the side tube till the liquid flows over. In siphons for commercial purposes, the suction is usually produced by a pump.

273. Siphon for Sulphuric Acid.—Fig. 177 represents a siphon used for transferring sulphuric acid from one vessel to another. The long branch is first filled with sulphuric acid. This is effected by means of two funnels (which can be plugged at pleasure) at the bend of the tube. One of these admits the liquid, and the other suffers the air to escape. The two funnels are then closed, and the tap at the lower end of the tube is opened so as to allow the liquid to escape. The air in the short branch follows the acid, and becomes rarefied; the acid behind it rises, and if it passes the bend, the siphon will be started, for each portion of the liquid which issues from the tube will draw an equal portion from the short to the long branch.

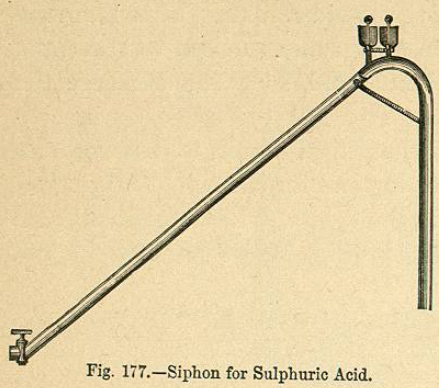

Fig. 177.—Siphon for Sulphuric Acid.

To insure the working of the sulphuric acid siphon, it is not sufficient to have the vertical height of the long branch greater than that of the short branch; it is farther necessary that it should exceed a certain limit, which depends upon the dimensions of the siphon in each particular case. In order to calculate this limit, we must remark that when the liquid begins to flow, its height diminishes in the long and increases in the short branch; if these two heights should become equal, there would be equilibrium. We see, then, that in order that the siphon may work, it is necessary that when the liquid rises to the bend of the tube, there should be in the long branch a column of liquid whose vertical height is at least equal to that of the short branch, which we shall denote by h, and the actual length of the short branch from the surface of the liquid in which it dips to the summit of the bend by h'. Then if α be the inclination of the long branch to the vertical, and L the length of the long branch, which we suppose barely sufficient, the length of the column of liquid remaining in the long branch will be $h \sec \alpha$. The air which at atmospheric pressure H occupied the length h', now under the pressure $H - h$ occupies a length $L - h \sec \alpha$; hence by Boyle's law, we have

$$Hh' = (H - h)(L - h \sec \alpha), \text{ whence } L = h \sec \alpha + \frac{Hh'}{H - h}.$$

In this formula H denotes the height of a column of sulphuric acid whose pressure equals that of the atmosphere.

274. Cup of Tantalus.—The siphon may be employed to produce the intermittent flow of a liquid. Suppose, for instance, that we have a cup (Fig. 178) in which is a bent tube rising to a height n, and with the short branch terminating near the bottom of the cup, while the long branch passes through the bottom. If liquid be poured into the cup, the level will gradually rise in the short branch of the bent tube, till it reaches the summit of the bend, when the siphon will begin to discharge the liquid. If the liquid then escapes by the siphon faster than it is poured into the vessel, the level of the liquid in the cup will gradually fall below the termination of the shorter branch. The siphon will then empty itself, and will not recommence its action till the liquid has again risen to the level of the bend.

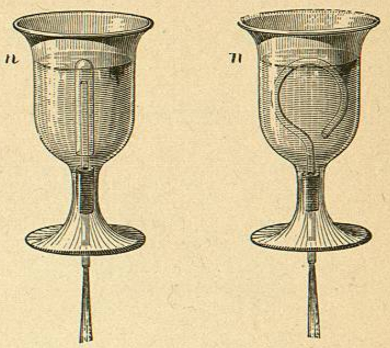

Fig. 178.—Vase of Tantalus.

The siphon may be concealed in the interior of the figure of a man whose mouth is just above the top of the siphon. If water be poured in very slowly, it will continually rise nearly to his lips and then descend again. Hence the name. Instead of a bent tube we may employ, as in the first figure, a straight tube covered by a bell-glass left open below; in this case the space between the tube and the bell takes the place of the shorter leg of the siphon.

It is to an action of this kind that natural intermittent springs are generally attributed. Suppose a reservoir (Fig. 179) to communicate with an outlet by a bent tube forming a siphon, and suppose it to be fed by a stream of water at a slower rate than the siphon is able to discharge it. When the water has reached the bend, the siphon will become charged, and the reservoir will be emptied; flow will then cease until it becomes charged again.

275. Mariotte's Bottle.—This is an apparatus often employed to obtain a uniform flow of water. Through the cork at the top of the bottle (Fig. 180) passes a straight vertical tube open at both ends, and in one side of the bottle near the bottom is a second opening furnished with a horizontal efflux tube *b* at a lower level than the lower end of the vertical tube. Suppose that both the bottle and the vertical tube are in the first instance full of water, and that the efflux tube is then opened. The liquid flows out, and the vertical tube is rapidly emptied. Air then enters the bottle through the vertical tube, and bubbles up from its lower end *a* through the liquid to the upper part of the bottle. As soon as this process begins, the velocity of efflux, which up to this point has been rapidly diminishing (as is shown by the diminished range of the

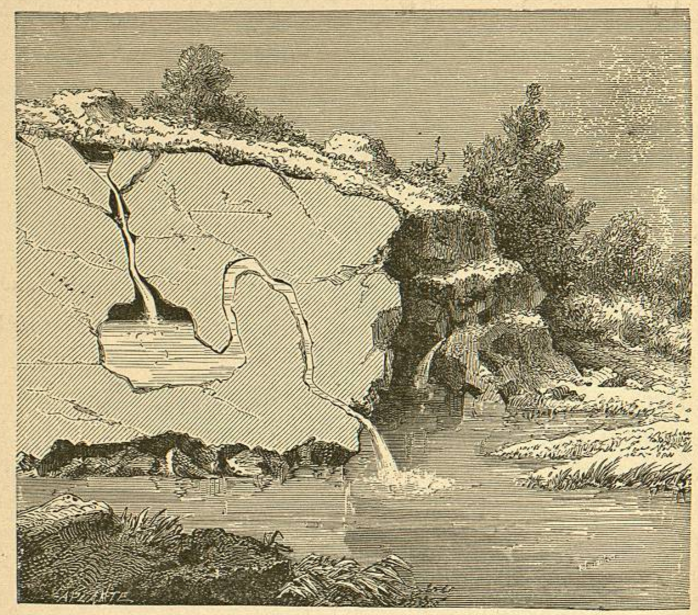

Fig. 179.—Intermittent Spring.

jet), becomes constant, and continues so till the level of the liquid has fallen to *a*, after which it again diminishes. During the time of constant flow, the velocity of efflux is that due to the height of *a* above *b*, and the air in the upper part of the bottle is at less than atmospheric pressure, the difference being measured by the height of the surface of the liquid above *a*. Strictly speaking, since the air enters not in a continuous stream but in bubbles, there must be slight oscillations of velocity, keeping time with the bubbles, but they are scarcely perceptible.

Instead of the vertical tube, we may have a second opening in the

side of the bottle, at a higher level than the first; as shown in Fig. 180. Air will enter through the pipe *a*, which is fitted in this upper opening, and the liquid will issue at the lower pipe *b*, with a constant velocity due to the height of *a* above *b*.

Mariotte's bottle is sometimes used in the laboratory to produce

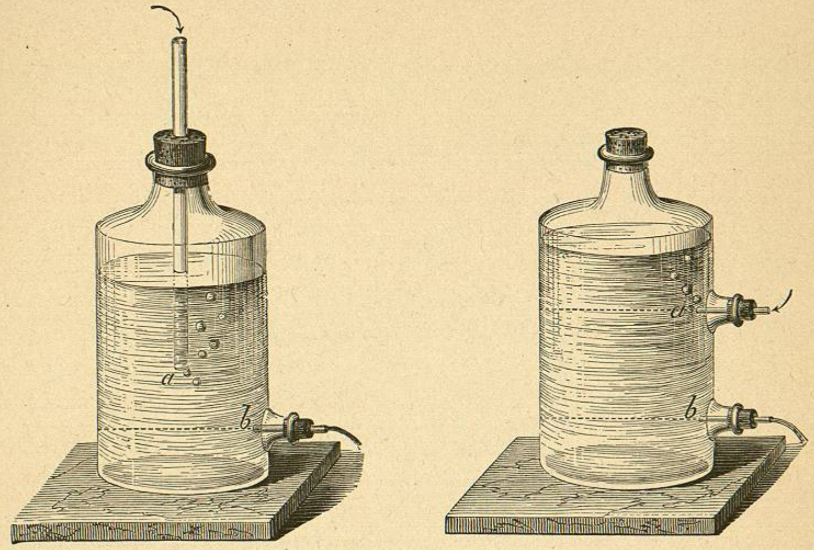

Fig. 180.—Mariotte's Bottle.

the uniform flow of a gas by employing the water which escapes to expel the gas. We may also draw in gas through the tube of Mariotte's bottle; in this case, the flow of the *water* is uniform, but the flow of the *gas* is continually accelerated, since the space occupied by it in the bottle increases uniformly, but the density of the gas in this space continually increases.

a U tube contains Hg to the height of 4 cu cms. in one leg and another liquid to the height of 54.4. cms in other Find density of other liquid Hg = 13.6

$4 \times 13.6 = 54.4 \times D.$

$D \quad D' \quad V \quad V'$

$p = wh$

$\frac{D}{D'} = \frac{V}{V'}$

$DV' = D'V$

4 = volume of mercury

54.4 = volume of liquid

13.6 = sp gr of mercury

X = sp — — of liquid

$4. \times 13.6 = 54.4. X$

$54.4 \overline{)54.4}$

$X = 1$

$p = p_1 \left(\frac{V}{V+v}\right)^n$

$p = p_1 \left(\frac{V}{V+v}\right)$

$pV = p_1(V+v)$

watter.

7.55
685
1.700 (7.38
5100 0.50

a ball
rate.
e nor-
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Find
point
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ween
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direc-
0, 15,
3 lbs.
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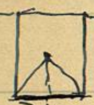
n the
t rod
eight

A vertical flood gate is 5 ft height & 3 feet wide Its upper edge is at the surface of the watter. Find pressure



$5 \times 3 = 15$ ~~sq feet~~

$P = VD$

5 | 2
10, 2.5

$P = 15 \times 1$

$P = 15 \times 62.5$

	62.5
	15
	3125
	625
	937.5

62.5 weight of a cubic pound of watter.

$P = 937.5 \times 5/2 = 2{,}343.75$ lbs.

P = height of centre of gravity times volume times weights of a cubic pound of watter

	937.5
	2.5
	46875
	18750
	2343.75 ans

EXAMPLES.

Parallelogram of Velocities, and Parallelogram of Forces.

1. A ship sails through the water at the rate of 10 miles per hour, and a ball rolls across the deck in a direction perpendicular to the course, at the same rate. Find the velocity of the ball relative to the water.

2. The wind blows from a point intermediate between N. and E. The northerly component of its velocity is 5 miles per hour, and the easterly component is 12 miles per hour. Find the total velocity.

3. The wind is blowing due N.E. with a velocity of 10 miles an hour. Find the northerly and easterly components.

4. Two forces of 6 and 8 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

5. Two equal forces of 100 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

6. A force of 100 units acts at an inclination of 45° to the horizon. Resolve it into a horizontal and a vertical component.

7. Two equal forces act in lines which meet in a point, and the angle between their directions is 120°. Show that the resultant is equal to either of the forces.

8. A body is pulled north, south, east, and west by four strings whose directions meet in a point, and the forces of tension in the strings are equal to 10, 15, 20, and 32 lbs. weight respectively. Show that the resultant is equal to 13 lbs. weight.

9. Five equal forces act at a point, in one place. The angles between the first and second, between the second and third, between the third and fourth, and between the fourth and fifth, are each 60°. Find their resultant.

10. If θ be the angle between the directions of two forces P and Q acting at a point, and R be their resultant, show that

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta.$$

11. Show that the resultant of two equal forces P, acting at an angle θ, is $2P \cos \frac{1}{2}\theta$.

Parallel Forces, and Centre of Gravity.

10*. A straight rod 10 ft. long is supported at a point 3 ft. from one end. What weight hung from this end will be supported by 12 lbs. hung from the other, the weight of the rod being neglected?

11*. Weights of 15 and 20 lbs. are hung from the two ends of a straight rod 70 in. long. Find the point about which the rod will balance, its own weight being neglected.