A nerticul flood gate is 5ft height A 3 feet wide Its upper edge is at the surface of the watter. Find prom 5 x 3 = 15 paperix P= ·VD 512 P=15.X1 3175 x 10, 2,5 P= 15. × 62.5 625 62.5 weight of a cubic found of water. P= height of centre of grandy times volume times weight of a cubic pourt of water 18750 18750 2343,75 aus

EXAMPLES.

PARALLELOGRAM OF VELOCITIES, AND PARALLELOGRAM OF FORCES.

1. A ship sails through the water at the rate of 10 miles per hour, and a ball rolls across the deck in a direction perpendicular to the course, at the same rate. Find the velocity of the ball relative to the water.

2. The wind blows from a point intermediate between N. and E. The northerly component of its velocity is 5 miles per hour, and the easterly component is 12 miles per hour. Find the total velocity.

3. The wind is blowing due N.E. with a velocity of 10 miles an hour. Find the northerly and easterly components.

4. Two forces of 6 and 8 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

5. Two equal forces of 100 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

6. A force of 100 units acts at an inclination of 45° to the horizon. Resolve it into a horizontal and a vertical component.

7. Two equal forces act in lines which meet in a point, and the angle between their directions is 120°. Show that the resultant is equal to either of the forces.

8. A body is pulled north, south, east, and west by four strings whose directions meet in a point, and the forces of tension in the strings are equal to 10, 15, 20, and 32 lbs. weight respectively. Show that the resultant is equal to 13 lbs. weight.

9. Five equal forces act at a point, in one place. The angles between the first and second, between the second and third, between the third and fourth, and between the fourth and fifth, are each 60°. Find their resultant.

10. If θ be the angle between the directions of two forces P and Q acting at a point, and R be their resultant, show that

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta.$$

11. Show that the resultant of two equal forces P, acting at an angle θ , is $2P\cos\frac{1}{2}\theta$.

PARALLEL FORCES, AND CENTRE OF GRAVITY.

10*. A straight rod 10 ft. long is supported at a point 3 ft. from one end. What weight hung from this end will be supported by 12 lbs. hung from the other, the weight of the rod being neglected?

11*. Weights of 15 and 20 lbs. are hung from the two ends of a straight rod 70 in. long. Find the point about which the rod will balance, its own weight being neglected.

12. A weight of 100 lbs. is slung from a pole which rests on the shoulders of two men, A and B. The distance between the points where the pole presses their shoulders is 10 ft., and the point where the weight is slung is 4 ft. from the point where the pole presses on A's shoulder. Find the weight borne by each, the weight of the pole being neglected.

13. A uniform straight lever 10 ft. long balances at a point 3 ft. from one end, when 12 lbs. are hung from this end and an unknown weight from the other.

The lever itself weighs 8 lbs. Find the unknown weight.

14. A straight lever 6 ft. long weighs 10 lbs., and its centre of gravity is 4 ft. from one end. What weight at this end will support 20 lbs. at the other, when the lever is supported at 1 ft. distance from the latter?

15. Two equal weights of 10 lbs. each are hung one at each end of a straight lever 6 ft. long, which weighs 5 lbs.; and the lever, thus weighted, balances about a point 3 in. distant from the centre of its length. Find its centre of gravity.

16. A uniform lever 10 ft. long balances about a point 1 ft. from one end,

when loaded at that end with 50 lbs. Find the weight of the lever.

17. A straight lever 10 ft. long, when unweighted, balances about a point 4 ft. from one end; but when loaded with 20 lbs. at this end and 4 lbs. at the other. it balances about a point 3 ft. from the end. Find the weight of the lever.

18. A lever is to be cut from a bar weighing 3 lbs. per ft. What must be its length that it may balance about a point 2 ft. from one end, when weighted at this end with 50 lbs.? (The solution of this question involves a quadratic equation.)

19. A lever is supported at its centre of gravity, which is nearer to one end than to the other. A weight P at the shorter arm is balanced by 2 lbs. at the longer; and the same weight P at the longer arm is balanced by 18 lbs. at the shorter. Find P.

20. Weights of 2, 3, 4 and 5 lbs. are hung at points distant respectively 1, 2, 3 and 4 ft. from one end of a lever whose weight may be neglected. Find the point about which the lever thus weighted will balance. (This and the following questions are best solved by taking moments round the end of the lever. The sum of the moments of the four weights is equal to the moment of their resul-

21. Solve the preceding question, supposing the lever to be 5 ft. long, uniform, and weighing 2 lbs.

22. Find, in position and magnitude, the resultant of two parallel and oppositely directed forces of 10 and 12 units, their lines of action being 1 yard apart.

23. A straight lever without weight is acted on by four parallel forces at the following distances from one end:-

> At 1 ft., a force of 2 units, acting upwards. At 2 ft., ,, 3 ,, downwards. At 3 ft., upwards. At 4 ft., " downwards.

Where must the fulcrum be placed that the lever may be in equilibrium, and what will be the pressure against the fulcrum?

24. A straight lever, turning freely about an axis at one end, is acted on by four parallel forces, namelyA downward force of 3 lbs. at 1 ft. from axis. A downward force of 5 ,, 3 ft.

An upward force of 4 ,, 2 ft. An upward force of 6 ,, 4 ft.

What must be the weight of the lever that it may be in equilibrium, its centre of gravity being 3 ft. from the axis?

25. In a pair of nut-crackers, the nut is placed one inch from the hinge, and the hand is applied at a distance of six inches from the hinge. How much pressure must be applied by the hand, if the nut requires a pressure of 13 lbs. to break it, and what will be the amount of the pressure on the hinges?

26. In the steelyard, if the horizontal distance between the fulcrum and the knife-edge which supports the body weighed be 3 in., and the movable weight be 7 lbs., how far must the latter be shifted for a difference of 1 lb. in the body

weighed?

27. The head of a hammer weighs 20 lbs. and the handle 2 lbs. The distance between their respective centres of gravity is 24 inches. Find the distance of the centre of gravity of the hammer from that of the head.

28. One of the four triangles into which a square is divided by its diagonals is removed. Find the distance of the centre of gravity of the remainder from the

intersection of the diagonals.

29. A square is divided into four equal squares and one of these is removed. Find the distance of the centre of gravity of the remaining portion from the centre of the original square.

30. Find the centre of gravity of a sphere 1 decimetre in radius, having in its interior a spherical excavation whose centre is at a distance of 5 centimetres from

the centre of the large sphere and whose radius is 4 centimetres.

31. Weights P, Q, R, S are hung from the corners A, B, C, D of a uniform square plate whose weight is W. Find the distances from the sides AB, AD of the point about which the plate will balance.

32. An isosceles triangle stands upon one side of a square as base, the altitude of the triangle being equal to a side of the square. Show that the distance of the centre of the whole figure from the opposite side of the square is $\frac{7}{9}$ of a side of the

33. A right cone stands upon one end of a right cylinder as base, the altitude of the cone being equal to the height of the cylinder. Show that the distance of the centre of the whole volume from the opposite end of the cylinder is $\frac{11}{16}$ of the height of the cylinder.

WORK AND STABILITY.

34. A body consists of three pieces, whose masses are as the numbers 1, 3, 9; and the centres of these masses are at heights of 2, 3, and 5 cm. above a certain level. Find the height of the centre of the whole mass above this level.

35. The body above-mentioned is moved into a new position, in which the heights of the centres of the three masses are 1, 3, and 7 cm. Find the new

height of the centre of the whole mass.

36. Find the work done against gravity in moving the body from the first position into the second; employing as the unit of work the work done in raising the smallest of the three pieces through 1 cm.

37. Find the portions of this work done in moving each of the three pieces.

38. The dimensions of a rectangular block of stone of weight W are AB = a, AC = b, AD = c, and the edges AB, AC are initially horizontal. How much work is done against gravity in tilting the stone round the edge AB until it balances.

39. A chain of weight W and length l hangs freely by its upper end which is attached to a drum upon which the chain can be wound, the diameter of the drum being small compared with l. Compute the work done against gravity in winding up two-thirds of the chain.

40. Two equal and similar cylindrical vessels with their bases at the same level contain water to the respective heights h and H centimetres, the area of either base being α sq. cm. Find, in gramme-centimetres, the work done by gravity in equalizing the levels when the two vessels are connected.

41. Two forces acting at the ends of a rigid rod without weight equilibrate each other. Show that the equilibrium is stable if the forces are pulling outwards

and unstable if they are pushing inwards.

42. Two equal weights hanging from the two ends of a string, which passes over a fixed pulley without friction, balance one another. Show that the equilibrium is neutral if the string is without weight, and is unstable if the string is heavy.

43. Show that a uniform hemisphere resting on a horizontal plane has two positions of stable equilibrium. Has it any positions of unstable equilibrium?

INCLINED PLANE, &c.

- 44. On an inclined plane whose height is 1/8 of its length, what power acting parallel to the plane will sustain a weight of 112 lbs. resting on the plane without
- 45. The height, base, and length of an inclined plane are as the numbers 3, 4, 5. What weight will be sustained on the plane without friction by a power of 100 lbs. acting (a) parallel to the base, (b) parallel to the plane?

46. Find the ratio of the power applied to the pressure produced in a screwpress without friction, the power being applied at the distance of 1 ft. from the

axis of the screw, and the distance between the threads being \frac{1}{8} in.

47. In the system of pulleys in which one cord passes round all the pulleys, its different portions being parallel, what power will sustain a weight of 2240 lbs. without friction, if the number of cords at the lower block be 6?

48. A balance has unequal arms, but the beam assumes the horizontal position when both scale-pans are empty. Show that if the two apparent weights of a body are observed when it is placed first in one pan and then in the other, the true weight will be found by multiplying these together and taking the square root.

Force, Mass, and Velocity.

The motion is supposed to be rectilinear.

- 49. A force of 1000 dynes acting on a certain mass for one second gives it a velocity of 20 cm. per sec. Find the mass in grammes.
- 50. A constant force acting on a mass of 12 gm. for one sec. gives it a velocity of 6 cm. per sec. Find the force in dynes.

- 51. A force of 490 dynes acts on a mass of 70 gm. for one sec. Find the velocity generated.
- 52. In the preceding example, if the time of action be increased to 5 sec., what will be the velocity generated?

In the following examples the unit of momentum referred to is the momentum of a gramme moving with a velocity of a centimetre per second.

- 53. What is the momentum of a mass of 15 gm. moving with a velocity of translation of 4 cm. per sec.?
- 54. What force, acting upon the mass for 1 sec., would produce this velocity?
- 55. What force, acting upon the mass for 10 sec., would produce the same velocity?
- 56. Find the force which, acting on an unknown mass for 12 sec., would produce a momentum of 84.
- 57. Two bodies initially at rest move towards each other in obedience to mutual attraction. Their masses are respectively 1 gm. and 100 gm. If the force of attraction be $\frac{1}{100}$ of a dyne, find the velocity acquired by each mass in 1 sec.
- 58. A gun is suspended by strings so that it can swing freely. Compare the velocity of discharge of the bullet with the velocity of recoil of the gun; the masses of the gun and bullet being given, and the mass of the powder being neglected.
- 59. A bullet fired vertically upwards, enters and becomes imbedded in a block of wood falling vertically overhead; and the block is brought to rest by the impact. If the velocities of the bullet and block immediately before collision were respectively 1500 and 100 ft. per sec., compare their masses.

FALLING BODIES AND PROJECTILES.

Assuming that a falling body acquires a velocity of 980 cm. per sec. by falling for 1 sec., find :-

- 60. The velocity acquired in $\frac{1}{10}$ of a second.
- 61. The distance passed over in \(\frac{1}{10}\) sec.
- 62. The distance that a body must fall to acquire a velocity of 980 cm. per sec.
- 63. The time of rising to the highest point, when a body is thrown vertically upwards with a velocity of 6860 cm. per sec.
- 64. The height to which a body will rise, if thrown vertically upwards with a velocity of 490 cm. per sec.
- 65. The velocity with which a body must be thrown vertically upwards that it may rise to a height of 200 cm.
- 66. The velocity that a body will have after $\frac{3}{10}$ sec., if thrown vertically upwards with a velocity of 300 cm. per sec.
 - 67. The point that the body in last question will have attained.
- 68. The velocity that a body will have after 2½ secs., if thrown vertically upwards with a velocity of 800 cm. per sec.
 - 69. The point that the body in last question will have reached.

Assuming that a falling body acquires a velocity of 32 ft. per sec. by falling for

- 70. The velocity acquired in 12 sec.
- 71. The distance fallen in 12 sec.

72. The distance that a body must fall to acquire a velocity of 10 ft. per sec.

73. The time of rising to the highest point, when a body is thrown vertically upwards with a velocity of 160 ft. per sec.

74. The height to which a body will rise, if thrown vertically upwards with a velocity of 32 ft. per sec.

75. The velocity with which a body must be thrown vertically upwards that it may rise to a height of 25 ft.

76. The velocity that a body will have after 3 sec., if thrown vertically upwards with a velocity of 100 ft. per sec.

77. The height that the body in last question will have ascended.

78. The velocity that a body will have after $1\frac{1}{2}$ sec., if thrown vertically downwards with a velocity of 30 ft. per sec.

79. The distance that the body in last question will have described.

80. A body is thrown horizentally from the top of a tower 100 m. high with a velocity of 30 metres per sec. When and where will it strike the ground?

81. Two bodies are successively dropped from the same point, with an interval of $\frac{1}{5}$ of a second. When will the distance between them be one metre?

82. Show that if x and y are the horizontal and vertical co-ordinates of a projectile referred to the point of projection as origin, their values after time t are

$$\alpha = Vt \cos \alpha$$
, $y = Vt \sin \alpha - \frac{1}{2} gt^2$.

83. Show that the equation to the trajectory is

$$y = x \tan \alpha - \frac{g x^2}{2V^2 \cos^2 \alpha},$$

and that if V and α can be varied at pleasure, the projectile can in general be made to traverse any two given points in the same vertical plane with the point of projection.

ATWOOD'S MACHINE.

Two weights are connected by a cord passing over a pulley as in Atwood's machine, friction being neglected, and also the masses of the pulley and cord; find:—

84. The acceleration when one weight is double of the other.

85. The acceleration when one weight is to the other as 20 to 21.

Taking g as 980, in terms of the cm. and sec., find:—

86. The velocity acquired in 10 sec., when one weight is to the other as 39 to 41.

87. The velocity acquired in moving through 50 cm., when the weights are as 19 to 21.

88. The distance through which the same weights must move that the velocity acquired may be double that in last question.

89. The distance through which two weights which are as 49 to 51 must move that they may acquire a velocity of 98 cm. per sec.

ENERGY AND WORK.

90. Express in ergs the kinetic energy of a mass of 50 gm, moving with a velocity of 60 cm, per sec.

91. Express in ergs the work done in raising a kilogram through a height of

1 metre, at a place where g is 981.

92. A mass of 123 gm. is at a height of 2000 cm. above a level floor. Find its energy of position estimated with respect to the floor as the standard level (g being 981).

93. A body is thrown vertically upwards at a place where g is 980. If the velocity of projection is 9800 cm. per sec. and the mass of the body is 22 gm., find the energy of the body's motion when it has ascended half way to its maximum height. Also find the work done against gravity in this part of the ascent.

94. The height of an inclined plane is 12 cm., and the length 24 cm. Find the work done by gravity upon a mass of 1 gm. in sliding down this plane (g being 980), and the velocity with which the body will reach the bottom if there be no friction.

95. If the plane in last question be not frictionless, and the velocity on reaching the bottom be 20 cm. per sec., find how much energy is consumed in function

96. Find the work expended in discharging a bullet whose mass is 30 gm. with a velocity of 40,000 cm. per sec.; and the number of such bullets that will be discharged with this velocity in a minute if the rate of working is 7460 million ergs per sec. (one horse-power).

97. One horse-power being defined as 550 foot-pounds per sec.; show that it is nearly equivalent to 8.8 cubic ft. of water lifted 1 ft. high per sec. (A cubic foot of water weighs $62\frac{1}{2}$ lbs. nearly. A foot-pound is the work done against gravity in lifting a pound through a height of 1 ft.)

98. How many cubic feet of water will be raised in one hour from a mine

200 ft. deep, if the rate of pumping be 15 horse-power?

CENTRIFUGAL FORCE.

99. What must be the radius of curvature, that the centrifugal force of a body travelling at 30 miles an hour may be one-tenth of the weight of the body; g being 981, and a mile an hour being 44.7 cm. per sec.?

100. A heavy particle moves freely along a frictionless tube which forms a vertical circle of radius a. Find the velocity which the particle will have at the lowest point, if it all but comes to rest at the highest. Also find its velocity at the lowest point if in passing the highest point it exerts no pressure against the tube. [Use the principle that what is lost in energy of position is gained in energy of motion.]

101. Show that the total intensity of centrifugal force due to the earth's rotation, at a place in latitude λ , is $\omega^2 \operatorname{R} \cos \lambda$, ω denoting $\frac{2\pi}{\mathrm{T}}$, and R the earth's radius; that the vertical component (tending to diminish gravity) is $\omega^2 \operatorname{R} \cos^2 \lambda$, and that the horizontal component (directed from the pole towards the equator) is $\omega^2 \operatorname{R} \cos \lambda \sin \lambda$.

EXAMPLES.

247

101*. The length of the seconds pendulum at Greenwich is 99:413 cm.; find the length of a pendulum which makes a single vibration in 11 sec.

102. The weight of a fly-wheel is M grammes, and the distance of the inside of the rim from the axis of revolution is R centims. Supposing this distance to be identical with k (§ 117), find the moment of inertia.

If a force of F dynes acts steadily upon the wheel at an arm of α centims. what will be the value of the angular velocity $\frac{2\pi}{n}$ after the lapse of t seconds from the commencement of motion?

103. For a uniform thin rod of length a, swinging about a point of suspension at one end, the moment of inertia is the mass of the rod multiplied by $\frac{1}{3}a^2$. Find the length of the equivalent simple pendulum; also the moment of inertia round a parallel axis through the centre of the rod.

104. At what point in its length must the rod in last question be suspended to give a minimum time of vibration: and at what point must it be suspended to give the same time of vibration as if suspended at one end?

105. Show that if P be the mass of the pulley in Atwood's machine, r its radius, and Pk2 its moment of inertia, the value of C in § 100 will be P plus the mass of the string. [The mass of the friction-wheels is neglected.]

106. A body moves with constant velocity in a vertical circle, going once round per second; and its shadow is cast upon level ground by a vertical sun. Find the value of μ (§ 111) for the shadow, using the centimetre and second as

107. What is the value of μ for one of the prongs of a C tuning-fork which makes 512 complete vibrations per second?

PRESSURE OF LIQUIDS.

Find, in gravitation measure (grammes per sq. cm.), atmospheric pressure being neglected :-

108. The pressure at the depth of a kilometre in sea-water of density 1.025.

109. The pressure at the depth of 65 cm. in mercury of density 13:59.

110. The pressure at the depth of 2 cm. in mercury of density 13:59 surmounted by 3 cm. of water of unit density, and this again by 12 cm. of oil of density 9.

Find, in centimetres of mercury of density 13.6, atmospheric pressure being included, and the barometer being supposed to stand at 76 cm.:-

111. The pressure at the depth of 10 metres in water of unit density.

112. The pressure at the depth of a mile in sea-water of density 1.026, a mile being 160933 cm.

Find, in dynes per square centimetre, taking g as 981:—

113. The pressure due to 1 cm. of mercury of density 13:596.

114. The pressure due to a foot of water of unit density, a foot being 30:48 cm.

115. The pressure due to the weight of a layer a metre thick, of air of density

116. At what depth, in brine of density 1.1, is the pressure the same as at a depth of 33 feet in water of unit density?

117. At what depth, in oil of density 9, is the pressure the same as at the depth of 10 inches in mercury of density 13:596?

118. With what value of g will the pressure of 3 cm. of mercury of density 13.596 be 4×10^4 ?

Find, in grammes weight, the amount of pressure (atmospheric pressure being neglected):-

119. On a triangular area of 9 sq. cm. immersed in naphtha of density 848; the centre of gravity of the triangle being at the depth of 6 cm.

120. On a rectangular area 12 cm. long, and 9 cm. broad, immersed in mercury of density 13.596; its highest and lowest corners being at depths of 3 cm. and 7 cm. respectively.

121. On a circular area of 10 cm. radius, immersed in alcohol of density '791, the centre of the circle being at the depth of 4 cm.

122. On a triangle whose base is 5 cm. and altitude 6 cm., the base being at the uniform depth of 9 cm., and the vertex at the depth of 7 cm., in water of unit

123. On a sphere of radius r centimetres, completely immersed in a liquid of density d; the centre of the sphere being at the depth of h centimetres. [The amount of pressure in this case is not the resultant pressure.]

DENSITY, AND PRINCIPLE OF ARCHIMEDES.

Densities are to be expressed in grammes per cubic centimetre.

124. A rectangular block of stone measures 86 × 37 × 16 cm., and weighs 120 kilogrammes. Find its density.

125. A specific-gravity bottle holds 100 gm. of water, and 180 gm. of sulphuric acid. Find the density of the acid.

126. A certain volume of mercury of density 13.6 weighs 216 gm., and the same volume of another liquid weighs 14.8 gm. Find the density of this liquid.

127. Find the mean section of a tube 16 cm. long, which holds 1 gm. of mercury of density 13.6.

128. A bottle filled with water, weighs 212 gm. Fifty grammes of filings are thrown in, and the water which flows over is removed, still leaving the bottle just filled. The bottle then weighs 254 gm. Find the density of the filings.

129. Find the density of a body which weighs 58 gm. in air, and 46 gm. in water of unit density.

130. Find the density of a body which weighs 63 gm. in air, and 35 gm. in a liquid of density '85.

EXAMPLES.

249

131. A glass ball loses 33 gm. when weighed in water, and loses 6 gm. more when weighed in a saline solution. Find the density of the solution.

132. A body, lighter than water, weighs 102 gm. in air; and when it is immersed in water by the aid of a sinker, the joint weight is 23 gm. The sinker alone weighs 50 gm. in water. Find the density of the body.

133. A piece of iron, when plunged in a vessel full of water, makes 10 grammes run over. When placed in a vessel full of mercury it floats, displacing 78 grammes of mercury. Required the weight, volume, and specific gravity of the iron.

 $134.\ {\rm Find}$ the volume of a solid which weighs $357\ {\rm gm}.$ in air, and $253\ {\rm gm}.$ in water of unit density.

135. Find the volume of a solid which weighs 458 gm, in air, and 409 gm, in brine of density 1.2.

136. How much weight will a body whose volume is 47 cubic cm. lose, by weighing in a liquid whose density is 2.5?

137. Find the weights in air, in water, and in mercury, of a cubic cm. of gold of density 19.3.

 $138.\ A$ wire 1293 cm. long loses 508 gm. by weighing in water. Find its mean section, and mean radius.

139. A copper wire 2156 cm. long weighs 158 gm. in air, and 140 gm. in water. Find its volume, density, mean section, and mean radius.

140. What will be the weights, in air and in water, of an iron wire 1000 cm. long and a millimetre in diameter, its density being 7.7?

141. How much water will be displaced by 1000 c.c. of oak of density 9, floating in equilibrium?

142. A ball, of density 20 and volume 3 c.c., is surmounted by a cylindrical stem, of density 2:5, of length 12 cm., and of cross section $\frac{1}{3}$ sq. cm. What length of the stem will be in air when the body floats in equilibrium in mercury of density 13:6?

143. A hollow closed cylinder, of mean density 4 (including the hollow space), is weighted with a ball of volume 5, and mean density 2. What must be the volume of the cylinder, that exactly half of it may be immersed, when the body is left to itself in water?

144. A long cylindrical tube, constructed of flint glass of density 3, is closed at both ends, and is found to have the property of remaining at whatever depth it is placed in water. If the mass of the ends can be neglected, show that the ratio of the internal to the external radius is $\sqrt{\frac{2}{3}}$

145. A glass bottle provided with a stopper of the same material weighs 120 gm. when empty. When it is immersed in water, its apparent weight is 10 gm., but when the stopper is loosened and the water let in, its apparent weight is 80 gm. Find the density of the glass and the capacity of the bottle.

146. A hydrometer sinks to a certain depth in a fluid of density 8; and if 100 gm. be placed upon it, it sinks to the same depth in water. Find the weight of the hydrometer.

147. Find the mean density of a combination of 8 parts by volume of a substance of density 7, with 19 of a substance of density 3.

148. Find the mean density of a combination of 8 parts by weight of a substance of density 7, with 19 of a substance of density 3.

149. What volume of fir, of density 5, must be joined to 3 c.c. of iron, of density 7·1, that the mean density of the whole may be unity?

150. What mass of fir, of density 5, must be joined to 300 gm. of iron, of density 7·1, that the mean density of the whole may be unity?

151. Two parts by volume of a liquid of density 8, are mixed with 7 of water, and the mixture shrinks in the ratio of 21 to 20. Find its density.

152. A piece of iron of density 7.5 floats in mercury of density 13.5, and is completely covered with water which rests on the top of the mercury. How much of the iron is immersed in the mercury?

153. Two liquids are mixed. The total volume is 3 litres, with a sp. gr. of 0.9. The sp. gr. of the first liquid is 1.3, of the second 0.7. Find their volumes.

154. What volume of platinum of density 21.5 must be attached to a litre of iron of density 7.5 that the system may float freely at all depths in mercury of density 13.5?

155. What must be the thickness of a hollow sphere of platinum with an external radius of 1 decim., that it may barely float in water?

156. A sphere of cork of density '24, 3 cm. in radius, is weighted with a sphere of gold of density 19.3. What must be the radius of the latter that the system may barely float in alcohol of density '8?

157. An alloy of gold and silver has density D. The density of gold is d, that of silver d'. Find the proportions by weight of the two metals in the alloy, supposing that neither expansion nor contraction occurs in its formation.

158. A mixture of gold, of density 19.3, with silver, of density 10.5, has the density 18. Assuming that the volume of the alloy is the sum of the volumes of its components, find how many parts of gold it contains for one of silver—(a) by volume; (b) by weight.

159. A body weighs gM dynes in air of density A, gm in water, and gx in vacuo. Find x in terms of M, m, and A.

CAPILLARITY.

160. A horizontal disc of glass is held up by means of a film of water between it and a similar disc of the same or a larger size above it.

If R denote the radius of the lower disc,

d the distance between the discs, which is very small compared with R,

T the surface tension of water,

show that the weight of the lower disc together with that of the water between the discs is approximately equal to $\frac{2T\pi R^2}{d}$.

[The disc of water will be concave at the edge, and the radius of curvature of the concavity may be taken as $\frac{1}{2}d$.]

161. The surface-tension of water at 20° C. is 81 dynes per linear centim. How high will water be elevated by capillary action in a wetted tube whose diameter is half a millimetre?