whether the surface of the solid has been recently washed by the liquid, and perhaps upon some other particulars), there is (at any specified temperature) a definite angle of contact, which is independent of the directions of the surfaces with regard to the vertical.

2. Every liquid behaves as if a thin film, forming its external layer, were in a state of tension, and exerting a constant effort to contract. This tension, or contractile force, is exhibited over the whole of the free surface (that is, the surface which is exposed to air); but wherever the liquid is in contact with a solid, its existence is masked by other molecular actions. It is uniform in all directions in the free surface, and at all points in this surface, being dependent only on the nature and temperature of the liquid. Its intensity for several specified liquids is given in tabular form further on (§ 192) upon the authority of Van der Mensbrugghe. Tension of this kind must of course be stated in units of force per linear unit, because by doubling the width of a band we double the force required to keep it stretched. Mensbrugghe considers that such tension really exists in the superficial layer; but the majority of authors (and we think with more justice) regard it rather as a convenient fiction, which accurately represents the effects of the real cause. Two of the most eminent writers on the cause of capillary phenomena are Laplace and Dr. Thomas Young. The subject presents difficulties which have not yet been fully surmounted

186. Application to Elevation in Tubes.—The law of diameters is a direct consequence of the two preceding principles; for if α denote the external angle of contact (which is acute in the case of mercury against glass), T the tension per unit length, and r the radius of the tube, then $2\pi r\text{T}$ will be the whole amount of force exerted at the margin of the surface; and as this force is exerted in a direction making an angle α with the vertical, its vertical component will be $2\pi r\text{T}\cos\alpha$, which is exerted in pulling the tube upwards and the liquid downwards.

If w be the weight of unit volume of the liquid, then $\pi r^2 w$ is the weight of as much as would occupy unit length of the tube; and if h denote the height of a column whose weight is equal to the force tending to depress the liquid, we have

$$\pi r^2 hw = 2\pi r\text{T}\cos\alpha;$$

whence $h = \frac{2\text{T}\cos\alpha}{r \cdot w}$, which, when the other elements are given, varies inversely as r, the radius of the tube.

Having regard to the fact that the surface is not of the same height in the centre as at the edges, it is obvious that h denotes the mean height.

If α be obtuse, h will be negative—that is to say, there will be elevation instead of depression. In the case of water against a tube which has been well wetted with that liquid, α is 180°—that is to say, the tube is tangential to the surface. For this case the formula for h gives

$$\text{elevation} = \frac{2\text{T}}{rw}.$$

Again, for two parallel vertical plates at distance u, the vertical force of capillarity for a unit of length is $2\text{T}\cos\alpha$, which must be equal to whu, being the weight of a sheet of liquid of height h, thickness u, and length unity. We have therefore

$$h = \frac{2\text{T}\cos\alpha}{uw},$$

which agrees with the expression for the depression or elevation in a circular tube whose radius is equal to the distance between these parallel plates.

The surface tension always tends to reduce the surface to the smallest area which can be inclosed by its actual boundary; and therefore always produces a normal force directed from the convex to the concave side of the superficial film. Hence, wherever there is capillary elevation the free surface must be concave; wherever there is depression it must be convex.

187. It follows from a well-known proposition in statics (Todhunter's *Statics*, § 194), that if a *cylindrical* film be stretched with a uniform tension T (so that the force tending to pull the film asunder across any short line drawn on the film, is T times the length of the line), the resultant normal pressure (which the film exerts, for example, against the surface of a solid internal cylinder over which it is stretched) is T divided by the radius of the cylinder.

It can be proved that a film of any form, stretched with uniform tension T, exerts at each point a normal pressure equal to the sum of the pressures which would be exerted by two overlapping cylindrical films, whose axes are at right angles to one another, and whose cross sections are circles of curvature of normal sections at the point. That is to say, if P be the normal force per unit area, and r, r' the radii of curvature in two mutually perpendicular normal sections at the point, then

$$\text{P} = \text{T}\left(\frac{1}{r} + \frac{1}{r'}\right).$$

At any point on a curved surface, the normal sections of greatest and least curvature are mutually perpendicular, and are called the principal normal sections at the point. If the corresponding radii of curvature be R, R′, we have

$$\text{P} = \text{T}\left(\frac{1}{\text{R}} + \frac{1}{\text{R}'}\right); \qquad (1)$$

or *the normal force per unit area is equal to the tension per unit length multiplied by the sum of the principal curvatures.*

In the case of capillary depressions and elevations, the superficial film at the free surface is to be regarded as pressing the liquid inwards, or pulling it outwards, according as this surface is convex or concave, with a force P given by the above formula. The value of P at any point of the free surface is equal to the pressure due to the height of a column of liquid extending from that point to the level of the general horizontal surface. It is therefore greatest at the edges of the elevated or depressed column in a tube, and least in the centre; and the curvature, as measured by $\frac{1}{\text{R}} + \frac{1}{\text{R}'}$, must vary in the same proportion. If the tube is so large that there is no sensible elevation or depression in the centre of the column, the centre of the free surface must be sensibly plane.

188. Another consequence of the formula is, that in circumstances where there can be no normal pressure towards either side of the surface,

$$\frac{1}{R} + \frac{1}{R'} = 0; \qquad (2)$$

which implies that either the surface is plane, in which case each of the two terms is separately equal to zero, or else

$$R = -R'; \qquad (3)$$

that is, the principal radii of curvature are equal, and lie on opposite sides of the surface. The formulæ (2), (3) apply to a film of soapy water attached to a loop of wire. If the loop be in one plane, the film will be in the same plane. If the loop be not in one plane, the film cannot be in one plane, and will in fact assume that form which gives the least area consistent with having the loop for its boundary. At every point it will be observed to be, if we may so say, concave towards both sides, and convex towards both sides, the concavity being precisely equal to the convexity—that is to say, equation (3) is satisfied at every point of the film.

In this case both sides of the film are exposed to atmospheric pressure. In the case of a common soap-bubble the outside is exposed to atmospheric pressure, and the inside to a pressure somewhat greater, the difference of the pressures being balanced by the tendency of the film to contract. Formula (1) becomes for either the outer or inner surface of a spherical bubble

$$P = \frac{2T}{R};$$

but this result must be doubled, because there are two free surfaces; hence the excess of pressure of the inclosed above the external air is $\frac{4T}{R}$, R denoting the radius of the bubble.

The value of T for soapy water is about 1 grain per linear inch; hence, if we divide 4 by the radius of the bubble expressed in inches, we shall obtain the excess of internal over external pressure *in grains per square inch.*

The value of T for any liquid may be obtained by observing the amount of elevation or depression in a tube of given diameter, and employing the formula

$$T = \frac{whr}{2\cos\alpha}, \qquad (4)$$

which follows immediately from the formula for h in § 186.

189. It is this uniform surface tension, of which we have been

speaking, which causes a drop of a liquid falling through the air either to assume the spherical form, or to oscillate about the spherical form. The phenomena of drops can be imitated on an enlarged scale, under circumstances which permit us to observe the actual motions, by a method devised by Professor Plateau of Ghent. Olive-oil is intermediate in density between water and alcohol. Let a mixture of alcohol and water be prepared, having precisely the density of olive-oil, and let about a cubic inch of the latter be gently introduced into it with the aid of a funnel or pipette. It will assume a spherical form, and if forced out of this form and then left free, will slowly oscillate about it; for example, if it has been compelled to assume the form of a prolate spheroid, it will pass to the oblate form, will then become prolate again, and so on alternately, becoming however more nearly spherical every time, because its movements are hindered by friction, until at last it comes to rest as a sphere.

190. Capillarity furnishes no exception to the principle that the pressure in a liquid is the same at all points at the same depth. When the free surface within a tube is convex, and is consequently depressed below the plane surface of the external liquid, the pressure becomes suddenly greater on passing downwards through the superficial layer, by the amount due to the curvature. Below this it increases regularly by the amount due to the depth of liquid passed through. The pressure at any point vertically under the convex meniscus[1] may be computed, either by taking the depth of the point below the general free surface, and adding atmospheric pressure to the pressure due to this depth, according to the ordinary principles of hydrostatics, or by taking the depth of the point below that point of the meniscus which is vertically over it, adding the pressure due to the curvature at this point, and also adding atmospheric pressure.

When the free surface of the liquid within a tube is concave, the pressure suddenly diminishes on passing downwards through the superficial layer, by the amount due to the curvature as given by formula (1); that is to say, the pressure at a very small depth is less than atmospheric pressure by this amount. Below this depth it goes on increasing according to the usual law, and becomes equal to

[1] The convex or concave surface of the liquid in a tube is usually denoted by the name *meniscus* (μηνίσκος, a crescent), which denotes a form approximately resembling that of a watch-glass.