162. How much will mercury be depressed by capillary action in a glass tube of half a millimetre diameter, the surface-tension of mercury at 20° C. being 418 dynes per cm., its density 13·54, and the cosine of the angle of contact ·703?

163. Show by the method of § 186 that the capillary elevation or depression will be the same in a square tube as in a circular tube whose diameter is equal to a side of the square.

164. Two equal discs in a vertical position have a film of water between them sustained by capillary action. Show that if the water at the lowest point is at atmospheric pressure, the water at the centre of the discs is at a pressure less than atmospheric by rg dynes per sq. cm., r being the common radius of the discs in cm.; and that the discs are pressed together with a force of $\pi r^3 g$ dynes.

Barometer, and Boyle's Law.

165. A bent tube, having one end open and the other closed, contains mercury which stands 20 cm. higher in the open than in the closed branch. Compare the pressure of the air in the closed branch with that of the external air; the barometer at the time standing at 75 cm.

166. The cross sections of the open and closed branches of a siphon barometer are as 6 to 1. What distance will the mercury move in the closed branch, when a normal barometer alters its reading by 1 inch?

167. If the section of the closed limb of a siphon barometer is to that of the open limb as a to b, show that a rise of 1 cm. in the mercury in the closed limb corresponds to a rise of $\frac{a+b}{b}$ cm. of the theoretical barometer.

168. Compute, in dynes per sq. cm., the pressure due to the weight of a column of mercury 76 cm. high at the equator, where g is 978, and at the pole, where g is 983.

169. The volumes of a given quantity of mercury at 0° C. and 100° C. are as 1 to 1·0182. Compute the height of a column of mercury at 100°, which will produce the same pressure as 76 cm. of mercury at 0°.

170. The volumes of a given mass of mercury, at 0° and 20°, are as 1 to 1·0036. Find the height reduced to 0°, when the actual height (in true centimetres), at a temperature of 20°, is 76·2.

171. In performing the Torricellian experiment a little air is left above the mercury. If this air expands a thousandfold, what difference will it make in the height of the column of mercury sustained when a normal barometer reads 76 cm.?

172. In performing the Torricellian experiment, an inch in length of the tube is occupied with air at atmospheric pressure, before the tube is inverted. After the inversion, this air expands till it occupies 15 inches, while a column of mercury 28 inches high is sustained below it. Find the true barometric height.

173. The mercury stands at the same level in the open and in the closed branch of a bent tube of uniform section, when the air confined at the closed end is at the pressure of 30 inches of mercury, which is the same as the pressure of the external air. Express, in atmospheres, the pressure which, acting on the surface of the mercury in the open branch, compresses the confined air to half its original volume, and at the same time maintains a difference of 5 inches in the levels of the two mercurial columns.

174. At what pressure (expressed in atmospheres) will common air have the same density which hydrogen has at one atmosphere; their densities when compared at the same pressure being as 1276 to 88·4?

175. Two volumes of oxygen, of density ·00141, are mixed with three of nitrogen, of density ·00124. Find the density of the mixture—(a) if it occupies five volumes; (b) if it is reduced to four volumes.

176. The mass of a cub. cm. of air, at the temperature 0° C., and at the pressure of a million dynes to the square cm., is ·0012759 gramme. Find the mass of a cubic cm. of air at 0° C., under the pressure of 76 cm. of mercury—(a) at the pole, where g is 983·1; (b) at the equator, where g is 978·1; (c) at a place where g is 981.

177. Show that the density of air at a given temperature, and under the pressure of a given column of mercury, is greater at the pole than at the equator by about 1 part in 196; and that the gravitating force of a given volume of it is greater at the pole than at the equator by about 1 part in 98.

178. A cylindrical test-tube, 1 decim. long, is plunged, mouth downwards, into mercury. How deep must it be plunged that the volume of the inclosed air may be diminished by one-half?

179. The pressure indicated by a siphon barometer whose vacuum is defective is 750 mm., and when mercury is poured into the open branch till the barometric chamber is reduced to half its former volume, the pressure indicated is 740 mm. Deduce the true pressure.

180. An open manometer, formed of a bent tube of iron whose two branches are parallel and vertical, and of a glass tube of larger size, contains mercury at the same level in both branches, this level being higher than the junction of the iron with the glass tube. What must be the ratio of the sections of the two tubes, that the mercury may ascend half a metre in the glass tube when a pressure of 6 atmospheres is exerted in the opposite branch?

181. A curved tube has two vertical legs, one having a section of 1 sq. cm., the other of 10 sq. cm. Water is poured in, and stands at the same height in both legs. A piston, weighing 5 kilogrammes, is then allowed to descend, and press with its own weight upon the surface of the liquid in the larger leg. Find the elevation thus produced in the surface of the liquid in the smaller leg.

Pumps, &c.

182. The sectional area of the small plunger in a Bramah press is 1 sq. cm., and that of the larger 100 sq. cm. The lever handle gives a mechanical advantage of 6. What weight will the large plunger sustain when 1 cwt. is hung from the handle?

183. The diameter of the small plunger is half an inch; that of the larger 1 foot. The arms of the lever handle are 3 in. and 2 ft. Find the total mechanical advantage.

184. Find, in grammes weight, the force required to sustain the piston of a suction-pump without friction, if the radius of the piston be 15 cm., the depth

from it to the surface of the water in the well 600 cm., and the height of the column of water above it 50 cm. Show that the answer does not depend on the size of the pipe which leads down to the well.

185. Two vessels of water are connected by a siphon. A certain point P in its interior is 10 cm. and 30 cm. respectively above the levels of the liquid in the two vessels. The pressure of the atmosphere is 1000 grammes weight per sq. cm. Find the pressure which will exist at P—(*a*) if the end which dips in the upper vessel be plugged; (*b*) if the end which dips in the lower vessel be plugged.

186. If the receiver has double the volume of the barrel, find the density of the air remaining after 10 strokes, neglecting leakage, &c.

187. Air is forced into a vessel by a compression pump whose barrel has $\frac{1}{10}$th of the volume of the vessel. Compute the density of the air in the vessel after 20 strokes.

188. In the pump of Fig. 136 show that the excess of the pressure on the upper above that on the lower side of the piston, at the end of the first up-stroke, is $\frac{V}{V+V'}$ of an atmosphere [in the notation of § 230]; and hence that the first stroke is more laborious with a small than with a large receiver.

189. In Tate's pump show that the pressure to be overcome in the first stroke is nearly equal to an atmosphere during the greater part of the stroke; and that, when half the air has been expelled from the receiver, the pressure to be overcome varies, in different parts of the stroke, from half an atmosphere to an atmosphere.

ANSWERS TO EXAMPLES.

Ex. 1. 14·14. Ex. 2. 13. Ex. 3. 7·07 each. Ex. 4. 10. Ex. 5. 141·4. Ex. 6. 70·7 each. Ex. 7. Introduce a force equal and opposite to the resultant. Then we have three forces making angles of 120° with each other. Ex. 9. Equal to one of the forces.

Ex. 10*. 28. Ex. 11*. 40 in. from smaller weight. Ex. 12. 60 lbs. by A, 40 lbs. by B. Ex. 13. $2\frac{6}{7}$ lbs. Ex. 14. 2 lbs. Ex. 15. 15 in. from centre. Ex. 16. $12\frac{1}{2}$ lbs. Ex. 17. 32 lbs. Ex. 18. 10·4 ft. nearly. Ex. 19. 6 lbs. Ex. 20. $2\frac{6}{7}$ ft. from end. Ex. 21. $2\frac{13}{16}$. Ex. 22. 2 units acting at distance of 5 yards from the greater force. Ex. 23. 6 ft. from the end; pressure 2 units. Ex. 24. $4\frac{2}{3}$ lbs. Ex. 25. $2\frac{1}{6}$ lbs., $10\frac{5}{6}$ lbs. Ex. 26. $\frac{3}{7}$ in. Ex. 27. $2\frac{2}{11}$ in. Ex. 28. $\frac{1}{9}$ of side of square. Ex. 29. $\frac{1}{12}$ of diagonal of large square. Ex. 30. $\frac{40}{117}$ cm. from centre of large sphere. Ex. 31. Denoting side of square by a, distance from AB is $\frac{\frac{1}{2}W+R+S}{W+P+Q+R+S}a$, distance from AD is $\frac{\frac{1}{2}W+Q+R}{W+P+Q+R}a$.

Ex. 34. $4\frac{4}{13}$ cm. Ex. 35. $5\frac{8}{13}$ cm. Ex. 36. 17. Ex. 37. $-1, 0, +18$. Ex. 38. $\frac{1}{2}W\left(\sqrt{(b^2+c^2)}-c\right)$. Ex. 39. $\frac{4}{9}Wl$. Ex. 40. $\frac{a}{4}(H-h)^2$.

Ex. 44. 14 lbs. Ex. 45. (*a*) $133\frac{1}{3}$ lbs.; (*b*) $166\frac{2}{3}$ lbs. Ex. 46. 1 to 603 nearly. Ex. 47. $373\frac{1}{3}$.

Ex. 49. 50. Ex. 50. 72. Ex. 51. 7 cm. per sec. Ex. 52. 35. Ex. 53. 60. Ex. 54. 60 dynes. Ex. 55. 6 dynes. Ex. 56. 7 dynes. Ex. 57. Smaller mass $\frac{1}{100}$, larger $\frac{1}{10000}$ cm. per sec. Ex. 58. Inversely as masses of bullet and gun. Ex. 59. Mass of bullet is $\frac{1}{15}$ of mass of block.

Ex. 60. 98 cm. per sec. Ex. 61. 4·9 cm. Ex. 62. 490 cm. Ex. 63. 7 sec. Ex. 64. $122\frac{1}{2}$ cm. Ex. 65. 626 cm. per sec. Ex. 66. 6 cm. per sec. upwards. Ex. 67. 45·9 cm. above point of projection. Ex. 68. 1650 cm. per sec. downwards. Ex. 69. $1062\frac{1}{2}$ cm. below starting point. Ex. 70. 384 ft. per sec. Ex. 71. 2304 ft. Ex. 72. $1\frac{9}{16}$ ft. Ex. 73. 5 sec. Ex. 74. 16 ft. Ex. 75. 40 ft. per sec. Ex. 76. 4 ft. per sec. upwards. Ex. 77. 156 ft. Ex. 78. 78 ft. per sec. Ex. 79. 81 ft. Ex. 80. After 4·52 sec. At 135·6 m. from tower. Ex. 81. After ·41 sec. from dropping of second body.

Ex. 84. $\frac{1}{3}g$. Ex. 85. $\frac{1}{41}g$. Ex. 86. 245 cm. per sec. Ex. 87. 70 cm. per sec. Ex. 88. 200 cm. Ex. 89. 245 cm.

Ex. 90. 90,000 ergs. Ex. 91. 98,100,000 ergs. Ex. 92. 241,326,000 ergs. Ex. 93. 528,220,000 ergs each. Ex. 94. 11,760 ergs; $\sqrt{23520}=153{\cdot}4$ cm. per sec. Ex. 95. 11,560 ergs. Ex. 96. 24×10^9 ergs in each discharge. Not quite 19 discharges per min. Ex. 98. 2376 nearly.

Ex. 99. 18330 cm. or about 600 ft. Ex. 100. $2\sqrt{ga}$, $\sqrt{5ga}$.

Ex. 101*. 223·679 cm. Ex. 102. MR^2, $\frac{Fat}{MR^2}$. Ex. 103. $\frac{2}{3}a$; mass of rod multiplied by $\frac{1}{12}a^2$. Ex. 104. At either of the two points distant $\frac{a}{2\sqrt{3}}$ from centre; at either of the two points distant $\frac{a}{6}$ from centre. Ex. 106. $(2\pi)^2=39{\cdot}48$. Ex. 107. $(1024\pi)^2=10350000$.

Ex. 108. 102500. Ex. 109. 883·35. Ex. 110. 31·53. Ex. 111. 149·5. Ex. 112. 12217. Ex. 113. 13338. Ex. 114. 29901. Ex. 115. 126·5. Ex. 116. 30. Ex. 117. 12 ft. 7 in. Ex. 118. 980·68. Ex. 119. 45·79. Ex. 120. 7342. Ex. 121. 994. Ex. 122. 125. Ex. 123. $4\pi r^2hd$.

Ex. 124. 2·357. Ex. 125. 1·8. Ex. 126. ·932. Ex. 127. ·0046 sq. cm. Ex. 128. 6·25. Ex. 129. $4\frac{5}{6}$. Ex. 130. 1·9125. Ex. 131. $1\frac{2}{11}$. Ex. 132. $\frac{34}{43}$. Ex. 133. 10 cub. cm., 78 gm., 7·8. Ex. 134. 104. Ex. 135. $40{\cdot}8\dot{3}$. Ex. 136. 117·5. Ex. 137. 19·3, 18·3, 5·7. Ex. 138. ·393 sq. cm., ·354 cm. Ex. 139. 18, $8{\cdot}\dot{7}7\dot{7}$, ·00835 sq. cm., ·0516 cm. Ex. 140. 60·48, 52·62. Ex. 141. 900 c.c. Ex. 142. 5·56 cm. Ex. 143. 50 c.c. Ex. 145. 3, 70 c.c. Ex. 146. 400 gm. Ex. 147. $4\frac{5}{27}=4{\cdot}185$. Ex. 148. $3\frac{96}{157}=3{\cdot}6115$. Ex. 149. 36·6 c.c. Ex. 150. 257·7 gm. Ex. 151. 1·0033. Ex. 152. $\frac{13}{25}$ of the iron. Ex. 153. 1 lit. of first, 2 lit. of second. Ex. 154. $\frac{3}{4}$ of a litre. Ex. 155. $1-\sqrt[3]{\frac{41}{43}}$ decim. $=$ ·158 cm. Ex. 156. $\sqrt[3]{\frac{15{\cdot}12}{18{\cdot}5}}=$ ·935 cm. Ex. 157. Gold : silver :: $\frac{1}{d'}-\frac{1}{D}:\frac{1}{D}-\frac{1}{d}$. Ex. 158. (*a*) 5·77, (*b*) 10·6. Ex. 159. $\frac{M-mA}{1-A}$.

Ex. 161. 6·6 cm. nearly. Ex. 162. 1·77 cm.

Ex. 165. $\frac{19}{15}$. Ex. 166. $\frac{6}{7}$ in. Ex. 168. 1010564, 1015730. Ex. 169. 77·3832. Ex. 170. 75·93. Ex. 171. ·076. Ex. 172. 30 in. Ex. 173. $2\frac{1}{6}$. Ex. 174. ·0693. Ex. 175. (*a*) ·001308, (*b*) ·001635. Ex. 176. (*a*) ·0012961, (*b*) ·0012895, (*c*) ·0012933. Ex. 177. d varies as g, and therefore gd varies as g^2. Ex. 178. Its top must be $76-5=71$ cm. deep. Ex. 179. 760 m. Ex. 180. 33 to 5. Ex. 181. $454\frac{6}{11}$ cm.

Ex. 182. 30 tons. Ex. 183. 4608. Ex. 184. 459500 nearly. Ex. 185. (*a*) 970. (*b*) 990 gm. wt. per sq. cm. Ex. 186. $\frac{1}{58}$ of an atmosphere, nearly. Ex. 187. 3 atmospheres.

INDEX TO PART I.