DESCHANEL'S

NATURAL PHILOSOPHY.

BY PROFESSOR EVERETT.

PART I.

MECHANICS, HYDROSTATICS AND PNEUMATICS.

Jenaro Dávila
Boston Mass.
October 1st 1896.

Standard Cu
0°C
Sea level
Latitude 45°

14

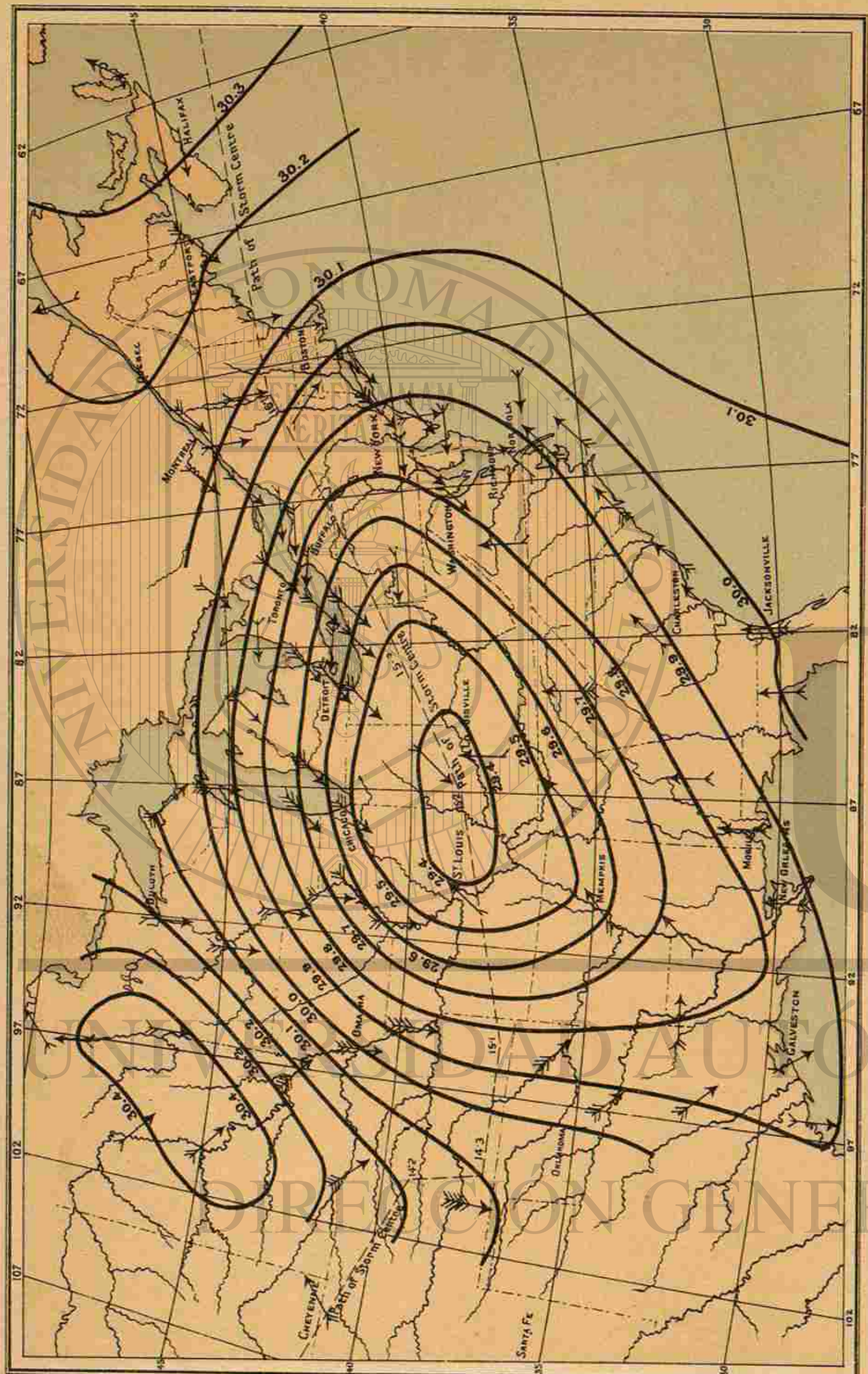


CHART OF PRESSURE AND WIND FOR THE STORM OF JAN. 15, 1877.

DRAWN BY PROFESSOR LOOMIS.

For explanation see page 165.

ELEMENTARY TREATISE

ON

NATURAL PHILOSOPHY

BASED ON THE TRAITÉ DE PHYSIQUE OF

A. PRIVAT DESCHANEL

FORMERLY PROFESSOR OF PHYSICS IN THE LYCÉE LOUIS-LE-GRAND,
INSPECTOR OF THE ACADEMY OF PARIS.

BY

J. D. EVERETT, M.A., D.C.L., F.R.S.

PROFESSOR OF NATURAL PHILOSOPHY IN THE
QUEEN'S COLLEGE, BELFAST.

PART I.

MECHANICS, HYDROSTATICS, AND PNEUMATICS.

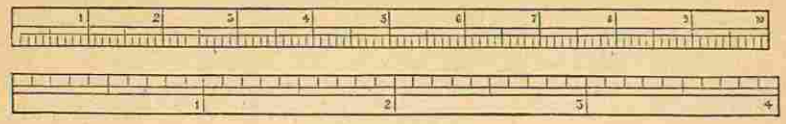
THIRTEENTH EDITION.

NEW YORK:
D. APPLETON AND COMPANY,
1895.

Authorised Edition.

FRENCH AND ENGLISH MEASURES.

A DECIMETRE DIVIDED INTO CENTIMETRES AND MILLIMETRES.



INCHES AND TENTHS.

REDUCTION OF FRENCH TO ENGLISH MEASURES.

Length.

1 millimetre = ·03937 inch, or about $\frac{1}{25}$ inch.
1 centimetre = ·3937 inch.
1 decimetre = 3·937 inch.
1 metre = 39·37 inch = 3·281 ft. = 1·0936 yd.
1 kilometre = 1093·6 yds., or about $\frac{5}{8}$ mile.

More accurately, 1 metre = 39·370432 in. = 3·2808693 ft. = 1·09362311 yd.

Area.

1 sq. millim. = ·00155 sq. in.
1 sq. centim. = ·155 sq. in.
1 sq. decim. = 15·5 sq. in.
1 sq. metre = 1550 sq. in. = 10·764 sq. ft. = 1·196 sq. yd.

Volume.

1 cub. millim. = ·000061 cub. in.
1 cub. centim. = ·061025 cub. in.
1 cub. decim. = 61·0254 cub. in.
cub. metre = 61025 cub. in. = 35·3156 cub. ft. = 1·308 cub. yd.

The Litre (used for liquids) is the same as the cubic decimetre, and is equal to 1·7617 pint, or ·22021 gallon.

Mass and Weight.

1 milligramme = ·01543 grain.
1 gramme = 15·432 grain.
1 kilogramme = 15432 grains = 2·205 lbs. avoir.

More accurately, the kilogramme is 2·20462125 lbs.

Miscellaneous.

1 gramme per sq. centim. = 2·0481 lbs. per sq. ft.
1 kilogramme per sq. centim. = 14·223 lbs. per sq. in.
1 kilogrammetre = 7·2331 foot-pounds.

1 force de cheval = 75 kilogrammetres per second, or 542½ foot-pounds per second nearly, whereas 1 horse-power (English) = 550 foot-pounds per second.

REDUCTION TO C.G.S. MEASURES. (See page 48.)

[*cm.* denotes centimetre(s); *gm.* denotes gramme(s).]

Length.

1 inch = 2·54 centimetres, nearly.
1 foot = 30·48 centimetres, nearly.
1 yard = 91·44 centimetres, nearly.
1 statute mile = 160933 centimetres, nearly.

More accurately, 1 inch = 2·5399772 centimetres.

Area.

1 sq. inch = 6·45 sq. cm., nearly.
1 sq. foot = 929 sq. cm., nearly.
1 sq. yard = 8361 sq. cm., nearly.
1 sq. mile = $2·59 \times 10^{10}$ sq. cm., nearly.

Volume.

1 cub. inch = 16·39 cub. cm., nearly.
1 cub. foot = 28316 cub. cm., nearly.
1 cub. yard = 764535 cub. cm., nearly.
1 gallon = 4541 cub. cm., nearly.

Mass.

1 grain = ·0648 gramme, nearly.
1 oz. avoir. = 28·35 gramme, nearly.
1 lb. avoir. = 453·6 gramme, nearly.
1 ton = $1·016 \times 10^6$ gramme, nearly.

More accurately, 1 lb. avoir. = 453·59265 gm.

Velocity.

1 mile per hour = 44·704 cm. per sec.
1 kilometre per hour = 27·7 cm. per sec.

Density.

1 lb. per cub. foot = ·016019 gm. per cub. cm.
62·4 lbs. per cub. ft. = 1 gm. per cub. cm.

FORCE (assuming $g=981$). (See p. 48.)

Weight of 1 grain $=63{\cdot}57$ dynes, nearly.
,, 1 oz. avoir. $=2{\cdot}78\times10^4$ dynes, nearly.
,, 1 lb. avoir. $=4{\cdot}45\times10^5$ dynes, nearly.
,, 1 ton $=9{\cdot}97\times10^8$ dynes, nearly.
,, 1 gramme $=981$ dynes, nearly.
,, 1 kilogramme $=9{\cdot}81\times10^5$ dynes, nearly.

WORK (assuming $g=981$). (See p. 48.)

1 foot-pound $=1{\cdot}356\times10^7$ ergs, nearly.
1 kilogrammetre $=9{\cdot}81\times10^7$ ergs, nearly.
Work in a second by one theoretical "horse." $=7{\cdot}46\times10^9$ ergs, nearly.

STRESS (assuming $g=981$).

1 lb. per sq. ft. $=479$ dynes per sq. cm., nearly.
1 lb. per sq. inch $=6{\cdot}9\times10^4$ dynes per sq. cm., nearly.
1 kilog. per sq. cm. $=9{\cdot}81\times10^5$ dynes per sq. cm., nearly.
760 mm. of mercury at 0°C. $=1{\cdot}014\times10^6$ dynes per sq. cm., nearly.
30 inches of mercury at 0° C. $=1{\cdot}0163\times10^6$ dynes per sq. cm., nearly.
1 inch of mercury at 0° C. $=3{\cdot}388\times10^4$ dynes per sq. cm., nearly.

TABLE OF DENSITIES, IN GRAMMES PER CUBIC CENTIMETRE.

LIQUIDS.

Pure water at 4° C.,	1·000
Sea water, ordinary,	1·026
Alcohol, pure,	·791
,, proof spirit,	·916
Ether,	·716
Mercury at 0° C.,	13·596
Naphtha,	·848

SOLIDS.

Brass, cast,	7·8 to 8·4
,, wire,	8·54
Bronze,	8·4
Copper, cast,	8·6
,, sheet,	8·8
,, hammered,	8·9
Gold,	19 to 19·6
Iron, cast,	6·95 to 7·3
,, wrought,	7·6 to 7·8
Lead,	11·4
Platinum,	21 to 22
Silver,	10·5
Steel,	7·8 to 7·9
Tin,	7·3 to 7·5
Zinc,	6·8 to 7·2
Ice,	·92
Basalt,	3·00
Brick,	2 to 2·17
Brickwork,	1·8
Chalk,	1·8 to 2·8
Clay,	1·92
Glass, crown,	2·5
,, flint,	3·0
Quartz (rock-crystal),	2·65
Sand,	1·42
Fir, spruce,	·48 to ·7
Oak, European,	·69 to ·99
Lignum-vitæ,	·65 to 1·33
Sulphur, octahedral,	2·05
,, prismatic,	1·98

GASES, at 0° C. and a pressure of a million dynes per sq. cm. (see p. 142).

Air, dry,	·0012759
Oxygen,	·0014107
Nitrogen,	·0012393
Hydrogen,	·00008837
Carbonic acid,	·0019509

ELEMENTARY TREATISE

ON

NATURAL PHILOSOPHY.

CHAPTER I.

INTRODUCTORY.

1. Natural Science, in the widest sense of the term, comprises all the phenomena of the material world. In so far as it merely describes and classifies these phenomena, it may be called Natural History; in so far as it furnishes accurate quantitative knowledge of the relations between causes and effects it is called Natural Philosophy. Many subjects of study pass through the natural history stage before they attain the natural philosophy stage; the phenomena being observed and compared for many years before the quantitative laws which govern them are disclosed.

2. There are two extensive groups of phenomena which are conventionally excluded from the domain of Natural Philosophy, and regarded as constituting separate branches of science in themselves; namely:—

First. Those phenomena which depend on vital forces; such phenomena, for example, as the growth of animals and plants. These constitute the domain of Biology.

Secondly. Those which depend on elective attractions between the atoms of particular substances, attractions which are known by the name of chemical affinities. These phenomena are relegated to the special science of Chemistry.

Again, Astronomy, which treats of the nature and movements of the heavenly bodies, is, like Chemistry, so vast a subject, that it forms a special science of itself; though certain general laws, which its phenomena exemplify, are still included in the study of Natural Philosophy.

FORCE (assuming $g=981$). (See p. 48.)

Weight of 1 grain	= 63·57 dynes, nearly.
,, 1 oz. avoir.	= $2·78 \times 10^4$ dynes, nearly.
,, 1 lb. avoir.	= $4·45 \times 10^5$ dynes, nearly.
,, 1 ton	= $9·97 \times 10^8$ dynes, nearly.
,, 1 gramme	= 981 dynes, nearly.
,, 1 kilogramme	= $9·81 \times 10^5$ dynes, nearly.

WORK (assuming $g=981$). (See p. 48.)

1 foot-pound	= $1·356 \times 10^7$ ergs, nearly.
1 kilogrammetre	= $9·81 \times 10^7$ ergs, nearly.
Work in a second by one theoretical "horse."	= $7·46 \times 10^9$ ergs, nearly.

STRESS (assuming $g=981$).

1 lb. per sq. ft.	= 479 dynes per sq. cm., nearly.
1 lb. per sq. inch	= $6·9 \times 10^4$ dynes per sq. cm., nearly.
1 kilog. per sq. cm.	= $9·81 \times 10^5$ dynes per sq. cm., nearly.
760 mm. of mercury at 0°C.	= $1·014 \times 10^6$ dynes per sq. cm., nearly.
30 inches of mercury at 0° C.	= $1·0163 \times 10^6$ dynes per sq. cm., nearly.
1 inch of mercury at 0° C.	= $3·388 \times 10^4$ dynes per sq. cm., nearly.

TABLE OF DENSITIES, IN GRAMMES PER CUBIC CENTIMETRE.

LIQUIDS.

Pure water at 4° C.,	1·000
Sea water, ordinary,	1·026
Alcohol, pure,	·791
,, proof spirit,	·916
Ether,	·716
Mercury at 0° C.,	13·596
Naphtha,	·848

SOLIDS.

Brass, cast,	7·8 to 8·4
,, wire,	8·54
Bronze,	8·4
Copper, cast,	8·6
,, sheet,	8·8
,, hammered,	8·9
Gold,	19 to 19·6
Iron, cast,	6·95 to 7·3
,, wrought,	7·6 to 7·8
Lead,	11·4
Platinum,	21 to 22
Silver,	10·5
Steel,	7·8 to 7·9
Tin,	7·3 to 7·5
Zinc,	6·8 to 7·2
Ice,	·92
Basalt,	3·00
Brick,	2 to 2·17
Brickwork,	1·8
Chalk,	1·8 to 2·8
Clay,	1·92
Glass, crown,	2·5
,, flint,	3·0
Quartz (rock-crystal),	2·65
Sand,	1·42
Fir, spruce,	·48 to ·7
Oak, European,	·69 to ·99
Lignum-vitæ,	·65 to 1·33
Sulphur, octahedral,	2·05
,, prismatic,	1·98

GASES, at 0° C. and a pressure of a million dynes per sq. cm. (see p. 142).

Air, dry,	·0012759
Oxygen,	·0014107
Nitrogen,	·0012393
Hydrogen,	·00008837
Carbonic acid,	·0019509

ELEMENTARY TREATISE ON NATURAL PHILOSOPHY.

CHAPTER I.

INTRODUCTORY.

1. Natural Science, in the widest sense of the term, comprises all the phenomena of the material world. In so far as it merely describes and classifies these phenomena, it may be called Natural History; in so far as it furnishes accurate quantitative knowledge of the relations between causes and effects it is called Natural Philosophy. Many subjects of study pass through the natural history stage before they attain the natural philosophy stage; the phenomena being observed and compared for many years before the quantitative laws which govern them are disclosed.

2. There are two extensive groups of phenomena which are conventionally excluded from the domain of Natural Philosophy, and regarded as constituting separate branches of science in themselves; namely:—

First. Those phenomena which depend on vital forces; such phenomena, for example, as the growth of animals and plants. These constitute the domain of Biology.

Secondly. Those which depend on elective attractions between the atoms of particular substances, attractions which are known by the name of chemical affinities. These phenomena are relegated to the special science of Chemistry.

Again, Astronomy, which treats of the nature and movements of the heavenly bodies, is, like Chemistry, so vast a subject, that it forms a special science of itself; though certain general laws, which its phenomena exemplify, are still included in the study of Natural Philosophy.

3. Those phenomena which specially belong to the domain of Natural Philosophy are called *physical;* and Natural Philosophy itself is called *Physics.* It may be divided into the following branches.

I. DYNAMICS, or the general laws of force and of the relations which exist between force, mass, and velocity. These laws may be applied to solids, liquids, or gases. Thus we have the three divisions, *Mechanics, Hydrostatics,* and *Pneumatics.*

II. THERMICS; the science of Heat.

III. The science of ELECTRICITY, with the closely related subject of MAGNETISM.

IV. ACOUSTICS; the science of Sound.

V. OPTICS; the science of Light.

The branches here numbered I. II. III. are treated in Parts I. II. III. respectively, of the present Work. The two branches numbered IV. V. are treated in Part IV.

CHAPTER II.

FIRST PRINCIPLES OF DYNAMICS. STATICS.

4. Force.—Force may be defined as that which tends to produce motion in a body at rest, or to produce change of motion in a body which is moving. A particle is said to have uniform or unchanged motion when it moves in a straight line with constant velocity; and every deviation of material particles from uniform motion is due to forces acting upon them.

5. Translation and Rotation.—When a body moves so that all lines in it remain constantly parallel to their original positions (or, to use the ordinary technical phrase, *move parallel to themselves*), its movement is called a *pure translation.* Since the lines joining the extremities of equal and parallel straight lines are themselves equal and parallel, it can easily be shown that, in such motion, all points of the body have equal and parallel velocities, so that the movement of the whole body is completely represented by the movement of any one of its points.

On the other hand, if one point of a rigid body be fixed, the only movement possible for the body is *pure rotation,* the axis of the rotation at any moment being some straight line passing through this point.

Every movement of a rigid body can be specified by specifying the movement of one of its points (any point will do) together with the rotation of the body about this point.

6. Force which acts uniformly on all the particles of a body, as gravity does sensibly in the case of bodies of moderate size on the earth's surface (equal particles being urged with equal forces and in parallel directions), tends to give the body a movement of pure translation.

In elementary statements of the laws of force, it is necessary, for

3. Those phenomena which specially belong to the domain of Natural Philosophy are called *physical;* and Natural Philosophy itself is called *Physics*. It may be divided into the following branches.

I. DYNAMICS, or the general laws of force and of the relations which exist between force, mass, and velocity. These laws may be applied to solids, liquids, or gases. Thus we have the three divisions, *Mechanics, Hydrostatics,* and *Pneumatics*.

II. THERMICS; the science of Heat.

III. The science of ELECTRICITY, with the closely related subject of MAGNETISM.

IV. ACOUSTICS; the science of Sound.

V. OPTICS; the science of Light.

The branches here numbered I. II. III. are treated in Parts I. II. III. respectively, of the present Work. The two branches numbered IV. V. are treated in Part IV.

CHAPTER II.

FIRST PRINCIPLES OF DYNAMICS. STATICS.

4. Force.—Force may be defined as that which tends to produce motion in a body at rest, or to produce change of motion in a body which is moving. A particle is said to have uniform or unchanged motion when it moves in a straight line with constant velocity; and every deviation of material particles from uniform motion is due to forces acting upon them.

5. Translation and Rotation.—When a body moves so that all lines in it remain constantly parallel to their original positions (or, to use the ordinary technical phrase, *move parallel to themselves*), its movement is called a *pure translation*. Since the lines joining the extremities of equal and parallel straight lines are themselves equal and parallel, it can easily be shown that, in such motion, all points of the body have equal and parallel velocities, so that the movement of the whole body is completely represented by the movement of any one of its points.

On the other hand, if one point of a rigid body be fixed, the only movement possible for the body is *pure rotation*, the axis of the rotation at any moment being some straight line passing through this point.

Every movement of a rigid body can be specified by specifying the movement of one of its points (any point will do) together with the rotation of the body about this point.

6. Force which acts uniformly on all the particles of a body, as gravity does sensibly in the case of bodies of moderate size on the earth's surface (equal particles being urged with equal forces and in parallel directions), tends to give the body a movement of pure translation.

In elementary statements of the laws of force, it is necessary, for the sake of simplicity, to confine attention to forces tending to produce pure translation.

7. Instruments for Measuring Force.—We obtain the idea of force through our own conscious exercise of muscular force, and we can approximately estimate the amount of a force (if not too great or too small) by the effort which we have to make to resist it; as when we try the weight of a body by lifting it.

Dynamometers are instruments in which force is measured by means of its effect in bending or otherwise distorting elastic springs, and the spring-balance is a dynamometer applied to the measurement of weights, the spring in this case being either a flat spiral (like the mainspring of a watch), or a helix (resembling a cork-screw).

A force may also be measured by causing it to act vertically downwards upon one of the scale-pans of a balance and counterpoising it by weights in the other pan.

8. Gravitation Units of Force.—In whatever way the measurement of a force is effected, the result, that is, the magnitude of the force, is usually stated in terms of weight; for example, in pounds or in kilogrammes. Such units of force (called gravitation units) are to a certain extent indefinite, inasmuch as gravity is not exactly the same over the whole surface of the earth; but they are sufficiently definite for ordinary commercial purposes.

9. Equilibrium, Statics, Kinetics.—When a body free to move is acted on by forces which do not move it, these forces are said to be *in equilibrium*, or to *equilibrate* each other. They may equally well be described as *balancing* each other. Dynamics is usually divided into two branches. The first branch, called *Statics*, treats of the conditions of equilibrium. The second branch, called *Kinetics*, treats of the movements produced by forces not in equilibrium.

10. Action and Reaction.—Experiment shows that force is always a mutual action between two portions of matter. When a body is urged by a force, this force is exerted by some other body, which is itself urged in the opposite direction with an equal force. When I press the table downwards with my hand, the table presses my hand upwards; when a weight hangs by a cord attached to a beam, the cord serves to transmit force between the beam and the weight, so that, by the instrumentality of the cord, the beam pulls the weight upwards and the weight pulls the beam downwards. Electricity and magnetism furnish no exception to this universal law. When a magnet attracts a piece of iron, the piece of iron attracts the magnet with a precisely equal force.

11. Specification of a Force acting at a Point.—Force may be applied over a finite area, as when I press the table with my hand; or may be applied through the whole substance of a body, as in the case of gravity; but it is usual to begin by discussing the action of forces applied to a *single particle*, in which case each force is supposed to act along a mathematical straight line, and the particle or material point to which it is applied is called its *point* of application. A force is completely specified when its *magnitude*, its *point of application*, and its *line of action* are all given.

12. Rigid Body. Fundamental Problem of Statics.—A force of finite magnitude applied to a mathematical point of any actual solid body would inevitably fracture the body. To avoid this complication and other complications which would arise from the bending and yielding of bodies under the action of forces, the fiction of a perfectly rigid body is introduced, a body which cannot bend or break under the action of any force however intense, but always retains its size and shape unchanged.

The fundamental problem of Statics consists in determining the conditions which forces must fulfil in order that they may be in equilibrium when applied to a rigid body.

13. Conditions of Equilibrium for Two Forces.—In order that two forces applied to a rigid body should be in equilibrium, it is necessary and sufficient that they fulfil the following conditions:—

1st. Their lines of action must be one and the same.

2nd. The forces must act in opposite directions along this common line.

3rd. They must be equal in magnitude.

It will be observed that nothing is said here about the points of application of the forces. They may in fact be anywhere upon the common line of action. *The effect of a force upon a rigid body is not altered by changing its point of application to any other point in its line of action.* This is called the principle of the *transmissibility of force.*

It follows from this principle that the condition of equilibrium for any number of forces with the same line of action is simply that the sum of those which act in one direction shall be equal to the sum of those which act in the opposite direction.

14. Three Forces Meeting in a Point. Triangle of Forces.—If three forces, not having one and the same line of action, are in equilibrium, their lines of action must lie in one plane, and must either meet in a point or be parallel. We shall first discuss the case in which they meet in a point.

From any point A (Fig. 1) draw a line AB parallel to one of the two given forces, and so that in travelling from A to B we should be travelling in the same direction in which the force acts (not in the opposite direction). Also let it be understood that the length of AB represents the magnitude of the force.

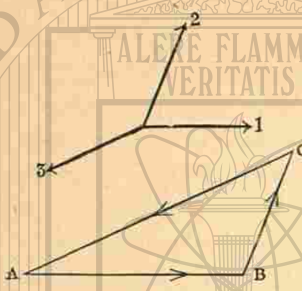


Fig. 1.—Triangle of Forces.

From the point B draw a line BC representing the second force in direction, and on the same scale of magnitude on which AB represents the first.

Then the line CA will represent both in direction and magnitude the third force which would equilibrate the first two.

The principle embodied in this construction is called the *triangle of forces*. It may be briefly stated as follows:—*The condition of equilibrium for three forces acting at a point is, that they be represented in magnitude and direction by the three sides of a triangle, taken one way round.* The meaning of the words "taken one way round" will be understood from an inspection of the arrows with which the sides of the triangle in Fig. 1 are marked. If the directions of all three arrows are reversed the forces represented will still be in equilibrium. The arrows must be so directed that it would be possible to travel completely round the triangle by moving along the sides in the directions indicated.

When a line is used to represent a force, it is always necessary to employ an arrow or some other mark of direction, in order to avoid ambiguity between the direction intended and its opposite. In naming such a line by means of two letters, one at each end of it, the order of the letters should indicate the direction intended. The direction of AB is from A to B; the direction of BA is from B to A.

15. Resultant and Components.—Since two forces acting at a point can be balanced by a single force, it is obvious that they are equivalent to a single force, namely, to a force equal and opposite to that which would balance them. This force to which they are equivalent

is called their *resultant*. Whenever one force acting on a rigid body is equivalent to two or more forces, it is called their resultant, and they are called its *components*. When any number of forces are in equilibrium, a force equal and opposite to any one of them is the resultant of all the rest.

The "triangle of forces" gives us the resultant of any two forces acting at a point. For example, in Fig. 1, AC (with the arrow in the figure reversed) represents the resultant of the forces represented by AB and BC.

16. Parallelogram of Forces.—The proposition called the "parallelogram of forces" is not essentially distinct from the "triangle of forces," but merely expresses the same fact from a slightly different point of view. It is as follows:—*If two forces acting upon the same rigid body in lines which meet in a point, be represented by two lines drawn from the point, and a parallelogram be constructed on these lines, the diagonal drawn from this point to the opposite corner of the parallelogram represents the resultant.*

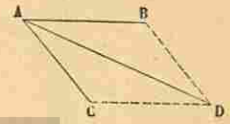


Fig. 2.—Parallelogram of Forces.

For example, if AB, AC, Fig. 2, represent the two forces, AD will represent their resultant.

To show the identity of this proposition with the triangle of forces, we have only to substitute BD for AC (which is equal and parallel to it). We have then two forces represented by AB, BD (two sides of a triangle) giving as their resultant a force represented by the third side AD. We might equally well have employed the triangle ACD, by substituting CD for AB.

17. Gravesande's Apparatus.—An apparatus for verifying the parallelogram of forces is represented in Fig. 3. ACDB is a light frame in the form of a parallelogram. A weight P″ can be hung at A, and weights P, P′ can be attached, by means of cords passing over pulleys, to the points B, C. When the weights P, P′, P″ are proportional to AB, AC and AD respectively, the strings attached at B and C will be observed to form prolongations of the sides, and the diagonal AD will be vertical.

18. Resultant of any Number of Forces at a Point.—To find the resultant of any number of forces whose lines of action meet in a point, it is only necessary to draw a crooked line composed of straight lines which represent the several forces. The resultant will be represented by a straight line drawn from the beginning to the end of this crooked line. For by what precedes, if ABCDE be a crooked line such that the straight lines AB, BC, CD, DE represent four forces acting at a point, we may substitute for AB and BC

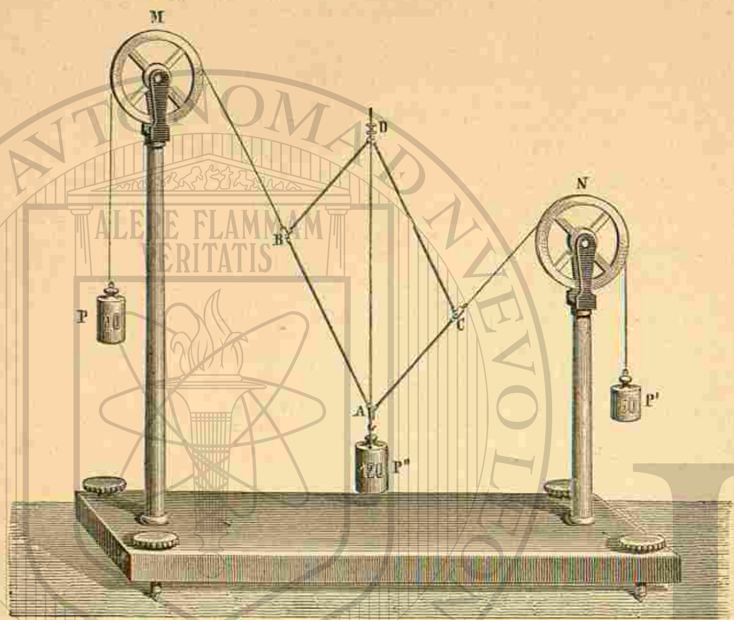


Fig. 3.—Gravesande's Apparatus.

the straight line AC, since this represents their resultant. We may then substitute AD for AC and CD, and finally AE for AD and DE.

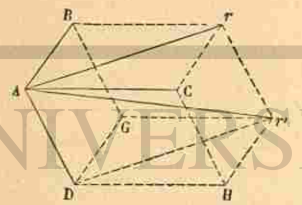


Fig. 4.—Parallelepiped of Forces.

One of the most important applications of this construction is to three forces not lying in one plane. In this case the crooked line will consist of three edges of a parallelepiped, and the line which represents the resultant will be the diagonal. This is evident from Fig. 4, in which AB, AC, AD represent three forces acting at A. The resultant of AB and AC is Ar, and the resultant of Ar and AD is Ar'. The crooked line whose parts represent the forces, may be either ABrr', or ABGr', or ADGr', &c., the total number of alternatives being six, since three things can be taken in six different orders. We have here an excellent illustration of the fact that the same final resultant is obtained in whatever order the forces are combined

19. Equilibrium of Three Parallel Forces.—If three parallel forces, P, Q, R, applied to a rigid body, balance each other, the following conditions must be fulfilled:—

1. The three lines of action AP, BQ, CR, Fig. 5, must be in one plane.

2. The two outside forces P, R, must act in the opposite direction to the middle force Q, and their sum must be equal to Q.

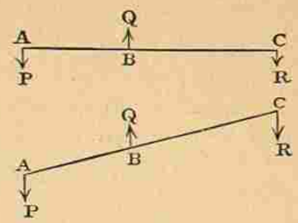


Fig. 5.

3. Each force must be proportional to the distance between the lines of action of the other two; that is, we must have

$$\frac{P}{BC} = \frac{Q}{AC} = \frac{R}{AB}. \qquad (1)$$

The figure shows that AC is the sum of AB and BC; hence it follows from these equations, that Q is equal to the sum of P and R, as above stated.

20. Resultant of Two Parallel Forces.—Any two parallel forces being given, a third parallel force which will balance them can be found from the above equations; and a force equal and opposite to this will be their resultant. We may distinguish two cases.

1. Let the two given forces be in the same direction. Then their resultant is equal to their sum, and acts in the same direction, along a line which cuts the line joining their points of application into two parts which are inversely as the forces.

2. Let the two given forces be in opposite directions. Then their resultant will be equal to their difference, and will act in the direction of the greater of the two forces, along a line which cuts the production of the line joining their points of application on the side of the greater force; and the distances of this point of section from the two given points of application are inversely as the forces.

21. Centre of Two Parallel Forces.—In both cases, if the points of application are not given, but only the magnitudes of the forces and their lines of action, the magnitude and line of action of the resultant are still completely determined; for all straight lines which are drawn across three parallel straight lines are cut by them in the same ratio; and we shall obtain the same result whatever points of application we assume.

If the points of application are given, the resultant cuts the line joining them, or this line produced, in a definite point, whose position depends only on the magnitudes of the given forces, and not at all on the angle which their direction makes with the joining line. This result is important in connection with centres of gravity. The point so determined is called the centre of the two parallel forces. If these two forces are the weights of two particles, the "centre" thus found is their centre of gravity, and the resultant force is the same as if the two particles were collected at this point.

22. Moments of Resultant and of Components Equal.—The following proposition is often useful. Let any straight line be drawn across the lines of action of two parallel forces P_1, P_2 (Fig. 6). Let O be any point on this line, and x_1, x_2 the distances measured from O to the points of section, distances measured in opposite directions being distinguished by opposite signs, and forces in opposite directions being also distinguished by opposite signs. Also let R denote the resultant of P_1 and P_2, and $\bar{x}$ the distance from O to its intersection with the line; then we shall have

Fig. 6.

$$P_1 x_1 + P_2 x_2 = R\bar{x}.$$

For, taking the standard case, as represented in Fig. 6, in which all the quantities are positive, we have $OA_1 = x_1$, $OA_2 = x_2$, $OB = \bar{x}$, and by § 19 or § 20 we have

$$P_1 . A_1B = P_2 . BA_2,$$

that is,

$$P_1 (\bar{x} - x_1) = P_2 (x_2 - \bar{x}),$$

whence

$$(P_1 + P_2)\bar{x} = P_1 x_1 + P_2 x_2,$$

that is,

$$R\bar{x} = P_1 x_1 + P_2 x_2. \qquad (2)$$

23. Any Number of Parallel Forces in One Plane.—Equation (2) affords the readiest means of determining the line of action of the resultant of several parallel forces lying in one plane. For let P_1, P_2, P_3, &c., be the forces, R_1 the resultant of the first two forces P_1, P_2, and R_2 the resultant of the first three forces P_1, P_2, P_3. Let a line be drawn across the lines of action, and let the distances of the points of section from an arbitrary point O on this line be expressed according to the following scheme:—

Force	P_1	P_2	P_3	R_1	R_2
Distance	x_1	x_2	x_3	$\bar{x}_1$	$\bar{x}_2$

Then, by equation (2) we have

$$R_1 \bar{x}_1 = P_1 x_1 + P_2 x_2.$$

Also since R_2 is the resultant of R_1 and P_3, we have

$$R_2 \bar{x}_2 = R_1 \bar{x}_1 + P_3 x_3,$$

and substituting for the term $R_1 \bar{x}_1$, we have

$$R_2 \bar{x}_2 = P_1 x_1 + P_2 x_2 + P_3 x_3.$$

This reasoning can evidently be extended to any number of forces, so that we shall have finally

$$R\bar{x} = \text{sum of such terms as } Px,$$

where R denotes the resultant of all the forces, and is equal to their algebraic sum; while $\bar{x}$ denotes the value of x for the point where the line of action of R cuts the fixed line. It is usual to employ the Greek letter Σ to denote "the sum of such terms as." We may therefore write

$$R = \Sigma (P)$$
$$R\bar{x} = \Sigma (Px)$$

whence

$$\bar{x} = \frac{\Sigma (Px)}{\Sigma (P)} \qquad (3)$$

24. Moment of a Force about a Point.—When the fixed line is at right angles to the parallel forces, the product Px is called the moment of the force P about the point O. More generally, the *moment of a force about a point* is the *product of the force by the length of the perpendicular dropped upon it from the point.* The above equations show that for parallel forces in one plane, the *moment of the resultant* about any point in the plane *is the sum of the moments of the forces* about the same point.

If the resultant passes through O, the distance $\bar{x}$ is zero; whence it follows from the equations that the algebraical sum of the moments vanishes.

The moment of a force about a point measures the tendency of the force to produce rotation about the point. If one point of a body be fixed, the body will turn in one direction or the other according as the resultant passes on one side or the other of this point (the direction of the resultant being supposed given). If the resultant passes through the fixed point, the body will be in equilibrium.

The moment Px of any force about a point, changes sign with P and also with x; thereby expressing (what is obvious in itself) that the direction in which the force tends to turn the body about the point will be reversed if the direction of P is reversed while its line of action remains unchanged, and will also be reversed if the line of action be shifted to the other side of the point while the direction of the force remains unchanged.

25. Experimental Illustration.—Fig. 7 represents a simple apparatus (called the *arithmetical lever*) for illustrating the laws of par-

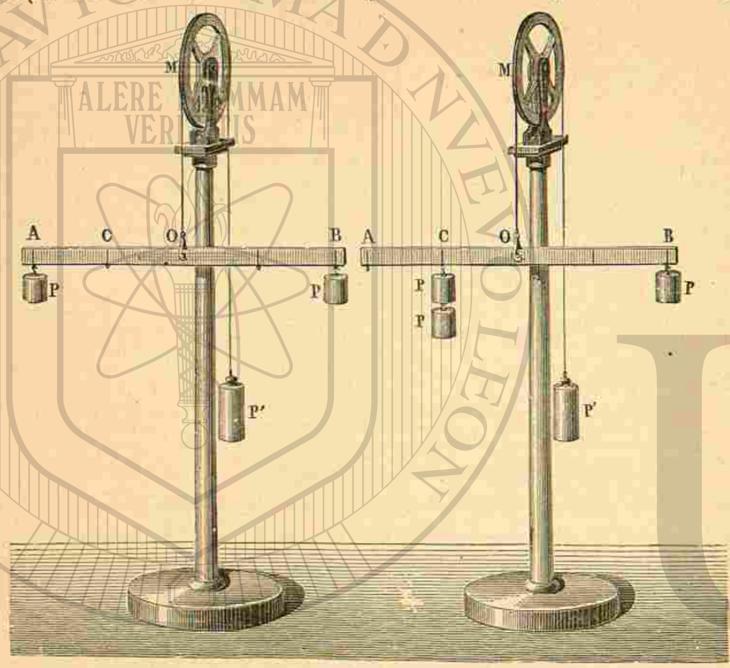


Fig. 7.—Composition of Parallel Forces.

allel forces. The lever AB is suspended at its middle point by a cord, so that when no weights are attached it is horizontal. Equal weights P, P are hung at points A and B equidistant from the centre, and the suspending cord after being passed over a very freely moving pulley M, has a weight P′ hung at its other end sufficient to produce equilibrium. It will be found that P′ is equal to the sum of the two weights P together with the weight required to counterpoise the lever itself.

In the second figure, the two weights hung from the lever are not equal, but one of them is double of the other, P being hung at B, and 2 P at C; and it is necessary for equilibrium that the distance OB be double of the distance OC. The weight P′ required

to balance the system will now be 3 P together with the weight of the lever.

26. Couple.—There is one case of two parallel forces in opposite directions which requires special attention; that in which the two forces are equal.

To obtain some idea of the effect of two such forces, let us first suppose them not exactly equal, but let their difference be very small compared with either of the forces. In this case, the resultant will be equal to this small difference, and its line of action will be at a great distance from those of the given forces. For in § 19 if Q is very little greater than P, so that Q-P, or R is only a small fraction of P, the equation $\frac{P}{BC}=\frac{R}{AB}$ shows that AB is only a small fraction of BC, or in other words that BC is very large compared with AB.

If Q gradually diminishes until it becomes equal to P, R will gradually diminish to zero; but while it diminishes, the product R . BC will remain constant, being always equal to P . AB.

A very small force R at a very great distance would have sensibly the same moment round all points between A and B or anywhere in their neighbourhood, and the moment of R is always equal to the algebraic sum of the moments of P and Q.

When Q is equal to P, they compose what is called *a couple*, and the algebraic sum of their moments about any point in their plane is constant, being always equal to P . AB, which is therefore called the moment of the couple.

A couple consists of two equal and parallel forces in opposite directions applied to the same body. The distance between their lines of action is called the arm of the couple, and the product of one of the two equal forces by this arm is called the moment of the couple.

27. Composition of Couples. Axis of Couple.—A couple cannot be balanced by a single force; but it can be balanced by any couple of equal moment, opposite in sign, if the plane of the second couple be either the same as that of the first or parallel to it.

Any number of couples in the same or parallel planes are equivalent to a single couple whose moment is the algebraic sum of theirs.

The laws of the composition of couples (like those of forces) can be illustrated by geometry.

Let a couple be represented by a line perpendicular to its plane, marked with an arrow according to the convention that if an ordinary screw were made to turn in the direction in which the couple tends to turn, it would advance in the direction in which the arrow points. Also let the length of the line represent the moment of the couple. Then the same laws of composition and resolution which hold for forces acting at a point will also hold for couples. A line thus drawn to represent a couple is called the *axis* of the couple.

Just as any number of forces acting at a point are either in equilibrium or equivalent to a single force, so any number of couples applied to the same rigid body (no matter to what parts of it) are either in equilibrium or equivalent to a single couple.

28. Resultant of Force and Couple in Same Plane.—The resultant of a force and a couple in the same plane is a single force. For the couple may be replaced by another of equal moment having its equal forces P, Q, each equal to the given force F, and the latter couple may then be turned about in its own plane and carried into such a position that one of its two forces destroys the force F, as represented in Fig. 8. There will then only remain the force P, which is equal and parallel to F.

Fig. 8.

By reversing this procedure, we can show that a force P which does not pass through a given point A is equivalent to an equal and parallel force F which does pass through it, together with a couple; the moment of the couple being the same as the moment of the force P about A.

29. General Resultant of any Number of Forces applied to a Rigid Body.—Forces applied to a rigid body in lines which do not meet in one point are not in general equivalent to a single force. By the process indicated in the concluding sentence of the preceding section, we can replace the forces by forces equal and parallel to them, acting at any assumed point, together with a number of couples. These couples can then be reduced (by the principles of § 27) to a single couple, and the forces at the point can be replaced by a single force; so that we shall obtain, as the complete resultant, a single force applied at any point we choose to select, and a couple.

We can in general make the couple smaller by resolving it into two components whose planes are respectively perpendicular and parallel to the force, and then compounding one of these components (the latter) with the force as explained in § 28, thus moving the

force parallel to itself without altering its magnitude. This is the greatest simplification that is possible. The result is that we have a single force and a couple whose plane is perpendicular to the force. Any combination of forces that can be applied to a rigid body is reducible to a force acting along one definite line and a couple in a plane perpendicular to this line. Such a combination of a force and couple is called a *wrench*, and the "one definite line" is called the *axis* of the wrench. The point of application of the force is not definite, but is any point of the axis.

30. Application to Action and Reaction.—Every action of force that one body can exert upon another is reducible to a wrench, and the law of reaction is that the second body will, in every case, exert upon the first an equal and opposite wrench. The two wrenches will have the same axis, equal and opposite forces along this axis, and equal and opposite couples in planes perpendicular to it.

31. Resolution the Inverse of Composition.—The process of finding the resultant of two or more forces is called *composition*. The inverse process of finding two or more forces which shall together be equivalent to a given force, is called *resolution*, and the two or more forces thus found are called *components*.

The problem to resolve a force into two components along two given lines which meet it in one point and are in the same plane with it, has a definite solution, which is obtained by drawing a triangle whose sides are parallel respectively to the given force and the required components. The given force and the required components will be proportional to the sides of this triangle, each being represented by the side parallel to it.

The problem to resolve a force into three components along three given lines which meet it in one point and are not in one plane, also admits of a definite solution.

32. Rectangular Resolution.—In the majority of cases which occur in practice the required components are at right angles to each other, and the resolution is then said to be rectangular. When "the component of a force along a given line" is mentioned, without anything in the context to indicate the direction of the other component or components, it is always to be understood that the resolution is rectangular. The process of finding the required component in this case is illustrated by Fig. 9. Let AB represent the given force F, and let AC be the line along which the component of F is required. It is only necessary to drop from B a perpendicular BC on this line; AC will represent the required component. CB represents the other component, which, along with AC, is equivalent to the given force. If the total number of rectangular components, of which AC represents one, is to be three, then the other two will lie in a plane perpendicular to AC, and they can be found by again resolving CB. The magnitude of AC will be the same whether the number of components be two or three, and the component along AC will be $F\frac{AC}{AB}$, or in trigonometrical language,

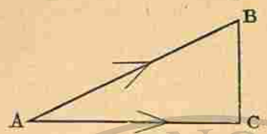


Fig. 9.—Component along a given Line.

$$F \cos . BAC.$$

We have thus the following rule:—*The component of a given force along a given line is found by multiplying the force by the cosine of the angle between its own direction and that of the required component.*

CHAPTER III.

CENTRE OF GRAVITY.

33. Gravity is the force to which we owe the names "up" and "down." The direction in which gravity acts at any place is called the downward direction, and a line drawn accurately in this direction is called *vertical;* it is the direction assumed by a plumb-line. A plane perpendicular to this direction is called *horizontal*, and is parallel to the surface of a liquid at rest. The verticals at different places are not parallel, but are inclined at an angle which is approximately proportional to the distance between the places. It amounts to 180° when the places are antipodal, and to about 1′ when their distance is one geographical mile, or to about 1″ for every hundred feet. Hence, when we are dealing with the action of gravity on a body a few feet or a few hundred feet in length, we may practically regard the action as consisting of parallel forces.

34. Centre of Gravity.—Let A and B be any two particles of a rigid body, let w_1 be the weight of the particle A, and w_2 the weight of B. These weights are parallel forces, and their resultant divides the line AB in the inverse ratio of the forces. As the body is turned about into different positions, the forces w_1 and w_2 remain unchanged in magnitude, and hence the resultant cuts AB always in the same point. This point is called the centre of the parallel forces w_1 and w_2, or the centre of gravity of the two particles A and B. The magnitude of the resultant will be w_1+w_2, and we may substitute it for the two forces themselves; in other words, we may suppose the two particles A and B to be collected at their centre of gravity. We can now combine this resultant with the weight of a third particle of the body, and shall thus obtain a resultant $w_1+w_2+w_3$, passing through a definite point in the line which joins perpendicular BC on this line; AC will represent the required component. CB represents the other component, which, along with AC, is equivalent to the given force. If the total number of rectangular components, of which AC represents one, is to be three, then the other two will lie in a plane perpendicular to AC, and they can be found by again resolving CB. The magnitude of AC will be the same whether the number of components be two or three, and the component along AC will be F $\frac{AC}{AB}$, or in trigonometrical language,

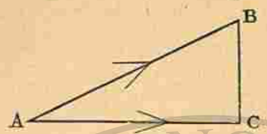


Fig. 9.—Component along a given Line.

$$F \cos . BAC.$$

We have thus the following rule:—*The component of a given force along a given line is found by multiplying the force by the cosine of the angle between its own direction and that of the required component.*

CHAPTER III.

CENTRE OF GRAVITY.

33. Gravity is the force to which we owe the names "up" and "down." The direction in which gravity acts at any place is called the downward direction, and a line drawn accurately in this direction is called *vertical;* it is the direction assumed by a plumb-line. A plane perpendicular to this direction is called *horizontal,* and is parallel to the surface of a liquid at rest. The verticals at different places are not parallel, but are inclined at an angle which is approximately proportional to the distance between the places. It amounts to 180° when the places are antipodal, and to about 1′ when their distance is one geographical mile, or to about 1″ for every hundred feet. Hence, when we are dealing with the action of gravity on a body a few feet or a few hundred feet in length, we may practically regard the action as consisting of parallel forces.

34. Centre of Gravity.—Let A and B be any two particles of a rigid body, let w_1 be the weight of the particle A, and w_2 the weight of B. These weights are parallel forces, and their resultant divides the line AB in the inverse ratio of the forces. As the body is turned about into different positions, the forces w_1 and w_2 remain unchanged in magnitude, and hence the resultant cuts AB always in the same point. This point is called the centre of the parallel forces w_1 and w_2, or the centre of gravity of the two particles A and B. The magnitude of the resultant will be w_1+w_2, and we may substitute it for the two forces themselves; in other words, we may suppose the two particles A and B to be collected at their centre of gravity. We can now combine this resultant with the weight of a third particle of the body, and shall thus obtain a resultant $w_1+w_2+w_3$, passing through a definite point in the line which joins the third particle to the centre of gravity of the first two. The first three particles may now be supposed to be collected at this point, and the same reasoning may be extended until all the particles have been collected at one point. This point will be the *centre of gravity* of the whole body. From the manner in which it has been obtained, it possesses the property that *the resultant of all the forces of gravity on the body passes through it, in every position in which the body can be placed.* The resultant force of gravity upon a rigid body is therefore a single force passing through its centre of gravity.

35. Centres of Gravity of Volumes, Areas, and Lines.—If the body is homogeneous (that is composed of uniform substance throughout), the position of the centre of gravity depends only on the figure, and in this sense it is usual to speak of the centre of gravity of a figure. In like manner it is customary to speak of the centres of gravity of areas and lines, an area being identified in thought with a thin uniform plate, and a line with a thin uniform wire.

It is not necessary that a body should be rigid in order that it may have a centre of gravity. We may speak of the centre of gravity of a mass of fluid, or of the centre of gravity of a system of bodies not connected in any way. The same point which would be the centre of gravity if all the parts were rigidly connected, is still called by this name whether they are connected or not.

36. Methods of Finding Centres of Gravity.—Whenever a homogeneous body contains a point which bisects all lines in the body that can be drawn through it, this point must be the centre of gravity. The centres of a sphere, a circle, a cube, a square, an ellipse, an ellipsoid, a parallelogram, and a parallelepiped, are examples.

Again, when a body consists of a finite number of parts whose weights and centres of gravity are known, we may regard each part as collected at its own centre of gravity.

When the parts are at all numerous, the final result will most readily be obtained by the use of the formula

$$\bar{x} = \frac{\Sigma\,(\mathrm{P}x)}{\Sigma\,(\mathrm{P})}, \qquad (3)$$

where P denotes the weight of any part, x the distance of its centre of gravity from any plane, and $\bar{x}$ the distance of the centre of gravity of the whole from that plane. We have already in § 23

proved this formula for the case in which the centres of gravity lie in one straight line and x denotes distance from a point in this line; and it is not difficult, by the help of the properties of similar triangles, to make the proof general.

37. Centre of Gravity of a Triangle.—To find the centre of gravity of a triangle ABC (Fig. 10), we may begin by supposing it divided into narrow strips by lines (such as bc) parallel to BC. It can be shown, by similar triangles, that each of these strips is bisected by the line AD drawn from A to D the middle point of BC. But each strip may be collected at its own centre of gravity, that is at its own middle point; hence the whole triangle may be collected on the line AD; its centre of gravity must therefore be situated upon this line. Similar reasoning shows that it must lie upon the line BE drawn from B to the middle point of AC. It is therefore the intersection of these two lines. If we join DE we can show that the triangles AGB, DGE, are similar, and that

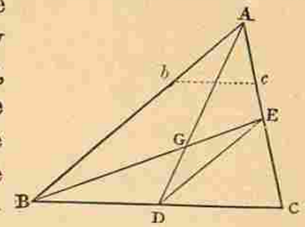


Fig. 10.

$$\frac{\mathrm{AG}}{\mathrm{GD}} = \frac{\mathrm{AB}}{\mathrm{DE}} = 2.$$

DG is therefore one third of DA. The centre of gravity of a triangle therefore lies upon the line joining any corner to the middle point of the opposite side, and is at one-third of the length of this line from the end where it meets that side.

It is worthy of remark that if three equal particles are placed at the corners of any triangle, they have the same centre of gravity as the triangle. For the two particles at B and C may be collected at the middle point D, and this double particle at D, together with the single particle at A, will have their centre of gravity at G, since G divides DA in the ratio of 1 to 2.

38. Centre of Gravity of a Pyramid.—If a pyramid or a cone be divided into thin slices by planes parallel to its base, and a straight line be drawn from the vertex to the centre of gravity of the base, this line will pass through the centres of gravity of all the slices, since all the slices are similar to the base, and are similarly cut by this line.

In a tetrahedron or triangular pyramid, if D (Fig. 11) be the centre of gravity of one face, and A be the corner opposite to this face, the centre of gravity of the pyramid must lie upon the line AD. In like manner, if E be the centre of gravity of one face, the centre of gravity of the pyramid must lie upon the line joining E with the opposite corner B. It must therefore be the intersection G of these two lines. That they do intersect is otherwise obvious, for the lines AE, BD meet in C, the middle point of one edge of the pyramid, E being found by taking CE one third of CA, and D by taking CD one third of CB.

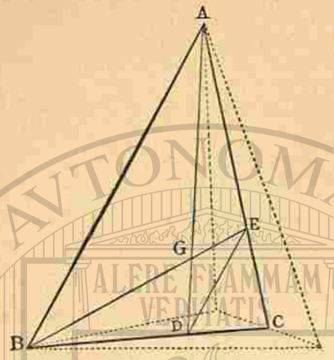


Fig. 11.—Centre of Gravity of Tetrahedron.

If D, E be joined, we can show that the joining line is parallel to BA, and that the triangles AGB, DGE are similar. Hence

$$\frac{AG}{GD} = \frac{AB}{DE} = \frac{BC}{DC} = 3.$$

That is, the line AD joining any corner to the centre of gravity of the opposite face, is cut in the ratio of 3 to 1 by the centre of gravity G of the triangle. DG is therefore one-fourth of DA, and the distance of the centre of gravity from any face is one-fourth of the distance of the opposite corner.

A pyramid standing on a polygonal base can be cut up into triangular pyramids standing on the triangular bases into which the polygon can be divided, and having the same vertex as the whole pyramid. The centres of gravity of these triangular pyramids are all at the same perpendicular distance from the base, namely at one-fourth of the distance of the vertex, which is therefore the distance of the centre of gravity of the whole from the base. The centre of gravity of any pyramid is therefore found by joining the vertex to the centre of gravity of the base, and cutting off one-fourth of the joining line from the end where it meets the base. The same rule applies to a cone, since a cone may be regarded as a polygonal pyramid with a very large number of sides.

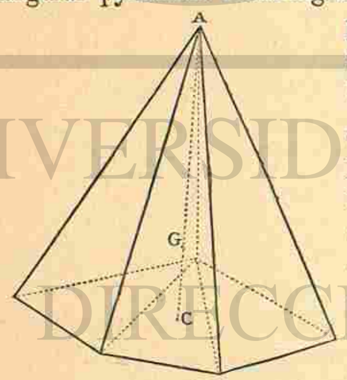


Fig. 12.—Centre of Gravity of Pyramid.

39. If four equal particles are placed at the corners of a triangular pyramid, they will have the same centre of gravity as the pyramid. For three of them may, as we have seen (§ 37) be collected at the centre of gravity of one face; and the centre of gravity of the four particles will divide the line which joins this point to the fourth, in the ratio of 1 to 3.

40. Condition of Standing or Falling.—When a heavy body stands on a base of finite area, and remains in equilibrium under the action of its own weight and the reaction of this base, the vertical through its centre of gravity must fall within the base. If the body is supported on three or more points, as in Fig. 13, we are to understand by the base the convex[1] polygon whose corners are the points of support; for if a body so supported turns over, it must turn about the line joining two of these points.


Fig. 13.—Equilibrium of a Body supported on a Horizontal Plane at three or more Points.

41. Body supported at one Point.—When a heavy body supported at one point remains at rest, the reaction of the point of support equilibrates the force of gravity. But two forces cannot be in equilibrium unless they have the same line of action; hence the vertical through the centre of gravity of the body must pass through the point of support. If instead of being supported at a point, the heavy body is supported by an axis about which it is free to turn, the vertical through the centre of gravity must pass through this axis.

42. Stability and Instability.—When the point of support, or axis of support, is vertically *below* the centre of gravity, it is easily seen that, if the body were displaced a little to either side, the forces acting upon it would turn it still further away from the position of equilibrium. On the other hand, when the point or axis of support is vertically *above* the centre of gravity, the forces which would

[1] The word *convex* is inserted to indicate that there must be no re-entrant angles. Any points of support which lie within the polygon formed by joining the rest, must be left out of account.

act upon it if it were slightly displaced would tend to restore it. In the latter case the equilibrium is said to be *stable*, in the former *unstable*.

When the centre of gravity coincides with the point of support, or lies upon the axis of support, the body will still be in equilibrium when turned about this point or axis into any other position. In this case the equilibrium is neither stable nor unstable but is called *neutral*.

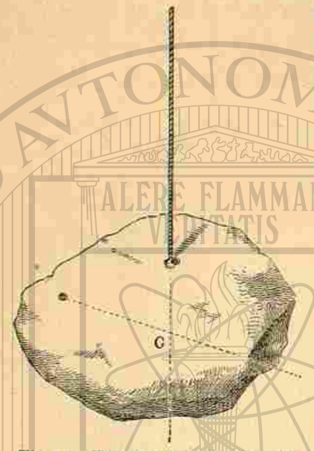


Fig. 14.—Experimental Determination of Centre of Gravity.

43. Experimental determination of Centre of Gravity.—In general, if we suspend a body by any point, in such a manner that it is free to turn about this point, it will come to rest in a position of stable equilibrium. The centre of gravity will then be vertically beneath the point of support. If we now suspend the body from another point, the centre of gravity will come vertically beneath this. The intersection of these two verticals will therefore be the centre of gravity (Fig. 14).

44. To find the centre of gravity of a flat plate or board (Fig. 15), we may suspend it from a point near its circumference, in such a manner that it sets itself in a vertical plane. Let a plumb-line be at the same time suspended from the same point, and made to leave its trace upon the board by chalking and "snapping" it. Let the board now be suspended from another point, and the operation be repeated. The two chalk lines will intersect each other at that point of the face which is opposite to the centre of gravity; the centre of gravity itself being of course in the substance of the board.

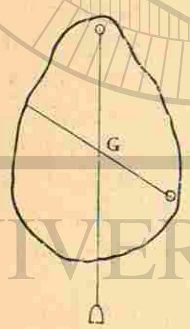


Fig. 15.—Centre of Gravity of Board.

45. Work done against Gravity.—When a heavy body is raised, work is said to be done against gravity, and the amount of this work is reckoned by multiplying together the weight of the body and the height through which it is raised. Horizontal movement does not count, and when a body is raised obliquely, only the vertical component of the motion is to be reckoned.

Suppose, now, that we have a number of particles whose weights

are w_1, w_2, w_3 &c., and let their heights above a given horizontal plane be respectively h_1, h_2, h_3 &c. We know by equation (3), § 23, that if $\bar{h}$ denote the height of their centre of gravity we have

$$(w_1 + w_2 + \&c.)\,\bar{h} = w_1 h_1 + w_2 h_2 + \&c. \qquad (4)$$

Let the particles now be raised into new positions in which their heights above the same plane of reference are respectively H_1, H_2, H_3 &c. The height $\bar{H}$ of their centre of gravity will now be such that

$$(w_1 + w_2 + \&c.)\,\bar{H} = w_1 H_1 + w_2 H_2 + \&c. \qquad (5)$$

From these two equations, we find, by subtraction

$$(w_1 + w_2 + \&c.)\,(\bar{H} - \bar{h}) = w_1 (H_1 - h_1) + w_2 (H_2 - h_2) + \&c. \qquad (6)$$

Now $H_1 - h_1$ is the height through which the particle of weight w_1 has been raised; hence the work done against gravity in raising it is $w_1 (H_1 - h_1)$ and the second member of equation (6) therefore expresses the whole amount of work done against gravity. But the first member expresses the work which would be done in raising all the particles through a uniform height $\bar{H} - \bar{h}$, which is the height of the new position of the centre of gravity above the old. The work done against gravity in raising any system of bodies will therefore be correctly computed by supposing all the system to be collected at its centre of gravity. For example, the work done in raising bricks and mortar from the ground to build a chimney, is equal to the total weight of the chimney multiplied by the height of its centre of gravity above the ground.

46. The Centre of Gravity tends to Descend.—When the forces which tend to move a system are simply the weights of its parts, we can determine whether it is in equilibrium by observing the path in which its centre of gravity would travel if movement took place. If we suppose this path to represent a hard frictionless surface, and the centre of gravity to represent a heavy particle placed upon it, the conditions of equilibrium will be the same as in the actual case. The centre of gravity tends to run down hill, just as a heavy particle does. There will be stable equilibrium if the centre of gravity is at the bottom of a valley in its path, and unstable equilibrium if it is at the top of a hill. When a rigid body turns about a horizontal axis, the path of its centre of gravity is a circle in a vertical plane. The highest and lowest points of this circle are the positions of the centre of gravity in unstable and stable equilibrium respectively;

except when the axis traverses the centre of gravity itself, in which case the centre of gravity can neither rise nor fall, and the equilibrium is neutral.

A uniform sphere or cylinder lying on a horizontal plane is in neutral equilibrium, because its centre of gravity will neither be raised nor lowered by rolling. An egg balanced on its end as in Fig. 16, is in unstable equilibrium, because its centre of gravity is at the top of a hill which it will descend when the egg rolls to one side. The position of equilibrium shown in Fig. 17 is stable as regards rolling to left or right, because the path of its centre of gravity in

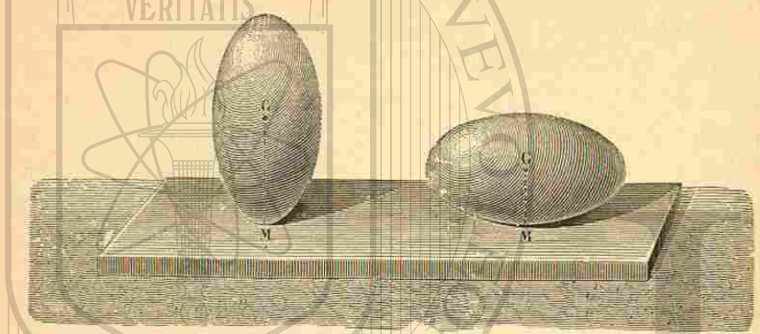


Fig. 16.—Unstable Equilibrium. Fig. 17.—Stable Equilibrium.

such rolling would be a curve whose lowest point is that now occupied by the centre of gravity. As regards rolling in the direction at right angles to this, if the egg is a true solid of resolution, the equilibrium is neutral.

47. Work done by Gravity.—When a heavy body is lifted, the lifting force does work against gravity. When it descends gravity does work upon it; and if it descends to the same position from which it was lifted, the work done by gravity in the descent is equal to the work done against gravity in the lifting; each being equal to the weight of the body multiplied by the vertical displacement of its centre of gravity. The tendency of the centre of gravity to descend is a manifestation of the tendency of gravity to do work; and this tendency is not peculiar to gravity.

48. Work done by any Force.—A force is said to do work when its point of application moves in the direction of the force, or in any direction making an acute angle with this, so as to give a component displacement in the direction of the force; and the amount of work done is the product of the force by this component. If F denote

the force, a the displacement, and θ the angle between the two, the work done by F is

$$F\, a \cos \theta.$$

which is what we obtain either by the above rule or by multiplying the whole displacement by the effective component of F, that is the component of F in the direction of the displacement. If the angle θ is obtuse, $\cos \theta$ is negative and the force F does negative work. If θ is a right angle F does no work. In this case F neither assists nor resists the displacement. When θ is acute, F assists the displacement, and would produce it if the body were constrained by guides which left it free to take this displacement and the directly opposite one, while preventing all others.

If θ is obtuse, F resists the displacement, and would produce the opposite displacement if the body were constrained in the manner just supposed.

49. Principle of Work.—If any number of forces act upon a body which is only free to move in a particular direction and its opposite, we can tell in which of these two directions it will move by calculating the work which each force would do. Each force would do positive work when the displacement is in one direction, and negative work when it is in the opposite direction, the absolute amounts of work being the same in both cases if the displacements are equal. The body will upon the whole be urged in that direction which gives an excess of positive work over negative. If no such excess exists, but the amounts of positive and negative work are exactly equal, the body is in equilibrium. This principle (which has been called the principle of *virtual velocities*, but is better called the *principle of work*) is often of great use in enabling us to calculate the ratio which two forces applied in given ways to the same body must have in order to equilibrate each other. It applies not only to the "mechanical powers" and all combinations of solid machinery, but also to hydrostatic arrangements; for example to the hydraulic press. The condition of equilibrium between two forces applied to any frictionless machine and tending to drive it opposite ways, is that in a small movement of the machine they would do equal and opposite amounts of work. Thus in the screw-press (Fig. 30) the force applied to one of the handles, multiplied by the distance through which this handle moves, will be equal to the pressure which this force produces at the foot of the screw, multiplied by the distance that the screw travels.

This is on the supposition of no friction. A frictionless machine gives out the same amount of work which is spent in driving it. The effect of friction is to make the work given out less than the work put in. Much fruitless ingenuity has been expended upon contrivances for circumventing this law of nature and producing a machine which shall give out more work than is put into it. Such contrivances are called "perpetual motions."

50. General Criterion of Stability.—If the forces which act upon a body and produce equilibrium remain unchanged in magnitude and direction when the body moves away from its position, and if the velocities of their points of application also remain unchanged in direction and in their ratio to each other, it is obvious that the equality of positive and negative work which subsists at the beginning of the motion will continue to subsist throughout the entire motion. The body will therefore remain in equilibrium when displaced. Its equilibrium is in this case said to be neutral.

If the forces which are in equilibrium in a given position of the body, gradually change in direction or magnitude as the body moves away from this position, the equality of positive and negative work will not in general continue to subsist, and the inequality will increase with the displacement. If the body be displaced with a constant velocity and in a uniform manner, the rate of doing work, which is zero at first, will not continue to be zero, but will have a value, whether positive or negative, increasing in simple proportion to the displacement. Hence it can be shown that the whole work done in a small movement is proportional to the square of the displacement, for when we double the displacement we, at the same time, double the mean working force.

If this work is positive, the forces assist the displacement and tend to increase it; the equilibrium must therefore have been unstable.

On the other hand, if the work is negative in all possible displacements from the position of equilibrium, the forces oppose the displacements and the equilibrium is stable.

51. Illustration of Stability.—A good example of stable equilibrium of this kind is furnished by Gravesande's apparatus (Fig. 3) simplified by removing the parallelogram and employing a string to support the three weights, one of them P″ being fastened to it at a point A near its middle, and the others P, P′ to its ends. The point A will take the same position as in the figure, and will return to it again when displaced. If we take hold of the point A and

move it in any direction whether in the plane of the string or out of it, we feel that at first there is hardly any resistance and the smallest force we can apply produces a sensible disturbance; but that as the displacement increases the resistance becomes greater. If we release the point A when displaced, it will execute oscillations, which will become gradually smaller, owing to friction, and it will finally come to rest in its original position of equilibrium.

The centre of gravity of the three weights is in its lowest position when the system is in equilibrium, and when a small displacement is produced the centre of gravity rises by an amount proportional to its square, so that a double displacement produces a quadruple rise of the centre of gravity.

In this illustration the three forces remain unchanged, and the directions of two of them change gradually as the point A is moved. Whenever the circumstances of stable equilibrium are such that the forces make no abrupt changes either in direction or magnitude for small displacements, the resistance will, as in this case, be proportional to the displacement (when small), and the work to the square of the displacement, and the system will oscillate if displaced and then left to itself.

52. Stability where Forces vary abruptly with Position.—There are other cases of stable equilibrium which may be illustrated by the example of a book lying on a table. If we displace it by lifting one edge, the force which we must exert does not increase with the displacement, but is sensibly constant when the displacement is small, and as a consequence the work will be simply proportional to the displacement. The reason is, that one of the forces concerned in producing equilibrium, namely, the upward pressure of the table, changes *per saltum* at the moment when the displacement begins. In applying the principle of work to such a case as this, we must employ, instead of the actual work done by the force which changes abruptly, the work which it would do if its magnitude and direction remained unchanged, or only changed gradually.

53. Illustrations from Toys.—The stability of the "balancer" (Fig. 18) depends on the fact that, owing to the weight of the two leaden balls, which are rigidly attached to the figure by stiff wires, the centre of gravity of the whole is below the point of support. If the figure be disturbed it oscillates, and finally comes to rest in a position in which the centre of gravity is vertically under the toe on which the figure stands.

The "tumbler" (Fig. 19) consists of a light figure attached to a hemisphere of lead, the centre of gravity of the whole being between the centre of gravity of the hemisphere and the centre of the sphere to which it belongs. When placed upon a level table, the lowest position of the centre of gravity is that in which the figure is upright, and it accordingly returns to this position when displaced.

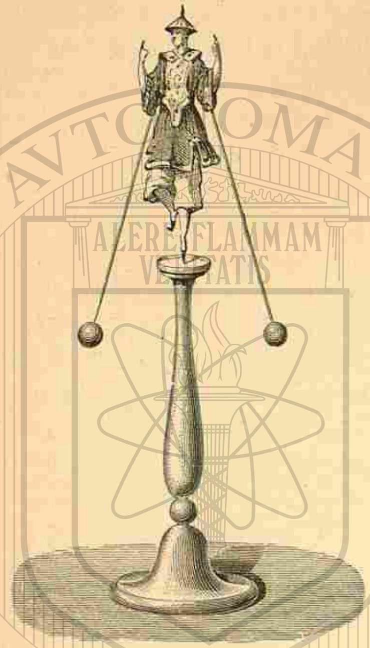


Fig. 18.—Balancer.

54. Limits of Stability.—In the foregoing discussion we have employed the term "stability" in its strict mathematical sense. But there are cases in which, though small displacements would merely produce small oscillations, larger displacements would cause the body, when left to itself, to fall entirely away from the given position of equilibrium. This may be expressed by saying that the equilibrium is stable for displacements lying within certain limits, but unstable for displacements beyond these limits. The equilibrium

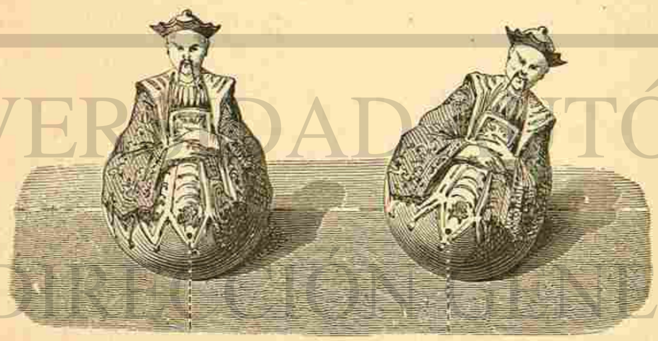


Fig. 19.—Tumblers.

of a system is *practically* unstable when the displacements which it is likely to receive from accidental disturbances lie beyond its limits of stability.

CHAPTER IV.

THE MECHANICAL POWERS.

55. We now proceed to a few practical applications of the foregoing principles; and we shall begin with the so-called "mechanical powers," namely, the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*.

56. Lever.—Problems relating to the lever are usually most conveniently solved by taking moments round the fulcrum. The general condition of equilibrium is, that the moments of the power and the weight about the fulcrum must be in opposite directions, and must be equal. When the power and weight act in parallel directions, the conditions of equilibrium are precisely those of three parallel forces (§ 19), the third force being the reaction of the fulcrum.

It is usual to distinguish three "orders" of lever. In levers of the first order (Fig. 20) the fulcrum is between the power and the

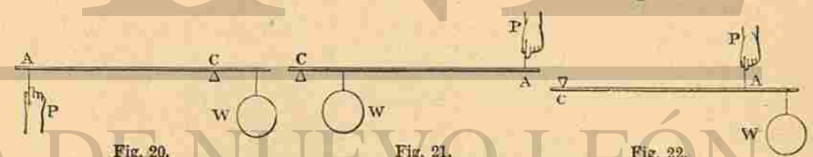


Fig. 20. Fig. 21. Fig. 22.
Three Orders of Lever.

weight. In those of the second order (Fig. 21) the weight is between the power and the fulcrum. In those of the third order (Fig. 22) the power is between the weight and the fulcrum.

In levers of the second order (supposing the forces parallel), the weight is equal to the sum of the power and the pressure on the fulcrum; and in levers of the third order, the power is equal to the sum of the weight and the pressure on the fulcrum; since the middle one of three parallel forces in equilibrium must always be equal to the sum of the other two.

The "tumbler" (Fig. 19) consists of a light figure attached to a hemisphere of lead, the centre of gravity of the whole being between the centre of gravity of the hemisphere and the centre of the sphere to which it belongs. When placed upon a level table, the lowest position of the centre of gravity is that in which the figure is upright, and it accordingly returns to this position when displaced.

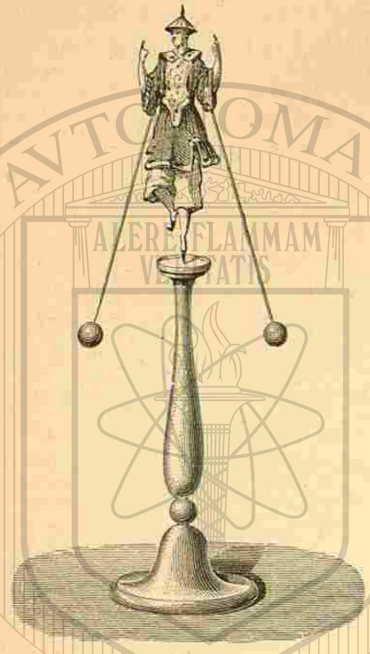


Fig. 18.—Balancer.

54. Limits of Stability.—In the foregoing discussion we have employed the term "stability" in its strict mathematical sense. But there are cases in which, though small displacements would merely produce small oscillations, larger displacements would cause the body, when left to itself, to fall entirely away from the given position of equilibrium. This may be expressed by saying that the equilibrium is stable for displacements lying within certain limits, but unstable for displacements beyond these limits. The equilibrium of a system is *practically* unstable when the displacements which it is likely to receive from accidental disturbances lie beyond its limits of stability.

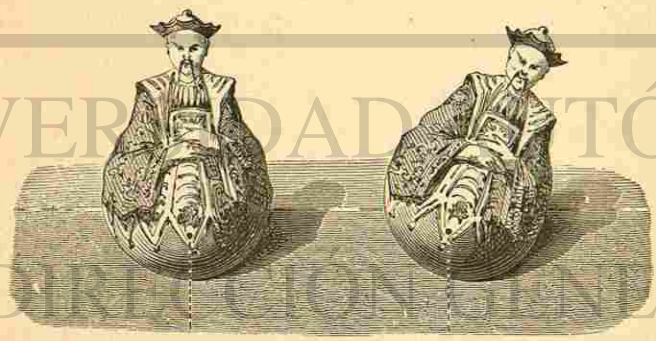


Fig. 19.—Tumblers.

CHAPTER IV.

THE MECHANICAL POWERS.

55. We now proceed to a few practical applications of the foregoing principles; and we shall begin with the so-called "mechanical powers," namely, the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*.

56. Lever.—Problems relating to the lever are usually most conveniently solved by taking moments round the fulcrum. The general condition of equilibrium is, that the moments of the power and the weight about the fulcrum must be in opposite directions, and must be equal. When the power and weight act in parallel directions, the conditions of equilibrium are precisely those of three parallel forces (§ 19), the third force being the reaction of the fulcrum.

It is usual to distinguish three "orders" of lever. In levers of the first order (Fig. 20) the fulcrum is between the power and the weight. In those of the second order (Fig. 21) the weight is between the power and the fulcrum. In those of the third order (Fig. 22) the power is between the weight and the fulcrum.

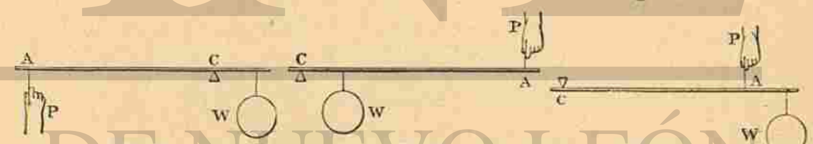


Fig. 20. Fig. 21. Fig. 22.
Three Orders of Lever.

In levers of the second order (supposing the forces parallel), the weight is equal to the sum of the power and the pressure on the fulcrum; and in levers of the third order, the power is equal to the sum of the weight and the pressure on the fulcrum; since the middle one of three parallel forces in equilibrium must always be equal to the sum of the other two.

57. Arms.—The *arms of a lever* are the two portions of it intermediate, respectively, between the fulcrum and the power, and between the fulcrum and the weight. If the lever is bent, or if, though straight, it is not at right angles to the lines of action of the power and weight, it is necessary to distinguish between the arms of the lever as above defined (which are parts of the lever), and the *arms of the power and weight* regarded as forces which have moments round the fulcrum. In this latter sense (which is always to be understood unless the contrary is evidently intended), the arms are the perpendiculars dropped from the fulcrum upon the lines of action of the power and weight.

58. Weight of Lever.—In the above statements of the conditions of equilibrium, we have neglected the weight of the lever itself. To take this into account, we have only to suppose the whole weight of the lever collected at its centre of gravity, and then take its moment round the fulcrum. We shall thus have three moments to take account of, and the sum of the two that tend to turn the lever one way, must be equal to the one that tends to turn it the opposite way.

59. Mechanical Advantage.—Every machine when in action serves to transmit *work* without altering its amount; but the *force* which the machine gives out (equal and opposite to what is commonly called the *weight*) may be much greater or much less than that by which it is driven (commonly called the *power*). When it is greater, the machine is said to confer *mechanical advantage*, and the mechanical advantage is measured by the ratio of the weight to the power for equilibrium. In the lever, when the power has a longer arm than the weight, the mechanical advantage is equal to the quotient of the longer arm by the shorter.

60. Wheel and Axle.—The wheel and axle (Fig. 23) may be regarded as an endless lever. The condition of equilibrium is at once given by taking moments round the common axis of the wheel and axle (§ 24). If we neglect the thickness of the ropes, the condition is that the power multiplied by the radius of the wheel must equal the weight multiplied by the radius of the axle; but it is more exact to regard the lines of action of the two forces as coinciding with the axes of the two ropes, so that each of the two radii should be increased by half the thickness of its own rope. If we neglect the thickness of the ropes, the

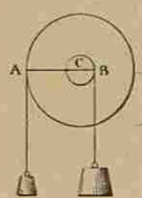


Fig. 23.

mechanical advantage is the quotient of the radius of the wheel by the radius of the axle.

61. Pulley.—A pulley, when fixed in such a way that it can only turn about a fixed axis (Fig. 24), confers no mechanical advantage. It may be regarded as an endless lever of the first order with its two arms equal.

The arrangement represented in Fig. 25 gives a mechanical advantage of 2; for the lower or movable pulley may be regarded as an endless lever of the second order, in which the arm of the power is the diameter of the pulley, and the arm of the weight is

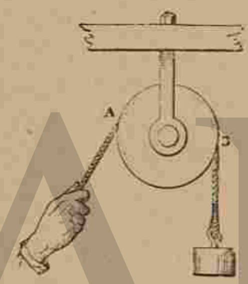


Fig. 24.


Fig. 25.


Fig. 26.

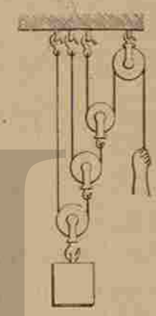


Fig. 27.

half the diameter. The same result is obtained by employing the principle of work; for if the weight rises 1 inch, 2 inches of slack are given over, and therefore the power descends 2 inches.

62. In Fig. 26 there are six pulleys, three at the upper and three at the lower block, and one cord passes round them all. All portions of this cord (neglecting friction) are stretched with the same force, which is equal to the power; and six of these portions, parallel to one another, support the weight. The power is therefore one-sixth of the weight, or the mechanical advantage is 6.

63. In the arrangement represented in Fig. 27, there are three movable pulleys, each hanging by a separate cord. The cord which supports the lowest pulley is stretched with a force equal to half the weight, since its two parallel portions jointly support the weight. The cord which supports the next pulley is stretched with a force half of this, or a quarter of the weight; and the next cord with a force half of this, or an eighth of the weight; but this cord is directly attached to the power. Thus the power is an eighth of the weight, or the mechanical advantage is 8. If the weight and the block[1] to which it is attached rise 1 inch, the next block rises 2 inches, the next 4, and the power moves through 8 inches. Thus, the work done by the power is equal to the work done upon the weight.

In all this reasoning we neglect the weights of the blocks themselves; but it is not difficult to take them into account when necessary.

64. Inclined Plane.—We now come to the inclined plane. Let AB (Fig. 28) be any portion of such a plane, and let AC and BC be drawn vertically and horizontally. Then AB is called the *length*, AC the *height*, and CB the *base* of the inclined plane. The force of gravity upon a heavy body M resting on the plane, may be represented by a vertical line MP, and may be resolved by the parallelogram of forces (§ 16) into two components, MT, MN, the former parallel and the latter perpendicular to the plane. A force equal and opposite to the component MT will suffice to prevent the body from slipping down the plane. Hence, if the power act parallel to the plane, and the weight be that of a heavy body resting on the plane, the power is to the weight as MT to MP; but the two triangles MTP and ACB are similar, since the angles at M and A are equal, and the angles at T and C are right angles; hence MT is to MP as AC to AB, that is, as the height to the length of the plane.

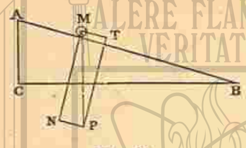


Fig. 28.

65. The investigation is rather easier by the principle of work (§ 49). The work done by the power in drawing the heavy body up the plane, is equal to the power multiplied by the length of the plane. But the work done upon the weight is equal to the weight multiplied by the height through which it is raised, that is, by the height of the plane. Hence we have

Power × length of plane = weight × height of plane; or
power : weight : : height of plane : length of plane.

66. If, instead of acting parallel to the plane, the power acted parallel to the base, the work done by the power would be the product of the power by the base; and this must be equal to the product of the weight by the height; so that in this case the condition of equilibrium would be—

[1] The "pulley" is the revolving wheel. The pulley, together with the frame in which it is inclosed, constitute the "block."

Power : weight : : height of plane : base of plane.

67. Wedge.—In these investigations we have neglected friction. The wedge may be regarded as a case of the inclined plane; but its practical action depends to such a large extent upon friction and impact[1] that we cannot profitably discuss it here.

68. Screw.—The screw (Fig. 29) is also a case of the inclined plane. The length of one convolution of the thread is the length of the corresponding inclined plane, the step of the screw, or distance between two successive convolutions (measured parallel to the axis of the screw), is the height of the plane, and the circumference of the screw is the base of the plane. This is easily shown by cutting out a right-angled triangle in paper, and bending it in cylindrical fashion so that its base forms a circle.

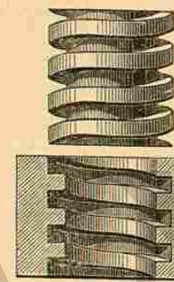


Fig 29.

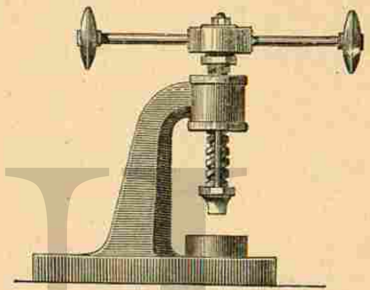


Fig. 30.

69. Screw Press.—In the screw press (Fig. 30) the screw is turned by means of a lever, which gives a great increase of mechanical advantage. In one complete revolution, the pressures applied to the two handles of the lever to turn it, do work equal to their sum multiplied by the circumference of the circle described (approximately) by either handle (we suppose the two handles to be equidistant from the axis of revolution); and the work given out by the machine, supposing the resistance at its lower end to be constant, is equal to this resistance multiplied by the distance between the threads. These two products must be equal, friction being neglected.

[1] An *impact* (for example a blow of a hammer) may be regarded as a very great (and variable) force acting for a very short time. The magnitude of an impact is measured by the momentum which it generates in the body struck.

CHAPTER V.

THE BALANCE.

70. General Description of the Balance.—In the common *balance* (Fig. 31) there is a stiff piece of metal, A B, called the *beam*, which turns about the sharp edge O of a steel wedge forming part of the beam and resting upon two hard and smooth supports. There are two other steel wedges at A and B, with their edges upwards, and upon these edges rest the hooks for supporting the scale pans. The three edges (called knife-edges) are parallel to one another and perpendicular to the length of the beam, and are very nearly in one plane.

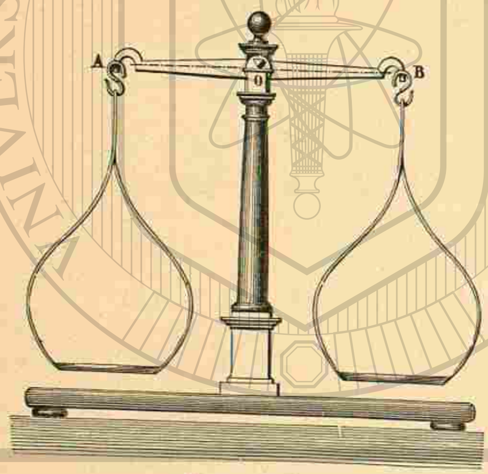


Fig. 31.—Balance.

71. Qualities Requisite.—The qualities requisite in a balance are:

1. That it be consistent with itself; that is, that it shall give the same result in successive weighings of the same body. This depends chiefly on the trueness of the knife-edges.

2. That it be just. This requires that the distances A O, O B, be equal, and also that the beam remain horizontal when the pans are empty. Any inequality in the distances A O, O B, can be detected by putting equal (and tolerably heavy) weights into the two pans. This adds equal moments if the distances are equal, but unequal

moments if they are unequal, and the greater moment will preponderate.

3. Delicacy or sensibility (that is, the power of indicating inequality between two weights even when their difference is very small).

This requires a minimum of friction, and a very near approach to neutral equilibrium (§ 40). In absolutely neutral equilibrium, the smallest conceivable force is sufficient to produce a displacement to the full limit of neutrality; and in barely stable equilibrium a small force produces a large displacement. The condition of stability is that if the weights supported at A and B be supposed collected at these edges, the centre of gravity of the system composed of the beam and these two weights shall be below the middle edge O. The equilibrium would be neutral if this centre of gravity exactly coincided with O; and it is necessary as a condition of delicacy that its distance below O be very small.

4. Facility for weighing quickly is desirable, but must sometimes be sacrificed when extreme accuracy is required.

The delicate balances used in chemical analysis are provided with a long pointer attached to the beam. The end of this pointer moves along a graduated arc as the beam vibrates; and if the weights in the two pans are equal, the excursions of the pointer on opposite sides of the zero point of this arc will also be equal. Much time is consumed in watching these vibrations, as they are very slow; and the more nearly the equilibrium approaches to neutrality, the slower they are. Hence quick weighing and exact weighing are to a certain extent incompatible.

72. Double Weighing.—Even if a balance be not just, yet if it be consistent with itself, a correct weighing can be made with it in the following manner:—Put the body to be weighed in one pan, and counterbalance it with sand or other suitable material in the other. Then remove the body and put in its place such weights as are just sufficient to counterpoise the sand. These weights are evidently equal to the weight of the body. This process is called *double weighing*, and is often employed (even with the best balances) when the greatest possible accuracy is desired.

73. Investigation of Sensibility.—Let A and B (Fig. 32) be the points from which the scale-pans are suspended, O the axis about which the beam turns, and G the centre of gravity of the beam. If when the scale-pans are loaded with equal weights, we put into one of them an excess of weight p, the beam will become inclined, and will take a position such as A′B′, turning through an angle which we will call α, and which is easily calculated.

In fact let the two forces P and $P + p$ act at A′ and B′ respectively, where P denotes the less of the two weights, including the weight of the pan. Then the two forces P destroy each other in consequence of the resistance of the axis O; there is left only the force p applied at B′, and the weight π of the beam applied at G′, the new position of the centre of gravity. These two forces are parallel, and are in equilibrium about the axis O, that is, their resultant passes through the point O. The distances of the points of application of the forces from a vertical through O are therefore inversely proportional to the forces themselves, which gives the relation

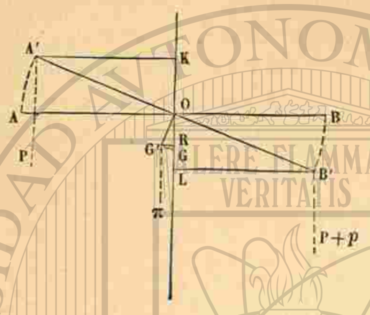


Fig. 32.

$$\pi.\ G'R = p.\ B'L.$$

But if we call half the length of the beam l, and the distance OG r we have

$$G'R = r \sin \alpha, \quad B'L = l \cos \alpha.$$

whence $\pi r \sin \alpha = pl \cos \alpha$, and consequently

$$\tan \alpha = \frac{pl}{\pi r}. \qquad (a)$$

The formula (a) contains the entire theory of the sensibility of the balance when properly constructed. We see, in the first place, that $\tan \alpha$ increases with the excess of weight p, which was evident beforehand. We see also that the sensibility increases as l increases and as π diminishes, or, in other words, as the beam becomes longer and lighter. At the same time it is obviously desirable that, under the action of the weights employed, the beam should be stiff enough to undergo no sensible change of shape. The problem of the balance then consists in constructing a beam of the greatest possible length and lightness, which shall be capable of supporting the action of given forces without bending.

Fortin, whose balances are justly esteemed, employed for his beams bars of steel placed edgewise; he thus obtained great rigidity, but

certainly not all the lightness possible. At present the makers of balances employ in preference beams of copper or steel made in the form of a frame, as shown in Fig 33. They generally give them the shape of a very elongated lozenge, the sides of which are connected by bars variously arranged. The determination of the best shape is, in fact, a special problem, and is an application on a small scale of that principle of applied mechanics which teaches us that hollow pieces have greater resisting power in proportion to their weight than solid pieces, and consequently, for equal resisting power, the former are lighter than the latter. Aluminium, which with a rigidity nearly equal to that of copper, has less than one-fourth of its density, seems naturally marked out as adapted to the construction of beams. It has as yet, however, been little used.

The formula (a) shows us, in the second place, that the sensibility increases as r diminishes; that is, as the centre of gravity approaches the centre of suspension. These two points, however, must not coincide, for in that case for any excess of weight, however small, the beam would deviate from the horizontal as far as the mechanism would permit, and would afford no indication of approach to equality in the weights. With equal weights it would remain in equilibrium in any position. In virtue of possessing this last property, such a balance is called *indifferent*. Practically the distance between the centre of gravity and the point of suspension must not be less than a certain amount depending on the use for which the balance is designed. The proper distance is determined by observing what difference of weights corresponds to a division of the graduated arc along which the needle moves. If, for example, there are 20 divisions on each side of zero, and if 2 milligrammes are necessary for the total displacement of the needle, each division will correspond to an excess of weight of $\frac{2}{20}$ or $\frac{1}{10}$ of a milligramme. That this may be the case we must evidently have a suitable value of r, and the maker is enabled to regulate this value with precision by means of the screw which is shown in the figure above the beam, and which enables him slightly to vary the position of the centre of gravity.

74. Weighing with Constant Load.—In the above analysis we have supposed that the three points of suspension of the beam and of the two scale-pans are in one straight line; in which case the value of $\tan \alpha$ does not include P, that is, the sensibility is independent of the weight in the pans. This follows from the fact that the resultant of the two forces P passes through O, and is thus destroyed, because the axis is fixed. This would not be the case if, for example, the points of suspension of the pans were above that of the beam; in this case the point of application of the common load is above the point O, and, when the beam is inclined, acts in the same direction as the excess of weight; whence the sensibility increases with the load up to a certain limit, beyond which the equilibrium becomes unstable.[1] On the other hand, when the points of suspension of the pans are below that of the beam, the sensibility increases as the load diminishes, and, as the centre of gravity of the beam may in this case be above the axis, equilibrium may become unstable when the load is less than a certain amount. This variation of the sensibility with the load is a serious disadvantage; for, as we have just shown, the displacement of the needle is used as the means of estimating weights, and for this purpose we must have the same displacement corresponding to the same excess of weight. If we wish to employ

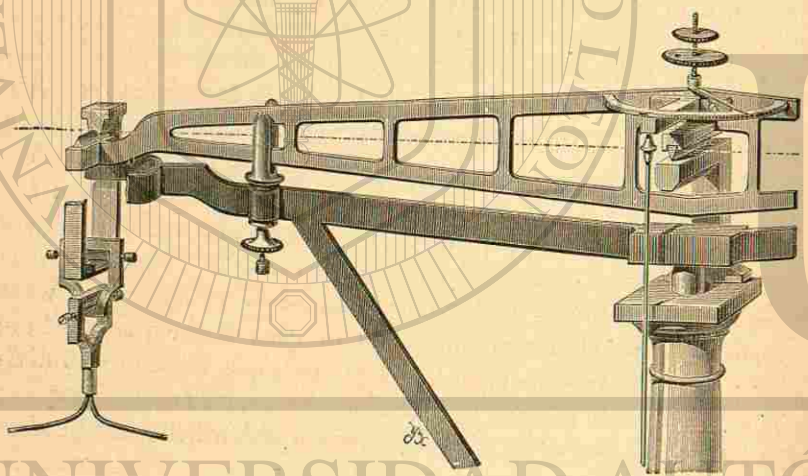


Fig. 33.—Beam of Balance.

either of the two above arrangements, we should weigh with a constant load. The method of doing so, which constitutes a kind of double weighing, consists in retaining in one of the pans a weight equal to this constant load. In the other pan is placed the same load subdivided into a number of marked weights. When the body

[1] This is an illustration of the general principle, applicable to a great variety of philosophical apparatus, that a maximum of sensibility involves a minimum of stability; that is, a very near approach to instability. This near approach is usually indicated by excessive slowness in the oscillations which take place about the position of equilibrium.

to be weighed is placed in this latter pan, we must, in order to maintain equilibrium, remove a certain number of weights, which evidently represent the weight of the body.

We may also remark that, strictly speaking, the sensibility always depends upon the load, which necessarily produces a variation in the friction of the axis of suspension. Besides, it follows from the nature

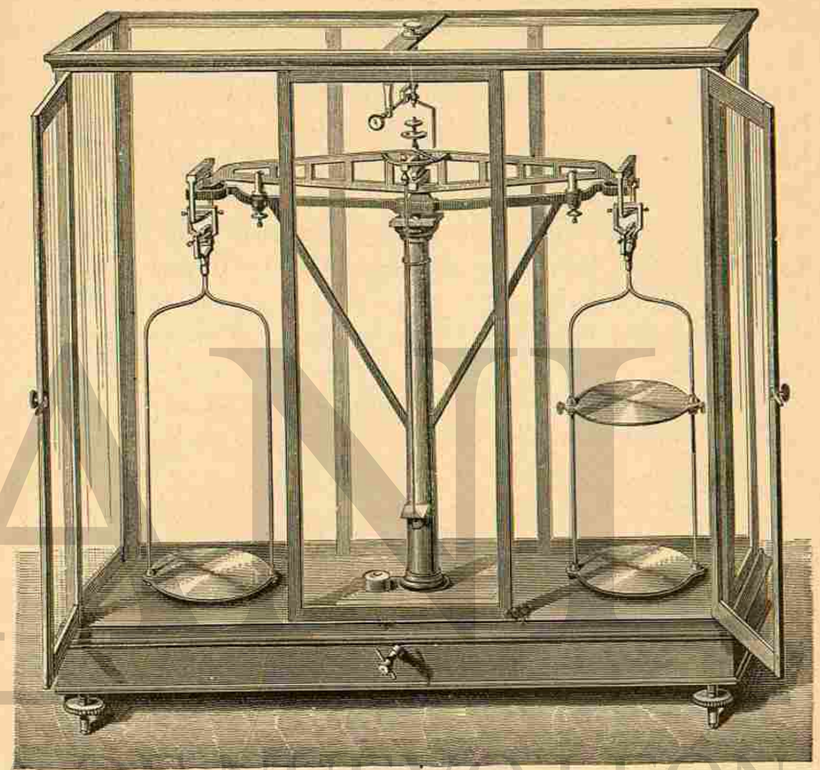


Fig. 34.—Balance for Purposes of Accuracy.

of bodies that there is no system that does not yield somewhat even to the most feeble action. For these reasons, there is a decided advantage in operating with constant load.

75. Details of Construction.—A fundamental condition of the correctness of the balance is, that the weight of each pan and of the load which it contains should always act exactly at the same point, and therefore at the same distance from the axis of suspension. This important result is attained by different methods. The arrangement represented in Fig. 33 is one of the most effectual. At the extremities of the beam are two knife-edges, parallel to the axis of rotation, and facing upwards. On these knife-edges rests, by a hard plane surface of agate or steel, a stirrup, the front of which has been taken away in the figure. On the lower part of the stirrup rests another knife-edge, at right angles to the former, the two being together equivalent to a universal joint supporting the scale-pan and its contents. By this arrangement, whatever may be the position of the weights, their action is always reduced to a vertical force acting on the upper knife-edge.

Fig. 34 represents a balance of great delicacy, with the glass case that contains it. At the bottom is seen the extremity of a lever, which enables us to raise the beam, and thus avoid wearing the knife-edge when not in use. At the top may be remarked an arrangement employed by some makers, consisting of a horizontal graduated circle, on which a small metallic index can be made to travel; its different displacements, whose value can be determined once for all, are used for the final adjustment to produce exact equilibrium.

76. **Steelyard.**—The steelyard (Fig. 35) is an instrument for weighing bodies by means of a single weight, P, which can be hung at any point of a graduated arm O B. As P is moved further from the fulcrum O, its moment round O increases, and therefore the weight which must be hung from the fixed point A to counterbalance it increases. Moreover, equal movements of P along the arm produce equal additions to its moment, and equal additions to the weight at A produce equal additions to the opposing moment. Hence the divisions on the arm (which indicate the weight in the pan at A) must be equidistant.

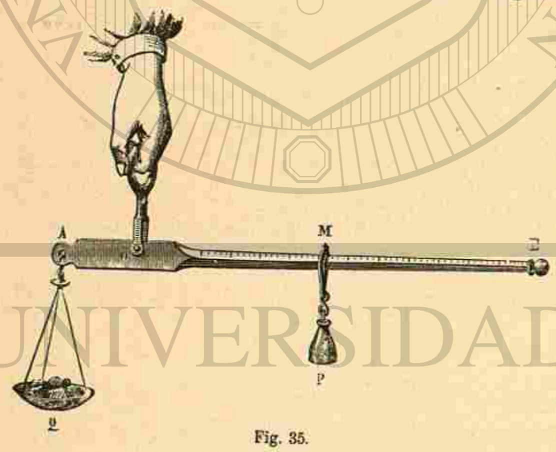


Fig. 35.

CHAPTER VI.

FIRST PRINCIPLES OF KINETICS.

77. **Principle of Inertia.**—A body not acted on by any forces, or only acted on by forces which are in equilibrium, will not commence to move; and if it be already in motion with a movement of pure translation, it will continue its velocity of translation unchanged, so that each of its points will move in a straight line with uniform velocity. This is Newton's first law of motion, and is stated by him in the following terms:—

"Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state."

The tendency to continue in a state of rest is manifest to the most superficial observation. The tendency to continue in a state of uniform motion can be clearly understood from an attentive study of facts. If, for example, we make a pendulum oscillate, the amplitude of the oscillations slowly decreases and at last vanishes altogether. This is because the pendulum experiences resistance from the air which it continually displaces; and because the axis of suspension rubs on its supports. These two circumstances combine to produce a diminution in the velocity of the apparatus until it is completely annihilated. If the friction at the point of suspension is diminished by suitable means, and the apparatus is made to oscillate *in vacuo*, the duration of the motion will be immensely increased.

Analogy evidently indicates that if it were possible to suppress entirely these two causes of the destruction of the pendulum's velocity, its motion would continue for an indefinite time unchanged.

This tendency to continue in motion is the cause of the effects which are produced when a carriage or railway train is suddenly stopped. The passengers are thrown in the direction of the motion, extremities of the beam are two knife-edges, parallel to the axis of rotation, and facing upwards. On these knife-edges rests, by a hard plane surface of agate or steel, a stirrup, the front of which has been taken away in the figure. On the lower part of the stirrup rests another knife-edge, at right angles to the former, the two being together equivalent to a universal joint supporting the scale-pan and its contents. By this arrangement, whatever may be the position of the weights, their action is always reduced to a vertical force acting on the upper knife-edge.

Fig. 34 represents a balance of great delicacy, with the glass case that contains it. At the bottom is seen the extremity of a lever, which enables us to raise the beam, and thus avoid wearing the knife-edge when not in use. At the top may be remarked an arrangement employed by some makers, consisting of a horizontal graduated circle, on which a small metallic index can be made to travel; its different displacements, whose value can be determined once for all, are used for the final adjustment to produce exact equilibrium.

76. **Steelyard.**—The steelyard (Fig. 35) is an instrument for weighing bodies by means of a single weight, P, which can be hung at any point of a graduated arm O B. As P is moved further from the fulcrum O, its moment round O increases, and therefore the weight which must be hung from the fixed point A to counterbalance it increases. Moreover, equal movements of P along the arm produce equal additions to its moment, and equal additions to the weight at A produce equal additions to the opposing moment. Hence the divisions on the arm (which indicate the weight in the pan at A) must be equidistant.

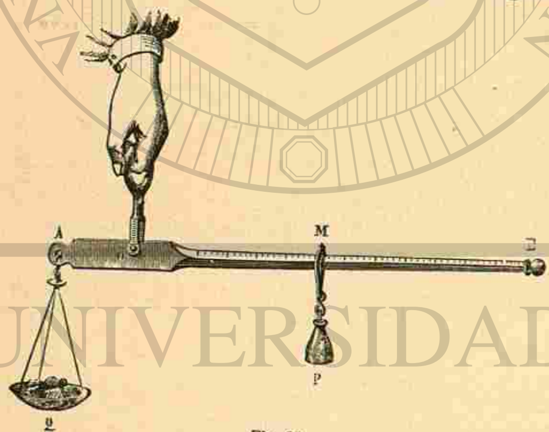


Fig. 35.

CHAPTER VI.

FIRST PRINCIPLES OF KINETICS.

77. **Principle of Inertia.**—A body not acted on by any forces, or only acted on by forces which are in equilibrium, will not commence to move; and if it be already in motion with a movement of pure translation, it will continue its velocity of translation unchanged, so that each of its points will move in a straight line with uniform velocity. This is Newton's first law of motion, and is stated by him in the following terms:—

"Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state."

The tendency to continue in a state of rest is manifest to the most superficial observation. The tendency to continue in a state of uniform motion can be clearly understood from an attentive study of facts. If, for example, we make a pendulum oscillate, the amplitude of the oscillations slowly decreases and at last vanishes altogether. This is because the pendulum experiences resistance from the air which it continually displaces; and because the axis of suspension rubs on its supports. These two circumstances combine to produce a diminution in the velocity of the apparatus until it is completely annihilated. If the friction at the point of suspension is diminished by suitable means, and the apparatus is made to oscillate *in vacuo*, the duration of the motion will be immensely increased.

Analogy evidently indicates that if it were possible to suppress entirely these two causes of the destruction of the pendulum's velocity, its motion would continue for an indefinite time unchanged.

This tendency to continue in motion is the cause of the effects which are produced when a carriage or railway train is suddenly stopped. The passengers are thrown in the direction of the motion, in virtue of the velocity which they possessed at the moment when the stoppage occurred. If it were possible to find a brake sufficiently powerful to stop a train suddenly at full speed, the effects of such a stoppage would be similar to the effects of a collision.

Inertia is also the cause of the severe falls which are often received in alighting incautiously from a carriage in motion; all the particles of the body have a forward motion, and the feet alone being reduced to rest, the upper portion of the body continues to move, and is thus thrown forward.

When we fix the head of a hammer on the handle by striking the end of the handle on the ground, we utilize the inertia of matter. The handle is suddenly stopped by the collision, and the head continues to move for a short distance in spite of the powerful resistances which oppose it.

78. **Second Law of Motion.**—Newton's second law of motion is that "Change of motion is proportional to the impressed force and is in the direction of that force."

Change of motion is here spoken of as a quantity, and as a directed quantity. In order to understand how to estimate change of motion, we must in the first place understand how to compound motions.

When a boat is sailing on a river, the motion of the boat relative to the shore is compounded of its motion relative to the water and the motion of the water relative to the shore. If a person is walking along the deck of the boat in any direction, his motion relative to the shore is compounded of three motions, namely the two above mentioned and his motion relative to the boat.

Let X, Y and Z be any three bodies or systems. The motion of X relative to Y, compounded with the motion of Y relative to Z, is the motion of X relative to Z. This is to be taken as the definition of what is meant by compounding two motions; and it leads very directly to the result that two rectilinear motions are compounded by the parallelogram law.

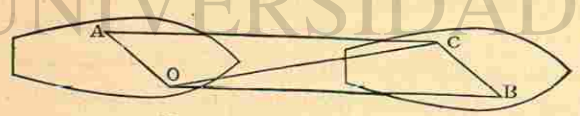


Fig. 36.—Composition of Motions.

For if a body moves along the deck of a ship from O to A (Fig. 36), and the ship in the meantime advances through the distance OB, it is obvious that, if we complete the parallelogram OBCA, the point A of the ship will be brought to C, and the movement of the body in space will be from O to C. If the motion along OA is uniform, and the motion of the ship is also uniform, the motion of the body through space will be a uniform motion along the diagonal OC. Hence, *if two component velocities be represented by two lines drawn from a point, and a parallelogram be constructed on these lines, its diagonal will represent the resultant velocity.*

It is obvious that if OA in the figure represented the velocity of the ship and OB the velocity of the body relative to the ship, we should obtain the same resultant velocity OC. This is a general law; the interchanging of velocities which are to be compounded does not affect their resultant.

Now suppose the velocity OB to be changed into the velocity OC, what are we to regard as the change of velocity? The change of velocity is that velocity which compounded with OB would give OC It is therefore OA. The same force which, in a given time, acting always parallel to itself, changes the velocity of a body from OB to OC, would give the body the velocity OA if applied to it for the same time commencing from rest. Change of motion, estimated in this way, depends only on the acting force and the body acted on by the force; it is entirely independent of any previous motion which the body may possess. No experiments on forces and motions inside a carriage or steamboat which is travelling with perfect smoothness in a straight course, will enable us to detect that it is travelling at all. We cannot even assert that there is any such thing as absolute rest, or that there is any difference between absolute rest and uniform straight movement of translation.

As change of motion is independent of the initial condition of rest or motion, so also is the change of motion produced by one force acting on a body independent of the change produced by any other force acting on the body, provided that each force remains constant in magnitude and direction. The actual motion will be that which is compounded of the initial motion and the motions due to the two forces considered separately. If AB represent one of these motions, BC another, and CD the third, the actual or resultant motion will be AD.

The change produced in the motion of a body by two forces acting jointly can therefore be found by compounding the changes which would be produced by each force separately. This leads at once to the "parallelogram of forces," since the changes of motion produced in one and the same body are proportional to the forces which produce them, and are in the directions of these forces.

In case any student should be troubled by doubt as to whether the "changes of motion" which are proportional to the forces, are to be understood as distances, or as velocities, we may remark that the law is equally true for both, and its truth for one implies its truth for the other, as will appear hereafter from comparing the formula for the distance $s = \frac{1}{2}ft^2$, with the formula for the velocity $v = ft$, since both of these expressions are proportional to f.

79. Explanation of Second Law continued.—It is convenient to distinguish between the *intensity* of a force and the *magnitude* or *amount* of a force. The intensity of a force is measured by the change of velocity which the force produces during the unit of time; and can be computed from knowing the motion of the body acted on, without knowing anything as to its mass. Two bodies are said to be of equal *mass* when the same change of motion (whether as regards velocity or distance) which is produced by applying a given force to one of them for a given time, would also be produced by applying this force to the other for an equal time. If we join two such bodies, we obtain a body of double the mass of either; and if we apply the same force as before for the same time to this double mass, we shall obtain only half the change of velocity or distance that we obtained before. Masses can therefore be compared by taking the inverse ratio of the changes produced in their velocities by equal forces.

The velocity of a body multiplied by its mass is called the *momentum* of the body, and is to be regarded as a directed magnitude having the same direction as the velocity. The change of velocity, when multiplied by the mass of the body, gives the change of momentum; and the second law of motion may be thus stated:—

The change of momentum produced in a given time is proportional to the force which produces it, and is in the direction of this force. It is independent of the mass; the change of velocity in a given time being inversely as the mass.

80. Proper Selection of Unit of Force.—If we make a proper selection of units, the change of momentum produced *in unit time* will be not only proportional but numerically *equal* to the force which produces it; and the change of momentum produced in any time will be the product of the force by the time. Suppose any units of length, time, and mass respectively to have been selected. Then the unit velocity will naturally be defined as the velocity with which unit length would be passed over in unit time; the unit momentum will be the momentum of the unit mass moving with this velocity; and the unit force will be that force which produces this momentum in unit time. We define the unit force, then, as *that force which acting for unit time upon unit mass produces unit velocity.*

81. Relation between Mass and Weight.—The *weight* of a body, strictly speaking, is the force with which the body tends towards the earth. This force depends partly on the body and partly on the earth. It is not exactly the same for one and the same body at all parts of the earth's surface, but is decidedly greater in the polar than in the equatorial regions. Bodies which, when weighed in a balance *in vacuo*, counterbalance each other, or counterbalance one and the same third body, have equal *weights* at that place, and will also be found to have equal weights at any other place. Experiments which we shall hereafter describe (§ 89) show that such bodies have equal masses; and this fact having been established, the most convenient mode of comparing masses is by weighing them. A pound of iron has the same mass as a pound of brass or of any other substance. A pound of any kind of matter tends to the earth with different forces at different places. The weight of a pound of matter is therefore not a definite standard of force. But the pound of matter itself is a perfectly definite standard of mass. If we weigh one and the same portion of matter in different states; for instance water in the states of ice, snow, liquid water, or steam; or compare the weight of a chemical compound with the weights of its components; we find an exact equality; hence it has been stated that the mass of a body is a measure of the quantity of matter which it contains; but though this statement expresses a simple fact when applied to the comparison of different quantities of one and the same substance, it expresses no known fact of nature when applied to the comparison of different substances. A pound of iron and a pound of lead tend to the earth with equal forces; and if equal forces are applied to them both their velocities are equally affected. We may if we please agree to measure "quantity of matter" by these tests; but we must beware of assuming that two things which are essentially different in kind can be equal in themselves.

82. Third Law of Motion. Action and Reaction.—Forces always occur in pairs, every exertion of force being a mutual action between two bodies. Whenever a body is acted on by a force, the body from which this force proceeds is acted on by an equal and opposite force. The earth attracts the moon, and the moon attracts the earth. A magnet attracts iron and is attracted by iron. When two boats are floating freely, a rope attached to one and hauled in by a person in the other, makes each boat move towards the other. Every exertion of force generates equal and opposite momenta in the two bodies affected by it, since these two bodies are acted on by equal forces for equal times.

If the forces exerted by one body upon the other are equivalent to a single force, the forces of reaction will also be equivalent to a single force, and these two equal and opposite resultants will have the same line of action. We have seen in § 29 that the general resultant of any set of forces applied to a body is a *wrench;* that is to say it consists of a force with a definite line of action (called the *axis*), accompanied by a couple in a perpendicular plane. The reaction upon the body which exerts these forces will always be an equal and opposite wrench; the two wrenches having the same axis, equal and opposite forces along this axis, and equal and opposite couples in the perpendicular plane.

83. Motion of Centre of Gravity Unaffected.—A consequence of the equality of the mutual forces between two bodies is, that these forces produce no movement of the common centre of gravity of the two bodies. For if A be the centre of gravity of a mass m_1, and B the centre of gravity of a mass m_2, their common centre of gravity C will divide AB inversely as the masses. Let the masses be originally at rest, and let them be acted on only by their mutual attraction or repulsion. The distances through which they are moved by these equal forces will be inversely as the masses, that is, will be directly as AC and BC; hence if A′ B′ are their new positions after any time, we have

$$\frac{AC}{BC} = \frac{AA'}{BB'} = \frac{AC \pm AA'}{BC \pm BB'} = \frac{A'C}{B'C}.$$

The line A′B′ is therefore divided at C in the same ratio in which the line AB was divided; hence C is still the centre of gravity.

84. Velocity of Centre of Gravity.—If any number of masses are moving with any velocities, and in any directions, but so that each of them moves uniformly in a straight line, their common centre of gravity will move uniformly in a straight line.

To prove this, we shall consider their component velocities in any one direction,

let these component velocities be $u_1 \quad u_2 \quad u_3$ &c.,
the masses being $m_1 \quad m_2 \quad m_3$ &c.,
and the distances of the bodies (strictly speaking the distances of their respective centres of gravity) from a fixed plane to which the given direction is normal, be $x_1 \quad x_2 \quad x_3$ &c.

The formula for the distance of their common centre of gravity from this plane is

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \&c.}{m_1 + m_2 + \&c.} \qquad (1)$$

In the time t, x_1 is increased by the amount $u_1 t$, x_2 by $u_2 t$, and so on; hence the numerator of the above expression is increased by

$$m_1 u_1 t + m_2 u_2 t + \&c.,$$

and the value of $\bar{x}$ is increased in each unit of time by

$$\frac{m_1 u_1 + m_2 u_2 + \&c.}{m_1 + m_2 + \&c.}, \qquad (2)$$

which is therefore the component velocity of the centre of gravity in the given direction. As this expression contains only given constant quantities, its value is constant; and as this reasoning applies to all directions, the velocity of the centre of gravity must itself be constant both in magnitude and direction.

We may remark that the above formula (2) correctly expresses the component velocity of the centre of gravity at the instant considered, even when u_1, u_2, &c., are not constant.

85. Centre of Mass.—The point which we have thus far been speaking of under the name of "centre of gravity," is more appropriately called the "centre of mass," a name which is at once suggested by formula (1) § 84. When gravity acts in parallel lines upon all the particles of a body, the resultant force of gravity upon the body is a single force passing through this point; but this is no longer the case when the forces of gravity upon the different parts of the body (or system of bodies) are not parallel.

86. Units of Measurement.—It is a matter of importance, in scientific calculations, to express the various magnitudes with which we have to deal in terms of units which have a simple relation to each other. The British weights and measures are completely at fault in this respect, for the following reasons:—

1. They are not a decimal system; and the reduction of a measurement (say) from inches and decimals of an inch to feet and decimals of a foot, cannot be effected by inspection.

2. It is still more troublesome to reduce gallons to cubic feet or inches.

3. The weight (properly the mass) of a cubic foot of a substance in lbs., cannot be written down by inspection, when the specific gravity of the substance (as compared with water) is given.

87. The C.G.S. System.—A committee of the British Association, specially appointed to recommend a system of units for general adoption in scientific calculation, have recommended that the *centimetre* be adopted as the unit of length, the *gramme* as the unit of mass, and the *second* as the unit of time. We shall first give the rough and afterwards the more exact definitions of these quantities.

The centimetre is approximately $\frac{1}{10^9}$ of the distance of either pole of the earth from the equator; that is to say 1 followed by 9 zeros expresses this distance in centimetres.

The gramme is approximately the mass of a cubic centimetre of cold water. Hence the same number which expresses the specific gravity of a substance referred to water, expresses also the mass of a cubic centimetre of the substance, in grammes.

The second is $\frac{1}{24 \times 60 \times 60}$ of a mean solar day.

More accurately, the centimetre is defined as one hundredth part of the length, at the temperature 0° Centigrade, of a certain standard bar, preserved in Paris, carefully executed copies of which are preserved in several other places; and the gramme is defined as one thousandth part of the mass of a certain standard which is preserved at Paris, and of which also there are numerous copies preserved elsewhere.

For brevity of reference, the committee have recommended that the system of units based on the Centimetre, Gramme, and Second, be called the C.G.S. system.

The unit of area in this system is the square centimetre.

The unit of volume is the cubic centimetre.

The unit of velocity is a velocity of a centimetre per second.

The unit of momentum is the momentum of a gramme moving with a velocity of a centimetre per second.

The unit force is that force which generates this momentum in one second. It is therefore that force which, acting on a gramme for one second, generates a velocity of a centimetre per second. This force is called the *dyne*, an abbreviated derivative from the Greek δύναμις (force).

The unit of work is the work done by a force of a dyne working through a distance of a centimetre. It might be called the dyne-centimetre, but a shorter name has been provided and it is called the *erg*, from the Greek ἔργον (work).

CHAPTER VII.

LAWS OF FALLING BODIES.

88. Effect of the Resistance of the Air.—In air, bodies fall with unequal velocities; a sovereign or a ball of lead falls rapidly, a piece of down or thin paper slowly. It was formerly thought that this difference was inherent in the nature of the materials; but it is easy to show that this is not the case, for if we compress a mass of down or a piece of paper by rolling it into a ball, and compare it with a piece of gold-leaf, we shall find that the latter body falls more slowly than the former. The inequality of the velocities which we observe is due to the resistance of the air, which increases with the extent of surface exposed by the body.

It was Galileo who first discovered the cause of the unequal rapidity of fall of different bodies. To put the matter to the test, he prepared small balls of different substances, and let them fall at the same time from the top of the tower of Pisa; they struck the ground almost at the same instant. On changing their forms, so as to give them very different extents of surface, he observed that they fell with very unequal velocities. He was thus led to the conclusion that gravity acts on all substances with the same intensity, and that in a vacuum all bodies would fall with the same velocity.

This last proposition could not be put to the test of experiment in the time of Galileo, the air-pump not having yet been invented. The experiment was performed by Newton, and is now well known as the "guinea and feather" experiment. For this purpose a tube from a yard and a half to two yards long is used, which can be exhausted of air, and which contains bodies of various densities, such as a coin, pieces of paper, and feathers. When the tube is full of air and is inverted, these different bodies are seen to fall with very unequal velocities; but if the experiment is repeated after the tube

87. The C.G.S. System.—A committee of the British Association, specially appointed to recommend a system of units for general adoption in scientific calculation, have recommended that the *centimetre* be adopted as the unit of length, the *gramme* as the unit of mass, and the *second* as the unit of time. We shall first give the rough and afterwards the more exact definitions of these quantities.

The centimetre is approximately $\frac{1}{10^9}$ of the distance of either pole of the earth from the equator; that is to say 1 followed by 9 zeros expresses this distance in centimetres.

The gramme is approximately the mass of a cubic centimetre of cold water. Hence the same number which expresses the specific gravity of a substance referred to water, expresses also the mass of a cubic centimetre of the substance, in grammes.

The second is $\frac{1}{24 \times 60 \times 60}$ of a mean solar day.

More accurately, the centimetre is defined as one hundredth part of the length, at the temperature 0° Centigrade, of a certain standard bar, preserved in Paris, carefully executed copies of which are preserved in several other places; and the gramme is defined as one thousandth part of the mass of a certain standard which is preserved at Paris, and of which also there are numerous copies preserved elsewhere.

For brevity of reference, the committee have recommended that the system of units based on the Centimetre, Gramme, and Second, be called the C.G.S. system.

The unit of area in this system is the square centimetre.

The unit of volume is the cubic centimetre.

The unit of velocity is a velocity of a centimetre per second.

The unit of momentum is the momentum of a gramme moving with a velocity of a centimetre per second.

The unit force is that force which generates this momentum in one second. It is therefore that force which, acting on a gramme for one second, generates a velocity of a centimetre per second. This force is called the *dyne*, an abbreviated derivative from the Greek δύναμις (force).

The unit of work is the work done by a force of a dyne working through a distance of a centimetre. It might be called the dyne-centimetre, but a shorter name has been provided and it is called the *erg*, from the Greek ἔργον (work).

CHAPTER VII.

LAWS OF FALLING BODIES.

88. Effect of the Resistance of the Air.—In air, bodies fall with unequal velocities; a sovereign or a ball of lead falls rapidly, a piece of down or thin paper slowly. It was formerly thought that this difference was inherent in the nature of the materials; but it is easy to show that this is not the case, for if we compress a mass of down or a piece of paper by rolling it into a ball, and compare it with a piece of gold-leaf, we shall find that the latter body falls more slowly than the former. The inequality of the velocities which we observe is due to the resistance of the air, which increases with the extent of surface exposed by the body.

It was Galileo who first discovered the cause of the unequal rapidity of fall of different bodies. To put the matter to the test, he prepared small balls of different substances, and let them fall at the same time from the top of the tower of Pisa; they struck the ground almost at the same instant. On changing their forms, so as to give them very different extents of surface, he observed that they fell with very unequal velocities. He was thus led to the conclusion that gravity acts on all substances with the same intensity, and that in a vacuum all bodies would fall with the same velocity.

This last proposition could not be put to the test of experiment in the time of Galileo, the air-pump not having yet been invented. The experiment was performed by Newton, and is now well known as the "guinea and feather" experiment. For this purpose a tube from a yard and a half to two yards long is used, which can be exhausted of air, and which contains bodies of various densities, such as a coin, pieces of paper, and feathers. When the tube is full of air and is inverted, these different bodies are seen to fall with very unequal velocities; but if the experiment is repeated after the tube has been exhausted of air, no difference can be perceived between the times of their descent.

89. Mass and Gravitation Proportional.—This experiment proves that bodies which have equal weights are equal in mass. For equal masses are defined to be those which, when acted on by equal forces, receive equal accelerations; and the forces, in this experiment, are the weights of the falling bodies.

Newton tested this point still more severely by experiments with pendulums (*Principia*, book III. prop. vi.). He procured two round wooden boxes of the same size and weight, and suspended them by threads eleven feet long. One of them he filled with wood, and he placed very accurately in the centre of oscillation of the other the same weight of gold. The boxes hung side by side, and, when set swinging in equal oscillations, went and returned together for a very long time. Here the forces concerned in producing and checking the motion, namely, the force of gravity and the resistance of the air, were the same for the two pendulums, and as the movements produced were the same, it follows that the masses were equal. Newton remarks that a difference of mass amounting to a thousandth part of the whole could not have escaped detection. He experimented in the same way with silver, lead, glass, sand, salt, water, and wheat, and with the same result. He therefore infers that universally bodies of equal mass gravitate equally towards the earth at the same place. He further extends the same law to gravitation generally, and establishes the conclusion that the mutual gravitating force between any two bodies depends only on their masses and distances, and is independent of their materials.

The time of revolution of the moon round the earth, considered in conjunction with her distance from the earth, shows that the relation between mass and gravitation is the same for the material of which the moon is composed as for terrestrial matter; and the same conclusion is proved for the planets by the relation which exists between their distances from the sun and their times of revolution in their orbits.

90. Uniform Acceleration.—The fall of a heavy body furnishes an illustration of the second law of motion, which asserts that the change of momentum in a body in a given time is a measure of the force which acts on the body. It follows from this law that if the same force continues to act upon a body the changes of momentum in successive equal intervals of time will be equal. When a heavy

body originally at rest is allowed to fall, it is acted on during the time of its descent by its own weight and by no other force, if we neglect the resistance of the air. As its own weight is a constant force, the body receives equal changes of momentum, and therefore of velocity, in equal intervals of time. Let g denote its velocity in centimetres per second, at the end of the first second. Then at the end of the next second its velocity will be $g+g$, that is $2g$; at the end of the next it will be $2g+g$, that is $3g$, and so on, the gain of velocity in each second being equal to the velocity generated in the first second. At the end of t seconds the velocity will therefore be tg. Such motion as this is said to be *uniformly accelerated*, and the constant quantity g is the measure of the acceleration. Acceleration is defined as the gain of velocity per unit of time.

91. Weight of a Gramme in Dynes. Value of g.—Let m denote the mass of the falling body in grammes. Then the change of momentum in each second is mg, which is therefore the measure of the force acting on the body. The weight of a body of m grammes is therefore mg dynes, and the weight of 1 gramme is g dynes. The value of g varies from 978·1 at the equator to 983·1 at the poles; and 981 may be adopted as its average value in temperate latitudes. Its value at any part of the earth's surface is approximately given by the formula

$$g = 980{\cdot}6056 - 2{\cdot}5028 \cos 2\lambda - {\cdot}000{,}003h,$$

in which λ denotes the latitude, and h the height (in centimetres) above sea-level.[1]

In § 79 we distinguished between the intensity and the amount of a force. The amount of the force of gravity upon a mass of m grammes is mg dynes. The intensity of this force is g dynes per gramme. The intensity of a force, in dynes per gramme of the body acted on, is always equal to the change of velocity which the force produces per second, this change being expressed in centimetres per second. In other words the intensity of a force is equal to the acceleration which it produces. The best designation for g is *the intensity of gravity*.

92. Distance fallen in a Given Time.—The distance described in a given time by a body moving with uniform velocity is calculated by multiplying the velocity by the time; just as the area of a rectangle is calculated by multiplying its length by its breadth. Hence if we draw a line such that its ordinates AA′, BB′, &c., represent the

[1] For the method of determination see § 120.

velocities with which a body is moving at the times represented by OA, OB (time being reckoned from the beginning of the motion), it can be shown that the whole distance described is represented by the area OB′B bounded by the curve, the last ordinate, and the base line. In fact this area can be divided into narrow strips (one of which is shown at AA′, Fig. 37) each of which may practically be regarded as a rectangle, whose height represents the velocity with which the body is moving during the very small interval of time represented by its base, and whose area therefore represents the distance described in this time.

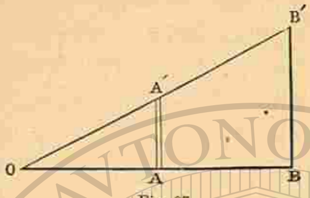


Fig. 37.

This would be true for the distance described by a body moving from rest with any law of velocity. In the case of falling bodies the law is that the velocity is simply proportional to the time; hence the ordinates AA′, BB′, &c., must be directly as the abscissæ OA, OB; this proves that the line OA′ B′ must be straight; and the figure OB′ B is therefore a triangle. Its area will be half the product of OB and BB′. But OB represents the time t occupied by the motion, and BB′ the velocity gt at the end of this time. The area of the triangle therefore represents half the product of t and gt, that is, represents $\frac{1}{2}gt^2$, which is accordingly the distance described in the time t. Denoting this distance by s, and the velocity at the end of time t by v, we have thus the two formulæ

$$v = gt, \quad (1)$$
$$s = \tfrac{1}{2}gt^2, \quad (2)$$

from which we easily deduce

$$gs = \tfrac{1}{2}v^2. \quad (3)$$

93. Work spent in Producing Motion.—We may remark, in passing, that the third of these formulæ enables us to calculate the work required to produce a given motion in a given mass. When a body whose mass is 1 gramme falls through a distance s, the force which acts upon it is its own weight, which is g dynes, and the work done upon it is gs ergs. Formula (3) shows that this is the same as $\frac{1}{2}v^2$ ergs. For a mass of m grammes falling through a distance s, the work is $\frac{1}{2}mv^2$ ergs. *The work required to produce a velocity* v (centimetres per second) *in a body of mass* m (grammes) originally at rest *is* $\frac{1}{2}mv^2$ (ergs).

94. Body thrown Upwards.—When a heavy body is projected vertically upwards, the formulæ (1) (2) (3) of § 92 will still apply to its motion, with the following interpretations:—

v denotes the velocity of projection.

t denotes the whole time occupied in the ascent.

s denotes the height to which the body will ascend.

When the body has reached the highest point, it will fall back, and its velocity at any point through which it passes twice will be the same in going up as in coming down.

95. Resistance of the Air.—The foregoing results are rigorously applicable to motion in vacuo, and are sensibly correct for motion in air as long as the resistance of the air is insignificant in comparison with the force of gravity. The force of gravity upon a body is the same at all velocities; but the resistance of the air increases with the velocity, and increases more and more rapidly as the velocity becomes greater; so that while at very slow velocities an increase of 1 per cent. in velocity would give an increase of 1 per cent. in the resistance, at a higher velocity it would give an increase of 2 per cent., and at the velocity of a cannon-ball an increase of 3 per cent.[1] The formulæ are therefore sensibly in error for high velocities. They are also in error for bodies which, like feathers or gold-leaf, have a large surface in proportion to their weight.

96. Projectiles.—If, instead of being simply let fall, a body is projected in any direction, its motion will be compounded of the motion of a falling body and a uniform motion in the direction of projection. Thus if OP (Fig. 38) is the direction of projection, and OQ the vertical through the point of projection, the body would move along OP keeping its original velocity unchanged, if it were not disturbed by gravity. To find where the body will be at any time t, we must lay off a length OP equal to Vt, V denoting the velocity of projection, and must then draw from P the vertical line PR downwards equal to $\frac{1}{2}gt^2$, which is the distance that the body would have fallen in the time if simply dropped. The point R thus determined, will be the actual position of the body. The velocity of the body at any time will in like manner be found by compounding the initial

Fig. 38.

[1] This is only another way of saying that the resistance varies approximately as the velocity when very small, and approximately as the cube of the velocity for velocities like that of a cannon-ball.

velocity with the velocity which a falling body would have acquired in the time.

The path of the body will be a curve, as represented in the figure, OP being a tangent to it at O, and its concavity being downwards. The equations above given, namely

$$OP = Vt, \ PR = \tfrac{1}{2}gt^2,$$

show that PR varies as the square of OP, and hence that the path (or *trajectory* as it is technically called) is a parabola, whose axis is vertical.

97. Time of Flight, and Range.—If the body is projected from a point at the surface of the ground (supposed level) we can calculate the time of flight and the range in the following way.

Let α be the angle which the direction of projection makes with the horizontal. Then the velocity of projection can be resolved into two components, $V \cos \alpha$ and $V \sin \alpha$, the former being horizontal, and the latter vertically upward. The horizontal component of the velocity of the body is unaffected by gravity and remains constant. The vertical velocity after time t will be compounded of $V \sin \alpha$ upwards and gt downwards. It will therefore be an upward velocity $V \sin \alpha - gt$, or a downward velocity $gt - V \sin \alpha$. At the highest point of its path, the body will be moving horizontally and the vertical component of its velocity will be zero; that is, we shall have

$$V \sin \alpha - gt = 0; \text{ whence } t = \frac{V \sin \alpha}{g}.$$

This is the time of attaining the highest point; and the time of flight will be double of this, that is, will be $\frac{2V \sin \alpha}{g}$.

As the horizontal component of the velocity has the constant value $V \cos \alpha$, the horizontal displacement in any time t is $V \cos \alpha$ multiplied by t. The range is therefore

$$\frac{2V^2 \sin \alpha \cos \alpha}{g} \text{ or } \frac{V^2 \sin 2\alpha}{g}.$$

The range (for a given velocity of projection) will therefore be greatest when $\sin 2\alpha$ is greatest, that is when $2\alpha = 90°$ and $\alpha = 45°$.

We shall now describe two forms of apparatus for illustrating the laws of falling bodies.

98. Morin's Apparatus.—Morin's apparatus consists of a wooden cylinder covered with paper, which can be set in uniform rotation about its axis by the fall of a heavy weight. The cord which sup-

ports the weight is wound upon a drum, furnished with a toothed wheel which works on one side with an endless screw on the axis of the cylinder, and on the other drives an axis carrying fans which serve to regulate the motion.

In front of the turning cylinder is a cylindro-conical weight of cast-iron carrying a pencil whose point presses against the paper, and having ears which slide on vertical threads, serving to guide it in its fall. By pressing a lever, the weight can be made to fall at a chosen moment. The proper time for this is when the motion of the cylinder has become sensibly uniform. It follows from this arrangement that during its vertical motion the pencil will meet in succession the different generating lines[1] of the revolving cylinder, and will consequently describe on its surface a certain curve, from the study of which we shall be able to gather the law of the fall of the body which has traced it. With this view, we describe (by turning the cylinder while the pencil is stationary) a circle passing through the commencement of the curve, and also draw a vertical line through this point. We cut the paper along this latter line and develop it (that is, flatten

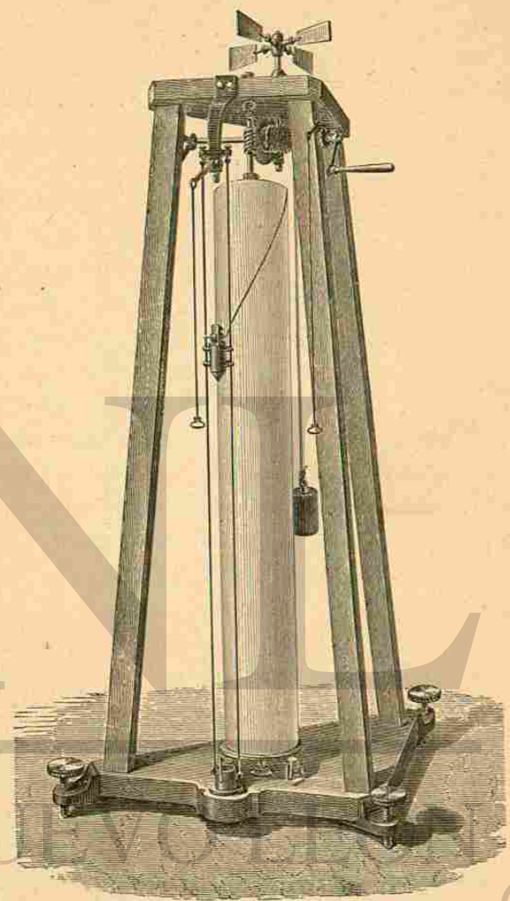


Fig. 39.—Morin's Apparatus.

[1] A cylindric surface could be swept out or "generated" by a straight line moving round the axis and remaining always parallel to it. The successive positions of this generating line are called the "generating lines of the cylinder."

it out into a plane). It then presents the appearance shown in Fig. 40.

If we take on the horizontal line equal distances at 1, 2, 3, 4, 5 . . . , and draw perpendiculars at their extremities to meet the curve, it is evident that the points thus found are those which were traced by the pencil when the cylinder had turned through the distances 1, 2, 3, 4, 5. . . . The corresponding verticals represent the spaces traversed in the times 1, 2, 3, 4, 5. . . . Now we find, as the figure shows, that these spaces are represented by the numbers 1, 4, 9, 16, 25 . . . , thus verifying the principle that the spaces described are proportional to the squares of the times employed in their description.

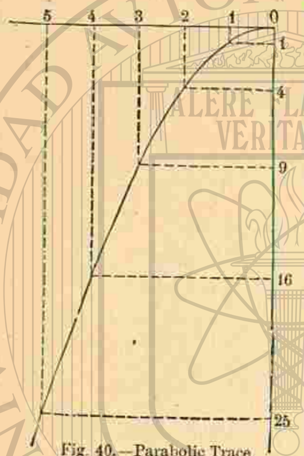


Fig. 40.—Parabolic Trace.

We may remark that the proportionality of the vertical lines to the squares of the horizontal lines shows that the curve is a parabola. The parabolic trace is thus the consequence of the law of fall, and from the fact of the trace being parabolic we can infer the proportionality of the spaces to the squares of the times.

The law of velocities might also be verified separately by Morin's apparatus; we shall not describe the method which it would be necessary to employ, but shall content ourselves with remarking that the law of velocities is a logical consequence of the law of spaces.[1]

99. Atwood's Machine.—Atwood's machine, which affords great facilities for illustrating the effects of force in producing motion, consists essentially of a very freely moving pulley over which a fine cord passes, from the ends of which two equal weights can be suspended. A small additional weight of flat and elongated form is laid upon one of them, which is thus caused to descend with uniform *acceleration*, and means are provided for suddenly removing

[1] Consider, in fact, the space traversed in any time t; this space is given by the formula $s = Kt^2$; during the time $t+\theta$ the space traversed will be $K(t+\theta)^2 = Kt^2 + 2Kt\theta + K\theta^2$, whence it follows that the space traversed during the time θ after the time t is $2Kt\,\theta + K\theta^2$. The average velocity during this time θ is obtained by dividing the space by θ, and is $2Kt + K\theta$, which, by making θ very small, can be made to agree as accurately as we please with the value $2Kt$. This limiting value $2Kt$ must therefore be the velocity at the end of time t.—*D.*

this additional weight at any point of the descent, so as to allow the motion to continue from this point onward with uniform *velocity*.

The machine is represented in Fig. 41. The pulley over which the string passes is the largest of the wheels shown at the top of the apparatus. In order to give it greater freedom of movement, the ends of its axis are made to rest, not on fixed supports, but on the circumferences of four wheels (two at each end of the axis) called friction-wheels, because their office is to diminish friction. Two small equal weights are shown, suspended from this pulley by a string passing over it. One of them P′ is represented as near the bottom of the supporting pillar, and the other P as near the top. The latter is resting upon a small platform, which can be suddenly dropped when it is desired that the motion shall commence. A little lower down and vertically beneath the platform, is seen a ring, large enough to let the weight pass through it without danger of

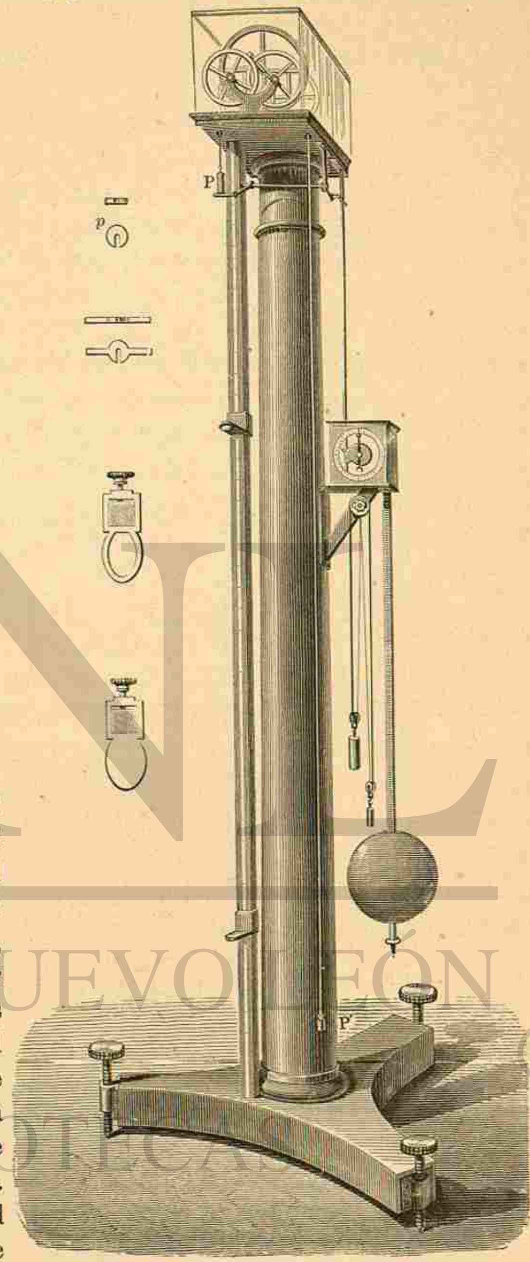


Fig. 41.—Atwood's Machine.

contact. This ring can be shifted up or down, and clamped at any height by a screw; it is represented on a larger scale in the margin. At a considerable distance beneath the ring, is seen the stop, which is also represented in the margin, and can like the ring be clamped at any height. The office of the ring is to intercept the additional weight, and the office of the stop is to arrest the descent. The upright to which they are both clamped is marked with a scale of equal parts, to show the distances moved over. A clock with a pendulum beating seconds, is provided for measuring the time; and there is an arrangement by which the movable platform can be dropped by the action of the clock precisely at one of the ticks. To measure the distance fallen in one or more seconds, the ring is removed, and the stop is placed by trial at such heights that the descending weight strikes it precisely at another tick. To measure the velocity acquired in one or more seconds, the ring must be fixed at such a height as to intercept the additional weight at one of the ticks, and the stop must be placed so as to be struck by the descending weight at another tick.

100. Theory of Atwood's Machine.—If M denote each of the two equal masses, in grammes, and m the additional mass, the whole moving mass (neglecting the mass of the pulley and string) is $2M+m$, but the moving force is only the weight of m. The acceleration produced, instead of being g, is accordingly only $\frac{m}{2M+m}g$. In order to allow for the inertia of the pulley and string, a constant quantity must be added to the denominator in the above formula, and the value of this constant can be determined by observing the movements obtained with different values of M and m. Denoting it by C, we have

$$\frac{m}{m+2M+C}g \qquad \text{(A)}$$

as the expression for the acceleration. As m is usually small in comparison with M, the acceleration is very small in comparison with that of a freely falling body, and is brought within the limits of convenient observation. Denoting the acceleration by a, and using v and s, as in § 92, to denote the velocity acquired and space described in time t, we shall have

$$v=at, \qquad (1)$$
$$s=\tfrac{1}{2}at^2, \qquad (2)$$
$$as=\tfrac{1}{2}v^2, \qquad (3)$$

and each of these formulæ can be directly verified by experiments with the machine.

Fig. 42.

101. Uniform Motion in a Circle.—A body cannot move in a curved path unless there be a force urging it towards the concave side of the curve. We shall proceed to investigate the intensity of this force when the path is circular and the velocity uniform. We shall denote the velocity by v, the radius of the circle by r, and the intensity of the force by f. Let AB (Figs. 42, 43) be a small portion of the path, and BD a perpendicular upon AD the tangent at A. Then, since the arc AB is small in comparison with the whole circumference, it is sensibly equal to AD, and the body would have been found at D instead of at B if no force had acted upon it since leaving A. DB is accordingly the distance due to the force; and if t denote the time from A to B, we have

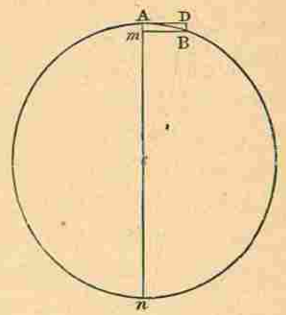


Fig 43.

$$AD = vt \qquad (1)$$
$$DB = \tfrac{1}{2}ft^2. \qquad (2)$$

The second of these equations gives

$$f = \frac{2DB}{t^2}$$

and substituting for t from the first equation, this becomes

$$f = \frac{2DB}{AD^2}v^2. \qquad (3)$$

But if An (Fig. 43) be the diameter at A, and Bm the perpendicular upon it from B, we have, by Euclid, $AD^2 = mB^2 = Am \, . \, mn = 2r \, . \, Am$ sensibly, $= 2r \, . \, DB$.

Therefore $\frac{2DB}{AD^2} = \frac{1}{r}$, and hence by (3)

$$f = \frac{v^2}{r}. \qquad (4)$$

Hence the force necessary for keeping a body in a circular path without change of velocity, is a force of intensity $\frac{v^2}{r}$ directed towards the centre of the circle. If m denote the mass of the body, the amount of the force will be $\frac{mv^2}{r}$. This will be in dynes, if m be in grammes, r in centimetres, and v in centimetres per second.

If the time of revolution be denoted by T, and π as usual denote the ratio of circumference to diameter, the distance moved in time

T is $2\pi r$; hence $v = \frac{2\pi r}{T}$, and another expression for the intensity of the force will be

$$f = \left(\frac{2\pi}{T}\right)^2 r. \qquad (5)$$

102. Deflecting Force in General.—In general, when a body is moving in any path, and with velocity either constant or varying, the force acting upon it at any instant can be resolved into two components, one along the tangent and the other along the normal. The intensity of the tangential component is measured by the rate at which the velocity increases or diminishes, and the intensity of the normal component is given by formula (4) of last article, if we make r denote the radius of curvature.

103. Illustrations of Deflecting Force.—When a stone is swung round by a string in a vertical circle, the tension of the string in the lowest position consists of two parts:—

(1) The weight of the stone, which is mg if m be the mass of the stone.

(2) The force $m\frac{v^2}{r}$ which is necessary for deflecting the stone from a horizontal tangent into its actual path in the neighbourhood of the lowest point.

When the stone is at the highest point of its path, the tension of the string is the difference of these two forces, that is to say it is

$$m\left(\frac{v^2}{r} - g\right),$$

and the motion is not possible unless the velocity at the highest point is sufficient to make $\frac{v^2}{r}$ greater than g.

The tendency of the stone to persevere in rectilinear motion and to resist deflection into a curve, causes it to exert a force upon the string, of amount $m\frac{v^2}{r}$, and this is called *centrifugal force*. It is not a force acting upon the stone, but a force exerted by the stone upon the string. Its direction is *from* the centre of curvature, whereas the deflecting force which acts upon the stone is *towards* the centre of curvature.

104. Centrifugal Force at the Equator.—Bodies on the earth's surface are carried round in circles by the diurnal rotation of the earth upon its axis. The velocity of this motion at the equator is about 46,500 centimetres per second, and the earth's equatorial radius is about $6{\cdot}38 \times 10^8$ centimetres. Hence the value of $\frac{v^2}{r}$ is found to be about 3·39. The case is analogous to that of the stone

at the highest point of its path in the preceding article, if instead of a string which can only exert a pull we suppose a stiff rod which can exert a push upon the stone. The rod will be called upon to exert a pull or a push at the highest point according as $\frac{v^2}{r}$ is greater or less than g. The force of the push in the latter case will be

$$m\left(g - \frac{v^2}{r}\right),$$

and this is accordingly the force with which the surface of the earth at the equator pushes a body lying upon it. The push, of course, is mutual, and this formula therefore gives the apparent weight or apparent gravitating force of a body at the equator, mg denoting its true gravitating force (due to attraction alone). A body falling in vacuo at the equator has an acceleration 978·10 relative to the surface of the earth in its neighbourhood; but this portion of the surface has itself an acceleration of 3·39, directed towards the earth's centre, and therefore in the same direction as the acceleration of the body. The absolute acceleration of the body is therefore the sum of these two, that is 981·49, which is accordingly the intensity of true gravity at the equator.

The apparent weight of bodies at the equator would be *nil* if $\frac{v^2}{r}$ were equal to g. Dividing 3·39 into 981·49, the quotient is approximately 289, which is $(17)^2$. Hence this state of things would exist if the velocity of rotation were about 17 times as fast as at present.

Since the movements and forces which we actually observe depend upon *relative* acceleration, it is usual to understand, by the value of g or the intensity of gravity at a place, the *apparent* values, unless the contrary be expressed. Thus the value of g at the equator is usually stated to be 978·10.

105. Direction of Apparent Gravity.—The total amount of centrifugal force at different places on the earth's surface, varies directly as their distance from the earth's axis; for this is the value of r in the formula (5) of § 101, and the value of T in that formula is the same for the whole earth. The direction of this force, being perpendicular to the earth's axis, is not vertical except at the equator; and hence, when we compound it with the force of true gravity, we obtain a resultant force of apparent gravity differing in direction as well as in magnitude from true gravity. What is always understood by a *vertical*, is the direction of *apparent* gravity; and a plane perpendicular to it is what is meant by a horizontal plane.

CHAPTER VIII.

THE PENDULUM.

106. The Pendulum.—When a body is suspended so that it can turn about a horizontal axis which does not pass through its centre of gravity, its only position of stable equilibrium is that in which its centre of gravity is in the same vertical plane with the axis and below it (§ 42). If the body be turned into any other position, and left to itself, it will oscillate from one side to the other of the position of equilibrium, until the resistance of the air and the friction of the axis gradually bring it to rest. A body thus suspended, whatever be its form, is called a pendulum. It frequently consists of a rod which can turn about an axis O (Fig. 44) at its upper end, and which carries at its lower end a heavy lens-shaped piece of metal M called the bob; this latter can be raised or lowered by means of the screw V. The applications of the pendulum are very important: it regulates our clocks, and it has enabled us to measure the intensity of gravity in different parts of the world; it is important then to know at least the fundamental points in its theory. For explaining these, we shall begin with the consideration of an ideal body called the *simple pendulum*.

Fig. 44.—Pendulum.

107. Simple Pendulum.—This is the name given to a pendulum consisting of a heavy particle M (Fig. 45) attached to one end of an inextensible thread without weight, the other end of the thread being fixed at A. When the thread is vertical, the weight of the particle acts in the direction of its length, and there is equilib-

rium. But suppose it is drawn aside into another position, as AM. In this case, the weight MG of the particle can be resolved into two forces MC and MH. The former, acting along the prolongation of the thread, is destroyed by the resistance of the thread; the other, acting along the tangent MH, produces the motion of the particle. This effective component is evidently so much the greater as the angle of displacement from the vertical position is greater. The particle will therefore move along an arc of a circle described from A as centre, and the force which urges it forward will continually diminish till it arrives at the lowest point M′. At M′ this force is zero, but, in virtue of the velocity acquired, the particle will ascend on the opposite side, the effective component of gravity being now opposed to the direction of its motion; and, inasmuch as the magnitude of this component goes through the same series of values in this part of the motion as in the former part, but in reversed order, the velocity will, in like manner, retrace its former values, and will become zero when the particle has risen to a point M″ at the same height as M. It then descends again and performs an oscillation from M″ to M precisely similar to the first, but in the reverse direction. It will thus continue to vibrate between the two points M, M″ (friction being supposed excluded), for an indefinite number of times, all the vibrations being of equal extent and performed in equal periods.

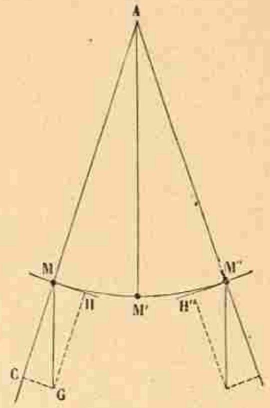


Fig. 45.—Motion of Simple Pendulum.

The distance through which a simple pendulum travels in moving from its lowest position to its furthest position on either side, is called its *amplitude*. It is evidently equal to half the complete arc of vibration, and is commonly expressed, not in linear measure, but in degrees of arc. Its numerical value is of course equal to that of the angle MAM′, which it subtends at the centre of the circle.

The *complete period* of the pendulum's motion is the time which it occupies in moving from M to M″ and back to M, or more generally, is the time from its passing through any given position to its next passing through the same position *in the same direction*.

What is commonly called the time of vibration, or the time of a single vibration, is the half of a complete period, being the time of passing from one of the two extreme positions to the other. Hence what we have above defined as a complete period is often called a double vibration.

When the amplitude changes, the time of vibration changes also, being greater as the amplitude is greater; but the connection between the two elements is very far from being one of simple proportion. The change of time (as measured by a ratio) is much less than the change of amplitude, especially when the amplitude is small; and when the amplitude is less than about 5°, any further diminution of it has little or no sensible effect in diminishing the time. *For small vibrations, then, the time of vibration is independent of the amplitude.* This is called the law of *isochronism.*

108. Law of Acceleration for Small Vibrations.—Denoting the length of a simple pendulum by l, and its inclination to the vertical at any moment by θ, we see from Fig. 45 that the ratio of the effective component of gravity to the whole force of gravity is $\frac{\text{MH}}{\text{MG}}$, that is $\sin\theta$; and when θ is small this is sensibly equal to θ itself as measured by $\frac{\text{arc}}{\text{radius}}$. Let s denote the length of the arc MM′ intervening between the lower end of the pendulum and the lowest point of its swing, at any time; then θ is equal to $\frac{s}{l}$, and the intensity of the effective force of gravity when θ is small is sensibly equal to $g\theta$, that is to $\frac{gs}{l}$. Since g and l are the same in all positions of the pendulum, this effective force varies as s. Its direction is always *towards* the position of equilibrium, so that it accelerates the motion during the approach to this position, and retards it during the recess; the acceleration or retardation being always in direct proportion to the distance from the position of equilibrium. This species of motion is of extremely common occurrence. It is illustrated by the vibration of either prong of a tuning-fork, and in general by the motion of any body vibrating in one plane in such a manner as to yield a simple musical tone.

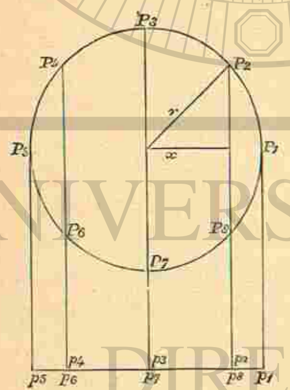


Fig. 46.—Projection of Circular Motion.

109. General Law for Period.—Suppose a point P to travel with uniform velocity round a circle (Fig. 46), and from its successive

positions P_1, P_2, &c., let perpendiculars P_1p_1, P_2p_2, &c., be drawn to a fixed straight line in the plane of the circle. Then while P travels once round the circle, its projection p executes a complete vibration.

The acceleration of P is always directed towards the centre of the circle, and is equal to $\left(\frac{2\pi}{\text{T}}\right)^2 r$ (§ 101). The component of this acceleration parallel to the line of motion of p, is the fraction $\frac{x}{r}$ of the whole acceleration (x denoting the distance of p from the middle point of its path), and is therefore $\left(\frac{2\pi}{\text{T}}\right)^2 x$. This is accordingly the acceleration of p, and as it is simply proportional to x we shall denote it for brevity by μx. To compute the periodic time T of a complete vibration, we have the equation $\mu = \left(\frac{2\pi}{\text{T}}\right)^2$, which gives

$$\text{T} = \frac{2\pi}{\sqrt{\mu}}. \qquad (1)$$

110. Application to the Pendulum.—For the motion of a pendulum in a small arc, we have

$$\text{acceleration} = \frac{g}{l}s,$$

where s denotes the displacement in linear measure. We must therefore put $\mu = \frac{g}{l}$, and we then have

$$\text{T} = 2\pi\sqrt{\frac{l}{g}} \qquad (2)$$

which is the expression for the time of a complete (or double) vibration. It is more usual to understand by the "time of vibration" of a pendulum the half of this, that is the time from one extreme position to the other, and to denote this time by T. In this sense we have

$$\text{T} = \pi\sqrt{\frac{l}{g}} \qquad (3)$$

To find the length of the seconds' pendulum we must put T = 1. This gives

$$\pi^2 \frac{l}{g} = 1, \quad l = \frac{g}{\pi^2} = \frac{g}{9{\cdot}87} \text{ nearly.}$$

If g were 987 we should have $l = 100$ centimetres or 1 metre. The actual value of g is everywhere a little less than this. The length of the seconds' pendulum is therefore everywhere rather less than a metre.

111. Simple Harmonic Motion.—Rectilinear motion consisting of vibration about a point with acceleration μx, where x denotes distance from this point, is called *Simple Harmonic Motion*, or Simple Harmonic Vibration. The above investigation shows that such vibration is isochronous, its period being $\frac{2\pi}{\sqrt{\mu}}$ whatever the amplitude may be.

To understand the reason of this isochronism we have only to remark that, if the amplitude be changed, the velocity at corresponding points (that is, points whose distances from the middle point are the same fractions of the amplitudes) will be changed in the same ratio. For example, compare two simple vibrations in which the values of μ are the same, but let the amplitude of one be double that of the other. Then if we divide the paths of both into the same number of small equal parts, these parts will be twice as great for the one as for the other; but if we suppose the two points to start simultaneously from their extreme positions, the one will constantly be moving twice as fast as the other. The number of parts described in any given time will therefore be the same for both.

In the case of vibrations which are not simple, it is easy to see (from comparison with simple vibration) that if the acceleration increases in a greater ratio than the distance from the mean position, the period of vibration will be shortened by increasing the amplitude; but if the acceleration increases in a less ratio than the distance, as in the case of the common pendulum vibrating in an arc of moderate extent, the period is increased by increasing the amplitude.

112. Experimental Investigation of the Motion of Pendulums.—The preceding investigation applies to the simple pendulum; that is to say to a purely imaginary existence; but it can be theoretically demonstrated that every rigid body vibrating about a horizontal axis under the action of gravity (friction and the resistance of the air being neglected), moves in the same manner as a simple pendulum of determinate length called the *equivalent simple pendulum*. Hence the above results can be verified by experiments on actual pendulums.

The discovery of the experimental laws of the motion of pendulums was in fact long anterior to the theoretical investigation. It was the earliest and one of the most important discoveries of Galileo, and dates from the year 1582, when he was about twenty years of age. It is related that on one occasion, when in the cathedral of Pisa, he was struck with the regularity of the oscillations of a lamp suspended from the roof, and it appeared to him

that these oscillations, though diminishing in extent, preserved the same duration. He tested the fact by repeated trials, which confirmed him in the belief of its perfect exactness. This law of isochronism can be easily verified. It is only necessary to count the vibrations which take place in a given time with different amplitudes. The numbers will be found to be exactly the same. This will be found to hold good even when some of the vibrations compared are so small that they can only be observed with a telescope.

By employing balls suspended by threads of different lengths, Galileo discovered the influence of length on the time of vibration. He ascertained that when the length of the thread increases, the time of vibration increases also; not, however, in proportion to the length simply, but to its square root.

113. Cycloidal Pendulum.—It is obvious from § 64 that the effective component of gravity upon a particle resting on a smooth inclined plane is proportional to the sine of the inclination. The acceleration of a particle so situated is in fact $g \sin \alpha$, if α denote the inclination of the plane. When a particle is guided along a smooth curve its acceleration is expressed by the same formula, α now denoting the inclination of the curve at any point to the horizon. This inclination varies from point to point of the curve, so that the acceleration $g \sin \alpha$ is no longer a constant quantity. The motion of a common pendulum corresponds to the motion of a particle which is guided to move in a circular arc; and if x denote distance from the lowest point, measured along the arc, and r the radius of the circle (or the length of the pendulum), the acceleration at any point is $g \sin \frac{x}{r}$. This is sensibly proportional to x so long as x is a small fraction of r; but in general it is not proportional to x, and hence the vibrations are not in general isochronous.

To obtain strictly isochronous vibrations we must substitute for the circular arc a curve which possesses the property of having an inclination whose sine is simply proportional to distance measured along the curve from the lowest point. The curve which possesses this property is the cycloid. It is the curve which is traced by a point in the circumference of a circle which rolls along a straight line. The cycloidal pendulum is constructed by suspending an ivory ball or some other small heavy body by a thread between two cheeks (Fig. 47), on which the thread winds as the ball swings to either side. The cheeks must themselves be the two halves of a cycloid whose length is double that of the thread, so that each cheek has the same length as the thread. It can be demonstrated[1] that under these circumstances the path of the ball will be a cycloid identical with that to which the cheeks belong. Neglecting friction and the rigidity of the thread, the acceleration in this case is proportional to distance measured along the cycloid from its lowest point, and hence the time of vibration will be strictly the same for large as for small amplitudes. It will, in fact, be the same as that of a simple pendulum having the same length as the cycloidal pendulum and vibrating in a small arc.

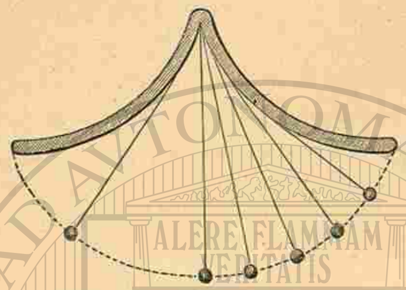


Fig 47.—Cycloidal Pendulum.

Attempts have been made to adapt the cycloidal pendulum to clocks, but it has been found that, owing to the greater amount of friction, its rate was less regular than that of the common pendulum. It may be remarked, that the spring by which pendulums are often suspended has the effect of guiding the pendulum bob in a curve which is approximately cycloidal, and thus of diminishing the irregularity of rate resulting from differences of amplitude.

114. Moment of Inertia.—Just as the mass of a body is the measure of the force requisite for producing unit acceleration when the movement is one of pure translation; so the *moment of inertia* of a rigid body turning about a fixed axis is the measure of the couple requisite for producing unit acceleration of angular velocity. We suppose angle to be measured by $\frac{\text{arc}}{\text{radius}}$, so that the angle turned by the body is equal to the arc described by any point of it divided by the distance of this point from the axis; and the angular velocity of the body will be the velocity of any point divided by its distance from the axis. The moment of inertia of the body round the axis is numerically equal to the couple which would produce unit change of angular velocity in the body in unit time. We shall now show how to express the moment of inertia in terms of the masses of the particles of the body and their distances from the axis.

[1] Since the evolute of the cycloid is an equal cycloid.

Let m denote the mass of any particle, r its distance from the axis, and ϕ the angular acceleration. Then $r\phi$ is the acceleration of the particle m, and the force which would produce this acceleration by acting directly on the particle along the line of its motion is $mr\phi$. The moment of this force round the axis would be $mr^2\phi$ since its arm is r. The aggregate of all such moments as this for all the particles of the body is evidently equal to the couple which actually produces the acceleration of the body. Using the sign Σ to denote "the sum of such terms as," and observing that ϕ is the same for the whole body, we have

$$\text{Applied couple} = \Sigma\,(mr^2\phi) = \phi\,\Sigma\,(mr^2). \qquad (1)$$

When ϕ is unity, the applied couple will be equal to $\Sigma\,(mr^2)$, which is therefore, by the foregoing definition, the moment of inertia of the body round the axis.

115. Moments of Inertia Round Parallel Axes.—The moment of inertia round an axis through the centre of mass is always less than that round any parallel axis.

For if r denote the distance of the particle m from an axis not passing through the centre of mass, and x and y its distances from two mutually rectangular planes through this axis, we have $r^2=x^2+y^2$.

Now let two planes parallel to these be drawn through the centre of mass; let ξ and η be the distances of m from them, and ρ its distance from their line of intersection, which will clearly be parallel to the given axis. Also let a and b be the distances respectively between the two pairs of parallel planes, so that a^2+b^2 will be the square of the distance between the two parallel axes, which distance we will denote by h. Then we have

$$x = \xi \pm a$$
$$y = \eta \pm b$$
$$x^2 = a^2 + \xi^2 \pm 2a\,\xi, \qquad y^2 = b^2 + \eta^2 \pm 2b\,\eta.$$

$$\begin{aligned}\Sigma\,(mr^2) &= \Sigma\left\{m\,(a^2+b^2)\right\} + \Sigma\left\{m\,(\xi^2+\eta^2)\right\} \\ &\quad \pm 2a\,\Sigma\,(m\xi) \pm 2b\,\Sigma\,(m\eta) \\ &= h^2\,\Sigma m + \Sigma\,(m\rho^2) \pm 2a\,\bar{\xi}\,\Sigma m \pm 2b\,\bar{\eta}\,\Sigma m.\end{aligned}$$

where $\bar{\xi}$ and $\bar{\eta}$ are the values of ξ and η for the centre of mass. But these values are both zero, since the centre of mass lies on both the planes from which ξ and η are measured. We have therefore

$$\Sigma\,(mr^2) = h^2\,\Sigma m + \Sigma\,(m\rho^2), \qquad (2)$$

that is to say, the moment of inertia round the given axis exceeds the moment of inertia round the parallel axis through the centre of mass by the product of the whole mass into the square of the distance between the axes.

116. Application to Compound Pendulum.—The application of this principle to the compound pendulum leads to some results of great interest and importance.

Let M be the mass of a compound pendulum, that is, a rigid body free to oscillate about a fixed horizontal axis. Let h, as in the preceding section, denote the distance of the centre of mass from this axis; let θ denote the inclination of h to the vertical, and ϕ the angular acceleration.

Then, since the forces of gravity on the body are equivalent to a single force Mg, acting vertically downwards at the centre of mass, and therefore having an arm $h \sin \theta$ with respect to the axis, the moment of the applied forces round the axis is $Mgh \sin \theta$; and this must, by § 114, be equal to $\phi \Sigma (mr^2)$. We have therefore

$$\frac{\Sigma (mr^2)}{Mh} = \frac{g \sin \theta}{\phi}. \qquad (3)$$

If the whole mass were collected at one point at distance l from the axis, this equation would become

$$\frac{Ml^2}{Ml} = l = \frac{g \sin \theta}{\phi}; \qquad (4)$$

and the angular motion would be the same as in the actual case if l had the value

$$l = \frac{\Sigma mr^2}{Mh}. \qquad (5)$$

Fig. 48.

l is evidently the length of the equivalent simple pendulum.

117. Convertibility of Centres.—Again, if we introduce a length k such that Mk^2 is equal to $\Sigma (m\rho^2)$, that is, to the moment of inertia round a parallel axis through the centre of mass, we have

$$\Sigma (mr^2) = \Sigma (m\rho^2) + h^2 \Sigma m = Mk^2 + Mh^2,$$

and equation (5) becomes

$$l = \frac{k^2 + h^2}{h} = \frac{k^2}{h} + h, \qquad (6)$$

$$\text{or } k^2 = (l - h)\, h. \qquad (7)$$

In the annexed figure (Fig. 48) which represents a vertical section through the centre of mass, let G be the centre of mass, A the "centre of suspension," that is, the point in which the axis cuts the plane of the figure, and O the "centre of oscillation," that is, the point at which the mass might be collected without altering the movement. Then, by definition, we have

$$l = AO,\ h = AG, \text{ therefore } l - h = GO,$$

so that equation (7) signifies

$$k^2 = AG \cdot GO. \qquad (8)$$

Since k^2 is the same for all parallel axes, this equation shows that when the body is made to vibrate about a parallel axis through O, the centre of oscillation will be the point A. That is to say; *the centres of suspension and oscillation are interchangeable, and the product of their distances from the centre of mass is k^2.*

118. If we take a new centre of suspension A′ in the plane of the figure, the new centre of oscillation O′ will lie in the production of A′G, and we must have

$$A'G \cdot GO' = k^2 = AG \cdot GO.$$

If A′G be equal to AG, GO′ will be equal to GO, and A′O′ to AO, so that the length of the equivalent simple pendulum will be unchanged. *A compound pendulum will* therefore *vibrate in the same time about all parallel axes which are equidistant from the centre of mass.*

When the product of two quantities is given, their sum is least when they are equal, and becomes continually greater as they depart further from equality. Hence the length of the equivalent simple pendulum AO or AG + GO is least when

$$AG = GO = k,$$

and increases continually as the distance of the centre of suspension from G is either increased from k to infinity or diminished from k to zero. Hence, when a body vibrates about an axis which passes very nearly through its centre of gravity, its oscillations are exceedingly slow.

119. Kater's Pendulum.—The principle of the convertibility of centres, established in § 117, was discovered by Huygens, and affords the most convenient practical method of constructing a pendulum of known length. In Kater's pendulum there are two parallel knife-edges about either of which the pendulum can be made to vibrate, and one of them can be adjusted to any distance from the other. The pendulum is swung first upon one of these edges and then upon the other, and, if any difference is detected in the times of vibration, it is corrected by moving the adjustable edge. When the difference has been completely destroyed, the distance between the two edges is the length of the equivalent simple pendulum. It is necessary, in any arrangement of this kind, that the two knife-edges should be in a plane passing through the centre of gravity; also that they should be on opposite sides of the centre of gravity, and at unequal distances from it.

120. Determination of the Value of *g*.—Returning to the formula for the simple pendulum $T=\pi\sqrt{\frac{l}{g}}$, we easily deduce from it $g=\frac{\pi^2 l}{T^2}$, whence it follows that the value of g can be determined by making a pendulum vibrate and measuring T and l. T is determined by counting the number of vibrations that take place in a given time; l can be calculated, when the pendulum is of regular form, by the aid of formulæ which are given in treatises on rigid dynamics, but its value is more easily obtained by Kater's method, described above, founded on the principle of the convertibility of the centres of suspension and oscillation.

It is from pendulum observations, taken in great numbers at different parts of the earth, that the approximate formula for the intensity of gravity which we have given at § 91 has been deduced. Local peculiarities prevent the possibility of laying down any general formula with precision; and the exact value of g for any place can only be ascertained by observations on the spot.

CHAPTER IX.

CONSERVATION OF ENERGY.

121. Definition of Kinetic Energy.—We have seen in § 93 that the work which must be done upon a mass of m grammes to give it a velocity of v centimetres per second is $\frac{1}{2}mv^2$ ergs. Though we have proved this only for the case of falling bodies, with gravity as the working force, the result is true universally, as is shown in advanced treatises on mathematical physics. It is true whether the motion be rectilinear or curvilinear, and whether the working force act in the line of motion or at an angle with it.

If the velocity of a mass increases from v_1 to v_2, the work done upon it in the interval is $\frac{1}{2}m\ (v_2^2-v_1^2)$; in other words, is the increase of $\frac{1}{2}mv^2$.

On the other hand, if a force acts in such a manner as to oppose the motion of a moving mass, the force will do negative work, the amount of which will be equal to the decrease in the value of $\frac{1}{2}mv^2$.

For example, during any portion of the ascent of a projectile, the diminution in the value of $\frac{1}{2}mv^2$ is equal to gm multiplied by the increase of height; and during any portion of its descent the increase in $\frac{1}{2}mv^2$ is equal to gm multiplied by the decrease of height.

The work which must have been done upon a body to give it its actual motion, supposing it to have been initially at rest, is called the *energy of motion* or the *kinetic energy* of the body. It can be computed by multiplying *half the mass by the square of the velocity.*

122. Definition of Static or Potential Energy.—When a body of mass m is at a height s above the ground, which we will suppose level, gravity is ready to do the amount of work gms upon it by making it fall to the ground. A body in an elevated position may therefore be regarded as a reservoir of work. In like manner a wound-up clock, whether driven by weights or by a spring, has from the other. The pendulum is swung first upon one of these edges and then upon the other, and, if any difference is detected in the times of vibration, it is corrected by moving the adjustable edge. When the difference has been completely destroyed, the distance between the two edges is the length of the equivalent simple pendulum. It is necessary, in any arrangement of this kind, that the two knife-edges should be in a plane passing through the centre of gravity; also that they should be on opposite sides of the centre of gravity, and at unequal distances from it.

120. Determination of the Value of g.—Returning to the formula for the simple pendulum $T=\pi\sqrt{\frac{l}{g}}$, we easily deduce from it $g=\frac{\pi^2 l}{T^2}$, whence it follows that the value of g can be determined by making a pendulum vibrate and measuring T and l. T is determined by counting the number of vibrations that take place in a given time; l can be calculated, when the pendulum is of regular form, by the aid of formulæ which are given in treatises on rigid dynamics, but its value is more easily obtained by Kater's method, described above, founded on the principle of the convertibility of the centres of suspension and oscillation.

It is from pendulum observations, taken in great numbers at different parts of the earth, that the approximate formula for the intensity of gravity which we have given at § 91 has been deduced. Local peculiarities prevent the possibility of laying down any general formula with precision; and the exact value of g for any place can only be ascertained by observations on the spot.

CHAPTER IX.

CONSERVATION OF ENERGY.

121. Definition of Kinetic Energy.—We have seen in § 93 that the work which must be done upon a mass of m grammes to give it a velocity of v centimetres per second is $\frac{1}{2}mv^2$ ergs. Though we have proved this only for the case of falling bodies, with gravity as the working force, the result is true universally, as is shown in advanced treatises on mathematical physics. It is true whether the motion be rectilinear or curvilinear, and whether the working force act in the line of motion or at an angle with it.

If the velocity of a mass increases from v_1 to v_2, the work done upon it in the interval is $\frac{1}{2}m\ (v_2^2-v_1^2)$; in other words, is the increase of $\frac{1}{2}mv^2$.

On the other hand, if a force acts in such a manner as to oppose the motion of a moving mass, the force will do negative work, the amount of which will be equal to the decrease in the value of $\frac{1}{2}mv^2$.

For example, during any portion of the ascent of a projectile, the diminution in the value of $\frac{1}{2}mv^2$ is equal to gm multiplied by the increase of height; and during any portion of its descent the increase in $\frac{1}{2}mv^2$ is equal to gm multiplied by the decrease of height.

The work which must have been done upon a body to give it its actual motion, supposing it to have been initially at rest, is called the *energy of motion* or the *kinetic energy* of the body. It can be computed by multiplying *half the mass by the square of the velocity.*

122. Definition of Static or Potential Energy.—When a body of mass m is at a height s above the ground, which we will suppose level, gravity is ready to do the amount of work gms upon it by making it fall to the ground. A body in an elevated position may therefore be regarded as a reservoir of work. In like manner a wound-up clock, whether driven by weights or by a spring, has work stored up in it. In all these cases there is force between parts of a system tending to produce relative motion, and there is room for such relative motion to take place. There is force ready to act, and space for it to act through. Also the force is always the same in the same relative position of the parts. Such a system possesses energy, which is usually called *potential.* We prefer to call it *statical,* inasmuch as its amount is computed on statical principles alone.[1] Statical energy depends jointly on mutual force and relative position. Its amount in any given position is the amount of work which would be done by the forces of the system in passing from this position to the standard position. When we are speaking of the energy of a heavy body in an elevated position above level ground, we naturally adopt as the standard position that in which the body is lying on the ground. When we speak of the energy of a wound-up clock, we adopt as the standard position that in which the clock has completely run down. Even when the standard position is not indicated, we can still speak definitely of the difference between the energies of two given positions of a system; just as we can speak definitely of the difference of level of two given points without any agreement as to the datum from which levels are to be reckoned.

123. Conservation of Mechanical Energy.—When a frictionless system is so constituted that its forces are always the same in the same positions of the system, the amount of work done by these forces during the passage from one position A to another position B will be independent of the path pursued, and will be equal to *minus* the work done by them in the passage from B to A. The earth and any heavy body at its surface constitute such a system; the force of the system is the mutual gravitation of these two bodies; and the work done by this mutual gravitation, when the body is moved by any path from a point A to a point B, is equal to the weight of the body multiplied by the height of A above B. When the system passes through any series of movements beginning with a given position and ending with the same position again, the algebraic total of work done by the forces of the system in this series of movements is zero. For instance, if a heavy body be carried by a roundabout path back to the point from whence it started, no work is done upon it by gravity upon the whole.

Every position of such a system has therefore a definite amount

[1] That is to say, the computation involves no reference to the laws of motion.

of statical energy, reckoned with respect to an arbitrary standard position. The work done by the forces of the system in passing from one position to another is (by definition) equal to the loss of static energy; but this loss is made up by an equal gain of kinetic energy. Conversely if kinetic energy is lost in passing from one position to another, the forces do negative work equal to this loss, and an equal amount of static energy is gained. The total energy of the system (including both static and kinetic) therefore remains unaltered.

An approximation to such a state of things is exhibited by a pendulum. In the two extreme positions it is at rest, and has therefore no kinetic energy; but its statical energy is then a maximum. In the lowest position its motion is most rapid; its kinetic energy is therefore a maximum, but its statical energy is zero. The difference of the statical energies of any two positions, will be the weight of the pendulum multiplied by the difference of levels of its centre of gravity, and this will also be the difference (in inverse order) between the kinetic energies of the pendulum in these two positions.

As the pendulum is continually setting the air in motion and thus doing external work, it gradually loses energy and at last comes to rest, unless it be supplied with energy from a clock or some other source. If a pendulum could be swung in a perfect vacuum, with an entire absence of friction, it would lose no energy, and would vibrate for an indefinite time without decrease of amplitude.

124. Illustration from Pile-driving.—An excellent illustration of transformations of energy is furnished by pile-driving. A large mass of iron called a *ram* is slowly hauled up to a height of several yards above the pile, and is then allowed to fall upon it. During the ascent, work must be supplied to overcome the force of gravity; and this work is represented by the statical energy of the ram in its highest position. While falling, it continually loses statical and gains kinetic energy; the amount of the latter which it possesses immediately before the blow being equal to the work which has been done in raising it. The effect of the blow is to drive the pile through a small distance against a resistance very much greater than the weight of the ram; the work thus done being nearly equal to the total energy which the ram possessed at any point of its descent. We say *nearly* equal, because a portion of the energy of the blow is spent in producing vibrations.

125. Hindrances to Availability of Energy.—There is almost always some waste in utilizing energy. When water turns a mill-wheel, it runs away from the wheel with a velocity, the square of which multiplied by half the mass of the water represents energy which has run to waste.

Friction again often consumes a large amount of energy; and in this case we cannot (as in the preceding one) point to any palpable motion of a mass as representing the loss. Heat, however, is produced, and the energy which has disappeared as regarded from a gross mechanical point of view, has taken a molecular form. Heat is a form of molecular energy; and we know, from modern researches, what quantity of heat is equivalent to a given amount of mechanical work. In the steam-engine we have the converse process; mechanical work is done by means of heat, and heat is destroyed in the doing of it, so that the amount of heat given out by the engine is less than the amount supplied to it.

The sciences of electricity and magnetism reveal the existence of other forms of molecular energy; and it is possible in many ways to produce one form of energy at the expense of another; but in every case there is an exact equivalence between the quantity of one kind which comes into existence and the quantity of another kind which simultaneously disappears. Hence the problem of constructing a self-driven engine, which we have seen to be impossible in mechanics, is equally impossible when molecular forms of energy are called to the inventor's aid.

Energy may be transformed, and may be communicated from one system to another; but it cannot be increased or diminished in total amount. This great natural law is called the *principle of the conservation of energy.*

CHAPTER X.

ELASTICITY.

126. Elasticity and its Limits.—There is no such thing in nature as an absolutely rigid body. All bodies yield more or less to the action of force; and the property in virtue of which they tend to recover their original form and dimensions when these are forcibly changed, is called *elasticity.* Most solid bodies possess almost perfect elasticity for small deformations; that is to say, when distorted, extended, or compressed, within certain small limits, they will, on the removal of the constraint to which they have been subjected, instantly regain almost completely their original form and dimensions. These limits (which are called the limits of elasticity) are different for different substances; and when a body is distorted beyond these limits, it takes a *set,* the form to which it returns being intermediate between its original form and that into which it was distorted.

When a body is distorted within the limits of its elasticity, the force with which it reacts is directly proportional to the amount of distortion. For example, the force required to make the prongs of a tuning-fork approach each other by a tenth of an inch, is double of that required to produce an approach of a twentieth of an inch; and if a chain is lengthened a twentieth of an inch by a weight of 1 cwt., it will be lengthened a tenth of an inch by a weight of 2 cwt., the chain being supposed to be strong enough to experience no permanent set from this greater weight. Also, within the limits of elasticity, equal and opposite distortions, if small, are resisted by equal reactions. For example, the same force which suffices to make the prongs of a tuning-fork approach by a twentieth of an inch, will, if applied in the opposite direction, make them separate by the same amount.

127. Isochronism of Small Vibrations.—An important consequence of these laws is, that when a body receives a slight distortion within the limits of its elasticity, the vibrations which ensue when the constraint is removed are isochronous. This follows from § 111, inasmuch as the accelerations are proportional to the forces, and are therefore proportional at each instant to the deformation at that instant.

128. Stress, Strain, and Coefficients of Elasticity.—A body which, like indian-rubber, can be subjected to large deformations without receiving a permanent set, is said to have wide limits of elasticity.

A body which, like steel, opposes great resistance to deformation, is said to have large coefficients of elasticity.

Any change in the shape or size of a body produced by the application of force to the body is called a *strain;* and an action of force tending to produce a strain is called a *stress.*

When a wire of cross-section A is stretched with a force F, the longitudinal stress is $\frac{F}{A}$; this being the intensity of force per unit area with which the two portions of the wire separated by any cross-section are pulling each other. If the length of the wire when unstressed is L and when stressed $L+l$, the longitudinal strain is $\frac{l}{L}$. A stress is always expressed in units of force per unit of area. A strain is always expressed as the ratio of two magnitudes of the same kind (in the above example, two lengths), and is therefore independent of the units employed.

The quotient of a stress by the strain (of a given kind) which it produces, is called a *coefficient* or *modulus of elasticity.* In the above example, the quotient $\frac{FL}{Al}$ is called *Young's modulus* of elasticity.

As the wire, while it extends lengthwise, contracts laterally, there will be another coefficient of elasticity obtained by dividing the longitudinal stress by the lateral strain.

It is shown, in special treatises, that a solid substance may have 21 independent coefficients of elasticity; but that when the substance is *isotropic*, that is, has the same properties in all directions, the number reduces to 2.

129. Volume-elasticity.—The only coefficient of elasticity possessed by liquids and gases is elasticity of volume. When a body of volume V is reduced by the application of uniform normal pressure over its whole surface to volume $V-v$, the volume-strain is $\frac{v}{V}$, and if this effect is produced by a pressure of p units of force per unit of area, the elasticity of volume is the quotient of the stress p by the strain $\frac{v}{V}$, or is $\frac{pV}{v}$. This is also called the *resistance to compression;* and its reciprocal $\frac{v}{pV}$ is called the *compressibility* of the substance. In dealing with gases, p must be understood as a pressure superadded to the original pressure of the gas.

Since a strain is a mere numerical quantity, independent of units, a coefficient of elasticity must be expressed, like a stress, in units of force per unit of area. In the C.G.S. system, stresses and coefficients of elasticity are expressed in dynes per square centimetre. The following are approximate values (thus expressed) of the two coefficients of elasticity above defined:—

	Young's Modulus.	Elasticity of Volume.
Glass (flint),	60×10^{10}	40×10^{10}
Steel,	210×10^{10}	180×10^{10}
Iron (wrought),	190×10^{10}	140×10^{10}
Iron (cast),	130×10^{10}	96×10^{10}
Copper,	120×10^{10}	160×10^{10}
Mercury,		54×10^{10}
Water,		2×10^{10}
Alcohol,		$1{\cdot}2 \times 10^{10}$

130. Œrsted's Piezometer.—The compression of liquids has been observed by means of Œrsted's piezometer, which is represented in Fig. 49. The liquid whose compression is to be observed is contained in a glass vessel b, resembling a thermometer with a very large bulb and short tube. The tube is open above, and a globule of mercury at the top of the liquid column serves as an index. This apparatus is placed in a very strong glass vessel a full of water. When pressure is exerted by means of the piston klh, the index of mercury is seen to descend, showing a diminution of volume of the liquid, and showing moreover that this diminution of volume exceeds that of the containing vessel b. It might at first

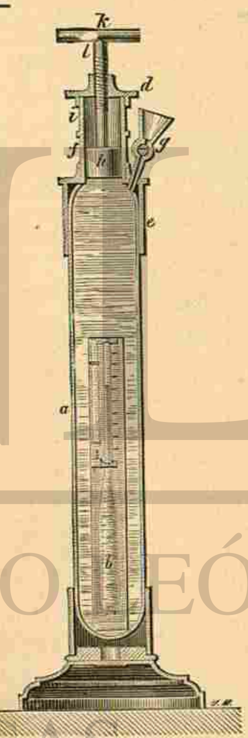


Fig. 49.—Œrsted's Piezometer.

sight appear that since this vessel is subjected to equal pressure within and without, its volume is unchanged; but in fact, its volume is altered to the same extent as that of a solid vessel of the same material; for the interior shells would react with a force precisely equivalent to that which is exerted by the contained liquid.

CHAPTER XI.

FRICTION.

131. Friction, Kinetical and Statical.—When two bodies are pressed together in such a manner that the direction of their mutual pressure is not normal to the surface of contact, the pressure can be resolved into two parts, one normal and the other tangential. The tangential component is called the *force of friction* between the two bodies. The friction is called *kinetical* or *statical* according as the bodies are or are not sliding one upon the other.

As regards kinetical friction, experiment shows that if the normal pressure between two given surfaces be changed, the tangential force changes almost exactly in the same proportion; in other words, the ratio of the force of friction to the normal pressure is nearly constant for two given surfaces. This ratio is called the *coefficient of kinetical friction* between the two surfaces, and is nearly independent of the velocity.

132. Statical Friction. Limiting Angle.—It is obvious that the statical friction between two given surfaces is zero when their mutual pressure is normal, and increases with the obliquity of the pressure if the normal component be preserved constant. The obliquity, however, cannot increase beyond a certain limit, depending on the nature of the bodies, and seldom amounting to so much as 45°. Beyond this limit sliding takes place. The limiting obliquity, that is, the greatest angle that the mutual force can make with the normal, is called the *limiting angle of friction* for the two surfaces; and the ratio of the tangential to the normal component when the mutual force acts at the limiting angle, is called the *coefficient of statical friction* for the two surfaces. The coefficient and limiting angle remain nearly constant when the normal force is varied.

The coefficient of statical friction is in almost every case greater than the coefficient of kinetical friction; in other words, friction offers more resistance to the commencement of sliding than to the continuance of it.

A body which has small coefficients of friction with other bodies is called slippery.

133. Coefficient=tan θ. Inclined Plane.—If θ be the inclination of the mutual force P to the common normal, the tangential component will be P sin θ, the normal component P cos θ, and the ratio of the former to the latter will be tan θ. Hence *the coefficient of statical friction is equal to the tangent of the limiting angle of friction.*

When a heavy body rests on an inclined plane, the mutual pressure is vertical, and the angle θ is the same as the inclination of the plane. Hence if an inclined plane is gradually tilted till a body lying on it slides under the action of gravity, the inclination of the plane at which sliding begins is the limiting angle of friction between the body and the plane, and the tangent of this angle is the coefficient of statical friction.

Again, if the inclination of a plane be such that the motion of a body sliding down it under the action of gravity is neither accelerated nor retarded, the tangent of this inclination will be the coefficient of kinetical friction.

CHAPTER XII.

HYDROSTATICS.

134. Hydrodynamics.—We shall now treat of the laws of force as applied to fluids. This branch of the general science of dynamics is called *hydrodynamics* (ὕδωρ, water), and is divided into *hydrostatics* and *hydrokinetics*. Our discussions will be almost entirely confined to hydrostatics.

FLUIDS.—TRANSMISSION OF PRESSURE.

The name *fluid* comprehends both liquids and gases.

135. No Statical Friction in Fluids.—A fluid at rest cannot exert any tangential force against a surface in contact with it; its pressure at every point of such a surface is entirely normal. A slight tangential force is exerted by fluids in motion; and this fact is expressed by saying that all fluids are more or less *viscous*. An imaginary perfect fluid would be perfectly free from viscosity; its pressure against any surface would be entirely normal, whether the fluid were in motion or at rest.

136. Intensity of Pressure.—When pressure is uniform over an area, the total amount of the pressure, divided by the area, is called the *intensity of the pressure*. The C.G.S. unit of intensity of pressure is a pressure of a *dyne on each square centimetre* of surface. A rough unit of intensity frequently used is the pressure of a pound per square inch. This unit varies with the intensity of gravity, and has an average value of about 69,000 C.G.S. units. Another rough unit of intensity of pressure frequently employed is "an atmosphere"—that is to say, the average intensity of pressure of the atmosphere at the surface of the earth. This is about 1,000,000 C.G.S. units.

than the coefficient of kinetical friction; in other words, friction offers more resistance to the commencement of sliding than to the continuance of it.

A body which has small coefficients of friction with other bodies is called slippery.

133. Coefficient=tan θ. Inclined Plane.—If θ be the inclination of the mutual force P to the common normal, the tangential component will be P sin θ, the normal component P cos θ, and the ratio of the former to the latter will be tan θ. Hence *the coefficient of statical friction is equal to the tangent of the limiting angle of friction.*

When a heavy body rests on an inclined plane, the mutual pressure is vertical, and the angle θ is the same as the inclination of the plane. Hence if an inclined plane is gradually tilted till a body lying on it slides under the action of gravity, the inclination of the plane at which sliding begins is the limiting angle of friction between the body and the plane, and the tangent of this angle is the coefficient of statical friction.

Again, if the inclination of a plane be such that the motion of a body sliding down it under the action of gravity is neither accelerated nor retarded, the tangent of this inclination will be the coefficient of kinetical friction.

CHAPTER XII.

HYDROSTATICS.

134. Hydrodynamics.—We shall now treat of the laws of force as applied to fluids. This branch of the general science of dynamics is called *hydrodynamics* (ὕδωρ, water), and is divided into *hydrostatics* and *hydrokinetics.* Our discussions will be almost entirely confined to hydrostatics.

FLUIDS.—TRANSMISSION OF PRESSURE.

The name *fluid* comprehends both liquids and gases.

135. No Statical Friction in Fluids.—A fluid at rest cannot exert any tangential force against a surface in contact with it; its pressure at every point of such a surface is entirely normal. A slight tangential force is exerted by fluids in motion; and this fact is expressed by saying that all fluids are more or less *viscous.* An imaginary perfect fluid would be perfectly free from viscosity; its pressure against any surface would be entirely normal, whether the fluid were in motion or at rest.

136. Intensity of Pressure.—When pressure is uniform over an area, the total amount of the pressure, divided by the area, is called the *intensity of the pressure.* The C.G.S. unit of intensity of pressure is a pressure of a *dyne on each square centimetre* of surface. A rough unit of intensity frequently used is the pressure of a pound per square inch. This unit varies with the intensity of gravity, and has an average value of about 69,000 C.G.S. units. Another rough unit of intensity of pressure frequently employed is "an atmosphere"—that is to say, the average intensity of pressure of the atmosphere at the surface of the earth. This is about 1,000,000 C.G.S. units.

The single word "pressure" is used sometimes to denote "amount of pressure" (which can be expressed in dynes) and sometimes "intensity of pressure" (which can be expressed in dynes per square centimetre). The context usually serves to show which of these two meanings is intended.

137. Pressure the Same in all Directions.—The intensity of pressure at any point of a fluid is the same in all directions; it is the same whether the surface which receives the pressure faces upwards, downwards, horizontally, or obliquely.

This equality is a direct consequence of the absence of tangential force between two contiguous portions of a fluid.

For in order that a small triangular prism of the fluid (its ends being right sections) may be in equilibrium, the pressures on its three faces must balance each other. But when three forces balance each other, they are proportional to the sides of a triangle to which they are perpendicular;[1] hence the *amounts* of pressure on the three faces are proportional to the faces, in other words the *intensities* of these three pressures are equal. As we can take two of the faces perpendicular to any two given directions, this proves that the pressures in all directions at a point are of equal intensity.

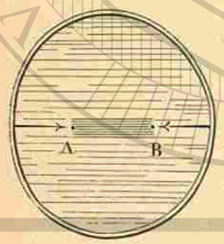


Fig. 50.

138. Pressure the Same at the Same Level.—In a fluid at rest, the pressure is the same at all points in the same horizontal plane. This appears from considering the equilibrium of a horizontal cylinder AB (Fig. 50), of small sectional area, its ends being right sections. The pressures on the sides are normal, and therefore give no component in the direction of the length; hence the pressures on the ends must be equal in amount; but they act on equal areas; therefore their intensities are equal.

A horizontal surface in a liquid at rest may therefore be called a "surface of equal pressure."

139. Difference of Pressure at Different Levels.—The increase of pressure with depth, in a fluid of uniform density, can be investigated as follows:—Consider the equilibrium of a vertical cylinder mm' (Fig. 51), its ends being right sections. The pressures on its

[1] This is an obvious consequence of the triangle of forces (art. 14); for if the sides of a triangle are parallel to three forces, we have only to turn the triangle through a right angle, and its sides will then be perpendicular to the forces.

sides are normal, and therefore horizontal. The only vertical forces acting upon it are its own weight and the pressures on its ends, of which it is to be observed that the pressure on the upper end acts downwards and that on the lower end upwards. The pressure on the lower end therefore exceeds that on the upper end by an amount equal to the weight of the cylinder. If a be the sectional area, w the weight of unit volume of the liquid, and h the length of the cylinder, the volume of the cylinder is ha, and its weight wha, which must be equal to $(p-p')\,a$ if p, p' are the intensities of pressure on the lower and upper ends respectively. We have therefore

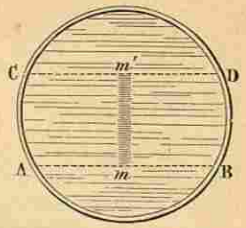


Fig. 51.

$$p - p' = wh,$$ = weight × depth

that is, *the increase of pressure in descending through a depth h is wh.*

The principles of this and the preceding section remain applicable whatever be the shape of the containing vessel, even if it be such as to render a circuitous route necessary in passing from one of two points compared to the other; for this route can always be made to consist of a succession of vertical and horizontal lines, and the preceding principles when applied to each of these lines separately, will give as the final result a difference of pressure wh for a difference of heights h.

If d denote the density of the liquid, in grammes per cub. cm., the weight of a cubic cm. will be gd dynes. The increase of pressure for an increase of depth h cm. is therefore ghd dynes per sq. cm. If there be no pressure at the surface of the liquid, this will be the actual pressure at the depth h.

140. Free Surface.—It follows from these principles that the free surface of a liquid at rest—that is, the surface in contact with the atmosphere—must be horizontal; since all points in this surface are at the same pressure. If the surface were not horizontal, but were higher at n than at n' (Fig. 52), the pressures at the two points m, m' vertically beneath them in any horizontal plane AB would be unequal, for they would be due to the weights

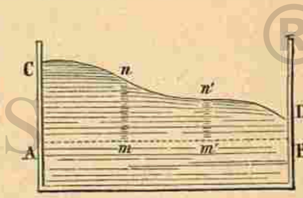


Fig. 52.

of unequal columns nm, $n'm'$, and motion would ensue from m towards m'.

The same conclusion can be deduced from considering the equilibrium of a particle at the surface, as M (Fig. 53). If the tangent plane at M were not horizontal there would be a component of gravity tending to make the particle slide down; and this tendency would produce motion, since there is no friction to oppose it.

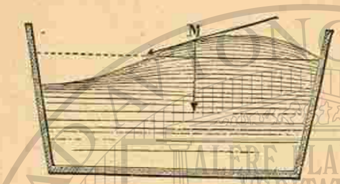


Fig. 53.

141. Transmissibility of Pressure in Fluids.—Since the difference of the pressures at two points in a fluid can be determined by the foregoing principles, independently of any knowledge of the absolute intensity of either, it follows that when increase or diminution of pressure occurs at one point, an equal increase or diminution must occur throughout the whole fluid. *A fluid in a closed vessel perfectly transmits through its whole substance whatever pressure we apply to any part.* The changes in *amount* of pressure will be equal for all equal areas. For unequal areas they will be proportional to the areas.

Thus if the two vertical tubes in Fig. 54 have sectional areas which are as 1 to 16, a weight of 1 kilogram acting on the surface of the liquid in the smaller tube will be balanced by 16 kilograms acting on the surface of the liquid in the larger.

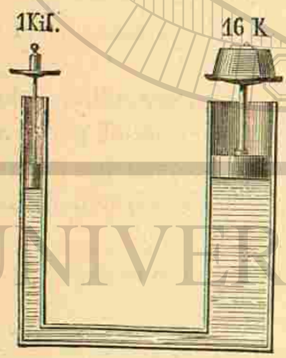


Fig. 54.—Principle of the Hydraulic Press.

This principle of the perfect transmission of pressure by fluids appears to have been first discovered and published by Stevinus; but it was rediscovered by Pascal a few years later, and having been made generally known by his writings is often called "Pascal's principle." In his celebrated treatise on the *Equilibrium of Liquids*, he says, "If a vessel full of water, closed on all sides, has two openings, the one a hundred times as large as the other, and if each be supplied with a piston which fits exactly, a man pushing the small piston will exert a force which will equilibrate that of a hundred men pushing the piston which is a hundred times as large,

and will overcome that of ninety-nine. And whatever may be the proportion of these openings, if the forces applied to the pistons are to each other as the openings, they will be in equilibrium."

142. Hydraulic Press.—This mode of multiplying force remained for a long time practically unavailable on account of the difficulty of making the pistons water-tight. The hydraulic press was first successfully made by Bramah, who invented the *cupped leather collar* illustrated in Fig. 166, § 264. Fig. 165 shows the arrangements of the press as a whole. Instead of pistons, *plungers* are employed; that is to say, solid cylinders of metal which can be pushed down into the liquid, or can be pushed up by the pressure of the liquid against their bases. The volume of liquid displaced by the advance of a plunger is evidently equal to that displaced by a piston of the same sectional area, and the above calculations for pistons apply to plungers as well. The plungers work through openings which are kept practically water-tight by means of the cup-leather arrangement. The cup-leather, which is shown both in plan and section in Fig. 166, consists of a leather ring bent so as to have a semicircular section. It is fitted into a hollow in the interior of the sides of the opening, so that water leaking up along the circumference of the plunger will fill the concavity of the leather, and, by pressing on it, will produce a packing which fits more tightly as the pressure on the plunger increases.

143. Principle of Work Applicable.—In Fig. 54, when the smaller piston advances and forces the other back, the volume of liquid driven out of the smaller tube is equal to the sectional area multiplied by the distance through which the piston advances. In like manner, the volume of liquid driven into the larger tube is equal to its sectional area multiplied by the distance that its piston is forced back. But these two volumes are equal, since the same volume of liquid that leaves one tube enters the other. The distances through which the two pistons move are therefore inversely as their sectional areas, and hence are inversely as the amounts of pressure applied to them. The *work done* in pushing forward the smaller piston is therefore equal to the work done by the liquid in pushing back the larger. This was remarked by Pascal, who says—

"It is, besides, worthy of admiration that in this new machine we find that constant rule which is met with in all the old ones such as the lever, wheel and axle, screw, &c., which is that the distance is increased in proportion to the force; for it is evident that as one of these openings is a hundred times as large as the other, if the man who pushes the small piston drives it forward one inch, he will drive the large piston backward only one-hundredth part of that length."

144. Experiment on Upward Pressure.—The upward pressure exerted by a liquid against a horizontal surface facing downwards can be exhibited by the following experiment. Take a tube open at both ends (Fig. 55), and keeping the lower end covered with a piece of card, plunge it into water. The liquid will press the card against the bottom of the tube with a force which increases as it is plunged deeper. If water be now poured into the tube, the card will remain in its place as long as the level of the liquid is lower within the tube than without; but at the moment when equality of levels is attained it will become detached.

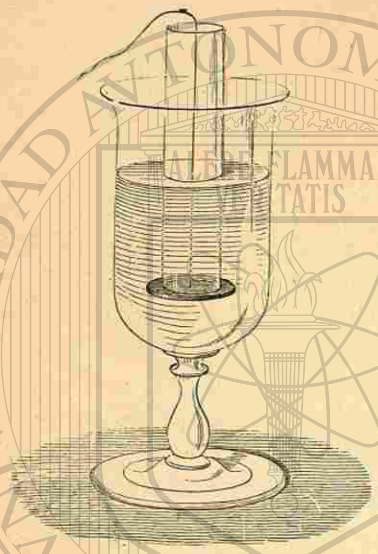


Fig. 55.—Upward Pressure.

145. Liquids in Superposition.—When one liquid rests on the top of another of different density, the foregoing principles lead to the result that the surface of demarcation must be horizontal. For the free surface of the upper liquid must, as we have seen, be horizontal. If now we take two small equal areas n and n' (Fig. 56) in a horizontal layer of the lower liquid, they must be subjected to equal pressures. But these pressures are measured by the weights of the liquid cylinders nrs, $n'tl$; and these latter cannot be equal unless the points r and t at the junction of the two liquids are at the same level. All points in the surface of demarcation are therefore in the same horizontal plane.

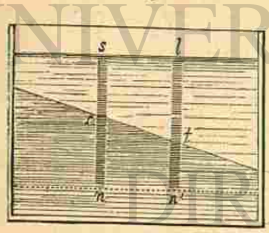


Fig. 56.

The same reasoning can be extended downwards to any number of liquids of unequal densities, which rest one upon another, and shows that all the surfaces of demarcation between them must be horizontal.

An experiment in illustration of this result is represented in Fig. 57. Mercury, water, and oil are poured into a glass jar. The mercury, being the heaviest, goes to the bottom; the oil, being the lightest, floats at the top; and the surfaces of contact of the liquids are seen to be horizontal.

Even when liquids are employed which gradually mix with one another, as water and alcohol, or fresh water and salt water, so that there is no definite surface of demarcation, but a gradual increase of density with depth, it still remains true that the density at all points in a horizontal plane is the same.

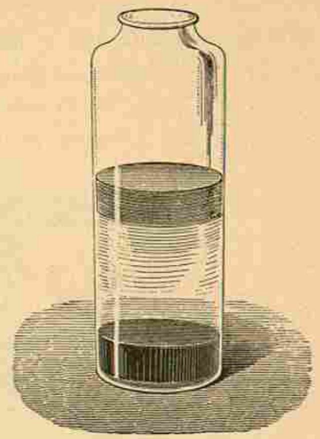


Fig. 57.
Phial of the Four Elements.

146. Two Liquids in Bent Tube.—If we pour mercury into a bent tube open at both ends (Fig. 58), and then pour water into one of the arms, the heights of the two liquids above the surface of junction will be very unequal, as shown in the figure. The general rule for the equilibrium of any two liquids in these circumstances is that *their heights above the surface of junction must be inversely as their densities*, since they correspond to equal pressures.

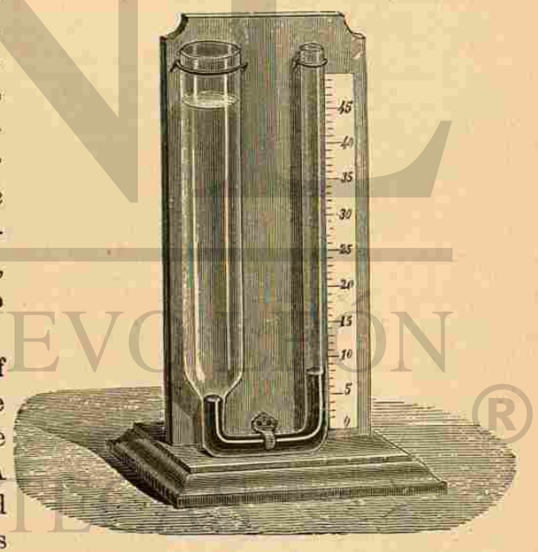


Fig. 58.—Equilibrium of Two Fluids in Communicating Vessels.

147. Experiment of Pascal's Vases.—Since the amount of pressure on a horizontal area A at the depth h in a liquid is whA, where w denotes the weight of unit volume of the liquid, it follows that the pressure on the bottom of a vessel containing liquid is not affected by the breadth or narrowness of the upper part of the vessel, provided the height of the free surface of the liquid be given. Pascal verified this fact by an experiment which is frequently exhibited in courses of physics. The apparatus employed (Fig. 59) is a tripod supporting a ring, into which can be screwed three vessels of different shapes, one widened upwards, another cylindrical, and the third tapering upwards. Beneath the ring is a movable disc

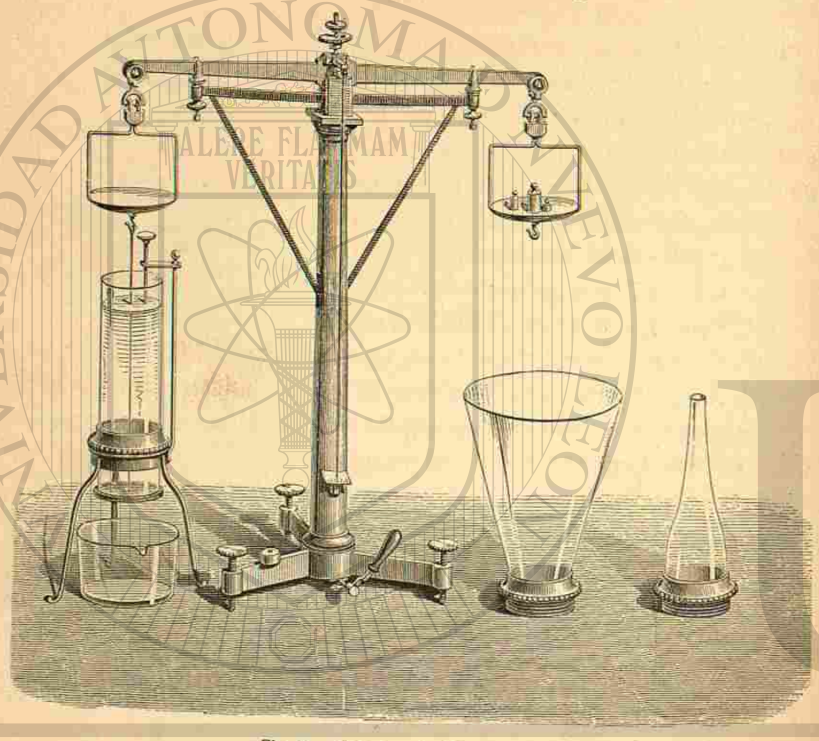


Fig. 59.—Experiment of Pascal's Vases.

supported by a string attached to one of the scales of a balance. Weights are placed in the other scale in order to keep the disc pressed against the ring. Let the cylindrical vase be mounted on the tripod, and filled up with water to such a level that the pressure is just sufficient to detach the disc from the ring. An indicator, shown in the figure, is used to mark the level at which this takes place. Let the experiment be now repeated with the two other vases, and the disc will be detached when the water has reached the same level as before.

In the case of the cylindrical vessel, the pressure on the bottom is evidently equal to the weight of the liquid. Hence in all three

cases the pressure on the bottom of the vessel is equal to the weight of a cylindrical column of the liquid, having the bottom as its base, and having the same height as the liquid in the vessel.

148. Resultant Pressure on Vessel.—The pressure exerted by the bottom of the vessel upon the stand on which it rests, consists of the weight of the vessel itself, together with the resultant pressure of the contained liquid against it. The actual pressure of the liquid against any portion of the vessel is normal to this portion, and if we resolve it into two components, one vertical and the other horizontal, only the vertical component need be attended to, in computing the resultant; for the horizontal components will always destroy one another. At such points as n, n' (Fig. 60) the vertical component is downwards; at s and s' it is upwards; at r and r' there is no vertical component; and at AB the whole pressure is vertical. It can be demonstrated mathematically that the resultant pressure is always equal to the total weight of the contained liquid; a conclusion which can also be deduced from the consideration that the pressure exerted by the vessel upon the stand on which it rests must be equal to its own weight together with that of its contents.

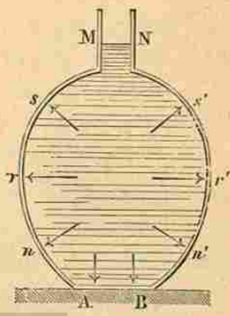


Fig. 60.—Total Pressure.

Some cases in which the proof above indicated becomes especially obvious, are represented in Fig. 61. In the cylindrical vessel ABDC, it is evident that the only pressure transmitted to the stand is that exerted upon the bottom, which is equal to the weight of the liquid. In the case of the vessel which is wider at the top, the stand is subjected to the weight of the liquid column ABSK, which presses on the bottom AB, together with the columns GHKC, RLDS, pressing on GH and RL; the sum of which weights composes the total weight of liquid contained in the vessel. Finally, in the third case, the pressure on the bottom AB, which is equal to the weight of a liquid column ABSK, must be diminished by the

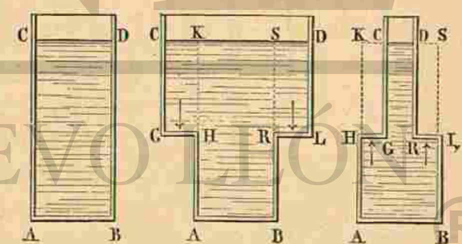


Fig. 61.—Hydrostatic Paradox.

upward pressures on HG and RL. These last being represented by liquid columns HGCK, RLSD, there is only left to be transmitted to the stand a pressure equal to the weight of the water in the vessel.

149. Back Pressure in Discharging Vessel.—The same analysis which shows that the resultant vertical pressure of a liquid against the containing vessel is equal to the weight of the liquid, shows also that the horizontal components of the pressures destroy one another. This conclusion is in accordance with everyday experience. However susceptible a vessel may be of horizontal displacement, it is not found to acquire any tendency to horizontal motion by being filled with a liquid.

When a system of forces are in equilibrium, the removal of one of them destroys the equilibrium, and causes the resultant of the system to be a force equal and opposite to the force removed. Accordingly if we remove an element of one side of the containing vessel, leaving a hole through which the liquid can flow out, the remaining pressure against this side will be insufficient to preserve equilibrium, and there will be an excess of pressure in the opposite direction.

This conclusion can be directly verified by the experiment represented in Fig. 62. A tall floating vessel of water is fitted with a horizontal discharge-pipe on one side near its base. The vessel is to be filled with water, and the discharge-pipe opened while the vessel is at rest. As the water flows out, the vessel will be observed to acquire a velocity, at first very slow, but continually increasing, in the opposite direction to that of the issuing stream.

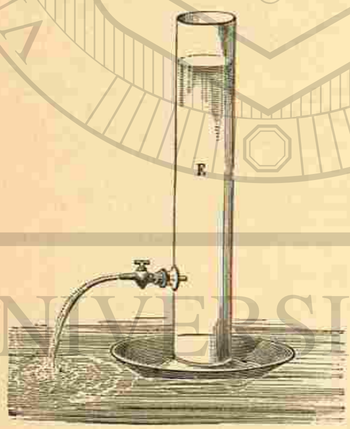


Fig. 62.—Backward Movement of Discharging Vessel.

This experiment may also be regarded as an illustration of the law of action and reaction, which asserts that momentum cannot be imparted to any body without equal and opposite momentum being imparted to some other body. The water in escaping from the vessel acquires horizontal momentum in one direction, and the vessel with its remaining contents acquires horizontal momentum in the opposite direction.

The movements of the vessel in this experiment are slow. More marked effects of the same kind can be obtained by means of the hydraulic tourniquet (Fig. 63), which when made on a larger scale is called Barker's mill. It consists of a vessel of water free to rotate about a vertical axis, and having at its lower end bent arms through which the water is discharged horizontally, the direction of discharge being nearly at right angles to a line joining the discharging orifice to the axis. The unbalanced pressures at the bends of the tube, opposite to the openings, cause the apparatus to revolve in the opposite direction to the issuing liquid.

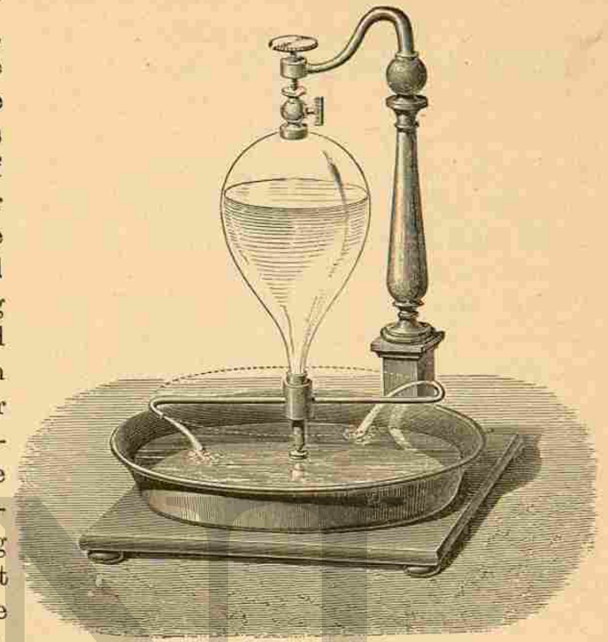


Fig. 63.—Hydraulic Tourniquet.

150. Total and Resultant Pressures. Centre of Pressure.—The intensity of pressure on an area which is not horizontal is greatest on those parts which are deepest, and the average intensity can be shown to be equal to the actual intensity at the centre of gravity of the area. Hence if A denote the area, h the depth of its centre of gravity, and w the weight of unit volume of the liquid, the total pressure will be $w\,Ah$. Strictly speaking, this is the pressure due to the weight of the liquid, the transmitted atmospheric pressure being left out of account.

In attaching numerical values to w, A, and h, the same unit of length must be used throughout. For example, if h be expressed in feet, A must be expressed in square feet, and w must stand for the weight of a cubic foot of the liquid.

When we employ the centimetre as the unit of length, the value of w will be sensibly 1 gramme if the liquid be water, so that the amount of pressure in grammes will be simply the product of the depth of the centre of gravity in centimetres by the area in square centimetres. For any other liquid, the pressure will be found by multiplying this product by the specific gravity of the liquid.

These rules for computing total pressure hold for areas of all forms, whether plane or curved; but the investigation of the total pressure on an area which is not plane is a mere mathematical exercise of no practical importance; for as the elementary pressures in this case are not parallel, their sum (which is the total pressure) is not the same thing as their resultant.

For a plane area, in whatever position, the elementary pressures, being everywhere normal to its plane, are parallel and give a resultant equal to their sum; and it is often a matter of interest to determine that point in the area through which the resultant passes. This point is called the *Centre of Pressure*. It is not coincident with the centre of gravity of the area unless the pressure be of equal intensity over the whole area. When the area is not horizontal, the pressure is most intense at those parts of it which are deepest, and the centre of pressure is accordingly lower down than the centre of gravity. For a horizontal area the two centres are coincident, and they are also sensibly coincident for any plane area whose dimensions are very small in comparison with its depth in the liquid, for the pressure over such an area is sensibly uniform.

151. Construction for Centre of Pressure.—If at every point of a plane area immersed in a liquid, a normal be drawn, equal to the depth of the point, the normals will represent the intensity of pressure at the respective points, and the volume of the solid constituted by all the normals will represent the total pressure. That normal which passes through the centre of gravity of this solid will be the line of action of the resultant, and will therefore pass through the centre of pressure.

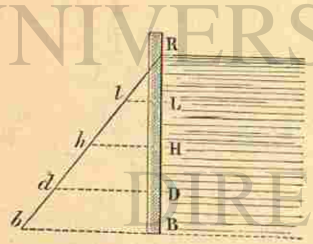


Fig. 64.—Centre of Pressure.

Thus, if RB (Fig. 64) be a rectangular surface (which we may suppose to be the surface of a flood-gate or of the side of a dam), its lower side B being at the bottom of the water and its upper side R at the top, the pressure is zero at R and goes on increasing uniformly to B. The normals Bb, Dd, Hh, Ll, equal to the depths of a series of points in the line BR will have their extremities b, d, h, l, in one straight line. To find the centre of pressure, we must find the centre of gravity of the triangle RBb and draw a normal through it. As the centre of gravity of a triangle is at one-third of its height, the centre of pressure will be at one-third of the height of BR. It will lie on the line joining the middle points of the upper and lower sides of the rectangle, and will be at one-third of the length of this line from its lower end.

The total pressure will be equal to the weight of a quantity of the liquid whose volume is equal to that of the triangular prism constituted by the aggregate of the normals, of which prism the triangle RBb is a right section. It is not difficult to show that the volume of this prism is equal to the product of the area of the rectangle by the depth of the centre of gravity of the rectangle, in accordance with the rule above given.

152. Whirling Vessel. D'Alembert's Principle.—If an open vessel of liquid is rapidly rotated round a vertical axis, the surface of the liquid assumes a concave form, as represented in Fig. 65, where the dotted line is the axis of rotation. When the rotation has been going on at a uniform rate for a sufficient time, the liquid mass rotates bodily as if its particles were rigidly connected together, and when this state of things has been attained the form of the surface is that of a paraboloid of revolution, so that the section represented in the figure is a parabola.

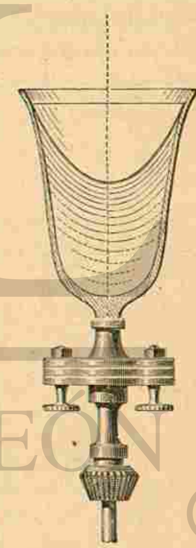


Fig. 65.—Rotating Vessel of Liquid.

We have seen in § 101 that a particle moving uniformly in a circle is acted on by a force directed towards the centre. In the present case, therefore, there must be a force acting upon each particle of the liquid urging it towards the axis. This force is supplied by the pressure of the liquid, which follows the usual law of increase with depth *for all points in the same vertical*. If we draw a horizontal plane in the liquid, the pressure at each point of it is that due to the height of the point of the surface vertically over it. The pressure is therefore least at the point where the plane is cut by the axis, and increases as we recede from this centre. Consequently each particle of liquid receives unequal pressures on two opposite sides, being more strongly pressed towards the axis than from it.

Another mode of discussing the case, is to treat it as one of statical equilibrium under the joint action of gravity and a fictitious force called centrifugal force, the latter force being, for each particle, equal and opposite to that which would produce the actual acceleration of the particle. This so-called centrifugal force is therefore to be regarded as a force directed radially outwards from the axis; and by compounding the centrifugal force of each particle with its weight we shall obtain what we are to treat as the resultant force on that particle. The form of the surface will then be determined by the condition that *at every point of the surface the normal must coincide with this resultant force;* just as in a liquid at rest, the normals must coincide with the direction of gravity.

The plan here adopted of introducing fictitious forces equal and opposite to those which if directly applied to each particle of a system would produce the actual accelerations, and then applying the conditions of statical equilibrium, is one of very frequent application, and will always lead to correct results. This principle was first introduced, or at least systematically expounded, by D'Alembert, and is known as D'Alembert's Principle.

CHAPTER XIII.

PRINCIPLE OF ARCHIMEDES.

153. Pressure of Liquids on Bodies Immersed.—When a body is immersed in a liquid, the different points of its surface are subjected to pressures which obey the rules laid down in the preceding chapter. As these pressures increase with the depth, those which tend to raise the body exceed those which tend to sink it, so that the resultant effect is a force in the direction opposite to that of gravity.

By resolving the pressure on each element into horizontal and vertical components, it can be shown that this resultant upward force is exactly equal to the weight of the liquid displaced by the body.

The reasoning is particularly simple in the case of a right cylinder (Fig. 66) plunged vertically in a liquid. It is evident, in the first place, that if we consider any point on the sides of the cylinder, the normal pressure on that point is horizontal and is destroyed by the equal and contrary pressure at the point diametrically opposite; hence, the horizontal pressures destroy each other. As regards the vertical pressures on the ends, one of them, that on the upper end AB, is in a downward direction, and equal to the weight of the liquid column ABNN; the other, that on the lower end CD, is in an upward direction, and equal to the weight of the liquid column CNND; this latter pressure exceeds the former by the weight of the liquid cylinder ABDC, so that the resultant effect of the pressure is to raise the body with a force equal to the weight of the liquid displaced.

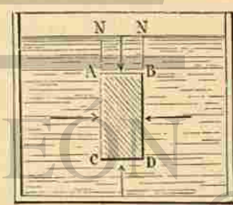


Fig. 66.—Principle of Archimedes.

Another mode of discussing the case, is to treat it as one of statical equilibrium under the joint action of gravity and a fictitious force called centrifugal force, the latter force being, for each particle, equal and opposite to that which would produce the actual acceleration of the particle. This so-called centrifugal force is therefore to be regarded as a force directed radially outwards from the axis; and by compounding the centrifugal force of each particle with its weight we shall obtain what we are to treat as the resultant force on that particle. The form of the surface will then be determined by the condition that *at every point of the surface the normal must coincide with this resultant force;* just as in a liquid at rest, the normals must coincide with the direction of gravity.

The plan here adopted of introducing fictitious forces equal and opposite to those which if directly applied to each particle of a system would produce the actual accelerations, and then applying the conditions of statical equilibrium, is one of very frequent application, and will always lead to correct results. This principle was first introduced, or at least systematically expounded, by D'Alembert, and is known as D'Alembert's Principle.

CHAPTER XIII.

PRINCIPLE OF ARCHIMEDES.

153. Pressure of Liquids on Bodies Immersed.—When a body is immersed in a liquid, the different points of its surface are subjected to pressures which obey the rules laid down in the preceding chapter. As these pressures increase with the depth, those which tend to raise the body exceed those which tend to sink it, so that the resultant effect is a force in the direction opposite to that of gravity.

By resolving the pressure on each element into horizontal and vertical components, it can be shown that this resultant upward force is exactly equal to the weight of the liquid displaced by the body.

The reasoning is particularly simple in the case of a right cylinder (Fig. 66) plunged vertically in a liquid. It is evident, in the first place, that if we consider any point on the sides of the cylinder, the normal pressure on that point is horizontal and is destroyed by the equal and contrary pressure at the point diametrically opposite; hence, the horizontal pressures destroy each other. As regards the vertical pressures on the ends, one of them, that on the upper end AB, is in a downward direction, and equal to the weight of the liquid column ABNN; the other, that on the lower end CD, is in an upward direction, and equal to the weight of the liquid column CNND; this latter pressure exceeds the former by the weight of the liquid cylinder ABDC, so that the resultant effect of the pressure is to raise the body with a force equal to the weight of the liquid displaced.

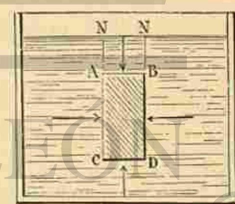


Fig. 66.—Principle of Archimedes.

By a synthetic process of reasoning, we may, without having recourse to the analysis of the different pressures, show that this conclusion is perfectly general. Suppose we have a liquid mass in equilibrium, and that we consider specially the portion M (Fig. 67); this portion is likewise in equilibrium. If we suppose it to become solid, without any change in its weight or volume, equilibrium will still subsist. Now this is a heavy mass, and as it does not fall, we must conclude that the effect of the pressures on its surface is to produce a resultant upward pressure exactly equal to its weight, and acting in a line which passes through its centre of gravity. If we now suppose M replaced by a body exactly occupying its place, the exterior pressures will remain the same, and their resultant effect will therefore be the same.

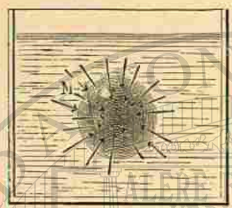


Fig. 67.—Principle of Archimedes.

The name *centre of buoyancy* is given to the centre of gravity of the liquid displaced,—that is, if the liquid be uniform, to the centre of gravity of the space occupied by the immersed body; and the above reasoning shows that the resultant pressure acts vertically upwards in a line which passes through this point. The results of the above explanations may thus be included in the following proposition: *Every body immersed in a liquid is subjected to a resultant pressure equal to the weight of the liquid displaced, and acting vertically upwards through the centre of buoyancy.*

This proposition constitutes the celebrated principle of Archimedes. The first part of it is often enunciated in the following form: *Every body immersed in a liquid loses a portion of its weight equal to the weight of the liquid displaced;* for when a body is immersed in a liquid, the force required to sustain it will evidently be diminished by a quantity equal to the upward pressure.

154. Experimental Demonstration of the Principle of Archimedes.—The following experimental demonstration of the principle of Archimedes is commonly exhibited in courses of physics:—

From one of the scales of a hydrostatic balance (Fig. 68) is suspended a hollow cylinder of brass, and below this a solid cylinder, whose volume is equal to the interior volume of the hollow cylinder; these are balanced by weights in the other scale. A vessel of water is then placed below the cylinders, in such a position that the lower cylinder shall be immersed in it. The equilibrium is immediately

destroyed, and the upward pressure of the water causes the scale with the weights to descend. If we now pour water into the hollow cylinder, equilibrium will gradually be re-established; and the beam

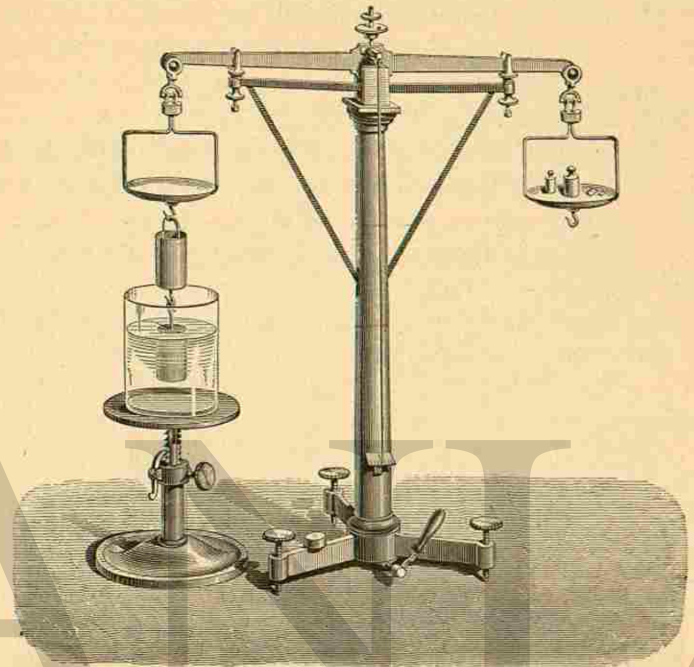


Fig. 68.—Experimental Verification of Principle of Archimedes.

will be observed to resume its horizontal position when the hollow cylinder is full of water, the other cylinder being at the same time completely immersed. The upward pressure upon this latter is thus equal to the weight of the water added, that is, to the weight of the liquid displaced.

155. Body Immersed in a Liquid.—It follows from the principle of Archimedes that when a body is immersed in a liquid, it is subjected to two forces: one equal to its weight and applied at its centre of gravity, tending to make the body descend; the other equal to the weight of the displaced liquid, applied at the centre of buoyancy, and tending to make it rise. There are thus three different cases to be considered:

(1.) The weight of the body may exceed the weight of the liquid displaced, or, in other words, the mean density of the body may be greater than that of the liquid; in this case, the body sinks in the liquid, as, for instance, a piece of lead dropped into water.

(2.) The weight of the body may be less than that of the liquid displaced; in this case the body will not remain submerged unless forcibly held down, but will rise partly out of the liquid, until the weight of the liquid displaced is equal to its own weight. This is what happens, for instance, if we immerse a piece of cork in water and leave it to itself.

(3.) The weight of the body may be equal to the weight of the liquid displaced; in this case, the two opposite forces being equal, the body takes a suitable position and remains in equilibrium.

These three cases are exemplified in the three following experiments (Fig. 69):—

(1.) An egg is placed in a vessel of water; it sinks to the bottom

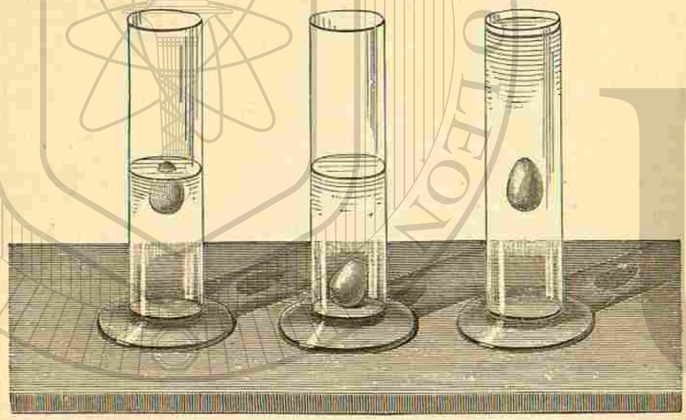


Fig. 69.—Egg Plunged in Fresh and Salt Water.

of the vessel, its mean density being a little greater than that of the liquid.

(2.) Instead of fresh water, salt water is employed; the egg floats at the surface of the liquid, which is a little denser than it.

(3.) Fresh water is carefully poured on the salt water; a mixture of the two liquids takes place where they are in contact; and if the egg is put in the upper part, it will be seen to descend, and, after a few oscillations, remain at rest at such a depth that it displaces its own weight of the liquid. In speaking of the liquid displaced in this case, we must imagine each horizontal layer of liquid surrounding the egg to be produced through the space which the egg occupies; and by the centre of buoyancy we must understand the centre of

gravity of the portion of liquid which would thus take the place of the egg. We may remark that, in this position the egg is in stable equilibrium; for, if it rises, the upward pressure diminishing, its weight tends to make it descend again; if, on the contrary, it sinks, the pressure increases and tends to make it reascend.

156. Cartesian Diver.—The experiment of the *Cartesian diver*, which is described in old treatises on physics, shows each of the different cases that can present themselves when a body is immersed. The diver (Fig. 70) consists of a hollow ball, at the bottom of which is a small opening O; a little porcelain figure is attached to the ball, and the whole floats upon water contained in a glass vessel, the mouth of which is closed by a strip of caoutchouc or a bladder. If we press with the hand on the bladder, the air is compressed, and the pressure, transmitted through the different horizontal layers, condenses the air in the ball, and causes the entrance of a portion of the liquid by the opening O; the floating body becomes heavier, and in consequence of this increase of weight the diver descends. When we cease to press upon the bladder, the pressure becomes what it was before, some water flows out and the diver ascends. It must be observed, however, that as the diver continues to descend, more and more water enters the ball, in consequence of the increase of pressure, so that if the depth of the water exceeded a certain limit, the diver would not be able to rise again from the bottom.

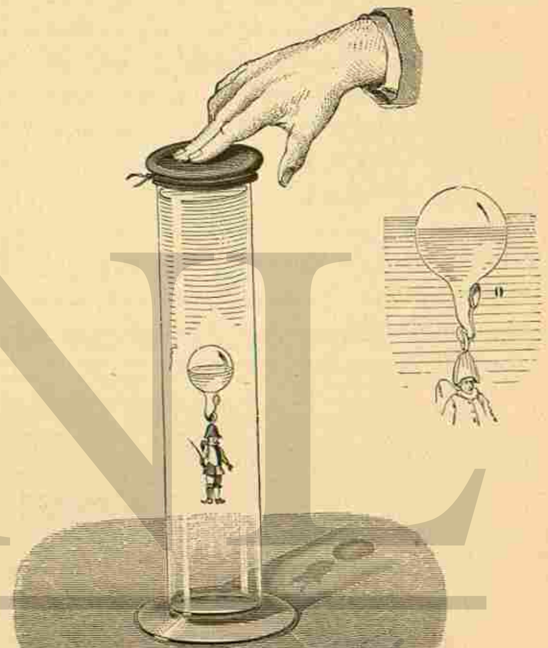


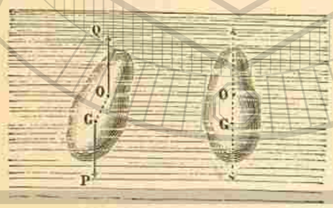
Fig. 70.—Cartesian Diver.

If we suppose that at a certain moment the weight of the diver becomes exactly equal to the weight of an equal volume of the liquid, there will be equilibrium; but, unlike the equilibrium in the experiment (3) of last section, this will evidently be *unstable*, for a slight movement either upwards or downwards will alter the resultant force so as to produce further movement in the same direction. As a consequence of this instability, if the diver is sent down below a certain depth he will not be able to rise again.

157. Relative Positions of the Centre of Gravity and Centre of Buoyancy.—In order that a floating body either wholly or partially immersed in a liquid, may be in equilibrium, it is necessary that its weight be equal to the weight of the liquid displaced.

This condition is however not sufficient; we require, in addition, that the action of the upward pressure should be exactly opposite to that of the weight; that is, that the centre of gravity and the centre of buoyancy be in the same vertical line; for if this were not the case, the two contrary forces would compose a couple, the effect of which would evidently be to cause the body to turn.

In the case of a body completely immersed, it is further necessary for stable equilibrium that *the centre of gravity should be below* the centre of buoyancy; in fact we see, by Fig. 71, that in any other



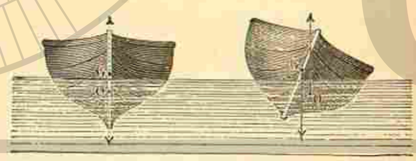


Fig. 71. Fig. 72.
Relative Positions of Centre of Gravity and Centre of Pressure.

position than that of equilibrium, the effect of the two forces applied at the two points G and O would be to turn the body, so as to bring the centre of gravity lower, relatively to the centre of buoyancy. But this is not the case when the body is only partially immersed, as most frequently happens. In this case it may indeed happen that, with stable equilibrium, the centre of gravity is below the centre of pressure; but this is not necessary, and in the majority of instances is not the case. Let Fig. 72 represent the lower part of a floating body—a boat, for instance. The centre of pressure is at O, the centre of gravity at G, considerably above; if the body

is displaced, and takes the position shown in the figure, it will be seen that the effect of the two forces acting at O and at G is to restore the body to its former position. This difference from what takes place when the body is completely immersed, depends upon the fact that, in the case of the floating body, the figure of the liquid displaced changes with the position of the body, and the centre of buoyancy moves towards the side on which the body is more deeply immersed. It will depend upon the form of the body whether this lateral movement of the centre of buoyancy is sufficient to carry it beyond the vertical through the centre of gravity. The two equal forces which act on the body will evidently turn it to or from the original position of equilibrium, according as the new centre of buoyancy lies beyond or falls short of this vertical.[1]

158. Advantage of Lowering the Centre of Gravity.—Although stable equilibrium may subsist with the centre of gravity above the centre of buoyancy, yet for a body of given external form the stability is always increased by lowering the centre of gravity; as we thus lengthen the arm of the couple which tends to right the body when displaced. It is on this principle that the use of ballast depends.

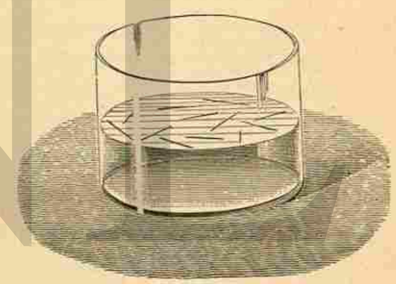


Fig. 73.—Steel Needles Floating on Water.

159. Phenomena in Apparent Contradiction to the Principle of Archimedes.—The principle of Archimedes seems at first sight to be contradicted by some well-known facts. Thus, for instance, if small needles are placed carefully on the surface of water, they will remain there in equilibrium (Fig. 73). It is on a similar principle

[1] If a vertical through the new centre of buoyancy be drawn upwards to meet that line in the body which in the position of equilibrium was a vertical through the centre of gravity, the point of intersection is called the *metacentre*. Evidently when the forces tend to restore the body to the position of equilibrium, the metacentre is above the centre of gravity; when they tend to increase the displacement, it is below. In ships the distance between these two points is usually nearly the same for all amounts of heeling, and this distance is a measure of the stability of the ship.

We have defined the metacentre as the intersection of two lines. When these lines lie in different planes, and do not intersect each other, there is no metacentre. This indeed is the case for most of the displacements to which a floating body of irregular shape can be subjected. There are in general only two directions of heeling to which metacentres correspond, and these two directions are at right angles to each other.

that several insects *walk* on water (Fig. 74), and that a great number of bodies of various natures, provided they be *very minute*, can, if we may so say, be placed on the surface of a liquid without penetrating into its interior. These curious facts depend on the circumstance that the small bodies in question are not wetted by the liquid, and hence, in virtue of principles which will be explained in connection with capillarity (Chap. xvi.), depressions are formed around them on the liquid surface, as represented in Fig. 75. The curvature of the liquid surface in the neighbourhood of the body is very distinctly shown by observing the shadow cast by the floating body, when it is illumined by the sun; it is seen to be bordered by luminous bands, which are owing to the refraction of the rays of light in the portion of the liquid bounded by a curved surface.

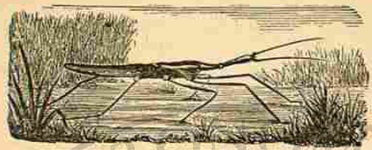


Fig. 74.—Insect Walking on Water.

The existence of the depression about the floating body enables us to bring the condition of equilibrium in this special case under the general enunciation of the principle of Archimedes. Let M (Fig. 75) be the body, CD the region of the depression, and AB the corresponding portion of any horizontal layer; since the pressure at each point of AB must be the same as in other parts of the same horizontal layer, the total weight above AB is the same as if M did not exist and the cavity were filled with the liquid itself.

Fig. 75.

We may thus say in this case also that the weight of the floating body is equal to the weight of the *liquid displaced*, understanding by these words the liquid which would occupy the whole of the depression due to the presence of the body.

CHAPTER XIV.

DENSITY AND ITS DETERMINATION.

160. Definitions.—By the *absolute density* of a substance is meant the mass of unit volume of it. By the *relative density* is meant the ratio of its absolute density to that of some standard substance, or, what amounts to the same thing, the ratio of the mass of any volume of the substance in question to the mass of an equal volume of the standard substance. Since equal masses gravitate equally, the comparison of masses can be effected by weighing, and the relative density of a substance is the ratio of its weight to that of an equal volume of the standard substance. Water at a specified temperature and under atmospheric pressure is usually taken as the standard substance, and the density of a substance relative to water is usually called the *specific gravity* of the substance.

Let V denote the volume of a substance, M its mass, and D its absolute density; then by definition, we have $M=VD$.

If s denote the specific gravity of a substance, and d the absolute density of water in the standard condition, then $D=sd$ and $M=Vsd$.

When masses are expressed in lbs. and volumes in cubic feet, the value of d is about 62·4, since a cubic foot of cold water weighs about 62·4 lbs.[1]

In the C.G.S. system, the value of d is sensibly unity, since a cubic centimetre of water, at a temperature which is nearly that of the maximum density of water, weighs exactly a gramme.[2]

The gramme is defined, not by reference to water, but by a standard kilogramme of platinum, which is preserved in Paris, and

[1] In round numbers, a cubic foot of water weighs 1000 oz., which is 62·5 lbs.

[2] According to the best determination yet published, the mass of a cubic centimetre of pure water at 4° is 1·000013, at 3° is 1·000004, and at 2° is ·999982.

that several insects *walk* on water (Fig. 74), and that a great number of bodies of various natures, provided they be *very minute*, can, if we may so say, be placed on the surface of a liquid without penetrating into its interior. These curious facts depend on the circumstance that the small bodies in question are not wetted by the liquid, and hence, in virtue of principles which will be explained in connection with capillarity (Chap. xvi.), depressions are formed around them on the liquid surface, as represented in Fig. 75. The curvature of the liquid surface in the neighbourhood of the body is very distinctly shown by observing the shadow cast by the floating body, when it is illumined by the sun; it is seen to be bordered by luminous bands, which are owing to the refraction of the rays of light in the portion of the liquid bounded by a curved surface.

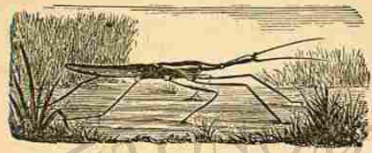


Fig. 74.—Insect Walking on Water.

The existence of the depression about the floating body enables us to bring the condition of equilibrium in this special case under the general enunciation of the principle of Archimedes. Let M (Fig. 75) be the body, CD the region of the depression, and AB the corresponding portion of any horizontal layer; since the pressure at each point of AB must be the same as in other parts of the same horizontal layer, the total weight above AB is the same as if M did not exist and the cavity were filled with the liquid itself.

Fig. 75.

We may thus say in this case also that the weight of the floating body is equal to the weight of the *liquid displaced*, understanding by these words the liquid which would occupy the whole of the depression due to the presence of the body.

CHAPTER XIV.

DENSITY AND ITS DETERMINATION.

160. Definitions.—By the *absolute density* of a substance is meant the mass of unit volume of it. By the *relative density* is meant the ratio of its absolute density to that of some standard substance, or, what amounts to the same thing, the ratio of the mass of any volume of the substance in question to the mass of an equal volume of the standard substance. Since equal masses gravitate equally, the comparison of masses can be effected by weighing, and the relative density of a substance is the ratio of its weight to that of an equal volume of the standard substance. Water at a specified temperature and under atmospheric pressure is usually taken as the standard substance, and the density of a substance relative to water is usually called the *specific gravity* of the substance.

Let V denote the volume of a substance, M its mass, and D its absolute density; then by definition, we have $M=VD$.

If s denote the specific gravity of a substance, and d the absolute density of water in the standard condition, then $D=sd$ and $M=Vsd$.

When masses are expressed in lbs. and volumes in cubic feet, the value of d is about 62·4, since a cubic foot of cold water weighs about 62·4 lbs.[1]

In the C.G.S. system, the value of d is sensibly unity, since a cubic centimetre of water, at a temperature which is nearly that of the maximum density of water, weighs exactly a gramme.[2]

The gramme is defined, not by reference to water, but by a standard kilogramme of platinum, which is preserved in Paris, and

[1] In round numbers, a cubic foot of water weighs 1000 oz., which is 62·5 lbs.

[2] According to the best determination yet published, the mass of a cubic centimetre of pure water at 4° is 1·000013, at 3° is 1·000004, and at 2° is ·999982.

of which several very carefully made copies are preserved in other places. In the above statements (as in all very accurate statements of weights), the weighings are supposed to be made in vacuo; for the masses of two bodies are not accurately proportional to their apparent gravitations in air, unless the two bodies happen to have the same density.

161. Ambiguity of the word "Weight."—Properly speaking, "the weight of a body" means the force with which the body gravitates towards the earth. This force, as we have seen, differs slightly according to the place of observation. If m denote the mass of the body, and g the intensity of gravity at the place, the weight of the body is mg. When the body is carried from one place to another without gain or loss of material, m will remain constant and g will vary; hence the weight mg will vary, and in the same ratio as g.

But the employment of gravitation units of force instead of absolute units, obscures this fact. The unit of measurement varies in the same ratio as the thing to be measured, and hence the numerical value remains unaltered. A body weighs the same number of pounds or grammes at one place as at another, because the weights of the pound and gramme are themselves proportional to g. Expressed in absolute units, the weight of unit mass is g, and the weight of a mass m is mg. The latter is m times the former; hence when the weight of unit mass is employed as the unit of weight, the same number m which denotes the mass of a body also denotes its weight. What are usually called standard weights—that is, standard pieces of metal used for weighing—are really standards of mass; and when the result of a weighing is stated in terms of these standards, (as it usually is,) the "weight," as thus stated, is really the *mass* of the body weighed. The standard "weights" which we use in our balances are really standard masses. In discussions relating to density, weights are most conveniently expressed in gravitation measure, and hence the words mass and weight can be used almost indiscriminately.

162. Determination of Density from Weight and Volume.—The absolute density of a substance can be directly determined by weighing a measured volume of it. Thus if v cubic centimetres of it weigh m grammes, its density (in grammes per cubic centimetre) is $\frac{m}{v}$. This method can be easily applied to solids of regular geometrical forms; since their volumes can be computed from their

linear measurements. It can also be applied to liquids, by employing a vessel of known content. The bottle usually employed for this purpose is a bottle of thin glass fitted with a perforated stopper, so that it can be filled and stoppered without leaving a space for air. The difference between its weights when full and empty is the weight of the liquid which fills it; and the quotient of this by the volume occupied (which can be determined once for all by weighing the bottle when filled with water) is the density of the liquid.

The advantage of employing a perforated stopper is that it enables us to ensure constancy of volume. If a wide-mouthed flask were employed, without a stopper, it would be difficult to pronounce when the flask was exactly full. This source of inaccuracy would be diminished by making the mouth narrower: but when it is very narrow, the filling and emptying of the flask are difficult, and there is danger of forcing in bubbles of air with the liquid. When a perforated stopper is employed, the flask is first filled, then the stopper is inserted and some of the liquid is thus forced up through the perforation, overflowing at the top. When the stopper has been pushed home, all the liquid outside is carefully wiped off, and the liquid which remains is as much as just fills the stoppered flask including the perforation in the stopper.

In accurate work, the temperature must be observed, and due allowance made for its effect upon volume.

163. Specific Gravity Flask for Solids.—The volume and density of a solid body of irregular shape, or consisting of a quantity of small pieces, can be determined by putting it into such a bottle (Fig. 76), and weighing the water which it displaces. The most convenient way of doing this is to observe (1) the weight of the solid; (2) the weight of the bottle full of water; (3) the weight of the bottle when it contains the solid, together with as much water as will fill it up. If the

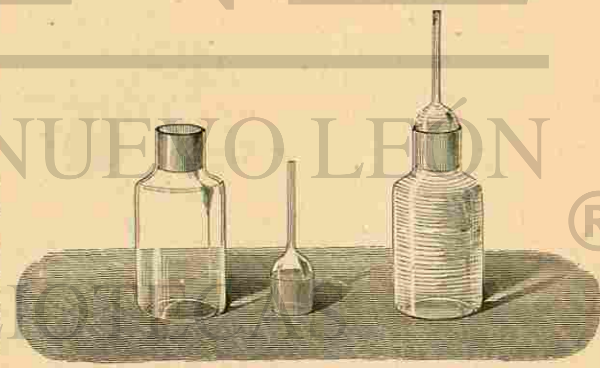


Fig. 76.—Specific Gravity Flask for Solids.

third of these results be subtracted from the sum of the first two, the remainder will be the weight of the water displaced; which, when expressed in grammes, is the same as the volume of the body expressed in cubic centimetres. The weight of the body, divided by this remainder, is the density of the body.

164. Method by Weighing in Water.—The methods of determining density which we are now about to describe depend upon the principle of Archimedes.

One of the commonest ways of determining the density of a solid body is to weigh it first in air and then in water (Fig. 77) the

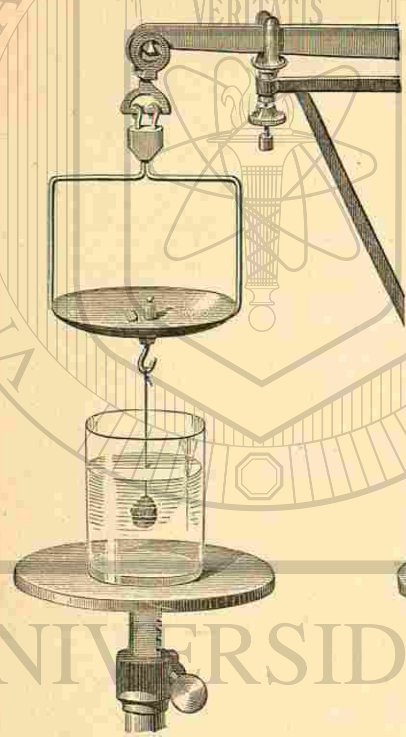


Fig. 77.—Specific Gravity of Solids.

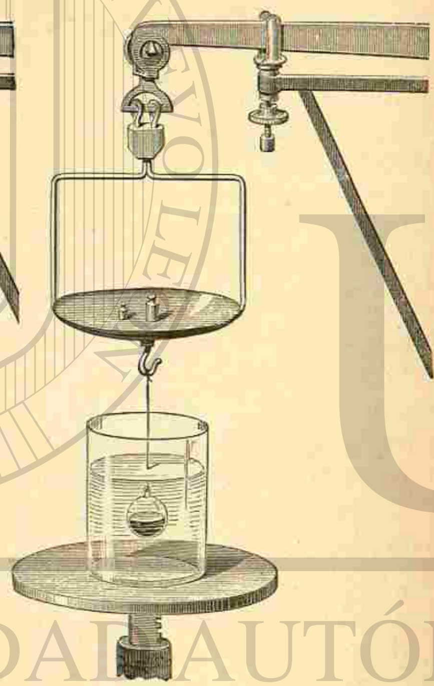


Fig. 78.—Specific Gravity of Liquids.

counterpoising weights being in air. Since the loss of weight due to its immersion in water is equal to the weight of the same volume of water, we have only to *divide the weight in air by this loss of weight*. We shall thus obtain the relative density of the body as compared with water—in other words, the specific gravity of the body.

Thus, from the observations

Weight in air,	125 *gm.*
Weight in water,	100 „
Loss of weight,	25 „

we deduce

$$\frac{125}{25} = 5 = \text{density.}$$

A very fine and strong thread or fibre should be employed for suspending the body, so that the volume of liquid displaced by this thread may be as small as possible.

165. Weighing in Water, with a Sinker.—If the body is lighter than water, we may employ a sinker—that is, a piece of some heavy material attached to it, and heavy enough to make it sink. It is not necessary to know the weight of the sinker in air, but we must observe its weight in water. Call this s. Let w be the weight of the body in air, and w' the weight of the body and sinker together in water. Then w' will be less than s. The body has an apparent upward gravitation in water equal to $s-w'$, showing that the resultant pressure upon it exceeds its weight by this amount. Hence the weight of the liquid displaced is $w+s-w'$, and the specific gravity of the body is $\frac{w}{w+s-w'}$.

If any other liquid than water be employed in the methods described in this and the preceding section, the result obtained will be the relative density as compared with that liquid. The result must therefore be multiplied by the density of the liquid, in order to obtain the absolute density.

166. Density of Liquid Inferred from Loss of Weight.—The densities of liquids are often determined by observing the loss of weight of a solid immersed in them, and dividing by the known volume of the solid or by its loss of weight in water.

Thus, from the observations

Weight in air,	200 *gm*
Weight in liquid,	120 „
Weight in water,	110 „

we deduce

Loss in liquid, 80. Loss in water, 90.

$$\text{Density of liquid, } \frac{80}{90} = \frac{8}{9}.$$

A glass ball (sometimes weighted with mercury, as in Fig. 78) is the solid most frequently employed for such observations.

167. Measurement of Volumes of Solids by Loss of Weight.—The volume of a solid body, especially if of irregular shape, can usually be determined with more accuracy by weighing it in a liquid than by any other method. If it weigh w grammes in air, and w' grammes in water, its volume is $w-w'$ cubic centimetres, since it displaces $w-w'$ grammes of water. The mean diameter of a wire can be very accurately determined by an observation of this kind for volume, combined with a direct measurement of length. The volume divided by the length will be the mean sectional area, which is equal to πr^2, where r is the radius.

168. Hydrometers.—The name hydrometer is given to a class of instruments used for determining the densities of liquids by observing either the depths to which they sink in the liquids or the

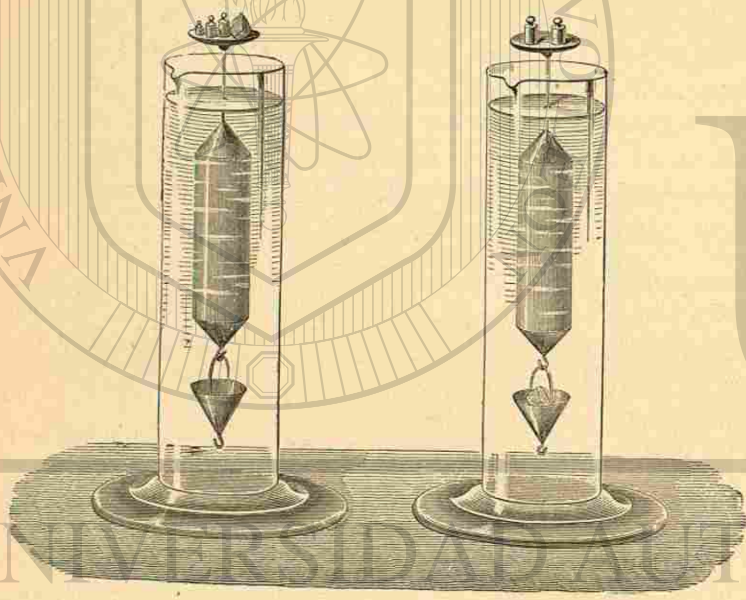


Fig. 79.—Nicholson's Hydrometer.

weights required to be attached to them to make them sink to a given depth. According as they are to be used in the latter or the former of these two ways, they are called hydrometers of constant or of variable immersion. The name areometer (from ἀραιός, rare) is used as synonymous with hydrometer, being probably borrowed from the French name of these instruments, *aréomètre.* The hydrometers of constant immersion most generally known are those of Nicholson and Fahrenheit.

169. Nicholson's Hydrometer.—This instrument, which is represented in Fig. 79, consists of a hollow cylinder of metal with conical ends, terminated above by a very thin rod bearing a small dish, and carrying at its lower end a kind of basket. This latter is of such weight that when the instrument is immersed in water a weight of 100 grammes must be placed in the dish above in order to sink the apparatus as far as a certain mark on the rod. By the principle of Archimedes, the weight of the instrument, together with the 100 grammes which it carries, is equal to the weight of the water displaced. Now, let the instrument be placed in another liquid, and the weights in the dish above be altered until they are just sufficient to make the instrument sink to the mark on the rod. If the weights in the dish be called w, and the weight of the instrument itself W, the weight of liquid displaced is now $W + w$, whereas the weight of the same volume of water was $W + 100$; hence the specific gravity of the liquid is $\frac{W + w}{W + 100}$.

This instrument can also be used either for weighing small solid bodies or for finding their specific gravities. To find the weight of a body (which we shall suppose to weigh less than 100 grammes), it must be placed in the dish at the top, together with weights just sufficient to make the instrument sink in water as far as the mark. Obviously these weights are the difference between the weight of the body and 100 grammes.

To find the specific gravity of a solid, we first ascertain its weight by the method just described; we then transfer it from the dish above to the basket below, so that it shall be under water during the observation, and observe what additional weights must now be placed in the dish. These additional weights represent the weight of the water displaced by the solid; and the weight of the solid itself divided by this weight is the specific gravity required.

170. Fahrenheit's Hydrometer.—This instrument, which is represented in Fig. 80, is generally constructed of glass, and differs from Nicholson's in having at its lower extremity a ball weighted with mercury instead of the basket. It resembles it in having a dish at the top, in which weights are to be placed sufficient to sink the instrument to a definite mark on the stem.

Hydrometers of constant immersion, though still described in text-books, have quite gone out of use for practical work.

171. Hydrometers of Variable Immersion.—These instruments are usually of the forms represented at A, B, C, Fig. 81. The lower end is weighted with mercury in order to make the instrument sink to a convenient depth and preserve an upright position. The stem is cylindrical, and is graduated, the divisions being frequently marked

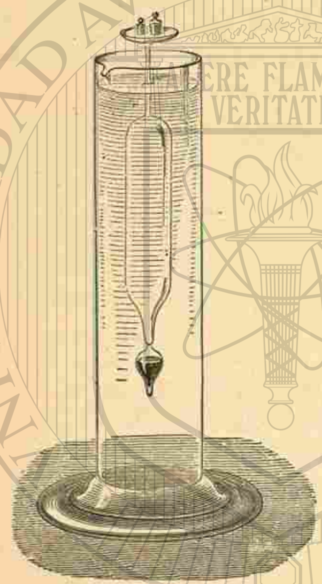


Fig. 80.—Fahrenheit's Hydrometer.

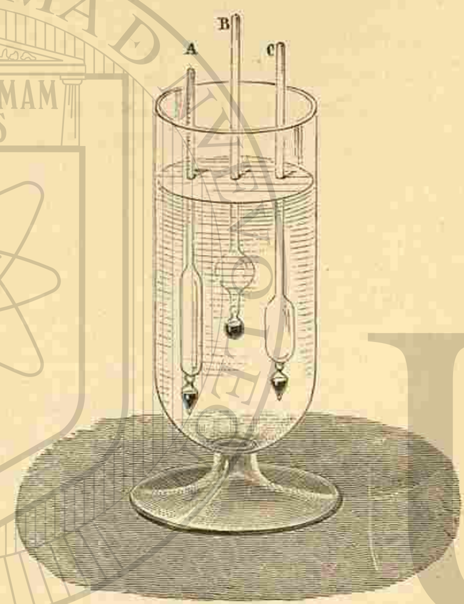


Fig. 81.—Forms of Hydrometers.

upon a piece of paper inclosed within the stem, which must in this case be of glass. It is evident that the instrument will sink the deeper the less is the specific gravity of the liquid, since the weight of the liquid displaced must be equal to that of the instrument. Hence if any uniform system of graduation be adopted, so that all the instruments give the same readings in liquids of the same densities, the density of a liquid can be obtained by a mere immersion of the hydrometer—an operation not indeed very precise, but very easy of execution. These instruments have thus come into general use for commercial purposes and in the excise.

172. General Theory of Hydrometers of Variable Immersion.—Let V be the volume of a hydrometer which is immersed when the instrument floats freely in a liquid whose density is d, then Vd represents the weight of liquid displaced, which by the principle of Archimedes is the same as the weight of the hydrometer itself. If V', d' be the corresponding values for another liquid, we have therefore

$$Vd = V'd', \text{ or } d : d' :: V' : V,$$

that is, the density varies inversely as the volume immersed. Let d_1, d_2, d_3...be a series of densities, and V_1, V_2, V_3...the corresponding volumes immersed, then we have

$$d_1, d_2, d_3 \ldots \text{ proportional to } \frac{1}{V_1}, \frac{1}{V_2}, \frac{1}{V_3} \ldots$$

$$\text{and } V_1, V_2, V_3 \ldots \text{ proportional to } \frac{1}{d_1}, \frac{1}{d_2}, \frac{1}{d_3} \ldots$$

Hence, if we wish the divisions to indicate equal differences of density, we must place them so that the corresponding volumes immersed form a harmonical progression. This implies that the distances between the divisions must diminish as the densities increase.

The following investigation shows how the density of a liquid may be computed from observations made with a hydrometer graduated with equal divisions. It is necessary first to know the divisions to which the instrument sinks in two liquids of known densities. Let these divisions be numbered n_1, n_2, reckoning from the top downwards, and let the corresponding densities be d_1, d_2. Now if we take for our unit of volume one of the equal parts on the stem, and if we take c to denote the volume which is immersed when the instrument sinks to the division marked zero, it is obvious that when the instrument sinks to the nth division (reckoned downwards on the stem from zero) the volume immersed is $c-n$, and if the corresponding density be called d, then $(c-n)\,d$ is the weight of the hydrometer. We have therefore

$$(c-n_1)\,d_1 = (c-n_2)\,d_2, \text{ whence } c = \frac{n_1 d_1 - n_2 d_2}{d_1 - d_2}.$$

This value of c can be computed once for all.

Then the density D corresponding to any other division N can be found from the equation

$$(c-N)\,D = (c-n_1)\,d_1 \text{ which gives } D = \frac{c-n_1}{c-N} d_1.$$

173. Beaumé's Hydrometers.—In these instruments the divisions are equidistant. There are two distinct modes of graduation, according as the instrument is to be used for determining densities greater or less than that of water. In the former case the instrument is called a salimeter, and is so constructed that when immersed in pure water of the temperature 12° Cent. it sinks nearly to the top of the stem, and the point thus determined is the zero of the scale. It is then immersed in a solution of 15 parts of salt to 85 of water, the density of which is about 1·116, and the point to which it sinks is marked 15. The interval is divided into 15 equal parts, and the graduation is continued to the bottom of the stem, the length of which varies according to circumstances; it generally terminates at the degree 66, which corresponds to sulphuric acid, whose density is commonly the greatest that it is required to determine. Referring to the formulæ of last section, we have here

Fig. 82. Baumé's Salimeter.

$$n_1=o,\ d_1=1,\ n_2=15,\ d_2=1·116;$$

whence

$$c=\frac{15\times 1·116}{·116}=144,\ \mathrm{D}=\frac{144}{144-\mathrm{N}}.$$

When the instrument is intended for liquids lighter than water, it is called an alcoholimeter. In this case the point to which it sinks in water is near the bottom of the stem, and is marked 10; the zero of the scale is the point to which it sinks in a solution of 10 parts of salt to 90 of water, the density of which is about 1·085, the divisions in this case being numbered upward from zero.

Fig. 83. Baumé's Fig 84. Alcoholimeters.

In order to adapt the formulæ of last section to the case of graduations numbered upwards, it is merely necessary to reverse the signs of n_1, n_2, and N; that is we must put

$$c=\frac{n_2d_2-n_1d_1}{d_1-d_2},\ \mathrm{D}=\frac{c+n_1}{c+\mathrm{N}}d_1;$$

and as we have now $n_1=10$, $d_1=1$, $n_2=0$, $d_2=1·085$ the formulæ give[1]

$$c=\frac{10}{·085}=118,\ \mathrm{D}=\frac{128}{118+\mathrm{N}}.$$

174. Twaddell's Hydrometer.—In this instrument the divisions are

[1] On comparing the two formulæ for D in this section with the tables in the Appendix to Miller's *Chemical Physics*, I find that as regards the salimeter they agree to two places of decimals and very nearly to three. As regards the alcoholimeter, the table in *Miller* implies that c is about 136, which would make the density corresponding to the zero of the scale about 1·074.

placed not as in Beaumé's, at equal distances, but at distances corresponding to equal differences of density. In fact the specific gravity of a liquid is found by multiplying the reading by 5, cutting off three decimal places, and prefixing unity. Thus the degree 1 indicates specific gravity 1·005, 2 indicates 1·010, &c.

175. Gay-Lussac's Centesimal Alcoholimeter.—When a hydrometer is to be used for a special purpose, it may be convenient to adopt a mode of graduation different in principle from any that we have described above, and adapted to give a direct indication of the proportion in which two ingredients are mixed in the fluid to be examined. It may indicate, for example, the quantity of salt in sea-water, or the quantity of alcohol in a spirit consisting of alcohol and water. Where there are three or more ingredients of different specific gravities the method fails. Gay-Lussac's alcoholimeter is graduated to indicate, at the temperature of 15° Cent., the percentage of pure alcohol in a specimen of spirit. At the top of the stem is 100, the point to which the instrument sinks in pure alcohol, and at the bottom is 0, to which it sinks in water. The position of the intermediate degrees must be determined empirically, by placing the instrument in mixtures of alcohol and water in known proportions, at the temperature of 15°. The law of density, as depending on the proportion of alcohol present, is complicated by the fact that, when alcohol is mixed with water, a diminution of volume (accompanied by rise of temperature) takes place.

Fig. 85. Centesimal Alcoholimeter.

176. Specific Gravity of Mixtures.—When two or more substances are mixed without either shrinkage or expansion (that is, when the volume of the mixture is equal to the sum of the volumes of the components), the density of the mixture can easily be expressed in terms of the quantities and densities of the components.

First, let the volumes $v_1, v_2, v_3 \ldots$ of the components be given, together with their densities $d_1, d_2, d_3 \ldots$

Then their masses (or weights) are $v_1d_1, v_2d_2, v_3d_3 \ldots$

The mass of the mixture is the sum of these masses, and its volume is the sum of the volumes $v_1, v_2, v_3 \ldots$; hence its density is

$$\frac{v_1d_1+v_2d_2+\ldots}{v_1+v_2+\ldots}.$$

Secondly, let the weights or masses $m_1, m_2, m_3 \ldots$ of the components be given, together with their densities $d_1, d_2, d_3 \ldots$

Then their volumes are $\frac{m_1}{d_1}, \frac{m_2}{d_2}, \frac{m_3}{d_3} \dots$

The volume of the mixture is the sum of these volumes, and its mass is $m_1 + m_2 + m_3 + \dots$; hence its density is

$$\frac{m_1 + m_2 + \dots}{\frac{m_1}{d_1} + \frac{m_2}{d_2} + \dots}$$

177. Graphical Method of Graduation.—When the points on the stem which correspond to some five or six known densities, nearly equidifferent, have been determined, the intermediate graduations can be inserted with tolerable accuracy by the graphical method of interpolation, a method which has many applications in physics besides that which we are now considering. Suppose A and B (Fig. 86) to represent the extreme points, and I, K, L, R intermediate points, all of which correspond to known densities. Erect ordinates (that is to say, perpendiculars) at these points, proportional to the respective densities, or (which will serve our purpose equally well) erect ordinates II′, KK′, LL′, RR′, BC proportional to the excesses of the densities at I, K, L, R, B above the density at A. Any scale of equal parts can be employed for

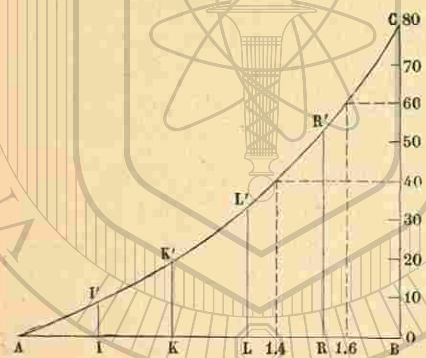


Fig. 86.—Graphical Method of Graduation.

laying off the ordinates, but it is convenient to adopt a scale which will make the greatest ordinate BC not much greater nor much less than the base-line AB. In the figure, the density at B is supposed to be 1·80, the density at A being 1. The difference of density is therefore ·80, as indicated by the figures 80 on the scale of equal parts. Having erected the ordinates, we must draw through their extremities the curve AI′K′L′R′C, making it as free from sudden bends as possible, as it is upon the regularity of this curve that the accuracy of the interpolation depends. Then to find the point on the stem AB at which any other density is to be marked—say 1·60, we must draw through the 60th division, on the line of equal parts, a horizontal line to meet the curve, and, through the point thus found on the curve,

draw an ordinate. This ordinate will meet the base-line AB in the required point, which is accordingly marked 1·6 in the figure. The curve also affords the means of solving the converse problem, that is, of finding the density corresponding to any given point on the stem. At the given point in AB, which represents the stem, we must draw an ordinate, and through the point where this meets the curve we must draw a horizontal line to meet the scale of equal parts. The point thus determined on the scale of equal parts indicates the density required, or rather the excess of this density above the density of A.

CHAPTER XV.

VESSELS IN COMMUNICATION—LEVELS.

178. Liquids tend to Find their own Level.—When a liquid is contained in vessels communicating with each other, and is in equilibrium, it stands at the same height in the different parts of the system, so that the free surfaces all lie in the same horizontal plane. This is obvious from the considerations pointed out in §§ 138, 139, being merely a particular case of the more general law that points of a liquid at rest which are at the same pressure are at the same level.

In the apparatus represented in Fig. 87, the liquid is seen to stand at the same height in the principal vessel and in the variously shaped tubes communicating with it. If one of these tubes is cut off at a height less than that of the liquid in the principal vessel, and is made to terminate in a narrow mouth, the liquid will be seen to spout up nearly to the level of that in the principal vessel.

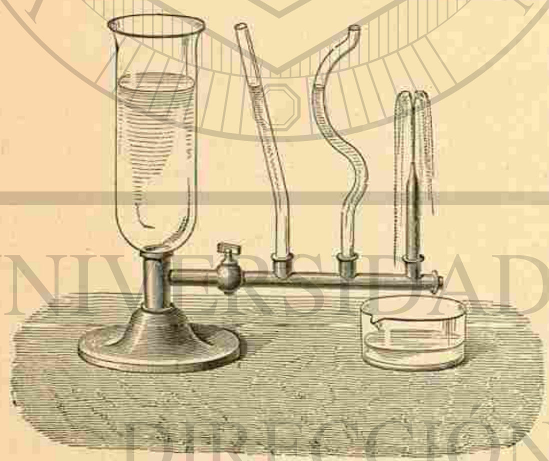


Fig. 87.—Vessels in Communication.

This tendency of liquids to find their own level is utilized for the water-supply of towns. Water will find its way from a reservoir through pipes of any length, provided that all parts of them are below the level of the water in the reservoir. It is necessary how-

ever to distinguish between the conditions of statical equilibrium and the conditions of flow. If no water were allowed to escape from the pipes in a town, their extremities might be carried to the height of the reservoir and they would still be kept full. But in practice there is a continual abstraction of energy, partly in the shape of the kinetic energy of the water which issues from taps, often with considerable velocity, and partly in the shape of work done against friction in the pipes. When there is a continual drawing off from various points of a main, the height to which the water will rise in the houses which it supplies is least in those which are most distant from the reservoir.

179. Water-level.—The instrument called the water-level is another illustration of the same principle. It consists of a metal tube *bb*, bent at right angles at its extremities. These carry two glass tubes

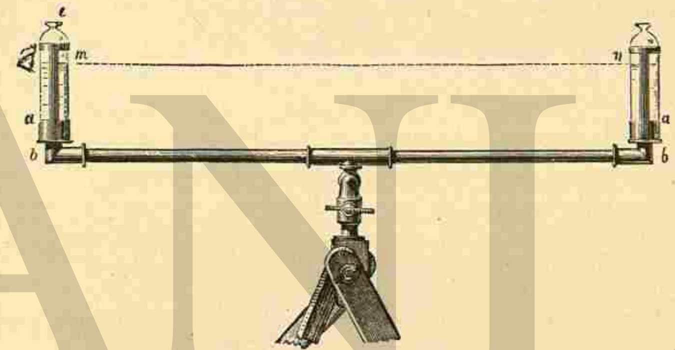


Fig. 88.—Water-level.

aa, very narrow at the top, and of the same diameter. The tube rests on a tripod stand, at the top of which is a joint that enables the observer to turn the apparatus and set it in any direction. The tube is placed in a position *nearly* horizontal, and water, generally coloured a little, is poured in until it stands at about three-fourths of the height of each of the glass tubes.

By the principle of equilibrium in vessels communicating with each other, the surfaces of the liquid in the two branches are in the same horizontal plane, so that if the line of the observer's sight just grazes the two surfaces it will be horizontal.

This is the principle of the operation called *levelling*, the object of which is to determine the difference of vertical height, or *difference of level*, between two given points. Suppose A and B to be the two points (Fig. 89). At each of these points is fixed a levelling-staff, that is, an upright rod divided into parts of equal length, on which slides a small square board whose centre serves as a mark for the observer.

The level being placed at an intermediate station, the observer directs the line of sight towards each levelling-staff, and the mark is raised or lowered till the line of sight passes through its centre. The marks on the two staves are in this way brought to the same level. The staff in the rear is then carried in advance of the other, the level is again placed between the two, and another observation taken. In this way, by noting the division of the staff at which the sliding mark stands in each case, the difference of levels of two distant stations can be deduced from observations at a number of intermediate points.

Fig. 89.—Levelling.

For more accurate work, a telescope with attached spirit-level (§ 181) is used, and the levelling staff has divisions upon it which are read off through the telescope.

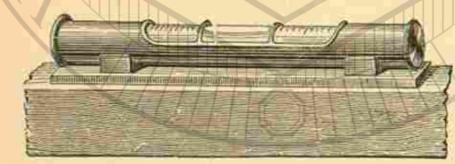


Fig. 90.—Spirit-level.

180. Spirit-level.—The spirit-level is composed of a glass tube slightly curved, containing a liquid, which is generally alcohol, and which fills the whole extent of the tube, except a small space occupied by an air-bubble. This tube is inclosed in a mounting which is firmly supported on a stand.

Suppose the tube to have been so constructed that a vertical section of its upper surface is an arc of a circle, and suppose the instrument placed upon a horizontal plane (Fig. 91). The air-bubble will take up a position MN at the highest part of the tube, such that the arcs MA and NB are equal. Hence it follows that if the level

Fig. 91.

be reversed end for end, the bubble will occupy the same position in the tube, the point N coming to M, and *vice versa*. This will not be the case if AB is inclined to the horizon (Fig. 92), for then the bubble will always stand nearest to that end of the tube which is highest, and will therefore change its place in the tube when the latter is reversed. The test, then, of the horizontality of the line on which the spirit-level rests is, that after this operation of reversal the bubble should remain between the same marks on the tube. The maker marks upon the tube two points equidistant from the centre, the distance between them being equal to the usual length of the bubble; and the instrument ought to be so adjusted that when the line on which it stands is horizontal, the ends of the bubble are at these marks.

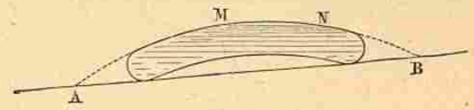


Fig 92.

In order that a plane surface may be horizontal, we must have two lines in it horizontal. This result may be attained in the following manner:—The body whose surface is to be levelled is made to rest on three levelling-screws which form the three vertices of an isosceles triangle; the level is first placed parallel to the base of the triangle, and, by means of one of the screws, the bubble is brought between the reference-marks. The instrument is then placed perpendicularly to its first position, and the bubble is brought between the marks by means of the third screw; this second operation cannot disturb the result of the first, since the plane has only been turned about a horizontal line as hinge.

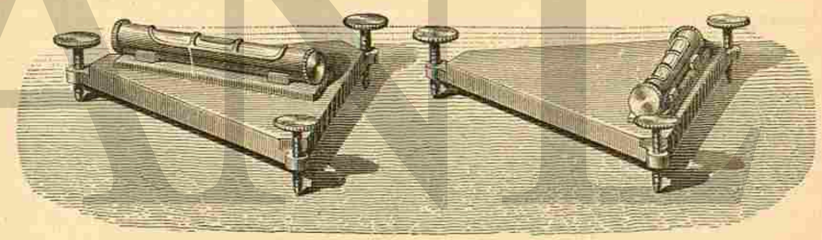


Fig. 93.—Testing the Horizontality of a Surface.

181. Telescope with Attached Level.—In order to apply the spirit-level to land-surveying, an apparatus such as that represented in

Fig. 94 is employed. Upon a frame AA, movable about a vertical axis B, are placed a spirit-level *nn*, and a telescope LL, in positions parallel to each other. The telescope is furnished at its focus with two fine wires crossing one another, whose point of intersection determines the line of sight with great precision. The apparatus, which is provided with levelling-screws H, rests on a tripod stand, and the observer is able, by turning it about its axis, to command the different points of the horizon. By a process of adjustment which need not here be described, it is known that when the bubble is between the marks the line of sight is horizontal. By furnishing the instrument with a graduated horizontal circle P, we may obtain the azimuths of the points observed, and thus map out contour lines.

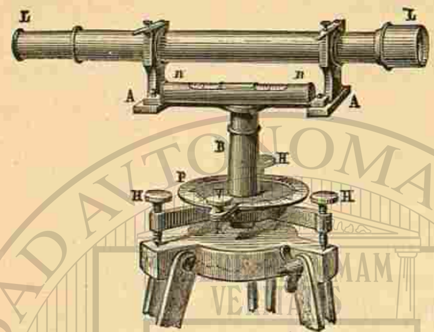


Fig. 94.—Spirit-level with Telescope.

Divisions are sometimes placed on each side of the reference-marks of the bubble, for measuring small deviations from horizontality. It is, in fact, easy to see, by reference to Fig. 91, that by tilting the level through any small angle, the bubble is displaced by a quantity proportional to this angle, at least when the curvature of the instrument is that of a circle.

For determining the angular value corresponding to each division

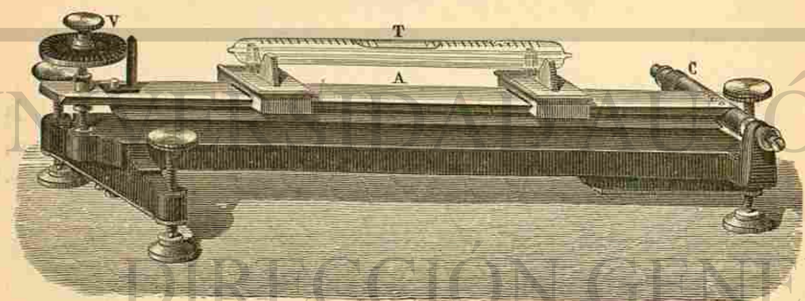


Fig. 95.—Graduation of Spirit-level.

of the tube, it is usual to employ an apparatus opening like a pair of compasses by a hinge C (Fig. 95), on one of the legs of which rests, by two V-shaped supports, the tube T of the level. The compass is opened by means of a micrometer screw V, of very regular action; and as the distance of the screw from the hinge is known, as well as the distance between the threads of the screw, it is easy to calculate beforehand the value of the divisions on the micrometer head. The levelling-screws of the instrument serve to bring the bubble between its reference-marks, so that the micrometer screw is only used to determine the value of the divisions on the tube.

CHAPTER XVI.

CAPILLARITY.

182. Capillarity—General Phenomena.—The laws which we have thus far stated respecting the levels of liquid surfaces are subject to remarkable exceptions when the vessels in which the liquids are contained are very narrow, or, as they are called, capillary (*capillus*, a hair); and also in the case of vessels of any size, when we consider the portion of the liquid which is in close proximity to the sides.

1. *Free Surface.*—The surface of a liquid is not horizontal in the neighbourhood of the sides of the vessel, but presents a very decided curvature. When the liquid wets the vessel, as in the case of water in a glass vessel (Fig. 96), the surface is concave; on the contrary

Fig. 96. Fig. 97.

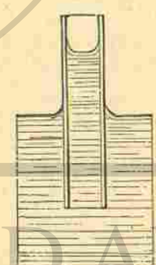


Fig. 98.

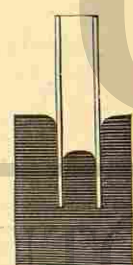


Fig. 99.

when the liquid does not wet the vessel, as in the case of mercury in a glass vessel (Fig. 97), the surface is, generally speaking, convex.

2. *Capillary Elevation and Depression.*—If a very narrow tube of glass be plunged in water, or any other liquid that will wet it (Fig. 98), it will be observed that the level of the liquid, instead of remaining at the same height inside and outside of the tube, stands perceptibly higher in the tube; a *capillary ascension* takes place, which varies in amount according to the nature of the liquid and

the diameter of the tube. It will also be seen that the liquid column thus raised terminates in a concave surface. If a glass tube be dipped in mercury, which does not wet it, it will be seen, by bringing the tube to the side of the vessel, that the mercury is depressed in its interior, and that it terminates in a convex surface (Fig. 99).

3. *Capillary Vessels in Communication with Others.*—If we take two bent tubes (Fig. 100), each having one branch of a considerable diameter and the other extremely narrow, and pour into one of them a liquid which wets it, and into the other mercury, the liquid will be observed in the former case to stand higher in the capillary than in the principal branch, and in the latter case to stand lower; the free surfaces being at the same time concave in the case of the liquid which wets the tubes, and convex in the case of the mercury.

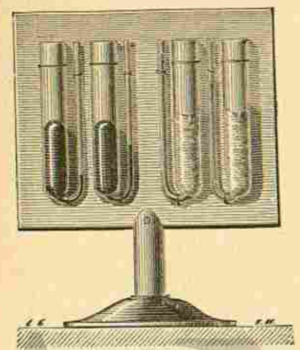


Fig. 100.

183. Circumstances which influence Capillary Elevation and Depression.—In wetted tubes the elevation depends upon the nature of the liquid; thus, at the temperature of 18° Cent., water rises 29·79^{mm} in a tube 1 millimetre in diameter, alcohol rises 12·18^{mm}, nitric acid 22·57^{mm}, essence of lavender 4·28^{mm}, &c. The nature of the tube is almost entirely immaterial, provided the precaution be first taken of wetting it with the liquid to be employed in the experiment, so as to leave a film of the liquid adhering to the sides of the tube.

Capillary depression, on the other hand, depends both on the nature of the liquid and on that of the tube. Both ascension and depression diminish as the temperature increases; for example, the elevation of water, which in a tube of a certain diameter is equal to 132^{mm} at 0° Cent., is only 106^{mm} at 100°.

184. Law of Diameters.—*Capillary elevations and depressions, when all other circumstances are the same, are inversely proportional to the diameters of the tubes.* As this law is a consequence of the mathematical theories which are generally accepted as explaining capillary phenomena, its verification has been regarded as of great importance.

The experiments of Gay-Lussac, which confirmed this law, have been repeated, with slight modifications, by several observers. The method employed consists essentially in measuring the capillary elevation of a liquid by means of a cathetometer (Fig. 101). The telescope ll is directed first to the top n of the column in the tube, and then to the end of a pointer b, which touches the surface of the

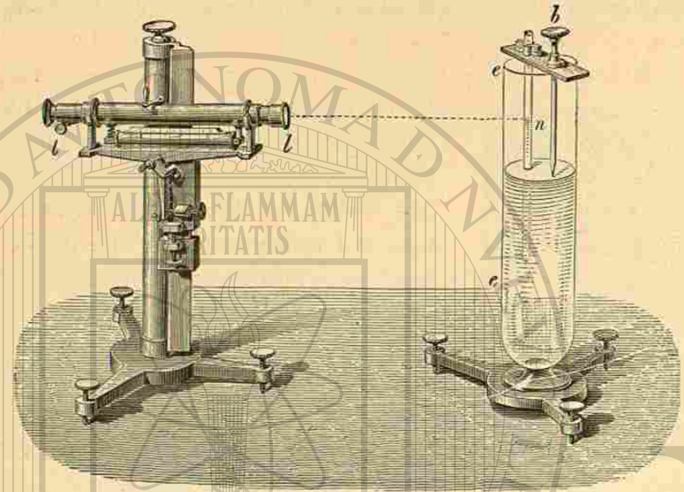


Fig. 101.—Verification of Law of Diameters.

liquid at a point where it is horizontal. In observing the depression of mercury, since the opacity of the metal prevents us from seeing the tube, we must bring the tube close to the side of the vessel e.

The diameter of the tube can be measured directly by observing its section through a microscope, or we may proceed by the method employed by Gay-Lussac. He weighed the quantity of mercury which filled a known length l of the tube; this weight w is that of a cylinder of mercury whose radius x is determined by the equation $13{\cdot}59\ \pi x^2 l = w$, where x and l are in centimetres, and w in grammes.

The result of these different experiments is, that in the case of wetted tubes the law is exactly fulfilled, provided that they be previously washed with the greatest care, so as to remove all foreign matters, and that the liquid on which the experiment is to be performed be first passed through them. When the liquid does not wet the tube, various causes combine to affect the form of the surface in which the liquid column terminates; and we cannot infer the depression from knowing the diameter, unless we also take into consideration some element connected with the form of the terminal surface, such as the length of the sagitta, or the angle made with the sides

of the tube by the extremities of the curved surface, which is called the *angle of contact.*

185. Fundamental Laws of Capillary Phenomena.—Capillary phenomena, as they take place alike in air and in vacuo, cannot be attributed to the action of the atmosphere. They depend upon molecular actions which take place between the particles of the liquid itself, and between the liquid and the solid containing it, the actions in question being purely superficial—that is to say, being confined to an extremely thin layer forming the external boundary of the liquid, and to an extremely thin superficial layer of the solid in contact with the liquid. For example, it is found in the case of glass tubes, that the amount of capillary elevation or depression is not at all affected by the thickness of the sides of the tube. The following are some of the principles which govern capillary phenomena.

1. For a given liquid in contact with a given solid, with a definite intimateness of contact (this last element being dependent upon the cleanness of the surface, upon whether the surface of the solid has been recently washed by the liquid, and perhaps upon some other particulars), there is (at any specified temperature) a definite angle of contact, which is independent of the directions of the surfaces with regard to the vertical.

2. Every liquid behaves as if a thin film, forming its external layer, were in a state of tension, and exerting a constant effort to contract. This tension, or contractile force, is exhibited over the whole of the free surface (that is, the surface which is exposed to air); but wherever the liquid is in contact with a solid, its existence is masked by other molecular actions. It is uniform in all directions in the free surface, and at all points in this surface, being dependent only on the nature and temperature of the liquid. Its intensity for several specified liquids is given in tabular form further on (§ 192) upon the authority of Van der Mensbrugghe. Tension of this kind must of course be stated in units of force per linear unit, because by doubling the width of a band we double the force required to keep it stretched. Mensbrugghe considers that such tension really exists in the superficial layer; but the majority of authors (and we think with more justice) regard it rather as a convenient fiction, which accurately represents the effects of the real cause. Two of the most eminent writers on the cause of capillary phenomena are Laplace and Dr. Thomas Young. The subject presents difficulties which have not yet been fully surmounted

186. Application to Elevation in Tubes.—The law of diameters is a direct consequence of the two preceding principles; for if α denote the external angle of contact (which is acute in the case of mercury against glass), T the tension per unit length, and r the radius of the tube, then $2\pi r$T will be the whole amount of force exerted at the margin of the surface; and as this force is exerted in a direction making an angle α with the vertical, its vertical component will be $2\pi r$T cos α, which is exerted in pulling the tube upwards and the liquid downwards.

If w be the weight of unit volume of the liquid, then $\pi r^2 w$ is the weight of as much as would occupy unit length of the tube; and if h denote the height of a column whose weight is equal to the force tending to depress the liquid, we have

$$\pi r^2 h w = 2\pi r \mathrm{T} \cos\alpha;$$

whence $h = \frac{2\mathrm{T}\cos\alpha}{r \cdot w}$, which, when the other elements are given, varies inversely as r, the radius of the tube.

Having regard to the fact that the surface is not of the same height in the centre as at the edges, it is obvious that h denotes the mean height.

If α be obtuse, h will be negative—that is to say, there will be elevation instead of depression. In the case of water against a tube which has been well wetted with that liquid, α is 180°—that is to say, the tube is tangential to the surface. For this case the formula for h gives

$$\text{elevation} = \frac{2\mathrm{T}}{rw}.$$

Again, for two parallel vertical plates at distance u, the vertical force of capillarity for a unit of length is 2T$\cos\alpha$, which must be equal to whu, being the weight of a sheet of liquid of height h, thickness u, and length unity. We have therefore

$$h = \frac{2\mathrm{T}\cos\alpha}{uw},$$

which agrees with the expression for the depression or elevation in a circular tube whose radius is equal to the distance between these parallel plates.

The surface tension always tends to reduce the surface to the smallest area which can be inclosed by its actual boundary; and therefore always produces a normal force directed from the convex to the concave side of the superficial film. Hence, wherever there is capillary elevation the free surface must be concave; wherever there is depression it must be convex.

187. It follows from a well-known proposition in statics (Todhunter's *Statics*, § 194), that if a *cylindrical* film be stretched with a uniform tension T (so that the force tending to pull the film asunder across any short line drawn on the film, is T times the length of the line), the resultant normal pressure (which the film exerts, for example, against the surface of a solid internal cylinder over which it is stretched) is T divided by the radius of the cylinder.

It can be proved that a film of any form, stretched with uniform tension T, exerts at each point a normal pressure equal to the sum of the pressures which would be exerted by two overlapping cylindrical films, whose axes are at right angles to one another, and whose cross sections are circles of curvature of normal sections at the point. That is to say, if P be the normal force per unit area, and r, r' the radii of curvature in two mutually perpendicular normal sections at the point, then

$$\mathrm{P} = \mathrm{T}\left(\frac{1}{r} + \frac{1}{r'}\right).$$

At any point on a curved surface, the normal sections of greatest and least curvature are mutually perpendicular, and are called the principal normal sections at the point. If the corresponding radii of curvature be R, R′, we have

$$\mathrm{P} = \mathrm{T}\left(\frac{1}{\mathrm{R}} + \frac{1}{\mathrm{R}'}\right); \qquad (1)$$

or *the normal force per unit area is equal to the tension per unit length multiplied by the sum of the principal curvatures.*

In the case of capillary depressions and elevations, the superficial film at the free surface is to be regarded as pressing the liquid inwards, or pulling it outwards, according as this surface is convex or concave, with a force P given by the above formula. The value of P at any point of the free surface is equal to the pressure due to the height of a column of liquid extending from that point to the level of the general horizontal surface. It is therefore greatest at the edges of the elevated or depressed column in a tube, and least in the centre; and the curvature, as measured by $\frac{1}{\mathrm{R}} + \frac{1}{\mathrm{R}'}$, must vary in the same proportion. If the tube is so large that there is no sensible elevation or depression in the centre of the column, the centre of the free surface must be sensibly plane.

188. Another consequence of the formula is, that in circumstances where there can be no normal pressure towards either side of the surface,

$$\frac{1}{R} + \frac{1}{R'} = 0; \qquad (2)$$

which implies that either the surface is plane, in which case each of the two terms is separately equal to zero, or else

$$R = -R'; \qquad (3)$$

that is, the principal radii of curvature are equal, and lie on opposite sides of the surface. The formulæ (2), (3) apply to a film of soapy water attached to a loop of wire. If the loop be in one plane, the film will be in the same plane. If the loop be not in one plane, the film cannot be in one plane, and will in fact assume that form which gives the least area consistent with having the loop for its boundary. At every point it will be observed to be, if we may so say, concave towards both sides, and convex towards both sides, the concavity being precisely equal to the convexity—that is to say, equation (3) is satisfied at every point of the film.

In this case both sides of the film are exposed to atmospheric pressure. In the case of a common soap-bubble the outside is exposed to atmospheric pressure, and the inside to a pressure somewhat greater, the difference of the pressures being balanced by the tendency of the film to contract. Formula (1) becomes for either the outer or inner surface of a spherical bubble

$$P = \frac{2T}{R};$$

but this result must be doubled, because there are two free surfaces; hence the excess of pressure of the inclosed above the external air is $\frac{4T}{R}$, R denoting the radius of the bubble.

The value of T for soapy water is about 1 grain per linear inch; hence, if we divide 4 by the radius of the bubble expressed in inches, we shall obtain the excess of internal over external pressure *in grains per square inch.*

The value of T for any liquid may be obtained by observing the amount of elevation or depression in a tube of given diameter, and employing the formula

$$T = \frac{whr}{2\cos a}, \qquad (4)$$

which follows immediately from the formula for h in § 186.

189. It is this uniform surface tension, of which we have been

speaking, which causes a drop of a liquid falling through the air either to assume the spherical form, or to oscillate about the spherical form. The phenomena of drops can be imitated on an enlarged scale, under circumstances which permit us to observe the actual motions, by a method devised by Professor Plateau of Ghent. Olive-oil is intermediate in density between water and alcohol. Let a mixture of alcohol and water be prepared, having precisely the density of olive-oil, and let about a cubic inch of the latter be gently introduced into it with the aid of a funnel or pipette. It will assume a spherical form, and if forced out of this form and then left free, will slowly oscillate about it; for example, if it has been compelled to assume the form of a prolate spheroid, it will pass to the oblate form, will then become prolate again, and so on alternately, becoming however more nearly spherical every time, because its movements are hindered by friction, until at last it comes to rest as a sphere.

190. Capillarity furnishes no exception to the principle that the pressure in a liquid is the same at all points at the same depth. When the free surface within a tube is convex, and is consequently depressed below the plane surface of the external liquid, the pressure becomes suddenly greater on passing downwards through the superficial layer, by the amount due to the curvature. Below this it increases regularly by the amount due to the depth of liquid passed through. The pressure at any point vertically under the convex meniscus[1] may be computed, either by taking the depth of the point below the general free surface, and adding atmospheric pressure to the pressure due to this depth, according to the ordinary principles of hydrostatics, or by taking the depth of the point below that point of the meniscus which is vertically over it, adding the pressure due to the curvature at this point, and also adding atmospheric pressure.

When the free surface of the liquid within a tube is concave, the pressure suddenly diminishes on passing downwards through the superficial layer, by the amount due to the curvature as given by formula (1); that is to say, the pressure at a very small depth is less than atmospheric pressure by this amount. Below this depth it goes on increasing according to the usual law, and becomes equal to

[1] The convex or concave surface of the liquid in a tube is usually denoted by the name *meniscus* (μηνίσκος, a crescent), which denotes a form approximately resembling that of a watch-glass.

atmospheric pressure at that depth which corresponds with the level of the plane external surface. The pressure at any point in the liquid within the tube can therefore be obtained either by subtracting from atmospheric pressure the pressure due to the elevation of the point above the general surface, or by adding to atmospheric pressure the pressure due to the depth below that point of the meniscus which is on the same vertical, and subtracting the pressure due to the curvature at this point.

These rules imply, as has been already remarked, that the curvature is different at different points of the meniscus, being greatest where the elevation or depression is greatest, namely at the edges of the meniscus; and least at the point of least elevation or depression, which in a cylindrical tube is the middle point.

The principles just stated apply to all cases of capillary elevation and depression.

They enable us to calculate the force with which two parallel vertical plates, partially immersed in a liquid which wets them, are urged towards each other by capillary action. The pressure in the portion of liquid elevated between them is less than atmospheric, and therefore is insufficient to balance the atmospheric pressure which is exerted on the outer faces of the plates. The average pressure in the elevated portion of liquid is equal to the actual pressure at the centre of gravity of the elevated area, and is less than atmospheric pressure by the pressure of a column of liquid whose height is the elevation of this centre of gravity.

Even if the liquid be one which does not wet the plates, they will still be urged towards each other by capillary action; for the inner faces of the plates are exposed to merely atmospheric pressure over the area of depression, while the corresponding portions of the external faces are exposed to atmospheric pressure increased by the weight of a portion of the liquid.

These principles explain the apparent attraction exhibited by bodies floating on a liquid which either wets them both or wets neither of them. When the two bodies are near each other they behave somewhat like parallel plates, the elevation or depression of the liquid between them being greater than on their remote sides.

If two floating bodies, one of which is wetted and the other unwetted by the liquid, come near together, the elevation and depression of the liquid will be less on the near than on the remote sides, and apparent repulsion will be exhibited.

In all cases of capillary elevation or depression, the solid is pulled downwards or upwards with a force equal to that by which the liquid is raised or depressed. In applying the principle of Archimedes to a solid partially immersed in a liquid, it is therefore necessary (as we have seen in § 159), when the solid produces capillary depression, to reckon the void space thus created as part of the displacement; and when the solid produces capillary elevation, the fluid raised above the general level must be reckoned as *negative* displacement, tending to *increase* the apparent weight of the solid.

191. Thus far all the effects of capillary action which we have mentioned are connected with the curvature of the superficial film, and depend upon the principle that a convex surface increases and a concave surface diminishes the pressure in the interior of the liquid. But there is good reason for maintaining that whatever be the form of the free surface there is always pressure in the interior due to the molecular action at this surface, and that the pressure due to the curvature of the surface is to be added to or subtracted from a definite amount of pressure which is independent of the curvature and depends only on the nature and condition of the liquid. This indeed follows at once from the fact that capillary elevation can take place in vacuo. As far as the principles of the preceding paragraphs are concerned, we should have, at points within the elevated column, a pressure less than that existing in the vacuum. This, however, cannot be; we cannot conceive of negative pressure existing in the interior of a liquid, and we are driven to conclude that the elevation is owing to the excess of the pressure caused by the plane surface in the containing vessel above the pressure caused by the concave surface in the capillary tube.

There are some other facts which seem only explicable on the same general principle of interior pressure due to surface action,—facts which attracted the notice of some of the earliest writers on pneumatics, namely, that siphons will work in vacuo, and that a column of mercury at least 75 inches in length can be sustained—as if by atmospheric pressure—in a barometer tube, the mercury being boiled and completely filling the tube.

192. We have now to notice certain phenomena which depend on the difference in the surface tensions of different liquids, or of the same liquid in different states.

Let a thin layer of oil be spread over the upper surface of a thin sheet of brass, and let a lamp be placed underneath. The oil will be observed to run away from the spot directly over the flame, even though this spot be somewhat lower than the rest of the sheet. This effect is attributable to the excess of surface tension in the cold oil above the hot.

In like manner, if a drop of alcohol be introduced into a thin layer of water spread over a nearly horizontal surface, it will be drawn away in all directions by the surrounding water, leaving a nearly dry spot in the space which it occupied. In this experiment the water should be coloured in order to distinguish it from the alcohol.

Again, let a very small fragment of camphor be placed on the surface of hot water. It will be observed to rush to and fro, with frequent rotations on its own axis, sometimes in one direction and sometimes in the opposite. These effects, which have been a frequent subject of discussion, are now known to be due to the diminution of the surface tension of the water by the camphor which it takes up. Superficial currents are thus created, radiating from the fragment of camphor in all directions; and as the camphor dissolves more quickly in some parts than in others, the currents which are formed are not equal in all directions, and those which are most powerful prevail over the others and give motion to the fragment.

The values of T, the apparent surface tension, for several liquids, are given in the following table, on the authority of Van der Mensbrugghe, in milligrammes (or thousandth parts of a gramme) per millimetre of length. They can be reduced to grains per inch of length by multiplying them by ·392; for example, the surface tension of distilled water is $7{\cdot}3 \times {\cdot}392 = 2{\cdot}86$ grains per inch.

Distilled water at 20° Cent., . . .	7·3	Solution of Marseilles soap, 1 part of soap to 40 of water, ...	2·83
Sulphuric ether, ...	1·88	Solution of saponine, ...	4·67
Absolute alcohol, ...	2·5	Saturated solution of carbonate of soda, ...	4·28
Olive-oil, ...	3·5	Water impregnated with camphor, .	4·5
Mercury, ...	49·1		
Bisulphide of carbon, ...	3·57		

193. **Endosmose.**—Capillary phenomena have undoubtedly some connection with a very important property discovered by Dutrochet, and called by him *endosmose*.

The *endosmometer* invented by him to illustrate this phenomenon consists of a reservoir *v* (Fig. 102) closed below by a membrane *ba*, and terminating above in a tube of considerable length. This reservoir is filled, suppose, with a solution of gum in water, and is kept

immersed in water. At the end of some time the level of the liquid in the tube will be observed to have risen to *n*, suppose, and at the same time traces of gum will be found in the water in which the reservoir is immersed. Hence we conclude that the two liquids have penetrated through the membrane, but in different proportions; and this is what is called endosmose.

If instead of a solution of gum we employed water containing albumen, sugar, or gelatine in solution, a similar result would ensue. The membrane may be replaced by a slab of wood or of porous clay. Physiologists have justly attached very great importance to this discovery of Dutrochet. It explains, in fact, the interchange of liquids which is continually taking place in the tissues and vessels of the animal system, as well as the absorption of water by the spongioles of roots, and several similar phenomena.

As regards the power of passing through porous diaphragms, Graham has divided substances into two classes—*crystalloids* and *colloids* (κόλλη, glue). The former are susceptible of crystallization, form solutions free from viscosity, are sapid, and possess great powers of diffusion through porous septa. The latter, including gum, starch, albumen, &c., are characterized by a remarkable sluggishness and indisposition both to diffusion and to crystallization, and when pure are nearly tasteless.

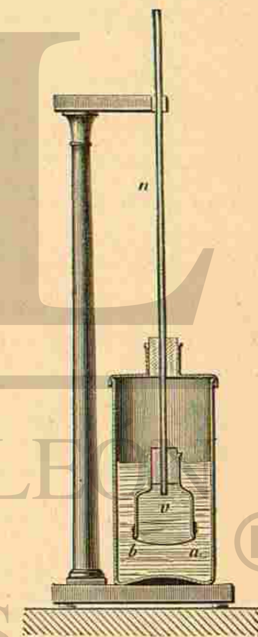


Fig. 102.—Endosmometer.

Diffusion also takes place through colloidal diaphragms which are not porous, the diaphragm in this case acting as a solvent, and giving out the dissolved material on the other side. In the important process of modern chemistry called *dialysis*, saline ingredients are separated from organic substances with which they are blended, by interposing a colloidal diaphragm (De La Rue's parchment paper) between the mixture and pure water. The organic matters, being colloidal, remain behind, while the salts pass through, and can be obtained in a nearly pure state by evaporating the water.

Gases are also capable of diffusion through diaphragms, whether porous or colloidal, the rate of diffusion being in the former case inversely as the square root of the density of the gas. Hydrogen diffuses so rapidly through unglazed earthenware as to afford opportunity for very striking experiments; and it shows its power of traversing colloids by rapidly escaping through the sides of india-rubber tubes, or through films of soapy water.

CHAPTER XVII.

THE BAROMETER.

194. Expansibility of Gases.—Gaseous bodies possess a number of properties in common with liquids; like them, they transmit pressures entire and in all directions, according to the principle of Pascal; but they differ essentially from liquids in the permanent repulsive force exerted between their molecules, in virtue of which a mass of gas always tends to expand.

This property, called the expansibility of gases, is commonly illustrated by the following experiment:—

A bladder, nearly empty of air, and tied at the neck, is placed under the receiver of an air-pump. At first the air which it contains and the external air oppose each other by their mutual pressure, and are in equilibrium. But if we proceed to exhaust the receiver, and thus diminish the external pressure, the bladder gradually becomes inflated, and thus manifests the tendency of the gas which it contains to occupy a greater volume.

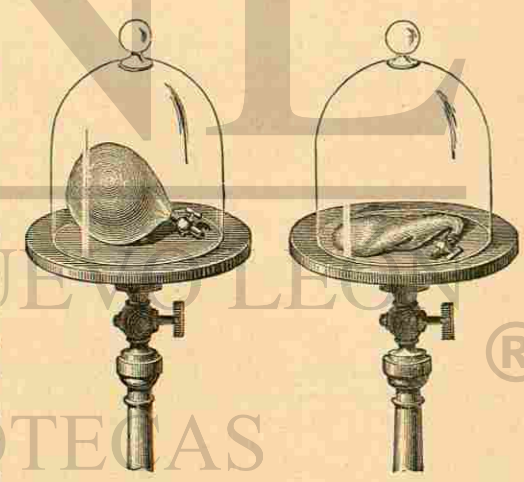


Fig. 103.—Expansibility of Gases.

However large a vessel may be, it can always be filled by *any quantity whatever* of a gas, which will always exert pressure against porous or colloidal, the rate of diffusion being in the former case inversely as the square root of the density of the gas. Hydrogen diffuses so rapidly through unglazed earthenware as to afford opportunity for very striking experiments; and it shows its power of traversing colloids by rapidly escaping through the sides of india-rubber tubes, or through films of soapy water.

CHAPTER XVII.

THE BAROMETER.

194. Expansibility of Gases.—Gaseous bodies possess a number of properties in common with liquids; like them, they transmit pressures entire and in all directions, according to the principle of Pascal; but they differ essentially from liquids in the permanent repulsive force exerted between their molecules, in virtue of which a mass of gas always tends to expand.

This property, called the expansibility of gases, is commonly illustrated by the following experiment:—

A bladder, nearly empty of air, and tied at the neck, is placed under the receiver of an air-pump. At first the air which it contains and the external air oppose each other by their mutual pressure, and are in equilibrium. But if we proceed to exhaust the receiver, and thus diminish the external pressure, the bladder gradually becomes inflated, and thus manifests the tendency of the gas which it contains to occupy a greater volume.

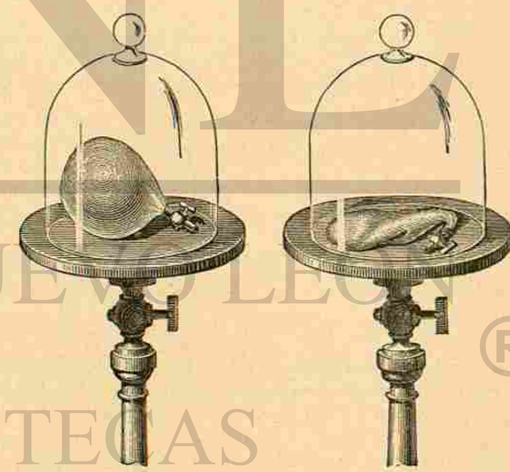


Fig. 103.—Expansibility of Gases.

However large a vessel may be, it can always be filled by *any quantity whatever* of a gas, which will always exert pressure against the sides. In consequence of this property, the name of *elastic fluids* is often given to gases.

195. Air has Weight.—The opinion was long held that the air was without weight; or, to speak more precisely, it never occurred to any of the philosophers who preceded Galileo to attribute any influence in natural phenomena to the weight of the air. And as this influence is really of the first importance, and comes into play in many of the commonest phenomena, it very naturally happened that the discovery of the weight of air formed the commencement of the modern revival of physical science.

It appears, however, that Aristotle conceived the idea of the possibility of air having weight, and, in order to convince himself on this point, he weighed a skin inflated and collapsed. As he obtained the same weight in both cases, he relinquished the idea which he had for the moment entertained. In fact, the experiment, as he performed it, could only give a negative result; for if the weight of the skin was increased, on the one hand, by the introduction of a fresh quantity of air, it was diminished, on the other, by the corresponding increase in the upward pressure of the air displaced. In order to draw a certain conclusion, the experiment should be performed with a vessel which could receive within it air of different degrees of density, without changing its own volume.

Galileo is said to have devised the experiment of weighing a globe filled alternately with ordinary air and with compressed air. As the weight is greater in the latter case, Galileo should have drawn the inference that air is heavy. It does not appear, however, that the importance of this conclusion made much impression on him, for he did not give it any of those developments which might have been expected to present themselves to a mind like his.

Otto Guericke, the illustrious inventor of the air-pump, in 1650 performed the following experiment, which is decisive:—

A globe of glass (Fig. 104), furnished with a stop-cock, and of a sufficient capacity (about twelve litres), is exhausted of air. It is then suspended from one of the scales of a balance, and a weight sufficient to produce equilibrium is placed in the other scale. The stop-cock is then opened, the air rushes into the globe, and the beam is observed gradually to incline, so that an additional weight is required in the other scale, in order to re-establish equilibrium. If the capacity of the globe is 12 litres, about 15·5 grammes will be

needed, which gives 1·3 gramme as the approximate weight of a litre (or 1000 cubic centimetres) of air.[1]

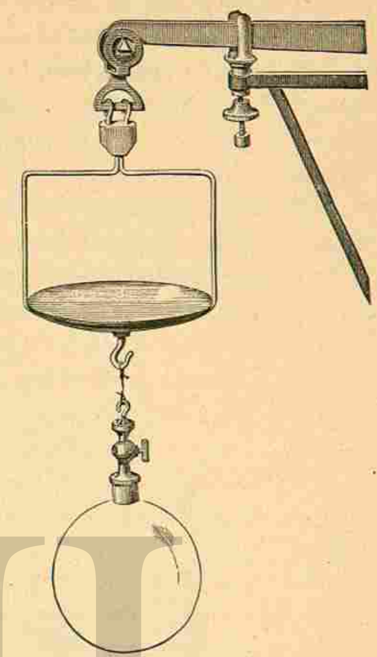


Fig. 104.—Weight of Air.

If, in performing this experiment, we take particular precautions to insure its precision, as we shall explain in the book on Heat, it will be found that, at the temperature of freezing water, and under the pressure of one atmosphere, a litre of perfectly dry air weighs 1·293 gramme.[2] Under these circumstances, the ratio of the weight of a volume of air to that of an equal volume of water is $\frac{1{\cdot}293}{1000}=\frac{1}{773}$. Air is thus 773 times lighter than water.

By repeating this experiment with other gases, we may determine their weight as compared with that of air, and the absolute weight of a litre of each of them. Thus it is found that a litre of oxygen weighs 1·43 gramme, a litre of carbonic acid 1·97 gramme, a litre of hydrogen 0·089 gramme, &c.

[1] A cubic foot of air in ordinary circumstances weighs about an ounce and a quarter.

[2] In strictness, the weight in grammes of a litre of air under the pressure of 760 millimetres of mercury is different in different localities, being proportional to the intensity of gravity—not because the force of gravity on the litre of air is different, for though this is true, it does not affect the numerical value of the weight when stated in grammes, but because the pressure of 760 millimetres of mercury varies as the intensity of gravity, so that more air is compressed into the space of a litre as gravity increases. (§ 201, 6.)

The *weight in grammes* is another name for the *mass*. The force of gravity on a litre of air under the pressure of 760 millimetres is proportional to the square of the intensity of gravity.

This is an excellent example of the ambiguity of the word *weight*, which sometimes denotes a mass, sometimes a force; and though the distinction is of no practical importance so long as we confine our attention to one locality, it cannot be neglected when different localities are compared.

Regnault's determination of the weight of a litre of dry air at 0° Cent. under the pressure of 760 millimetres at Paris is 1·293187 gramme. Gravity at Paris is to gravity at Greenwich as 3456 to 3457. The corresponding number for Greenwich is therefore 1·293561.

196. Atmospheric Pressure.—The atmosphere encircles the earth with a layer some 50 or 100 miles in thickness; this heavy fluid mass exerts on the surface of all bodies a pressure entirely analogous both in nature and origin to that sustained by a body wholly immersed in a liquid. It is subject to the fundamental laws mentioned in §§ 137–139. The pressure should therefore diminish as we ascend from the surface of the earth, but should have the same value for all points in the same horizontal layer, provided that the air is in a state of equilibrium. On account of the great compressibility of gas, the lower layers are much more dense than the upper ones; but the density, like the pressure, is constant in value for the

Fig. 105.—Torricellian Experiment.

same horizontal layer, throughout any portion of air in a state of equilibrium. Whenever there is an inequality either of density or pressure at a given level, wind must ensue.

We owe to Torricelli an experiment which plainly shows the pressure of the atmosphere, and enables us to estimate its intensity with great precision. This experiment, which was performed in 1643, one year after the death of Galileo, at a time when the weight and pressure of the air were scarcely even suspected, has immortalized the name of its author, and has exercised a most important influence upon the progress of natural philosophy.

197. Torricellian Experiment.—A glass tube (Fig. 105) about a quarter or a third of an inch in diameter, and about a yard in length, is completely filled with mercury; the extremity is then stopped with the finger, and the tube is inverted in a vessel containing mercury. If the finger is now removed, the mercury will descend in the tube, and after a few oscillations will remain stationary at a height which varies according to circumstances, but which is generally about 76 centimetres, or nearly 30 inches.[1]

The column of mercury is maintained at this height by the pressure of the atmosphere upon the surface of the mercury in the vessel. In fact, the pressure at the level ABCD (Fig. 106) must be the same within as without the tube; so that the column of mercury BE exerts a pressure equal to that of the atmosphere.

Fig. 106.

Accordingly, we conclude from this experiment of Torricelli that *every surface exposed to the atmosphere sustains a normal pressure equal, on an average, to the weight of a column of mercury whose base is this surface, and whose height is 30 inches.*

It is evident that if we performed a similar experiment with water, whose density is to that of mercury as 1 : 13·59, the height of the column sustained would be 13·59 times as much; that is, 30 × 13·59 inches, or about 34 feet. This is the maximum height to which water can be raised in a pump; as was observed by Galileo.

In general the heights of columns of different liquids equal in weight to a column of air on the same base, are inversely proportional to their densities.

198. Pressure of one Atmosphere.—What is usually adopted in accurate physical discussions as the standard "atmosphere" of pressure is the pressure due to a height of 76 centimetres of pure mercury at the temperature zero Centigrade, gravity being supposed to have

[1] 76 centimetres are 29·922 inches.

the same intensity which it has at Paris. The density of mercury at this temperature is 13·596; hence, when expressed in gravitation measure, this pressure is $76 \times 13{\cdot}596 = 1033{\cdot}3$ grammes per square centimetre.[1] To reduce this to absolute measure, we must multiply by the value of g (the intensity of gravity) at Paris, which is 980·94; and the result is 1013600, which is the intensity of pressure in dynes per square centimetre. In some recent works, the round number a million dynes per square centimetre has been adopted as the standard atmosphere.

199. Pascal's Experiments.—It is supposed, though without any decisive proof, that Torricelli derived from Galileo the definite conception of atmospheric pressure.[2] However this may be, when the experiment of the Italian philosopher became known in France in 1644, no one was capable of giving the correct explanation of it, and the famous doctrine that "nature abhors a vacuum," by which the rising of water in a pump was accounted for, was generally accepted. Pascal was the first to prove incontestably the falsity of this old doctrine, and to introduce a more rational belief. For this purpose, he proposed or executed a series of ingenious experiments, and discussed minutely all the phenomena which were attributed to nature's abhorrence of a vacuum, showing that they were necessary consequences of the pressure of the atmosphere.

We may cite in particular the observation, made at his suggestion, that the height of the mercurial column decreases in proportion as we ascend. This beautiful and decisive experiment, which is repeated as often as heights are measured by the barometer, and which leaves no doubt as to the nature of the force which sustains the mercurial column, was performed for the first time at Clermont, and on the top of the mountain Puy-de-Dôme, on the 19th September, 1648.

200. The Barometer.—By fixing the Torricellian tube in a permanent position, we obtain a means of measuring the amount of the atmospheric pressure at any moment; and this pressure may be expressed by the height of the column of mercury which it supports. Such an instrument is called a *barometer*. In order that its indications may be accurate, several precautions must be observed. In the first place, the liquid used in different barometers must be identical; for the height of the column supported naturally depends upon the density of the liquid employed, and if this varies, the observations made with different instruments will not be comparable.

[1] This is about 14·7 pounds per square inch.

[2] In the fountains of the Grand-duke of Tuscany some pumps were required to raise water from a depth of from 40 to 50 feet. When these were worked, it was found that they would not draw. Galileo determined the height to which the water rose in their tubes, and found it to be about 32 feet; and as he had observed and proved that air has weight, he readily conceived that it was the weight of a column of the atmosphere which maintained the water at this height in the pumps. No very useful results, however, were expected from this discovery, until, at a later date, Torricelli adopted and greatly extended it. Desiring to repeat the experiment in a more convenient form, he conceived the idea of substituting for water a liquid that is 14 times as heavy, namely, mercury, rightly imagining that a column of one-fourteenth of the length would balance the force which sustained 32 feet of water (Biot, *Biographie Universelle*, article "Torricelli").—*D.*

Fig. 107.—Barometer in its simplest form.

The mercury employed is chemically pure, being generally made so by washing with a dilute acid and by subsequent distillation. The barometric tube is filled nearly full, and is then placed upon a sloping furnace, and heated till the mercury boils. The object of this process is to expel the air and moisture which may be contained in the mercurial column, and which, without this precaution, would gradually ascend into the vacuum above, and cause a downward pressure of uncertain amount, which would prevent the mercury from rising to the proper height.

The next step is to fill up the tube with pure mercury, taking care not to introduce any bubble of air. The tube is then inverted in a cistern likewise containing pure mercury recently boiled, and is firmly fixed in a vertical position, as shown in Fig. 107.

We have thus a fixed barometer; and in order to ascertain the atmospheric pressure at any moment, it is only necessary to measure the height of the top of the column of mercury above the surface of the mercury in the cistern. One method of doing this is to employ an iron rod, working in a screw, and fixed vertically above the surface of the mercury in the dish. The extremities of this rod are pointed, and the lower extremity being brought down to touch the surface of the liquid below, the distance of the upper extremity from the top of the column of mercury is measured. Adding to this the length of the rod, which has previously been determined once for all, we have the barometric height. This measurement may be effected with great precision by means of the cathetometer.

201. Cathetometer.—This instrument, which is so frequently employed in physics to measure the vertical distance between two points, was invented by Dulong and Petit.

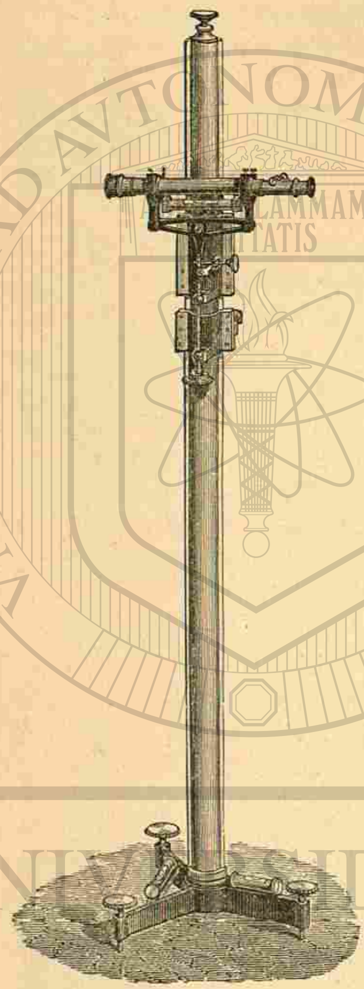


Fig. 108.—Cathetometer.

It consists essentially (Fig. 108) of a vertical scale divided usually into half millimetres. This scale forms part of a brass cylinder capable of turning very easily about a strong steel axis. This axis is fixed on a pedestal provided with three levelling screws, and with two spirit-levels at right angles to each other. Along the scale moves a sliding frame carrying a telescope furnished with cross-wires, that is, with two very fine threads, usually spider lines, in the focus of the eye-piece, whose point of intersection serves to determine the line of vision. By means of a clamp and slow-motion screw, the telescope can be fixed with great precision at any required height. The telescope is also provided with a spirit-level and adjusting screw. When the apparatus is in correct adjustment, the line of vision of the telescope is horizontal, and the graduated scale is vertical. If then we wish to measure the difference of level between two points, we have only to sight them successively, and measure the distance passed over on the scale, which is done by means of a vernier attached to the sliding frame.

202. Fortin's Barometer.—The barometer just described is intended to be fixed; when portability is required, the construction devised by Fortin (Fig. 109) is usually employed. It is also frequently employed for fixed barometers. The cistern, which is formed of a tube of boxwood, surmounted by a tube of glass, is closed below by a piece of leather, which can be raised or lowered by means of a screw. This screw works in the bottom of a brass case, which incloses the cistern except at the middle, where it is cut away in front and at the back, so as to leave the surface of the mercury open to view. The barometric tube is encased in a tube of brass with two slits at opposite sides (Fig. 110); and it is on this tube that the divisions are engraved, the zero point from which they are reckoned being the lower extremity of an ivory point fixed in the covering of the cistern. The temperature of the mercury, which is required for one of the corrections mentioned in next section, is given by a thermometer with its bulb resting against the tube. A cylindrical sliding piece (shown in Fig. 110) furnished with a vernier,[1] moves along the tube and enables us to determine the height with great precision. Its lower edge is the zero of the vernier. The way in which the barometric tube is fixed upon the cistern is worth notice. In the centre of the upper surface of the copper casing there is an opening, from which rises a short tube of the same metal, lined with a tube of boxwood. The barometric tube is pushed inside, and fitted in with a piece of chamois leather, which prevents the mercury from issuing, but does not exclude the air, which, passing through the pores of the leather, penetrates into the cistern, and so transmits its pressure.

Fig. 110. Upper portion of Barometer.

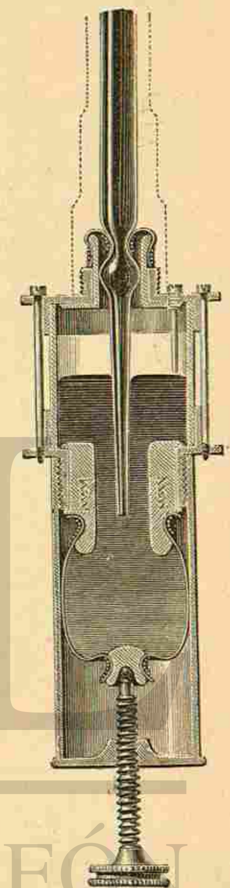


Fig. 109. Cistern of Fortin's Barometer.

Before taking an observation, the surface of the mercury is ad-

[1] The vernier is an instrument very largely employed for measuring the fractions of a unit of length on any scale. Suppose we have a scale divided into inches, and another scale containing nine inches divided into ten equal parts. If now we make the end of this

justed, by means of the lower screw, to touch the ivory point. The observer knows when this condition is fulfilled by seeing the extremity of the point touch its image in the mercury. The sliding piece which carries the vernier is then raised or lowered, until its base is seen to be tangential to the upper surface of the mercurial column, as shown in Fig. 110. In making this adjustment, the back of the instrument should be turned towards a good light, in order that the observer may be certain of the position in which the light is just cut off at the summit of the convexity.

When the instrument is to be carried from place to place, precautions must be taken to prevent the mercury from bumping against the top of the tube and breaking it. The screw at the bottom is to be turned until the mercury reaches the top of the tube, and the instrument is then to be inverted and carried upside down.

We may here remark that the goodness of the vacuum in a barometer, can be tested by the sound of the mercury when it strikes the top of the tube, which it can be made to do either by screwing

latter scale, which is called the vernier, coincide with one of the divisions in the scale of inches, as each division of the vernier is $\frac{9}{10}$ of an inch, it is evident that the first division on the scale will be $\frac{1}{10}$ of an inch beyond the first division on the vernier, the second on the scale $\frac{2}{10}$ beyond the second on the vernier, and so on until the ninth on the scale, which

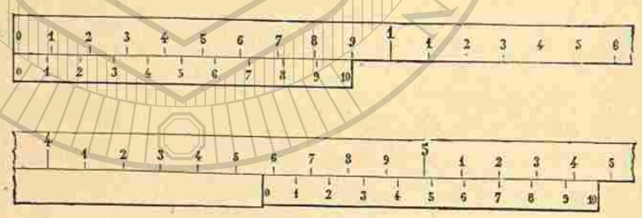


Fig. 111.—Vernier.

will exactly coincide with the tenth on the vernier. Suppose next that in measuring any length we find that its extremity lies between the degrees 5 and 6 on the scale; we bring the zero of the vernier opposite the extremity of the length to be measured, and observe what division on the vernier coincides with one of the divisions on the scale. We see in the figure that it is the seventh, and thus we conclude that the fraction required is $\frac{7}{10}$ of an inch.

If the vernier consisted of 19 inches divided into 20 equal parts, it would read to the $\frac{1}{20}$ of an inch; but there is a limit to the precision that can thus be obtained. An exact coincidence of a division on the vernier with one on the scale seldom or never takes place, and we merely take the division which approaches nearest to this coincidence; so that when the difference between the degrees on the vernier and those on the scale is very small, there may be so much uncertainty in this selection as to nullify the theoretical precision of the instrument. Verniers are also employed to measure angles; when a circle is divided into half degrees, a vernier is used which gives $\frac{1}{30}$ of a division on the circle, that is, $\frac{1}{30}$ of a half degree, or one minute.—*D.*

up or by inclining the instrument to one side. If the vacuum is good, a metallic clink will be heard, and unless the contact be made very gently, the tube will be broken by the sharpness of the collision. If any air be present, it acts as a cushion.

In making observations in the field, a barometer is usually suspended from a tripod stand (Fig. 112) by gimbals[1], so that it always takes a vertical position.

Fig. 112.—Barometer with Tripod Stand.

203. Float Adjustment.—In some barometers the ivory point for indicating the proper level of the mercury in the cistern is replaced by a float. F (Fig. 113) is a small ivory piston, having the float attached to its foot, and moving freely up and down between the two ivory guides I. A horizontal line (interrupted by the piston) is engraved on the two guides, and another is engraved on the piston, at such a height that the three lines form one straight line when the surface of the mercury in the cistern stands at the zero point of the scale.

204. Barometric Corrections.—In order that barometric heights

[1] A kind of universal joint, in common use on board ship for the suspension of compasses, lamps, &c. It is seen in Fig. 112, at the top of the tripod stand.

may be comparable as measures of atmospheric pressure, certain corrections must be applied.

1. *Correction for Temperature.* As mercury expands with heat, it follows that a column of warm mercury exerts less pressure than a column of the same height at a lower temperature; and it is usual to reduce the actual height of the column to the height of a column at the temperature of freezing water which would exert the same pressure.

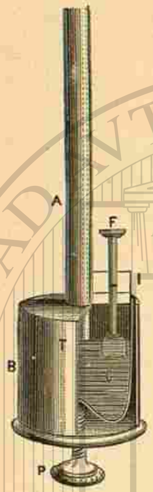


Fig. 113. Float Adjustment.

Let h be the observed height at temperature $t°$ Centigrade, and h_0 the height reduced to freezing-point. Then, if m be the coefficient of expansion of mercury per degree Cent., we have

$$h_0(1+mt)=h, \text{ whence } h_0=h-hmt \text{ nearly.}$$

The value of m is $\frac{1}{5550}=\cdot 00018018$. For temperatures Fahrenheit, we have

$$h_0\left\{1+m(t-32)\right\}=h,\ h_0=h-hm(t-32),$$

where m denotes $\frac{1}{9990}=\cdot 0001001$.

But temperature also affects the length of the divisions on the scale by which the height of the mercurial column is measured. If these divisions be true inches at 0° Cent., then at $t°$ the length of n divisions will be $n(1+lt)$ inches, l denoting the coefficient of linear expansion of the scale, the value of which for brass, the usual material, is ·00001878. If then the observed height h amounts to n divisions of the scale, we have

$$h_0(1+mt)=h=n(1+lt);$$

whence

$$h_0=\frac{n(1+lt)}{1+mt}=n-nt(m-l), \text{ nearly;}$$

that is to say, if n be the height read off on the scale, it must be diminished by the correction $nt(m-l)$, t denoting the temperature of the mercury in degrees Centigrade. The value of $m-l$ is ·0001614.

For temperatures Fahrenheit, assuming the scale to be of the correct length at 32° Fahr., the formula for the correction (which is still subtractive), is $n(t-32)(m-l)$, where $m-l$ has the value ·00008967.[1]

[1] The correction for temperature is usually made by the help of tables, which give its amount for all ordinary temperatures and heights. These tables, when intended for English barometers, are generally constructed on the assumption that the scale is of the correct length not at 32° Fahr., but at 62° Fahr., which is (by act of Parliament) the temperature at which the British standard yard (preserved in the office of the Exchequer) is correct. On this supposition, the length of n divisions of the scale at temperature $t°$ Fahr., is

$$n\left\{1+l(t-62)\right\};$$

and by equating this expression to

$$h_0\left\{1+m(t-32)\right\}$$

we find

$$h_0=n\left\{1-m(t-32)+l(t-62)\right\}$$
$$=n\left\{1-(m-l)t+(32m-62l)\right\}$$
$$=n\left\{1-\cdot 00008967\,t+\cdot 00255654\right\};$$

which, omitting superfluous decimals, may conveniently be put in the form—

$$n-\frac{n}{1000}(\cdot 09\,t-2\cdot 56).$$

The correction vanishes when

$$\cdot 09\,t-2\cdot 56=0;$$

that is, when $t=\frac{256}{9}=28\cdot 5$.

For all temperatures higher than this the correction is subtractive.

2. *Correction for Capillarity.*—In the preceding chapter we have seen that mercury in a glass tube undergoes a capillary depression: whence it follows that the observed barometric height is too small, and that we must add to it the amount of this depression. In all tubes of internal diameter less than about ¾ of an inch this correction is sensible; and its amount, for which no simple formula can be given, has been computed, from theoretical considerations, for various sizes of tube, by several eminent mathematicians, and recorded in tables, from which that given below is abridged. These values are applicable on the assumption that the meniscus which forms the summit of the mercurial column is decidedly convex, as it always is when the mercury is rising. When the meniscus is too flat, the mercury must be lowered by the foot-screw, and then screwed up again.

It is found by experiment, that the amount of capillary depression is only half as great when the mercury has been boiled in the tube as when this precaution has been neglected.

For purposes of special accuracy, tables have been computed, giving the amount of capillary depression for different degrees of convexity, as determined by the sagitta (or height) of the meniscus, taken in conjunction with the diameter of the tube. Such tables, however, are seldom used in this country.[1]

[1] The most complete collection of meteorological and physical tables, is that edited by Professor Guyot, and published under the auspices of the Smithsonian Institution, Washington.

TABLE OF CAPILLARY DEPRESSIONS IN UNBOILED TUBES.

(*To be halved for Boiled Tubes.*)

Diameter of tube in inches.	Depression in inches.	Diameter.	Depression.	Diameter.	Depression.
·10	·140	·20	·058	·40	·015
·11	·126	·22	·050	·42	·013
·12	·114	·24	·044	·44	·011
·13	·104	·26	·038	·46	·009
·14	·094	·28	·033	·48	·008
·15	·086	·30	·029	·50	·007
·16	·079	·32	·026	·55	·005
·17	·073	·34	·023	·60	·004
·18	·068	·36	·020	·65	·003
·19	·063	·38	·017	·70	·002

3. *Correction for Capacity.*—When there is no provision for adjusting the level of the mercury in the cistern to the zero point of the scale, another correction must be applied. It is called the correction for *capacity*. In barometers of this construction, which were formerly much more common than they are at present, there is a certain point in the scale at which the mercurial column stands when the mercury in the cistern is at the correct level. This is called the neutral point. If A be the interior area of the tube, and C the area of the cistern (exclusive of the space occupied by the tube and its contents), when the mercury in the tube rises by the amount x, the mercury in the cistern falls by an amount $y = \frac{A}{C}x$; for the volume of the mercury which has passed from the cistern into the tube is $Cy = Ax$. The change of atmospheric pressure is correctly measured by $x + y = \left(1 + \frac{A}{C}\right)x$; and if we now take x to denote the distance of the summit of the mercurial column from the neutral point, the corrected distance will be $\left(1 + \frac{A}{C}\right)x$, and the correction to be applied to the observed reading will be $\frac{A}{C}x$, which is additive if the observed reading be above the neutral point, subtractive if below.

It is worthy of remark that the neutral point depends upon the volume of mercury. It will be altered if any mercury be lost or added; and as temperature affects the volume, a special temperature-correction must be applied to barometers of this class. The investigation will be found in a paper by Professor Swan in the *Philosophical Magazine* for 1861.

In some modern instruments the correction for capacity is avoided, by making the divisions on the scale less than true inches, in the

ratio $\frac{C}{A+C}$, and the effect of capillarity is at the same time compensated by lowering the zero point of the scale. Such instruments, if correctly made, simply require to be corrected for temperature.

4. *Index Errors.*—Under this name are included errors of graduation, and errors in the position of the zero of the graduations. An error of zero makes all readings too high or too low by the same amount. Errors of graduation (which are generally exceedingly small) are different for different parts of the scale.

Barometers intended for accurate observation are now usually examined at Kew Observatory before being sent out; and a table is furnished with each, showing its index error at every half inch of the scale, errors of capillarity and capacity (if any) being included as part of the index error. We may make a remark here once for all respecting the signs attached to errors and corrections. The sign of an error is always opposite to that of its correction. When a reading is too high the index error is one of excess, and is therefore positive; whereas the correction needed to make the reading true is subtractive, and is therefore negative.

5. *Reduction to Sea-level.*—In comparing barometric observations taken over an extensive district for meteorological purposes, it is usual to apply a correction for difference of level. Atmospheric pressure, as we have seen, diminishes as we ascend; and it is usual to add to the observed height the difference of pressure due to the elevation of the place above sea-level. The amount of this correction is proportional to the observed pressure. The law according to which it increases with the height will be discussed in the next chapter.

6. *Correction for Unequal Intensity of Gravity.*—When two barometers indicate the same height, at places where the intensity of gravity is different (for example, at the pole and the equator), the same mass of air is superincumbent over both; but the pressures are unequal, being proportional to the intensity of gravity as measured by the values of g (§ 91) at the two places.

If h be the height, in centimetres, of the mercurial column at the temperature 0° Cent., the absolute pressure, in dynes per square centimetre, will be $gh \times 13{\cdot}596$; since $13{\cdot}596$ is the density of mercury at this temperature.

205. Other kinds of Mercurial Barometer.—The *Siphon Barometer*, which is represented in Fig. 114, consists of a bent tube, generally of uniform bore, having two unequal legs. The longer leg, which must be more than 30 inches long, is closed, while the shorter leg is open. A sufficient quantity of mercury having been introduced to fill the longer leg, the instrument is set upright (after boiling to expel air), and the mercury takes such a position that the difference of levels in the two legs represents the pressure of the atmosphere.

Fig. 114. Siphon Barometer.

Supposing the tube to be of uniform section, the mercury will always fall as much in one leg as it rises in the other. Each end of the mercurial column therefore rises or falls through only half the height corresponding to the change of atmospheric pressure.

In the best siphon barometers there are two scales, one for each leg, as indicated in the figure, the divisions on one being reckoned upwards, and on the other downwards, from an intermediate zero point, so that the sum of the two readings is the difference of levels of the mercury in the two branches.

Inasmuch as capillarity tends to depress both extremities of the mercurial column, its effect is generally neglected in siphon barometers; but practically it causes great difficulty in obtaining accurate observations, for according as the mercury is rising or falling its extremity is more or less convex, and a great deal of tapping is usually required to make both ends of the column assume the same form, which is the condition necessary for annihilating the effect of capillary action.

Wheel Barometer.—The wheel barometer, which is in more general use than its merits deserve, consists of a siphon barometer, the two branches of which have usually the same diameter. On the surface of the mercury of the open branch floats a small piece of iron or glass suspended by a thread, the other extremity of which is fixed to a pulley, on which the thread is partly rolled. Another thread, rolled parallel to the first, supports a weight which balances the float. To the axis of the pulley is fixed a needle which moves on a dial. When the level of the mercury varies in either direction, the float follows its movement through the same distance; by the action of the counterpoise the pulley turns, and with it the needle, the extremity of which points to the figures on the dial, marking the barometric heights. The mounting of the dial is usually placed

in front of the tube, so as to conceal its presence. The wheel barometer is a very old invention, and was introduced by the celebrated Hooke in 1683. The pulley and strings are sometimes replaced by a rack and pinion, as represented in the figure (Fig. 115).

Besides the faults incidental to the siphon barometer, the wheel

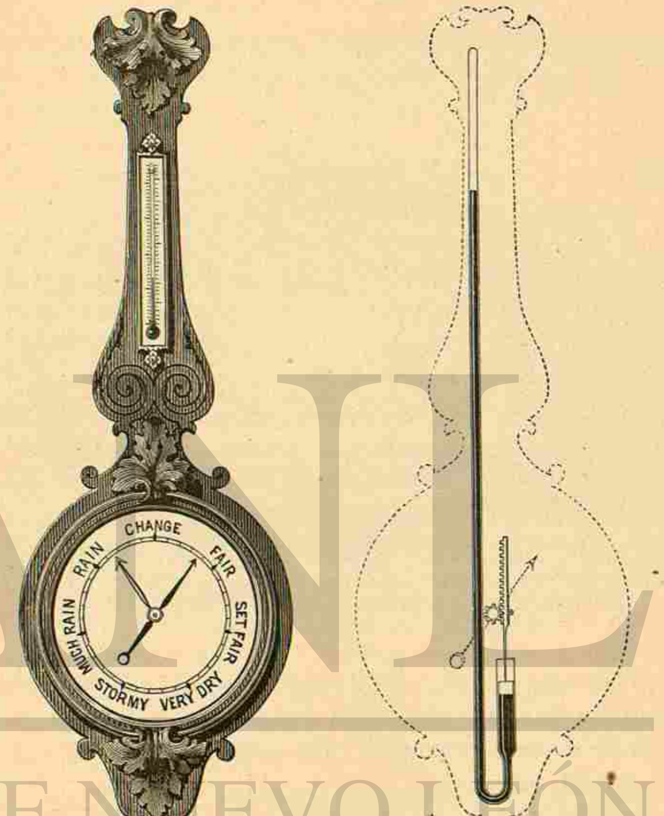


Fig. 115.—Wheel Barometer.

barometer is encumbered in its movements by the friction of the additional apparatus. It is quite unsuitable for measuring the exact amount of atmospheric pressure, and is slow in indicating changes.

Marine Barometer.—The ordinary mercurial barometer cannot be used at sea on account of the violent oscillations which the mercury would experience from the motion of the vessel. In order to meet this difficulty, the tube is contracted in its middle portion nearly to capillary dimensions, so that the motion of the mercury in either direction is hindered. An instrument thus constructed is called a marine barometer. When such an instrument is used on land it is always too slow in its indications.

206. Aneroid Barometer (α, νηρος).—This barometer depends upon the changes in the form of a thin metallic vessel partially exhausted

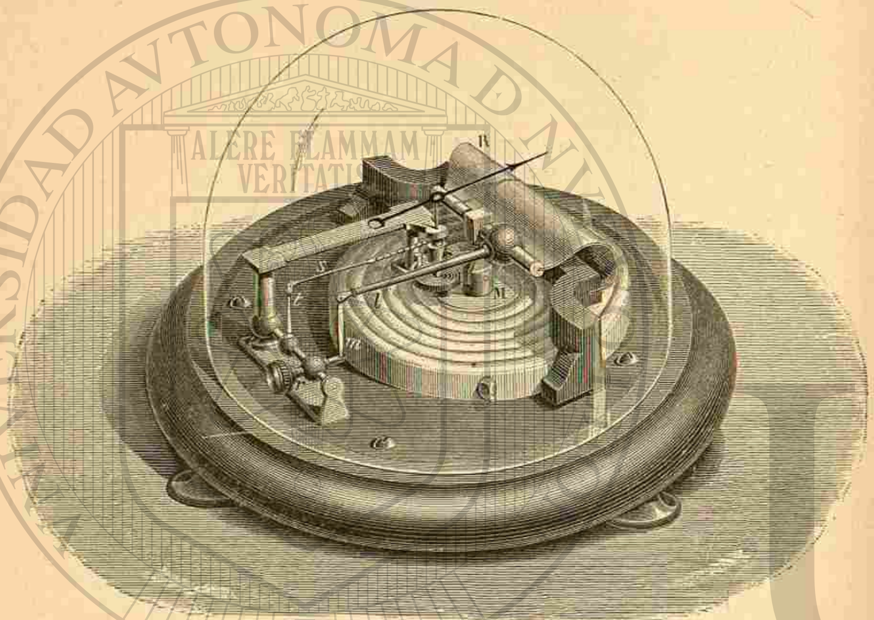


Fig. 116.—Aneroid Barometer.

of air, as the atmospheric pressure varies. M. Vidie was the first to overcome the numerous difficulties which were presented in the construction of these instruments. We subjoin a figure of the model which he finally adopted.

The essential part is a cylindrical box partially exhausted of air, the upper surface of which is corrugated in order to make it yield more easily to external pressure. At the centre of the top of the box is a small metallic pillar M, connected with a powerful steel spring R. As the pressure varies, the top of the box rises or falls, transmitting its movement by two levers *l* and *m*, to a metallic axis *r*. This latter carries a third lever *t*, the extremity of which is attached to a chain *s* which turns a drum, the axis of which bears the index needle. A spiral spring keeps the chain constantly stretched, and thus makes the needle always take a position corresponding to the shape of the box at the time. The graduation is performed empirically by comparison with a mercurial barometer. The aneroid barometer is very quick in indicating changes, and is much more portable than any form of mercurial barometer, being both lighter and less liable to injury. It is sometimes made small enough for the waistcoat pocket. It has the drawback of being affected by temperature to an extent which must be determined for each instrument separately, and of being liable to gradual changes which can only be checked by occasional comparison with a good mercurial barometer.

In the *metallic barometer*, which is a modification of the aneroid, the exhausted box is crescent-shaped, and the horns of the crescent separate or approach according as the external pressure diminishes or increases.

207. Old Forms Revived.—There are two ingenious modifications of the form of the barometer, which, after long neglect, have recently been revived for special purposes.

Counterpoised Barometer.—The invention of this instrument is attributed to Samuel Morland, who constructed it about the year 1680. It depends upon the following principle:—If the barometric tube is suspended from one of the scales of a balance, there will be required to balance it in the other scale a weight equal to the weight of the tube and the mercury contained in it, minus the upward pressure due to the liquid displaced in the cistern.[1] If the atmospheric pressure increases, the mercury will rise in the tube, and consequently the weight of the floating body will increase, while the sinking of the mercury in the cistern will diminish the upward pressure due to the displacement. The beam will thus incline to

[1] A complete investigation based on the assumption of a constant upward pull at the top of the suspended tube shows that the sensitiveness of the instrument depends only on the internal section of the upper part of the tube and the external section of its lower part. Calling the former A and the latter B, it is necessary for stability that B be greater than A (which is not the case in the figure in the text) and the movement of the tube will be to that of the mercury in a standard barometer as A is to $B - A$. The directions of these movements will be opposite. If $B - A$ is very small compared with A, the instrument will be exceedingly sensitive; and as $B - A$ changes sign, by passing through zero, the equilibrium becomes unstable.

A curious result of the investigation is that the level of the mercury in the cistern remains constant.

In the instrument represented in the figure, stability is probably obtained by the weight of the arm which carries the pencil.

In King's barograph, B is made greater than A by fixing a hollow iron drum round the lower end of the tube.

the side of the barometric tube, and the reverse movement would occur if the pressure diminished. For the balance may be substituted, as in Fig. 117, a lever carrying a counterpoise; the variations of pressure will be indicated by the movements of this lever.

Such an instrument may very well be used as a *barograph* or recording barometer; for this purpose we have only to attach to the lever an arm with a pencil, which is constantly in contact with a sheet of paper moved uniformly by clock-work. The result will be a continuous trace, whose form corresponds to the variations of pressure. It is very easy to determine, either by calculation or by comparison with a standard barometer, the pressure corresponding to a given position of the pencil on the paper; and thus, if the paper is ruled with twenty-four equidistant lines, corresponding to the twenty-four hours of the day, we can see at a glance what was the pressure at any given time. An arrangement of this kind has been adopted by the Abbé Secchi for the meteorograph of the observatory at Rome. The first successful employment of this kind of barograph appears to be due to Mr. Alfred King, a gas engineer of Liverpool, who invented and constructed such an instrument in 1853, for the use of the Liverpool Observatory, and subsequently designed a larger one, which is still in use, furnishing a very perfect record, magnified five-and-a-half times.

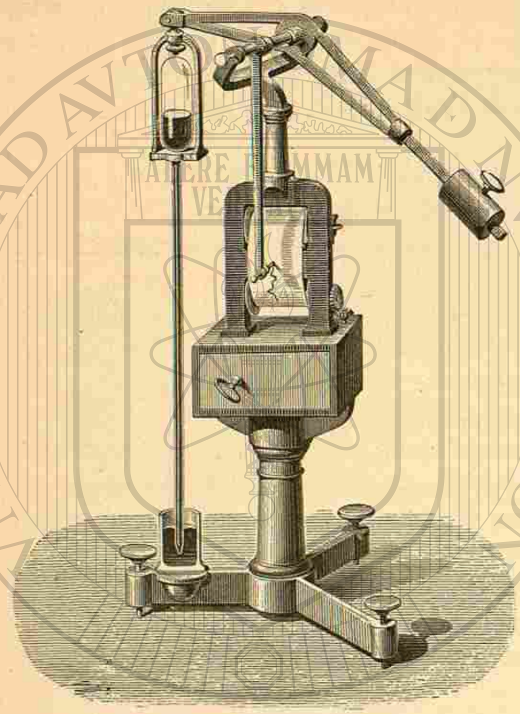


Fig. 117.—Counterpoised Barometer.

Fahrenheit's Barometer.—Fahrenheit's barometer consists of a tube bent several times, the lower portions of which contain mercury; the upper portions are filled with water, or any other liquid, usually

coloured. It is evident that the atmospheric pressure is balanced by the sum of the differences of level of the columns of mercury, diminished by the sum of the corresponding differences for the columns of water; whence it follows that, by employing a considerable number of tubes, we may greatly reduce the height of the barometric column. This circumstance renders the instrument interesting as a scientific curiosity, but at the same time diminishes its sensitiveness, and renders it unfit for purposes of precision. It is therefore never used for the measurement of atmospheric pressure; but an instrument upon the same principle has recently been employed for the measurement of very high pressures, as will be explained in Chap. xix.

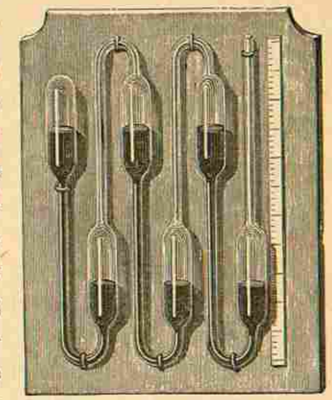


Fig. 118.—Fahrenheit's Barometer.

208. Photographic Registration.—Since the year 1847 various meteorological instruments at the Royal Observatory, Greenwich, have been made to yield continuous traces of their indications by the aid of photography, and the method is now generally employed at meteorological observatories in this country. The Greenwich system is fully described in the *Greenwich Magnetical and Meteorological Observations* for 1847, pp. lxiii.–xc. (published in 1849).

The general principle adopted for all the instruments is the same. The photographic paper is wrapped round a glass cylinder, and the axis of the cylinder is made parallel to the direction of the movement which is to be registered. The cylinder is turned by clock-work, with uniform velocity. The spot of light (for the magnets and barometer), or the boundary of the line of light (for the thermometers), moves, with the movements which are to be registered, backwards and forwards in the direction of the axis of the cylinder, while the cylinder itself is turned round. Consequently (as in Morin's machine, Chap. vii.), when the paper is unwrapped from its cylindrical form, there is traced upon it a curve of which the abscissa is proportional to the time, while the ordinate is proportional to the movement which is the subject of measure.

The barometer employed in connection with this system is a large siphon barometer, the bore of the upper and lower extremities of its arms being about 1·1 inch. A glass float in the quicksilver of the lower extremity is partially supported by a counterpoise acting on a light lever (which turns on delicate pivots), so that the wire supporting the float is constantly stretched, leaving a definite part of the weight of the float to be supported by the quicksilver. This lever is lengthened to carry a vertical plate of opaque mica with a small aperture, whose distance from the fulcrum is eight times the distance of the point of attachment of the float-wire, and whose movement, therefore (§ 205), is four times the movement of the column of a cistern barometer. Through this hole the light of a lamp, collected by a cylindrical lens, shines upon the photographic paper.

Every part of the cylinder, except that on which the spot of light falls, is covered with a case of blackened zinc, having a slit parallel to the axis of the cylinder; and by means of a second lamp shining through a small fixed aperture, and a second cylindrical lens, a base line is traced upon the paper, which serves for reference in subsequent measurements.

The whole apparatus, or any other apparatus which serves to give a continuous trace of barometric indications, is called a *barograph;* and the names *thermograph, magnetograph, anemograph,* &c., are similarly applied to other instruments for automatic registration. Such registration is now employed at a great number of observatories; and curves thus obtained are regularly published in the Quarterly Reports of the Meteorological Office.

CHAPTER XVIII.

VARIATIONS OF THE BAROMETER.

209. Measurement of Heights by the Barometer.—As the height of the barometric column diminishes when we ascend in the atmosphere, it is natural to seek in this phenomenon a means of measuring heights. The problem would be extremely simple, if the air had everywhere the same density as at the surface of the earth. In fact, the density of the air at sea-level being about 10,500 times less than that of mercury, it follows that, on the hypothesis of uniform density, the mercurial column would fall an inch for every 10,500 inches, or 875 feet that we ascend. This result, however, is far from being in exact accordance with fact, inasmuch as the density of the air diminishes very rapidly as we ascend, on account of its great compressibility.

210. Imaginary Homogeneous Atmosphere.—If the atmosphere were of uniform and constant density, its height would be approximately obtained by multiplying 30 inches by 10,500, which gives 26,250 feet, or about 5 miles.

More accurately, if we denote by H the height (in centimetres) of the atmosphere at a given time and place, on the assumption that the density throughout is the same as the observed density D (in grammes per cubic centimetre) at the base, and if we denote by P the observed pressure at the base (in dynes per square centimetre), we must employ the general formula for liquid pressure (§ 139)

$$P = g\,HD, \text{ which gives } H = \frac{P}{gD}. \qquad (1)$$

The height H, computed on this imaginary assumption, is usually called the *height of the homogeneous atmosphere,* corresponding to the pressure P, density D, and intensity of gravity g. It is sometimes called the *pressure-height.* The *pressure-height* at any point lower extremity is partially supported by a counterpoise acting on a light lever (which turns on delicate pivots), so that the wire supporting the float is constantly stretched, leaving a definite part of the weight of the float to be supported by the quicksilver. This lever is lengthened to carry a vertical plate of opaque mica with a small aperture, whose distance from the fulcrum is eight times the distance of the point of attachment of the float-wire, and whose movement, therefore (§ 205), is four times the movement of the column of a cistern barometer. Through this hole the light of a lamp, collected by a cylindrical lens, shines upon the photographic paper.

Every part of the cylinder, except that on which the spot of light falls, is covered with a case of blackened zinc, having a slit parallel to the axis of the cylinder; and by means of a second lamp shining through a small fixed aperture, and a second cylindrical lens, a base line is traced upon the paper, which serves for reference in subsequent measurements.

The whole apparatus, or any other apparatus which serves to give a continuous trace of barometric indications, is called a *barograph;* and the names *thermograph, magnetograph, anemograph,* &c., are similarly applied to other instruments for automatic registration. Such registration is now employed at a great number of observatories; and curves thus obtained are regularly published in the Quarterly Reports of the Meteorological Office.

CHAPTER XVIII.

VARIATIONS OF THE BAROMETER.

209. Measurement of Heights by the Barometer.—As the height of the barometric column diminishes when we ascend in the atmosphere, it is natural to seek in this phenomenon a means of measuring heights. The problem would be extremely simple, if the air had everywhere the same density as at the surface of the earth. In fact, the density of the air at sea-level being about 10,500 times less than that of mercury, it follows that, on the hypothesis of uniform density, the mercurial column would fall an inch for every 10,500 inches, or 875 feet that we ascend. This result, however, is far from being in exact accordance with fact, inasmuch as the density of the air diminishes very rapidly as we ascend, on account of its great compressibility.

210. Imaginary Homogeneous Atmosphere.—If the atmosphere were of uniform and constant density, its height would be approximately obtained by multiplying 30 inches by 10,500, which gives 26,250 feet, or about 5 miles.

More accurately, if we denote by H the height (in centimetres) of the atmosphere at a given time and place, on the assumption that the density throughout is the same as the observed density D (in grammes per cubic centimetre) at the base, and if we denote by P the observed pressure at the base (in dynes per square centimetre), we must employ the general formula for liquid pressure (§ 139)

$$P = g\,HD, \text{ which gives } H = \frac{P}{gD}. \qquad (1)$$

The height H, computed on this imaginary assumption, is usually called the *height of the homogeneous atmosphere,* corresponding to the pressure P, density D, and intensity of gravity g. It is sometimes called the *pressure-height.* The *pressure-height* at any point in a liquid or gas is the height of a column of fluid, having the same density as at the point, which would produce, by its weight, the actual pressure at the point. This element frequently makes its appearance in physical and engineering problems.

The expression for H contains P in the numerator and D in the denominator; and by Boyle's law, which we shall discuss in the ensuing chapter, these two elements vary in the same proportion, when the temperature is constant. Hence H is not affected by changes of pressure, but has the same value at all points in the air at which the temperature and the value of g are the same.

211. Geometric Law of Decrease.—The change of pressure as we ascend or descend *for a short distance* in the actual atmosphere, is sensibly the same as it would be in this imaginary "homogeneous atmosphere;" hence an ascent of 1 centimetre takes off $\frac{1}{H}$ of the total pressure, just as an ascent of one foot from the bottom of an ocean 60,000 feet deep takes off $\frac{1}{60000}$ of the pressure.

Since H is the same at all heights in any portion of the air which is at uniform temperature, it follows that in ascending by successive steps of 1 centimetre in air at uniform temperature, each step takes off the same fraction $\frac{1}{H}$ of the current pressure. The pressures therefore form a geometrical progression whose ratio is $1-\frac{1}{H}$. *In an atmosphere of uniform temperature,* neglecting the variation of g with height, *the densities and pressures diminish in geometrical progression as the heights increase in arithmetical progression.*

212. Computation of Pressure-height.—For perfectly dry air at 0° Cent., we have the data (§§ 195, 198),

$$D = \cdot 0012932 \text{ when } P = 1013600;$$

which give

$$\frac{P}{D} = 783800000 \text{ nearly.}$$

Taking g as 981, we have

$$H = \frac{783800000}{981} = 799000 \text{ centimetres nearly.}$$

This is very nearly 8 kilometres, or about 5 miles. At the temperature $t°$ Cent., we shall have

$$H = 799000\,(1 + \cdot 00366\,t). \qquad (2)$$

Hence in air at the the temperature 0° Cent., the pressure diminishes by 1 per cent. for an ascent of about 7990 centimetres or, say, 80 metres. At 20° Cent., the number will be 86 instead of 80

213. Formula for determining Heights by the Barometer.—To obtain an accurate rule for computing the difference of levels of two stations from observations of the barometer, we must employ the integral calculus.

Denote height above a fixed level by x, and pressure by p. Then we have

$$\frac{dx}{H} = -\frac{dp}{p};$$

and if p_1, p_2 are the pressures at the heights x_1, x_2, we deduce by integration

$$x_2 - x_1 = H\,(\log_e p_1 - \log_e p_2).$$

Adopting the value of H from (2), and remembering that Napierian logarithms are equal to common logarithms multiplied by 2·3026, we finally obtain

$$x_2 - x_1 = 1840000\,(1 + \cdot 00366\,t)\,(\log p_1 - \log p_2)$$

as the expression for the difference of levels, in centimetres. It is usual to put for t the arithmetical mean of the temperatures at the two stations.

The determination of heights by means of atmospheric pressure, whether the pressure be observed directly by the barometer, or indirectly by the boiling-point thermometer (which will be described in Part II.), is called *hypsometry* (ὕψος, height).

As a rough rule, it may be stated that, in ordinary circumstances, the barometer falls an inch in ascending 900 feet.

214. Diurnal Oscillation of the Barometer.—In these latitudes, the mercurial column is in a continual state of irregular oscillation; but in the tropics it rises and falls with great regularity according to the hour of the day, attaining two maxima in the twenty-four hours.

It generally rises from 4 A.M. to 10 A.M., when it attains its first maximum; it then falls till 4 P.M., when it attains its first minimum; a second maximum is observed at 10 P.M., and a second minimum at 4 A.M. The hours of maxima and minima are called the tropical hours (τρεπω, to turn), and vary a little with the season of the year. The difference between the highest maximum and lowest minimum is called the diurnal[1] *range*, and the half of this is called the *ampli-*

[1] The epithets *annual* and *diurnal*, when prefixed to the words *variation*, *range*, *amplitude*, denote the *period* of the variation in question; that is, the time of a complete oscillation. Diurnal variation does not denote variation from one day to another, but the variation which goes through its cycle of values in one day of twenty-four hours. Annual

tude of the diurnal oscillation. The amount of the former does not exceed about a tenth of an inch.

The character of this diurnal oscillation is represented in Fig. 119. The vertical lines correspond to the hours of the day; lengths have been measured upwards upon them proportional to the barometric heights at the respective hours, diminished by a constant quantity; and the points thus determined have been connected by a continuous curve. It will be observed that the two lower curves, one of which relates to Cumana, a town of Venezuela, situated in about 10° north latitude, show strongly marked oscillations corresponding to the maxima and minima. In our own country, the regular diurnal oscillation is masked by irregular fluctuations, so that a single day's observations give no clue to its existence. Nevertheless, on taking observations at regular hours for a number of consecutive days, and comparing the mean heights for the different hours, some indications of the law will be found. A month's observations will be sufficient for an approximate indication of the law; but observations extending over some years will be required, to establish with anything like precision the hours of maxima and the amplitude of the oscillation.

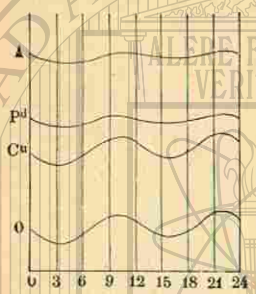


Fig. 119.
Curves of Diurnal Variation.

The two upper curves represent the diurnal variation of the barometer at Padua (lat. 45° 24′) and Abo (lat. 60° 56′), the data having been extracted from Kaemtz's *Meteorology*. We see, by inspection of the figure, that the oscillation in question becomes less strongly marked as the latitude increases. The range at Abo is less than half a millimetre. At about the 70th degree of north latitude it becomes insensible; and in approaching still nearer to the pole, it appears from observations, which however need further confirmation, that the oscillation is reversed; that is to say, that the maxima here are contemporaneous with the minima in lower latitudes.

There can be little doubt that the diurnal oscillation of the barometer is in some way attributable to the heat received from the sun, which produces expansion of the air, both directly, as a mere

range denotes the range that occurs within a year. This rule is universally observed by writers of high scientific authority.

A table, exhibiting the values of an element for each month in the year, is a table of annual (not monthly) variation; or it may be more particularly described as a table of variations from month to month.

consequence of heating, and indirectly, by promoting evaporation; but the precise nature of the connection between this cause and the diurnal barometric oscillation has not as yet been satisfactorily established.

215. Irregular Variations of the Barometer.—The height of the barometer, at least in the temperate zones, depends on the state of the atmosphere; and its variations often serve to predict the changes of weather with more or less certainty. In this country the barometer generally falls for rain or S.W. wind, and rises for fine weather or N.E. wind.

Barometers for popular use have generally the words—

Set fair.	Fair.	Change.	Rain.	Much rain.	Stormy.

marked at the respective heights

30·5	30	29·5	29	28·5	28 inches.

These words must not, however, be understood as absolute predictions. A low barometer rising is generally a sign of fine, and a high barometer falling of wet weather. Moreover, it is to be borne in mind that the barometer stands about a tenth of an inch lower for every hundred feet that we ascend above sea-level.

The connection between a low or falling barometer and wet weather is to be found in the fact that moist air is specifically lighter than dry, even at the same temperature, and still more when, as usually happens, moist air is warmer than dry.

Change of wind usually begins in the upper regions of the air and gradually extends downwards to the ground; hence the barometer, being affected by the weight of the whole superincumbent atmosphere, gives early warning.

216. Weather Charts. Isobaric Lines.—The probable weather can be predicted with much greater certainty if the height of the barometer at surrounding places is also known. The weather forecasts issued daily from the Meteorological Office in London are based on reports received twice a day from about sixty stations scattered over the west of Europe, from the north of Norway to Lisbon, and from the west of Ireland to Berlin. The reading of the barometer reduced to sea-level at each place is recorded on a chart, and curves called *isobaric lines* or *isobars* are drawn through places at which the pressure has given values, proceeding usually by steps of a tenth of an inch. Curves called *isothermal lines* or *isotherms* are also drawn through places of equal temperature. The strength and direction of wind, and the state of weather and of sea are also entered. The charts are compared with those of the previous day, and from the changes in progress the ensuing weather can be inferred with a fair probability of success.

The isobars furnish the most important aid in these forecasts; for from their form and distribution the direction and strength of the wind in each district can be inferred, and to a certain extent the state of the weather generally. As a rule the wind blows from places of higher to places of lower pressure, but not in the most direct line. It deviates more than 45° to the right of the direct line in the northern hemisphere, and to the left in the southern. This is known as Buys Ballot's law, and is a consequence of the earth's rotation.[1]

Very frequently a number of isobars form closed curves, encircling an area of low pressure, to which, in accordance with the above law, the wind blows spirally inwards, in the direction of watch-hands in the southern hemisphere, and against watch-hands in the northern. This state of things is called a *cyclone*. Cyclones usually approach the British Islands from the Atlantic, travelling in a north-easterly direction with a velocity of from ten to twenty miles an hour; sometimes disappearing within a day of their formation, and sometimes lasting for several days. They are the commonest type of distribution of pressure in western Europe, and are usually accompanied by unsettled weather.

The opposite state of things,—that is, a centre of maximum pressure from which the wind blows out spirally with watch-hands in the northern and against watch-hands in the southern hemisphere is called an *anticyclone*. It is usually associated with light winds and fine weather, and is favourable to frost in winter. Anticyclones usually move and change slowly.

The names *cyclone* and *anticyclone* are frequently applied to the distributions of pressure above indicated without taking account of the wind.

The strength of wind generally bears some proportion to the

[1] The influence of the earth's rotation in modifying the direction of winds is discussed in a paper "On the General Circulation and Distribution of the Atmosphere," by the editor of this work, in the *Philosophical Magazine* for September, 1871. Some of the results are stated in the last chapter of Part II. of the present work.

steepness of the barometric gradient, in other words to the closeness of the isobars. Violent storms of wind are usually cyclones, and it was to these that the name was first applied. The phenomenon reaches its extreme form in the tornadoes of tropical regions. The persistence of a cyclone can be explained by the fact that the centrifugal force of the spirally moving air tends to increase the original central depression.

The frontispiece of this volume is a chart of pressure and wind for the United States of America at 4·35 P.M. Washington time on the 15th of January, 1877, when a great storm was raging. The figures marked against the isobars are the pressures to tenths of an inch. They exhibit a very steep gradient on the north-west side of the central depression—a tenth of an inch for about forty-three nautical miles. The direction of the wind is shown by arrows, and the number of feathers on each arrow multiplied by five gives the velocity of the wind in miles per hour. It will be seen that the strongest winds are in or near the region of steepest gradient, and that the directions of the winds are for the most part in accordance with Buys Ballot's law.

CHAPTER XIX.

BOYLE'S (OR MARIOTTE'S) LAW.[1]

217. Boyle's Law.—All gases exhibit a continual tendency to expand, and thus exert pressure against the vessels in which they are confined. The intensity of this pressure depends upon the volume which they occupy, increasing as this volume diminishes. By a number of careful experiments upon this point, Boyle and Mariotte independently established the law that this pressure varies inversely as the volume, provided that the temperature remain constant. As the density also varies inversely as the volume, we may express the law in other words by saying that at the same temperature the density varies directly as the pressure.

If V and V′ be the volumes of the same quantity of gas, P and P′, D and D′, the corresponding pressures and densities, Boyle's law will be expressed by either of the equations

$$\frac{P}{P'} = \frac{V'}{V}, \quad \frac{P}{P'} = \frac{D}{D'}.$$

218. Boyle's Tube.—The correctness of this law may be verified by means of the following apparatus, which was employed by both the experimenters above named. It consists (Fig. 120) of a bent tube with branches of unequal length; the long branch is open, and the short branch closed. The tube is fastened to a board provided with two scales, one by the side of each branch. The

[1] Boyle, in his *Defence of the Doctrine touching the Spring and Weight of the Air against the Objections of Franciscus Linus*, appended to *New Experiments, Physico-mechanical*, &c. (second edition, 4to, Oxford, 1662), describes the two kinds of apparatus represented in Figs. 120, 121 as having been employed by him, and gives in tabular form the lengths of tube occupied by a body of air at various pressures. These observed lengths he compares with the theoretical lengths computed on the assumption that volume varies reciprocally as pressure, and points out that they agree within the limits of experimental error.

Mariotte's treatise, *De la Nature de l'Air*, is stated in the *Biographie Universelle* to have been published in 1679. (See Preface to Tait's *Thermodynamics*, p. iv.)

graduation of both scales begins from the same horizontal line through 0, 0. Mercury is first poured in at the extremity of the long branch, and by inclining the apparatus to either side, and cautiously adding more of the liquid if required, the mercury can be made to stand at the same level in both branches, and at the zero of both scales. Thus we have, in the short branch, a quantity of air separated from the external air, and at the same pressure. Mercury is then poured into the long branch, so as to reduce the volume of this inclosed air by one-half; it will then be found that the difference of level of the mercury in the two branches is equal to the height of the barometer at the time of the experiment; the compressed air therefore exerts a pressure equal to that of two atmospheres. If more mercury be poured in so as to reduce the volume of the air to one-third or one-fourth of the original volume, it will be found that the difference of level is respectively two or three times the height of the barometer; that is, that the compressed air exerts a pressure equal respectively to that of three or four atmospheres. This experiment therefore shows that if the volume of the gas becomes two, three, or four times as small, the pressure becomes two, three, or four times as great. This is the principle expressed in Boyle's law.

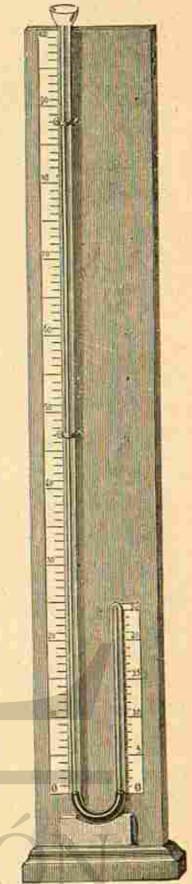


Fig. 120. Boyle's Tube.

The law may also be verified in the case where the gas expands, and where its pressure consequently diminishes. For this purpose a barometric tube (Fig. 121), partially filled with mercury, is inverted in a tall vessel, containing mercury also, and is held in such a position that the level of the liquid is the same in the tube and in the vessel. The volume occupied by the gas is marked, and the tube is raised; the gas expands, its pressure diminishes, and, in virtue of the excess of the atmospheric pressure, a column of mercury *ab* rises in the tube, such that its height, added to the pressure of the expanded air, is equal to the atmospheric pressure. It will then be seen that if the volume of air becomes double what it was before, the height of the column raised is one-half that of the barometer; that is, the expanded air exerts a pressure equal to half that of the atmosphere. If the volume is trebled, the height of the column is two-thirds that of the barometer; that is, the pressure of the expanded air is one-third that of the atmosphere, a result in accordance with Boyle's law.

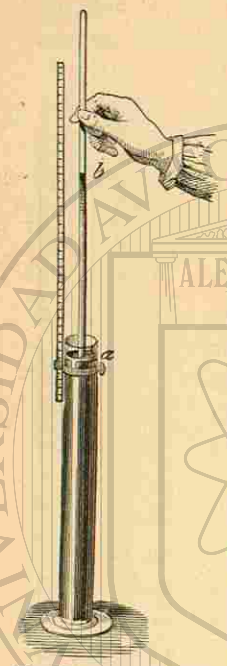


Fig. 121.—Proof of Boyle's Law for Expanding Air.

219. Despretz's Experiments.—The simplicity of Boyle's law, taken in conjunction with its apparent agreement with facts, led to its general acceptance as a rigorous truth of nature, until in 1825 Despretz published an account of experiments, showing that different gases are unequally compressible. He inverted in a cistern of mercury several cylindrical tubes of equal height, and filled them with different gases. The whole apparatus was then inclosed in a strong glass vessel filled with water, and having a screw piston as in Œrsted's piesometer (§ 130). On pressure being applied, the mercury rose to unequal heights in the different tubes, carbonic acid for example being more reduced in volume than air. These experiments proved that even supposing Boyle's law to be true for one of the gases employed, it could not be rigorously true for more than one.

In 1829 Dulong and Arago undertook a laborious series of experiments with the view of testing the accuracy of the law as applied to air; and the results which they obtained, even when the pressure was increased to twenty-seven atmospheres, agreed so nearly with it as to confirm them in the conviction that, for air at least, it was rigorously true. When re-examined, in the light of later researches, the results obtained by Dulong and Arago seem to point to a different conclusion.

220. Unequal Compressibility of Different Gases.—The unequal compressibility of different gases, which was first established by Despretz's experiments above described, is now usually exhibited by the aid of an apparatus designed by Pouillet (Fig. 122). A is a cast-iron reservoir, containing mercury surmounted by oil. In this latter liquid dips a bronze plunger P, the upper part of which has a thread cut upon it, and works in a nut, so that the plunger can be screwed up or down by means of the lever L. The reservoir A communicates

by an iron tube with another cast-iron vessel, into which are firmly fastened two tubes TT about six feet in length and $\frac{1}{10}$th of an inch in internal diameter, very carefully calibrated. Equal volumes of two gases, perfectly dry, are introduced into these tubes through their upper ends, which are then hermetically sealed. The plunger is then made to descend, and a gradually increasing pressure is exerted, the volumes occupied by the gases are measured, and it is ascertained that no two gases follow precisely the same law of compression. The difference, however, is almost insensible when the gases employed are those which are very difficult to liquefy, as air, oxygen, hydrogen, nitrogen, nitric oxide, and marsh-gas. But when we compare any one of these with one of the more liquefiable gases, such as carbonic acid, cyanogen, or ammonia, the difference is rapidly and distinctly manifested. Thus, under a pressure of twenty-five atmospheres, carbonic acid occupies a volume which is only $\frac{4}{5}$ths of that occupied by air.

221. Regnault's Experiments.—Boyle's law, therefore, is not to be considered as rigorously exact; but it is so nearly exact that to demonstrate its inaccuracy for one of the more permanent gases, and still more to determine the law of deviation for each gas, very precise methods of measurement are necessary. In ordinary experiments on compression, and even in the elaborate investigations of Dulong and Arago, a definite portion of gas is taken and successively diminished in volume by the application of continually increasing pressure. In experiments of this kind, as the pressure increases, the volume under measurement becomes smaller, and the precision with which it can be measured consequently diminishes.

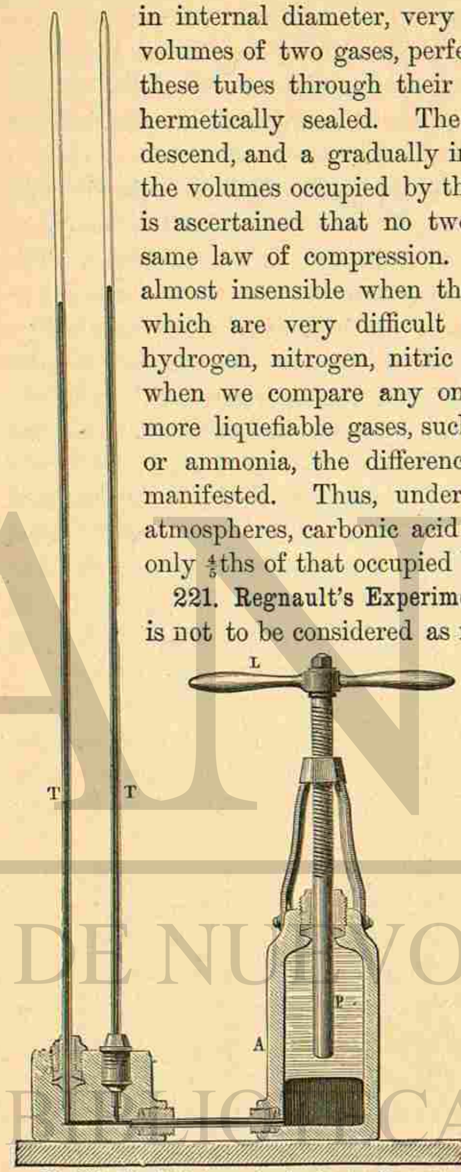


Fig. 122.—Pouillet's Apparatus for showing Unequal Compressibility of Different Gases.

Regnault adopted the plan of operating in all cases upon the same volume of gas, which being initially at different pressures, was always reduced to one-half. The pressure was observed before and after this operation, and, if Boyle's law were true, its value should be found to be doubled. In this way the same precision of measurement is obtained at high as at low pressures.

A general view of Regnault's apparatus is given in Fig. 123. There is an iron reservoir containing mercury, furnished at the top with a force-pump for water. The lower part of this reservoir communicates with a cylinder which is also of iron, and in which are two openings to admit tubes. Communication between the reservoir and the cylinder can be established or interrupted by means of a stop-cock R, of very exact workmanship. Into one of the openings is fitted the lowest of a series of glass tubes A, which are placed end to end, and firmly joined to each other by metal fittings, so as to form a vertical column of about twenty-five metres in height.

The height of the mercurial column in this long manometric tube could be exactly

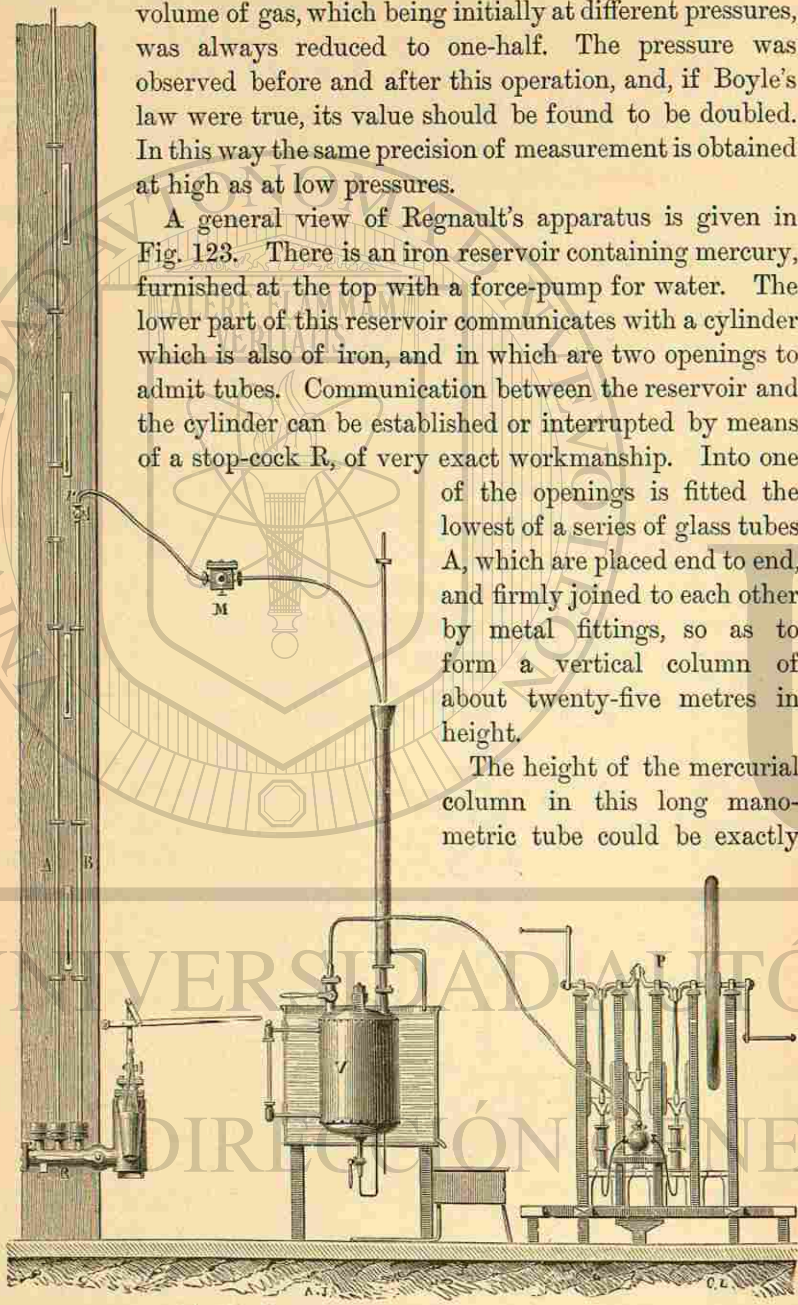


Fig. 123.—Regnault's Apparatus for Testing Boyle's Law.

determined by means of reference marks placed at distances of about ·95 of a metre, and by the graduation on the tubes forming the upper part of the column. The mean temperature of the mercurial column was given by thermometers placed at different heights. Into the second opening in the cylinder fits the lower extremity of the tube B, which is divided into millimetres, and also gauged with great accuracy. This tube has at its upper end a stop-cock *r* which can open communication with the reservoir V, into which the gas to be operated on is forced and compressed by means of the pump P.

An outer tube, which is not shown in the figure, envelops the tube B, and, being kept full of water, which is continually renewed, enables the operator to maintain the tube at a temperature sensibly constant, which is indicated by a very delicate thermometer. Before fixing the tube in its place, the point corresponding to the middle of its volume is carefully ascertained, and after the tube has been permanently fixed, the distance of this point from the nearest of the reference marks is observed.[1]

After these explanatory remarks we may describe the mode of conducting the experiments. The gas to be operated on, after being first thoroughly dried, was introduced at the upper part of the tube B, the stop-cock of the pump being kept open, so as to enable the gas to expel the mercury and occupy the entire length of the tube. The force-pump was then brought into play, and the gas was reduced to about half of its former volume; the pressure in both cases being ascertained by observing the height of the mercury in the long tube above the nearest mark. It is important to remark that it is not at all necessary to operate always upon exactly the same initial volume, and reduce it exactly to one-half, which would be a very tedious operation; these two conditions are approximately fulfilled, and the graduation of the tube enables the observer always to ascertain the actual volumes.

222. Results.—The general result of the investigations of Regnault

[1] Regnault's apparatus was fixed in a small square tower of about fifteen metres in height, forming part of the buildings of the Collége de France, and which had formerly been built by Savart for experiments in hydraulics. The tower could therefore contain only the lower part of the manometric column; the upper part rose above the platform at the top of the tower, resting against a sort of mast which could be ascended by the observer. The readings inside the tower could be made by means of a cathetometer, but this was impossible in the upper portion of the column, and for this reason the tubes forming this portion were graduated.—*D.*

is, that Boyle's law does not exactly represent the compressibility even of air, hydrogen, or nitrogen, which, with carbonic acid, were the gases operated on by him. He found that for all the gases on which he operated, except hydrogen, the product VP of the volume and pressure, instead of remaining constant, as it would if Boyle's law were exact, diminished as the compression was increased. This diminution is particularly rapid in the cases of the more liquefiable gases, such as carbonic acid, at least when the experiments are conducted at ordinary atmospheric temperatures. The lower the temperature, the greater is the departure from Boyle's law in the case of these gases. For hydrogen, he found the departure from Boyle's law to be in the opposite direction;—the product VP increased as the gas was more compressed.

223. Manometers or Pressure-gauges.—Manometers or pressure-gauges are instruments for measuring the elastic force of a gas or vapour contained in the interior of a closed space. This elastic force is generally expressed in units called atmospheres (§ 198), and is often measured by means of a column of mercury.

When one end of the column of mercury is open to the air, as in Regnault's experiments above described, the gauge is called an open mercurial gauge.

The open mercurial pressure-gauge is often used in the arts to measure pressures which are not very considerable. Fig. 124 represents one of its simplest forms. The apparatus consists of a box, generally of iron, at the top of which is an opening closed by a screw stopper, which is traversed by the tube *b*, open at both ends, and dipping into the mercury in the box. The air or vapour whose elastic force is to be measured enters by the tube *a*, and presses upon the mercury. It is evident that if the level of the liquid in the box is the same as in the tube, the pressure in the box must be exactly equal to that of the atmosphere. If the mercury in the tube rises above that in the box, the pressure of the air in the box must exceed that of the atmosphere by a pressure corresponding to the height of the column raised. The pressures are generally marked in atmospheres upon a scale beside the tube.

224. Multiple Branch Manometer.—When the pressures to be measured are considerable, as in the boiler of a high-pressure steam-engine, the above instrument, if employed at all, must be of a length corresponding to the pressure. If, for instance, the pressure in question is eight atmospheres, the length of the tube must be at least

8 × 30 inches = 20 feet. Such an arrangement is inconvenient even for stationary machines, and is entirely inapplicable to movable machines.

Without departing from the principle of the open mercurial pressure-gauge, namely, the balancing of the pressure to be observed against the weight of a liquid increased by one atmosphere, we may reduce the length of the instrument by an artifice already employed by Fahrenheit in his barometer (§ 207).

The apparatus for this purpose consists of an iron tube ABCD

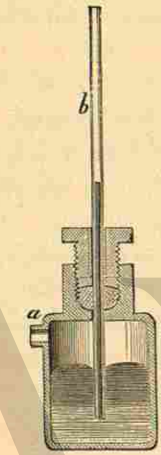


Fig. 124.—Open Mercurial Manometer.

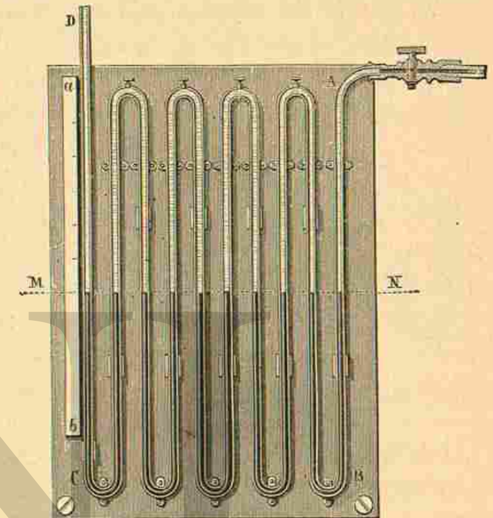


Fig. 125.—Multiple Branch Manometer.

(Fig. 125) bent back upon itself several times. The extremity A communicates with the boiler by a stop-cock, and the last branch CD is of glass, with a scale by its side.

The first step is to fill the tube with mercury as far as the level MN. At this height are holes by which the mercury escapes when it reaches them, and which are afterwards hermetically sealed. The upper portions are filled with water through openings which are also stopped after the tube has been filled. If the mercury in the first tube, which is in communication with the reservoir of gas, falls through a distance h, it will alternately rise and fall through the same distance in the other tubes. The difference of pressure between the two ends of the gauge is represented by the weight of a column of mercury of height $10h$ diminished by the weight of a column of water of height $8h$. Reduced to mercury, the difference of pressure is therefore $10h - \frac{8h}{13{\cdot}5} = 9{\cdot}4h$.

225. Compressed-air Manometer.—This instrument, which may assume different forms, sometimes consists, as in Fig. 126, of a bent tube AB closed at one end a, and containing within the space Aa a quantity of air, which is cut off from external communication by a column of mercury. The apparatus has been so constructed, that when the pressure on B is equal to that of the atmosphere, the mercury stands at the same height in both branches; so that, under these circumstances, the inclosed air is exactly at atmospheric pressure. But if the pressure increases, the mercury is forced into the left branch, so that the air in that branch is compressed, until equilibrium is established. The pressure exerted by the gas at B is then equal to the pressure of the compressed air, together with that of a column of mercury equal to the difference of level of the liquid in the two branches. This pressure is usually expressed in atmospheres on the scale ab.

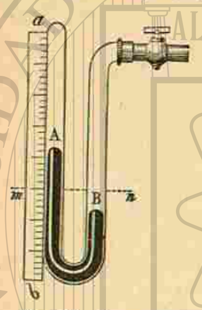


Fig. 126.—Compressed-air Manometer.

The graduation of this scale is effected empirically in practice, by placing the manometer in communication with a reservoir of compressed air whose pressure is given by an open mercurial gauge, or by a standard manometer of any kind.

If the tube AB be supposed cylindrical, the graduation can be calculated by an application of Boyle's law.

Let l be the length of the tube occupied by the inclosed air when its pressure is equal to that of one atmosphere; at the point to which the level of the mercury rises is marked the number 1. It is required to find what point the end of the liquid column should reach when a pressure of n atmospheres is exerted at B. Let x be the height of this point above 1; then the volume of the air, which was originally l, has become $l-x$, and its pressure is therefore equal to $H\frac{l}{l-x}$, H being the mean height of the barometer. This pressure, together with that due to the difference of level $2x$, is equivalent to n atmospheres. We have thus the equation—

$$H\frac{l}{l-x}+2x=nH,$$

whence

$$2x^2-(nH+2l)x+(n-1)Hl=0.$$

$$x=\frac{nH+2l\pm\sqrt{(nH+2l)^2-8(n-1)Hl}}{4}.$$

We thus find two values of x; but that given by taking the positive

sign of the radical is inadmissible; for if we put $n=1$, we ought to have $x=o$, which will not be the case unless the sign of the radical is negative.

By giving n the successive values $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, &c., in this expression for x, we find the points on the scale corresponding to pressures of one atmosphere and a half, two atmospheres, &c.

As the pressure increases, the distance traversed by the mercury for an increment of pressure equal to one atmosphere becomes continually less, and the sensibility of the instrument accordingly decreases. This inconvenience is partly avoided by the arrangement shown in Fig. 127. The branch containing the air is made tapering so that, as the mercury rises, equal changes of volume correspond to increasing lengths.

226. Metallic Manometers.—The fragility of glass tubes, and the fact that they are liable to become opaque by the mercury clinging

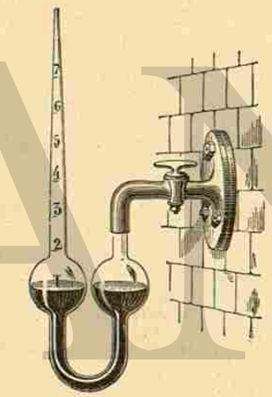


Fig. 127.—Compressed air Manometer.

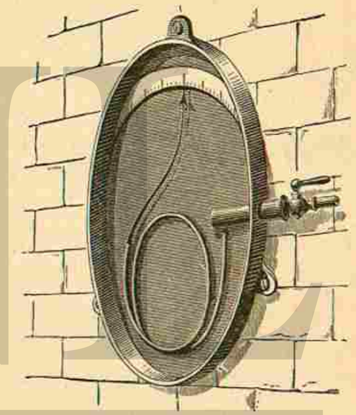


Fig. 128.—Bourdon's Pressure gauge.

to their sides, are serious drawbacks to their use, especially in machines in motion. Accordingly, metallic manometers are often employed, their indications depending upon changes of form effected by the pressure of gas on its containing vessel. We shall here mention only Bourdon's gauge (Fig. 128). It consists essentially of a copper tube of elliptic section, which is bent through about 540°, as represented in the figure. One of the extremities communicates by a stop-cock with the reservoir of steam or compressed gas; to the other extremity is attached a steel needle which traverses a scale. When the pressure is the same within and without the tube the end of the needle stands at the mark 1; but if the pressure within the tube increases, the curvature diminishes, the free extremity of the tube moves away from the fixed extremity, and the needle traverses the scale.

227. Mixture of Gases.—When gases of different densities are inclosed in the same space, experiment shows that, even under the most unfavourable circumstances, an intimate mixture takes place, so that each gas becomes uniformly diffused through the entire space. This fact has been shown by a decisive experiment due to Berthollet. He took two globes (Fig. 129) which could be screwed together, and placed them in a cellar. The lower globe was filled with carbonic acid, the upper globe with hydrogen. Communication was established between them, and after some time it was ascertained that the gases had become uniformly mixed; the proportions being the same in both globes. Gaseous diffusion is a comparatively rapid process. The diffusion of liquids, when not assisted by gravity, is, on the other hand, exceedingly slow.

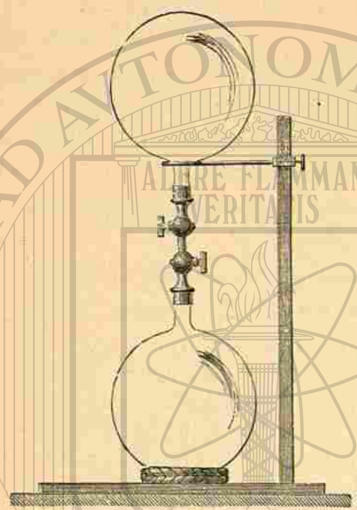


Fig. 129.—Mixture of Gases.

If several gases are inclosed in the same space, each of them exerts the same pressure as if the others were absent, in other words, the pressure exerted by the mixture is equal to the sum of the pressures which each would exert separately. This is known as "Dalton's law for gaseous mixtures." The separate pressures can easily be calculated by Boyle's law, when the original pressure and volume of each gas are known.

For example, let V and P, V′ and P′, V″ and P″ be the volumes and pressures of the gases which are made to pass into a vessel of volume U. The first gas exerts, when in this vessel, a pressure equal to $\frac{VP}{U}$, the second a pressure equal to $\frac{V'P'}{U}$, the third a pressure equal to $\frac{V''P''}{U}$, and so on, so that the total pressure M is equal to $\frac{VP}{U}+\frac{V'P'}{U}+\frac{V''P''}{U}$, whence $MU = VP + V'P' + V''P''$.

This law can easily be verified by passing different volumes of

gas into a graduated glass jar inverted over mercury, after having first measured their volumes and pressures. It may be observed that Boyle's law is merely a particular case of this. It is what this law becomes when applied to a mixture of two portions of the same gas.

228. Absorption of Gases by Liquids and Solids.—All gases are to a greater or less extent soluble in water. This property is of considerable importance in the economy of nature; thus the life of aquatic animals and plants is sustained by the oxygen of the air which the water holds in solution. The *volume* of a given gas that can be dissolved in water at a given temperature is generally found to be approximately the same at all pressures,[1] and the ratio of this volume to that of the water which dissolves it is called the *coefficient of solubility, or of absorption.* At the temperature 0° Cent., the coefficient of solubility for carbonic acid is 1, for oxygen ·04, and for ammonia 1150.

If a mixture of two or more gases be placed in contact with water, each gas will be dissolved to the same extent as if it were the only gas present.

Other liquids as well as water possess the power of absorbing gases, according to the same laws, but with coefficients of solubility which are different for each liquid.

Increase of temperature diminishes the coefficient of solubility, which is reduced to zero when the liquid boils.

Some solids, especially charcoal, possess the power of absorbing gases. Boxwood charcoal absorbs about nine times its volume of oxygen, and about ninety times its volume of ammonia. When saturated with one gas, if put into a different gas, it gives up a portion of that which it first absorbed, and takes up in its place a quantity of the second. Finely-divided platinum condenses on the surface of its particles a large quantity of many gases, amounting in the case of oxygen to many times its own volume. If a jet of hydrogen gas be allowed to fall, in air, upon a ball of spongy platinum, the gas combines rapidly, in the pores of the metal, with the oxygen of the air, giving out an amount of heat which renders the platinum incandescent and usually sets fire to the jet of hydrogen.

Most solids have in ordinary circumstances a film of air adhering

[1] Hence the *mass* of gas absorbed is directly as the pressure.

to their surfaces. Hence iron filings, if carefully sprinkled on water, will not be wetted, but will float on the surface, and hence also the power which many insects have of running on the surface of water without wetting their feet. The film of air in these cases prevents wetting, and hence, by the principles of capillarity, produces increased buoyancy.

CHAPTER XX.

AIR-PUMP.

229. Air-pump.—The air-pump was invented by Otto Guericke about 1650, and has since undergone some improvements in detail which have not altered the essential parts of its construction.

Fig. 130 represents the pattern most commonly adopted in France. It contains a glass or metal cylinder called the barrel, in which a piston works. This piston has an opening through it which is closed at the lower end by a valve S opening upwards. The barrel

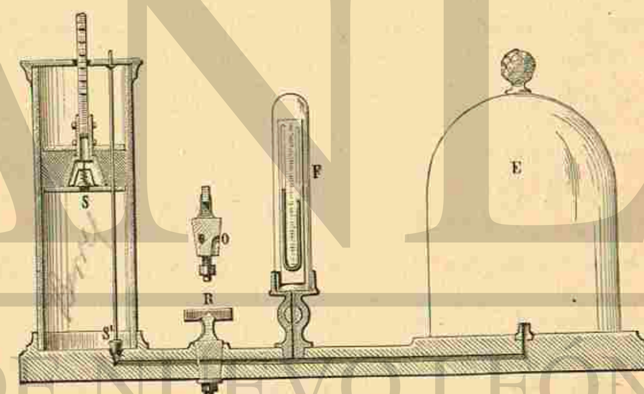


Fig. 130.—Air pump

communicates with a passage leading to the centre of a brass surface carefully polished, which is called the *plate* of the air-pump. The entrance to the passage is closed by a conical stopper S′, at the extremity of a metal rod which passes through the piston-head and works in it tightly, so as to be carried up and down with the motion of the piston. A catch at the upper part of the rod confines its motion within very narrow limits, and only permits the stopper to rise a small distance above the opening.

to their surfaces. Hence iron filings, if carefully sprinkled on water, will not be wetted, but will float on the surface, and hence also the power which many insects have of running on the surface of water without wetting their feet. The film of air in these cases prevents wetting, and hence, by the principles of capillarity, produces increased buoyancy.

CHAPTER XX.

AIR-PUMP.

229. Air-pump.—The air-pump was invented by Otto Guericke about 1650, and has since undergone some improvements in detail which have not altered the essential parts of its construction.

Fig. 130 represents the pattern most commonly adopted in France. It contains a glass or metal cylinder called the barrel, in which a piston works. This piston has an opening through it which is closed at the lower end by a valve S opening upwards. The barrel

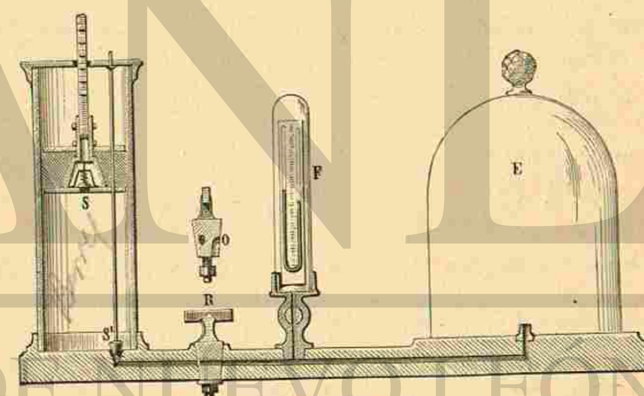


Fig. 130.—Air pump

communicates with a passage leading to the centre of a brass surface carefully polished, which is called the *plate* of the air-pump. The entrance to the passage is closed by a conical stopper S′, at the extremity of a metal rod which passes through the piston-head and works in it tightly, so as to be carried up and down with the motion of the piston. A catch at the upper part of the rod confines its motion within very narrow limits, and only permits the stopper to rise a small distance above the opening.

Suppose now that the piston is at the bottom of the cylinder, and is raised. The valve S′ is opened, and air from the receiver E rushes into the cylinder. On lowering the piston, the valve S′ closes its opening, the air which has entered the cylinder cannot return into the receiver, and, on being compressed, raises the valve S in the piston, and escapes into the air outside. On raising the piston again, a portion of the air remaining in the receiver will pass into the cylinder, whence it will escape on pushing down the piston, and so on.

We see, then, that if this motion be continued, a fresh portion of the air in the receiver will be removed at each successive stroke. But as the quantity of air removed at each stroke is only a fraction of the quantity which was in the receiver at the beginning of the stroke, we can never produce a perfect vacuum, though we might approach as near to it as we pleased if this were the only obstacle.

230. Theoretical Rate of Exhaustion.—It is easy to calculate the quantity of air left in the receiver after a given number of strokes of the piston. Let V be the volume of the barrel, V′ that of the receiver, and M the mass of air in the receiver at first. On raising the piston, the air which occupied the volume V′ occupies a volume V′ + V; of the air thus expanded the volume V is removed, and the volume V′ left, being $\frac{V'}{V'+V}$ of the whole quantity or mass M. The quantity remaining after the second stroke is $\frac{V'}{V'+V}$ of that after the first, or is $\left(\frac{V'}{V'+V}\right)^2 M$; and after n strokes $\left(\frac{V'}{V'+V}\right)^n M$. Hence the density and (by Boyle's law) the pressure are each reduced by n strokes to $\left(\frac{V'}{V'+V}\right)^n$ of their original values.

This calculation gives the theoretical rate of exhaustion for a perfect pump. Ordinary pumps come nearly up to this standard during the earlier part of the process of exhaustion; but as further progress is made, the imperfections of the apparatus become more sensible, and set a limit to the exhaustion attainable.

231. Mercurial Gauges.—To enable the operator to observe the progress of the exhaustion, the instrument is usually provided with a mercurial gauge. Sometimes, as in Fig. 130, this consists of a short siphon-barometer, the difference of levels between its two columns being the measure of the pressure in the receiver. Another plan is to have a straight tube open at both ends, and more than 30

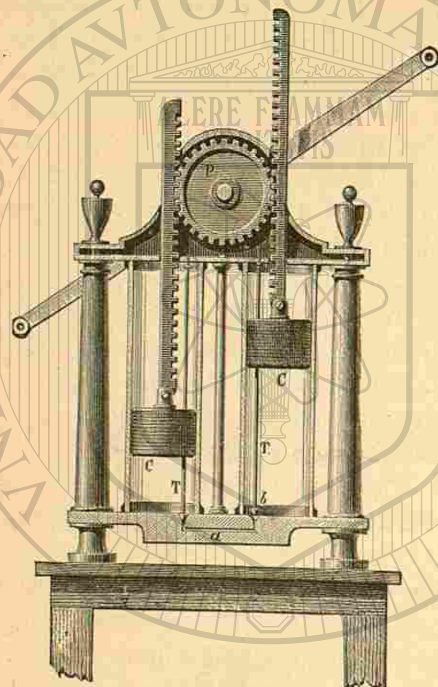
inches long; its upper end being connected with the receiver, while its lower end dips into a cistern of mercury. As exhaustion proceeds, the mercury rises in this tube, and its height above the mercury in the cistern measures the difference between the pressure in the receiver and that in the external air.

232. Admission Stop-cock.—After the receiver has been exhausted of air, if it were required to raise it from the plate, a very considerable force would be necessary, amounting to as many times fifteen pounds as the base of the receiver contained square inches. This difficulty is obviated by having an admission stop-cock R, which is shown in section above. It is perforated by a straight channel, which, when the machine is being worked, forms part of the communicating passage. At 90° from the extremities of this channel is another opening O, forming the mouth of a bent passage, leading to the external air. When we wish to admit the air into the receiver, we have only to turn the stop-cock so as to bring the opening O to the side next the receiver; if, on the contrary, we turn it towards the pump-barrel, all communication between the pump and the receiver is stopped, the risk of air entering is diminished, and the vacuum remains good for a greater length of time. This precaution is taken when we wish to leave bodies in a vacuum for a considerable time. Another method is to employ a separate plate, which can be detached so as to leave the machine available for other purposes.

233. Double-barrelled Air-pump.—The machine just described has only a single pump-barrel; air-pumps of this kind are sometimes employed, and are usually worked by a lever like a pump-handle. With this arrangement, it is evident that no air is expelled in the down-stroke; and that the piston, after having expelled the air from the barrel in the up-stroke, must descend idle in order to prepare for the next stroke.

Double-barrelled pumps are more frequently used. An idea of their general arrangement may be formed from Figs. 131, 132, and 133. Fig. 133 gives the machine in perspective, Fig. 131 is a section through the axes of the pump-barrels, and Fig. 132 shows the manner in which communication is established between the receiver and the two barrels. It will be observed that the two passages from the barrels unite in a single passage to the centre of the plate p.

Two racks carrying the pistons CC work with the pinion P. This pinion is turned by a double-handed lever, which is moved alternately in opposite directions. In this arrangement, when one piston ascends the other descends, and consequently in each single stroke the air of the receiver passes into one or other pump-barrel. A vacuum is thus produced by half the number of strokes which would be required with a single-barrelled pump. It has besides another advantage, as compared with the single-barrelled pump above described. In that pump the force required to raise the piston increases as the exhaustion proceeds, and when it is nearly completed there is the resistance of almost an atmosphere to be overcome. In the



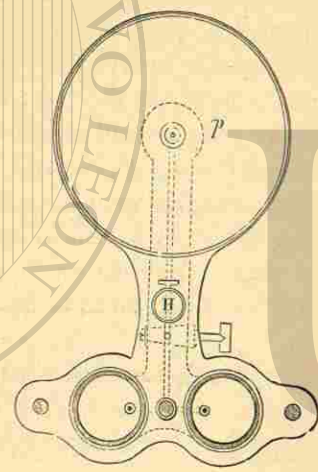


Fig. 131. Double-barrelled Air-pump. Fig. 132.

double-barrelled pump, with the same construction of barrel, the force opposing the ascent of one piston is precisely equal, at the beginning of each stroke, to that which assists the descent of the other. This equality, however, exists only at the beginning of the stroke; for the air below the descending piston is compressed, and its tension increases till it becomes equal to that of the atmosphere and raises the piston valve. During the remainder of the stroke, the resistance to the ascent of the other piston is entirely uncompensated, and up to this point the compensation has been gradually diminishing. But the more nearly we approach to a perfect vacuum, the later in the stroke does this compensation occur.

The pump, accordingly, becomes easier to work as the exhaustion proceeds.

234. Single-barrelled Pumps with Double Action.—We do not, however, require two pump-barrels in order to obtain double action,

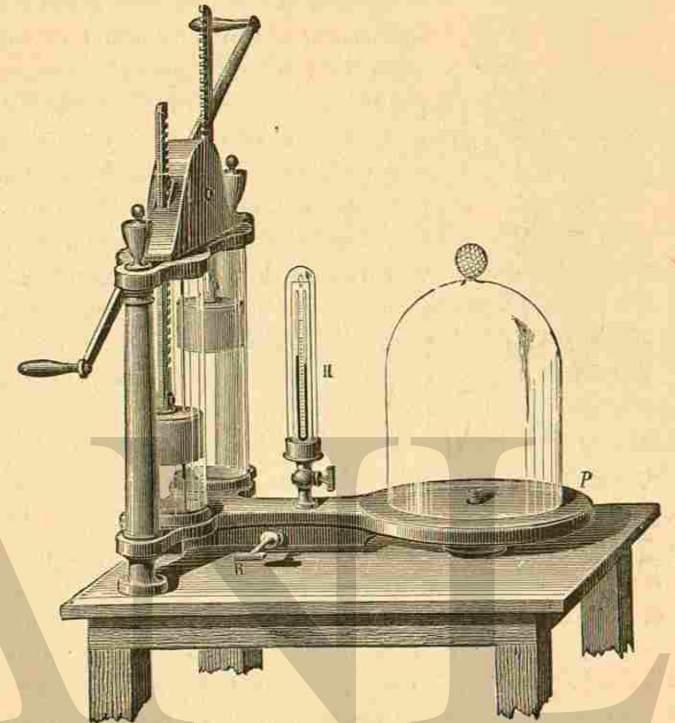


Fig. 133.—Air-pump.

as the same effect may be obtained with a single barrel. An arrangement for this purpose was long ago suggested by Delahire for water-pumps; but the principle has only lately been applied to the construction of air-pumps.

Fig. 134 represents the single barrel of the double-acting pump of Bianchi. It will be seen that the piston-valve opens into the hollow piston-rod; a second valve, also opening upwards, is placed at the top of the pump-barrel. Two other openings, one above, the other below, serve to establish communication, by means of a bent vertical tube, between the pump-barrel and the passage to the plate. These openings are closed alternately by two conical stoppers at the two extremities of a metal rod passing through the piston, and carried with it in its vertical movement by means of friction. When the piston ascends, as in the figure, the upper opening is closed and the lower one is open; when the piston begins to descend, the opposite effect is immediately produced. Accordingly we see that, whichever be the direction in which the piston is moving, the receiver is being exhausted of air. In fact, when the piston ascends, air from the receiver will enter by the lower opening, and the air above the piston will be gradually compressed, and will finally escape by the valve above. In the descending movement, air will enter by the upper opening, and the compressed air beneath the piston will escape by the piston-valve. The movement of the piston is produced by a peculiar arrangement shown in Fig. 135, which gives a general view of the apparatus.

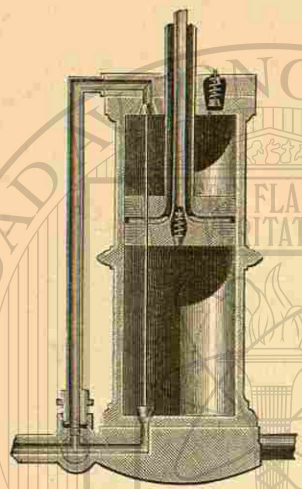


Fig. 134.
Barrel of Bianchi's Air-pump.

The pump-barrel, which is composed entirely of cast-iron, oscillates about an axis passing through its base. On the top are guides in which the end of a crank travels. The pump is worked by turning a heavy fly-wheel of cast-iron, on the axis of which is a pinion which drives a toothed wheel on the axis of the crank. The end of the crank is attached to the extremity of the piston-rod. It is evident that on turning the fly-wheel the pump-barrel will oscillate from side to side, following the motions of the crank, and the piston will alternately ascend and descend in the barrel, the length of which should be equal to the diameter of the circle described by the end of the crank.

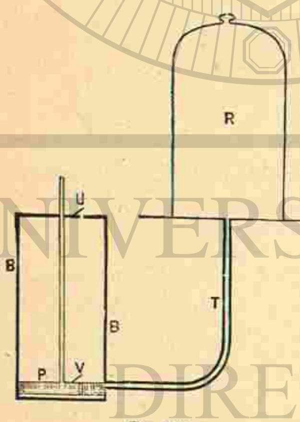


Fig. 136

235. English forms of Air-pump.—Some of the drawbacks to the single-barrelled pump are obviated by inserting a valve, opening upwards, in the top of the barrel as at U, Fig. 136. The top of the piston is thus relieved from atmospheric pressure, and the operation of pumping does not become more laborious as

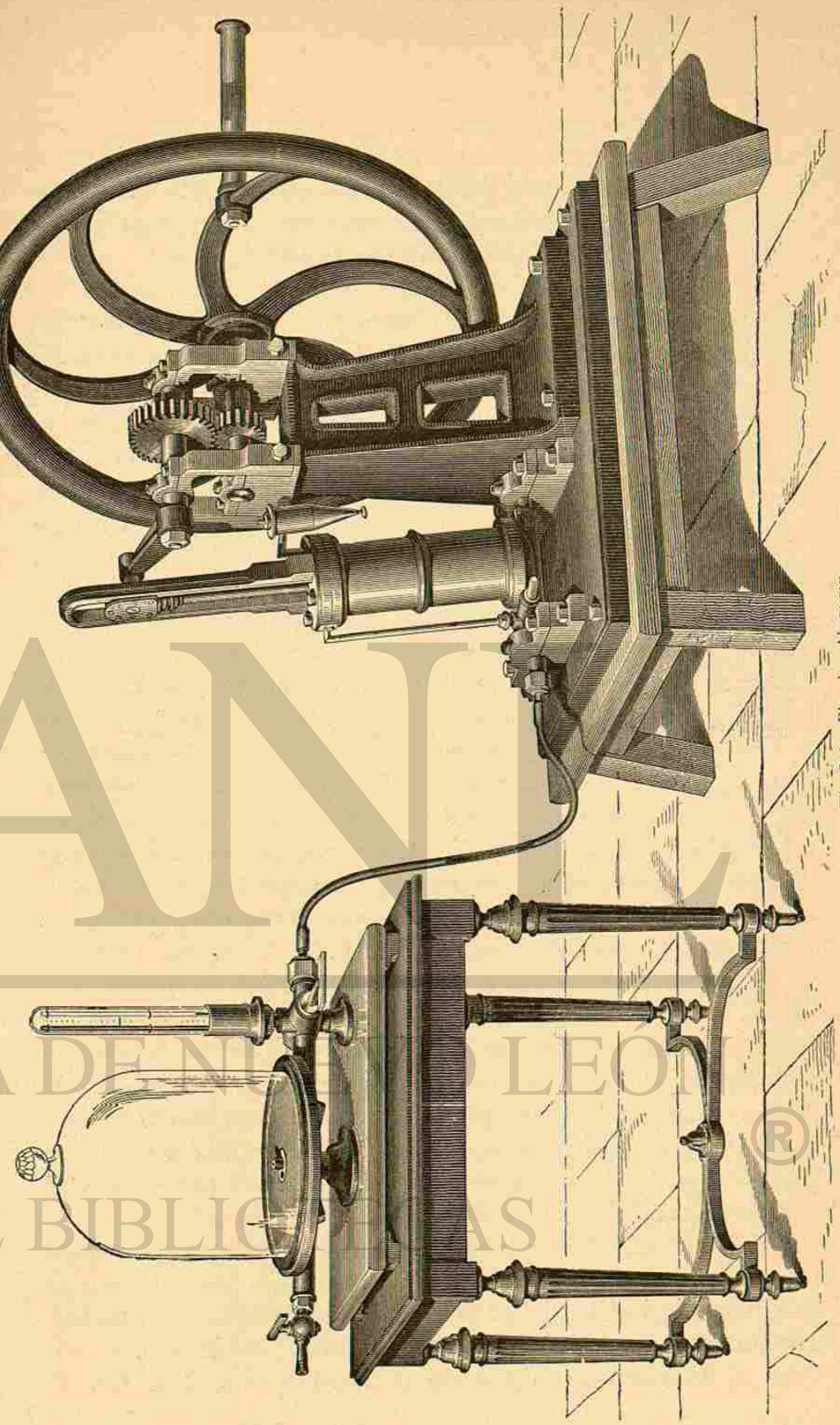


Fig. 135.—Bianchi's Air-pump.

the exhaustion proceeds, but less laborious, the difference being most marked when the receiver is small.

In the up-stroke, the piston-valve V keeps shut, and the air above the piston is pushed out of the barrel through the valve U. In the down-stroke, U is kept closed by the preponderance of atmospheric pressure outside, and V opens, allowing the air to pass up through it as the piston descends to the bottom of the barrel. When the exhaustion is far advanced, U does not open till the piston has nearly reached the top. This is a simple and good form of pump.

Another form very much in use in this country is the double-acting pump of Professor T. Tate, the working parts of which are shown in Fig. 137. CD is the barrel; A and B are two solid pistons rigidly connected by a rod, and moved by the piston-rod AH, which passes through a stuffing-box S. VV are valves in the two ends of the barrel, both opening outwards, and R is a passage leading from the middle of the cylinder to the receiver. The distance between the extreme faces of the pistons is about $\frac{3}{8}$ths of an inch less than half the length of the cylinder. The volume of air expelled at each single stroke is thus about half the volume of the cylinder.

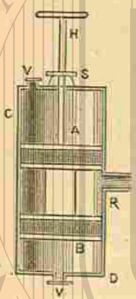


Fig. 137.
Tate's Pump.

This figure and description are in accordance with the original account of the pump given by the inventor in the *Philosophical Magazine*. It is now usual to replace the two pistons by a single piston of great thickness, its two faces being as far apart as the extreme faces of the two pistons in the figure. It is also usual to make the barrel horizontal.

The valves of these pumps, and of most English pumps are "silk valves." They consist of a short and narrow slit in a thin plate of brass, with a flap of oiled silk secured at both ends to the plate, in such a position that its central portion covers the slit. When the pressure of the air is greater on the further side of the plate than on the side where the silk is, the flap is slightly lifted and the air gets through; but excess of pressure on the near side presses the flap down over the slit and makes it air-tight.

236. Various Experiments with the Air-pump.—At the time when the air-pump was invented, several experiments were devised to show the effects of a vacuum, some of which have become classical, and are usually repeated in courses of experimental physics.

Burst Bladder.—On the plate of an air-pump (Fig. 138) is

placed a glass cylinder open at the bottom, and having a piece of bladder or thin indian-rubber tightly stretched over the top. As the exhaustion proceeds, this bends inwards in consequence of the atmospheric pressure above it, and finally bursts with a loud report.

Magdeburg Hemispheres.—We take two hemispheres (Fig. 139), which can be exactly fitted on each other; their exact adjustment is further assisted by a projecting internal rim, which is smeared with lard. The apparatus is exhausted of air through the medium of the stop-cock attached to one of the hemispheres; and when a vacuum has been produced, it will be found that a considerable force is required to separate the two parts, this force increasing with the size of the hemispheres.

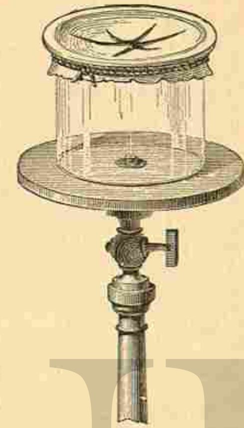


Fig. 138.
Burst Bladder.


Fig. 139.
Magdeburg Hemisphere.

This resistance to separation is due to the normal exterior pressure of the air on every point of the surface, a pressure which is counterbalanced by only a very feeble pressure from the interior. In order to estimate the resultant effect of these different pressures, let us suppose that one hemisphere is vertically over the other, and that the external surface is cut into a series of steps,—that is to say, of alternate vertical and horizontal elements. It is evident that the pressure urging either hemisphere towards the other will be simply the sum of the pressures upon its horizontal elements; and this sum is identical with the pressure which would be exerted upon a circular area equal to the common base of the hemispheres. For example, if this area is 10 square inches, and the external pressure exceeds the internal by 14 lbs. to the inch, the hemispheres will be pressed together with a force of 140 lbs.

Fountain in Vacuo.—The apparatus for this experiment consists of a bell-shaped vessel of glass (Fig. 140), the base of which is pierced by a tube fitted with a stop-cock which enables us to exhaust the vessel of air. If, after a vacuum has been produced, we place the lower end of the tube in a vessel of water, and open the stop-cock, the liquid, being pressed externally by the atmosphere, mounts up the tube and ascends in a jet into the interior of the vessel. This experiment is often made in the opposite manner. Under the receiver of the air-pump is placed a vial partly filled with water, and having its cork pierced by a tube open at both ends, the lower end being beneath the surface of the water. As the exhaustion proceeds, the air in the vial, by its excess of pressure, acts upon the liquid and makes it issue in a jet.

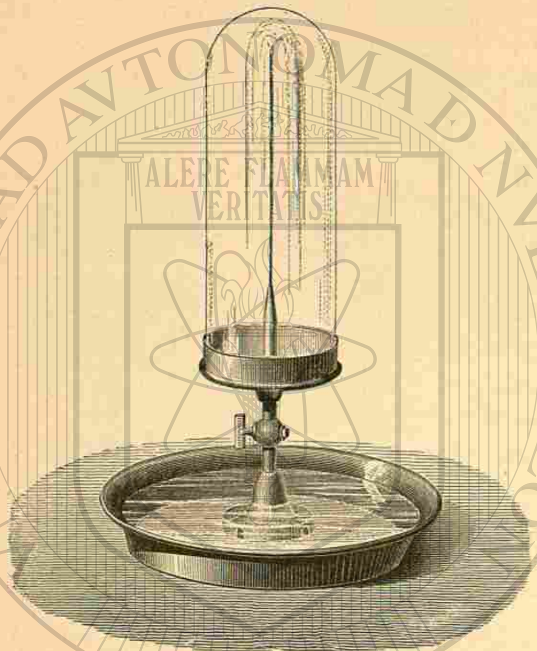


Fig. 140.—Fountain in Vacuo.

237. Limit to the Action of the Air-pump.—We have said above (§ 230) that the air-pump does not continue the process of rarefaction indefinitely, but that at a certain stage its effect ceases, and the pressure of the air in the receiver undergoes no further diminution. If the pump is very badly made, this pressure is considerable; but even with the most perfect machines it is always sensible. A pump such as we have described may be considered good if it reduces the pressure of the air in the receiver to a tenth of an inch of mercury. A fiftieth of an inch is perhaps the lowest limit.

Leakage.—This limit to the action of the machine is due to various causes. In the first place, there is frequently leakage at different parts of the apparatus; and although at the beginning of the operation the quantity of air which thus enters is small in comparison with that which is pumped out, still, as the exhaustion proceeds, the air enters faster, on account of the diminished internal pressure, and at the same time the quantity expelled at each stroke becomes less,

so that at length a point is reached at which the inflow and outflow are equal.

In order to prevent leakage as far as possible, the plate of the pump and the base of the receiver must be truly plane so as to fit accurately; the base of the receiver must be ground (that is roughened) and must be well greased before pressing it down on the plate. The piston must also be well lubricated with oil.

Space untraversed by Piston.—Another reason of imperfect exhaustion is that, after all possible precautions, a space is still left between the bottom of the pump-barrel and the lower surface of the piston when the latter is at the end of its downward stroke. It is evident that at this moment the air contained in this *untraversed space* is of the same tension as the atmosphere. On raising the piston, this air is indeed rarefied; but it still preserves a certain tension, and it is evident that when the air in the receiver has been brought to this stage of rarefaction, the machine will cease to produce any effect.

If v is the volume of this space, and V the volume of the pump-barrel, the air, which at volume v has a pressure H equal to that of the atmosphere, will have, at volume V, a pressure $H\frac{v}{V}$. This gives the limit to the action of the machine as deduced from the consideration of the untraversed space.

Air given out by Oil.—Finally, perhaps the most important cause, and the most difficult to remedy, is the absorption of air by the oil used for lubricating the pistons. This oil is poured on the top of the piston, but the pressure of the external air forces it between the piston and the barrel, whence it falls in greater or less quantity to the bottom of the barrel, where it absorbs air, and partially yields it up at the moment when the piston begins to rise, thus evidently tending to derange the working of the machine. It has been attempted to get rid of untraversed space by employing a kind of piston of mercury. This has also the advantage of fitting the barrel more accurately, and thus preventing the entrance of air. The use of oil is at the same time avoided, and we thus escape the injurious effects mentioned above. We proceed to describe two machines founded upon this principle.

238. Kravogl's Air-pump.—This contains a hollow glass cylinder AB (Fig. 141) tapering at the upper end, and surmounted by a kind of funnel. The piston is of the same shape as the cylinder, and is covered with a layer of mercury, whose depth over the point of the piston is about $\frac{1}{50}$th of an inch when the piston is at the bottom of its stroke, but is nearly an inch when the piston rises and fills the

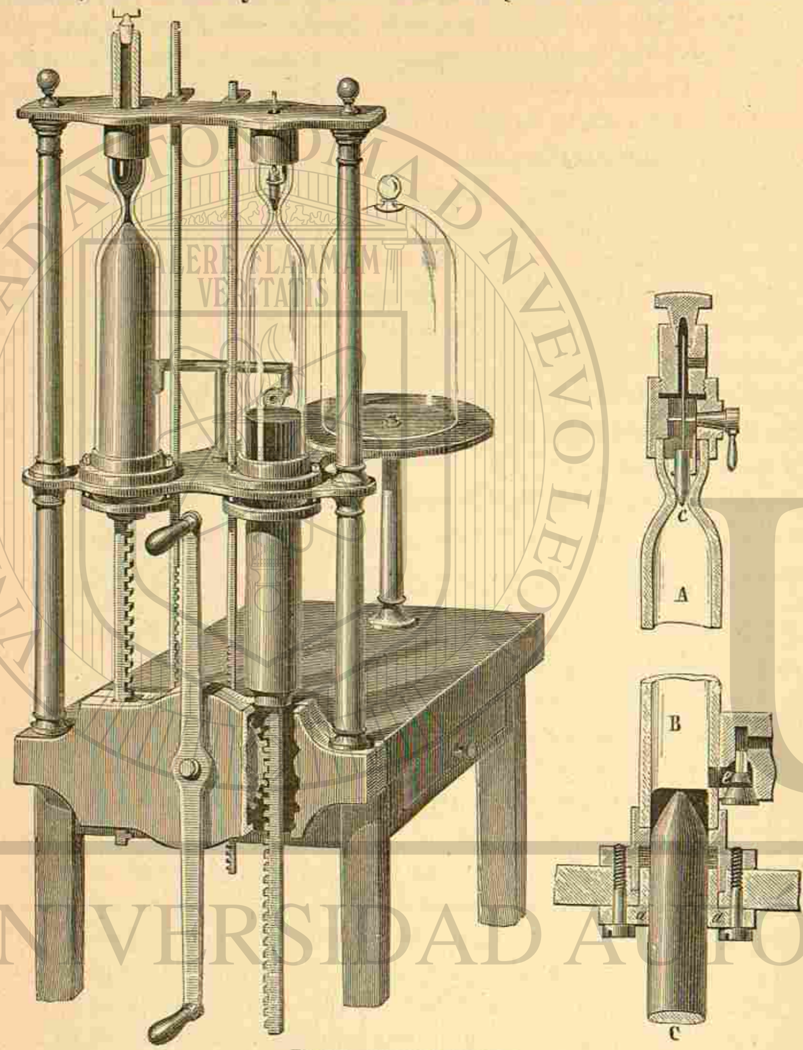


Fig. 141.—Kravogl's Air-pump.

funnel-shaped cavity in which the pump-barrel terminates. A small interval, filled by the liquid, is left between the barrel and the piston; but at the bottom of the barrel the piston passes through a leather box carefully made, so as to be perfectly air-tight.

The air from the receiver enters through the lateral opening *e*, and

is driven before the mercury into the funnel above. With the air passes a certain quantity of mercury, which is detained by a steel valve *c* at the narrowest part of the funnel. This valve rises automatically when the surface of the mercury is at a distance of about half an inch from the funnel, and falls back into its former position when the piston is at the end of its upward stroke. In the downward stroke, when the mercury is again half an inch from the funnel. the valve opens again and allows a portion of the mercury to pass.

The effect of this arrangement is easily understood; there is no "untraversed space," the presence of the mercury above and around the piston causes a very complete fit, and excludes the external air; and hence the machine, when well made, is very effective.

When this is the case, and when the mercury used in the apparatus is perfectly dry, a vacuum of about $\frac{1}{250}$th of an inch can be obtained. The dryness of the mercury is a very important condition, for at ordinary temperatures the elastic force of the vapour of water has a very sensible value. If we wish to employ the full powers of the machine, we must have, between the vessel to be exhausted of air and the pump-barrel, a desiccating apparatus.

The arrangement of the valve *e* is peculiar. It is of a conical form, so as, in its lowest position, to permit the passage of air coming from the receiver. Its ascent is produced by the pressure of the mercury, which forces it against the conical extremity of the passage, and the liquid is thus prevented from escaping.

The figure represents a double-barrelled machine analogous to the ordinary air-pump. Besides the pinion working with the racks of the pistons, there is a second smaller pinion, not shown in the figure, which governs the movements of the valves *c*. All the parts of this machine, as the stop-cocks, valves, pipes, &c., must be of steel, to avoid the action which the mercury would have upon any other metal.

239. Geissler's Machine.—Geissler, of Bonn, invented a mercurial air-pump, in which the vacuum is produced by communication of the receiver with a Torricellian vacuum. Fig. 142 represents this machine as constructed by Alvergniat. It consists of a vertical tube, serving as a barometric tube, and communicating at the bottom, by means of a caoutchouc tube, with a globe which serves as the cistern.

At the top of the tube is a three-way stop-cock, by which communication can be established either with the receiver to the left, or with a funnel to the right, which latter has an ordinary stop-cock at the bottom. By means of another stop-cock on the left, communication with the receiver can be opened or closed. These stop-cocks are made entirely of glass. The machine works in the following manner; communication being established with the funnel, the globe which serves as cistern is raised, and placed, as shown in the figure, at a higher level than the stop-cock of the funnel. By the law of equilibrium in communicating vessels, the mercury fills the barometric tube, the neck of the funnel, and part of the funnel itself. If the communication between the funnel and tube be now stopped, and the globe lowered, a Torricellian vacuum is produced in the upper part of the vertical tube.

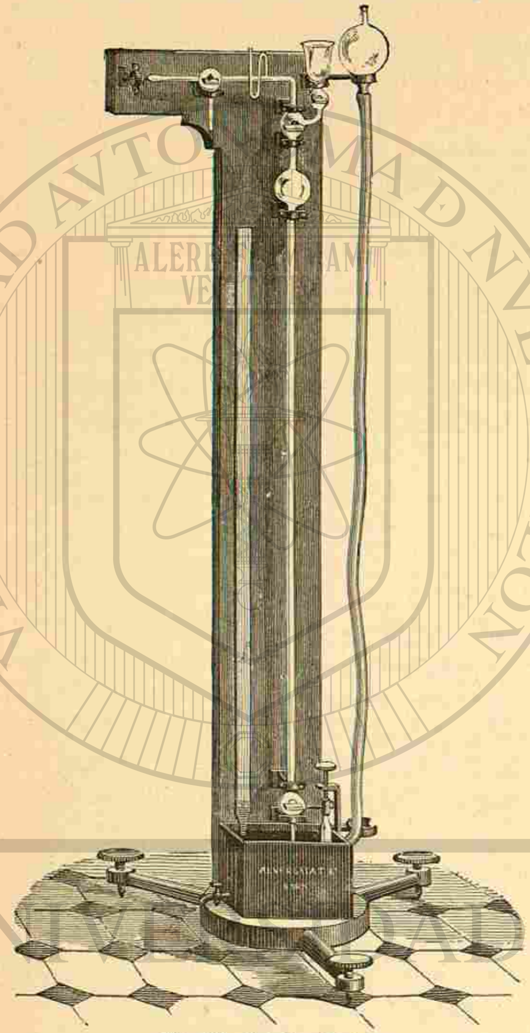


Fig. 142.—Geissler's Machine.

Communication is now opened with the receiver; the air rushes into the vacuum, and the column of mercury falls a little. Communication is now stopped between the tube and receiver, and opened between the tube and the funnel, the simple stop-cock of the funnel being, however, left shut. If at this moment the globe is replaced in the position shown in the figure, the air tends to escape by the funnel, and it is easy to allow it to do so. Thus, a part of the air of the receiver has been removed, and the apparatus is in the same position as at the beginning. The operation described is equivalent to a stroke of the piston in the ordinary machine, and this process must be repeated till the receiver is exhausted.

As the only mechanical parts of this machine are glass stop-cocks, which are now executed with great perfection, it is capable of giving very good results. With dry mercury a vacuum of $\frac{1}{250}$th of an inch may very easily be obtained. The working of the machine, however, is inconvenient, and becomes exceedingly laborious when the receiver is large. It is therefore employed directly only for producing a vacuum in very small vessels; when the spaces to be exhausted of air are at all large, the operation is begun with the ordinary machine, and the mercurial air-pump is only employed to render the vacuum thus obtained more perfect.

240. Sprengel's Air-pump.—This instrument, which may be regarded as an improvement upon Geissler's, is represented in its simplest form in Fig. 143. *cd* is a glass tube longer than a barometer tube, down which mercury is allowed to fall from the funnel A. Its lower end dips into the glass vessel B, into which it is fixed by means of a cork. This vessel has a spout at its side, a few millimetres higher than the lower end of the tube. The first portions of mercury which run down will consequently close the tube, and prevent the possibility of air entering it from below. The upper part of *cd* branches off at *x* into a lateral tube communicating with the receiver R, which it is required to exhaust. A convenient height for the whole instrument is 6 feet. The funnel A is supported by a ring as shown in the figure, or by a board with a hole cut in it. The tube *cd* consists of two parts, connected by a piece of india-rubber tubing, which can be compressed by a clamp so as to keep the tube closed when desired. As soon as the mercury is allowed to run down, the exhaustion begins, and the whole length of the tube, from *x* to *d*, is seen to be filled with cylinders of mercury separated by cylinders of air, all moving downwards. Air and mercury escape through the spout of the bulb B, which is above the basin H, where the mercury is collected. This has to be poured back from time to time into the funnel A, to pass through the tube again and again until the exhaustion is completed.

As the exhaustion is progressing, it will be noticed that the inclosed air between the mercury cylinders becomes less and less, until the lower part of *cd* presents the aspect of a continuous column of mercury about 30 inches high. Towards this stage of the operation a considerable noise begins to be heard, similar to that of a shaken water-hammer, and common to all liquids shaken in a vacuum. The operation may be considered completed when the column of mercury does not inclose any air, and when a drop of mercury falls upon the top of this column without inclosing the slightest air-bubble. The height of this column now corresponds exactly with the height of the column of mercury in a barometer; or, what is the same, it represents a barometer whose vacuum is the receiver R and connecting tube.

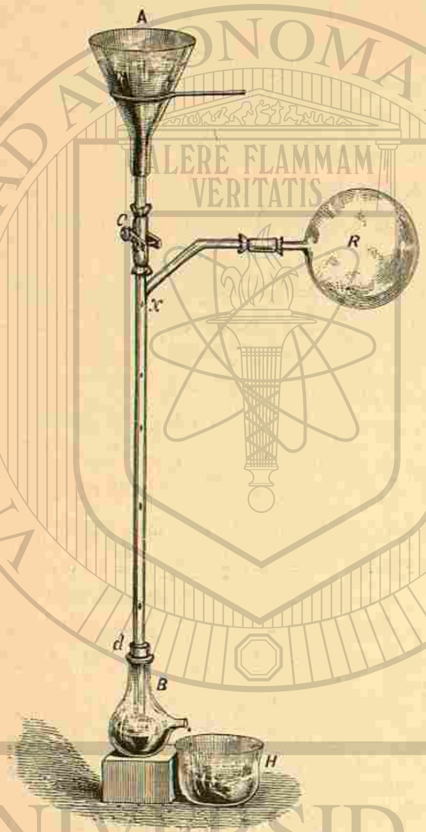


Fig. 143.—Sprengel's Air-pump.

Dr. Sprengel recommends the employment of an auxiliary air-pump of the ordinary kind, to commence the exhaustion, when time is an object, as without this from 20 to 30 minutes are required to exhaust a receiver of the capacity of half a litre. As, however, the employment of the auxiliary pump involves additional connections and increased leakage, it should be avoided when the best possible exhaustion is desired. The fall tube must not exceed about a tenth of an inch in diameter, and special precautions must be employed to make the india-rubber connections air-tight. (See *Chemical Journal* for 1865, p. 9.)

By this instrument air has been reduced to $\frac{1}{1300000}$th of atmospheric density, and the average exhaustion attainable by its use is about one-millionth, which is equivalent to ·00003 of an inch of mercury.

241. Double Exhaustion.—In the mercurial machines just described there is no "untraversed space," as the liquid completely expels all the air from the pump-barrel. These machines are of very recent

invention. Babinet long before introduced an arrangement for the purpose, not of getting rid of this space, but of exhausting it of air.

For this purpose, when the machine ceases to work with the ordinary arrangement, the communication of the receiver with one of the pump-barrels is shut off, and this barrel is employed to exhaust the air from the other. This change is effected by means of a stop-cock at the point of junction of the passages leading from the two barrels (Fig. 144). The stop-cock has a T-shaped aperture, the point of intersection of the two branches being in constant communication with the receiver. In a different plane from that of the T-shaped aperture is another aperture *mn*, which, by means of the tube *l*, establishes communication between the pump-barrel B and the communicating passage of the pump-barrel A. From this explanation it will be seen that if the stop-cock be turned as shown in the first figure, the two pump-barrels both communicate with the receiver, and the operation proceeds in the ordinary manner. But if the stop-cock be turned through a quarter of a revolution, as shown in the second figure, the pump-barrel B alone communicates with the receiver, while it is itself exhausted of air by the barrel A.

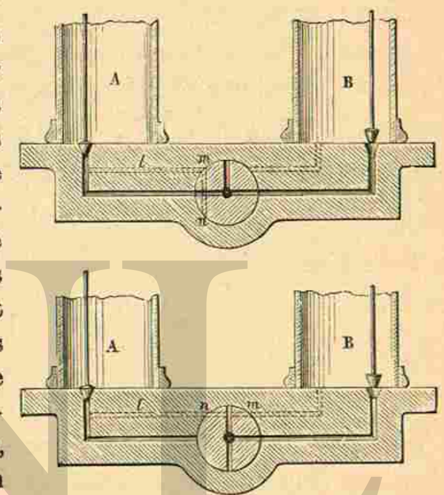


Fig. 144.—Babinet's Doubly-exhausting Stop-cock.

It is easy to express by a formula the effect of this double exhaustion. Suppose the pump to have ceased, under the ordinary method of working, to produce any farther exhaustion, the air in the receiver has therefore reached a tension nearly equal to $H\frac{v}{V}$ (§ 237). At this moment the stop-cock is turned into its second position. When the piston B descends, the piston A rises, and the air of the "untraversed space" in B is drawn into A and rarefied. During the inverse operation, the air in A is prevented from returning to B, and thus the rarefied air from B, becoming still further rarefied, will draw a fresh quantity of air from the receiver. This air will then be driven into A, where it will be compressed by the descending movement of the piston, and will find its way into the air outside.[1]

This double exhaustion will itself cease to work when air ceases to pass from the pump-barrel B into the pump-barrel A. Now when the piston in this latter is raised, the elastic force of the air which was contained in its "untraversed space" is equal to $H\frac{v}{V}$, for, on the last opening of the valve, the air in this space escaped into the atmosphere. On the other hand, when the piston in B is at the end of its upward stroke, the tension of the air is the same as in the receiver. Let this be denoted by x. When the piston in B descends, the air is compressed into the "untraversed space" and the passage leading to A. Let the volume of this passage be l. Then the tension will increase, and become $x\frac{V+l}{v+l}$. When the machine ceases to produce any farther effect, this tension cannot be greater than that in the pump-barrel A, which is $H\frac{v}{V}$; we have thus, to determine the limit to the action of the pump, the equation

$$x\frac{V+l}{v+l} = H\frac{v}{V}, \text{ whence}$$

$$x = H.\frac{v}{V}.\frac{v+l}{V+l}$$

242. Air-pump with Free Piston.—We shall describe one more air-pump (Fig. 145), constructed by Deleuil, and founded upon an interesting principle. We know that gases possess a remarkable power of adhesion for solids, so that a body placed in the atmosphere may be considered as covered with a very thin coat of air, forming, so to speak, a permanent envelope. On account of this circumstance, gases find very great difficulty in moving in very narrow spaces. This is the principle of the "air-pump with free piston."

The piston P (Fig. 146), which is composed entirely of metal, is of considerable length; and on its outer surface is a series of parallel circular grooves very close together. It does not touch the pump-barrel at any point; but the distance between the two is very small, about ·001 of an inch. This free piston is surrounded by a cushion of air, which forms its only stuffing, and is sufficient to enable the machine to work in the ordinary manner, notwithstanding the per-

[1] It will be observed that during the process of double exhaustion the piston of B behaves like a solid piston; its valve never opens, because the pressure below it is always less than atmospheric.

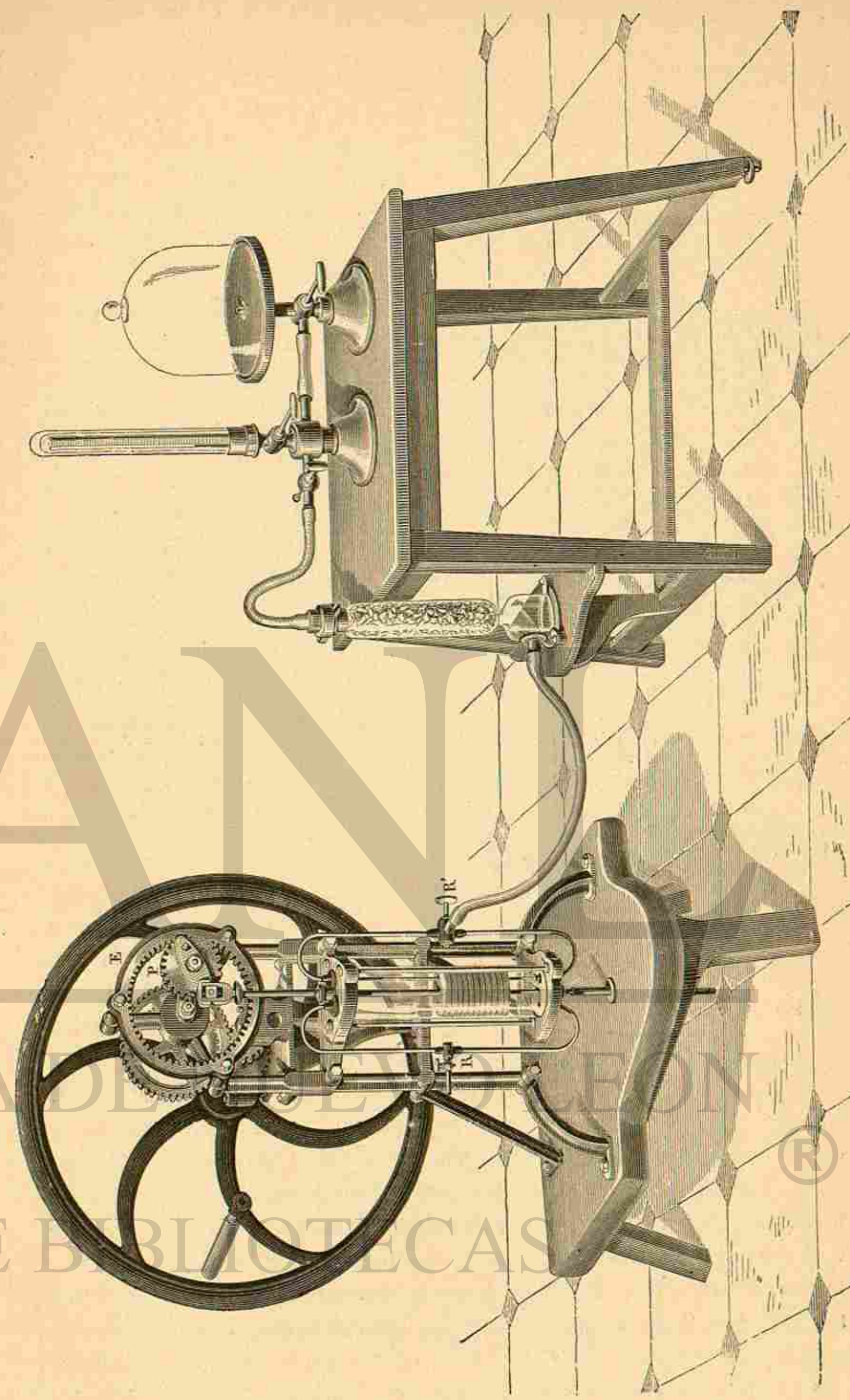


Fig. 145.—Deleuil's Air-pump

manent communication between the upper and lower surfaces of the piston. This machine gives a vacuum about as good as is obtainable by ordinary pumps, and it has the important advantages of not requiring oil, and of having less friction. It consequently wears better, and is less liable to the development of heat, which is a frequent source of annoyance in air-pumps. It is single-barrelled with double action, like Bianchi's. The two openings S and S′ are to admit air from the receiver; they are closed and opened alternately by conical stoppers at the end of the rod T, which passes through the piston, and is carried with it by friction in its movement. They communicate with tubes which unite, at R′, with a tube leading from the receiver. A and A′ are valves for the expulsion of the air, which escapes by tubes uniting at R. The alternate movement of the piston is produced by what is called Delahire's gearing. This depends on the principle, that *when a circle rolls without sliding in the interior of another circle of double the diameter, any point on the circumference of the rolling circle describes a diameter of the fixed circle.* In order to utilize this property, the end of the piston-rod is jointed to the extremity of a piece of metal which is rigidly attached to the pinion P, the joint being exactly opposite the circumference of the pinion. This latter is driven by a fly-wheel with suitable gearing, and works with the fixed wheel E, which is toothed on the inside. Thus the piston will freely, and without any lateral effort, describe a vertical line, the length of the stroke being equal to the diameter of the fixed wheel.

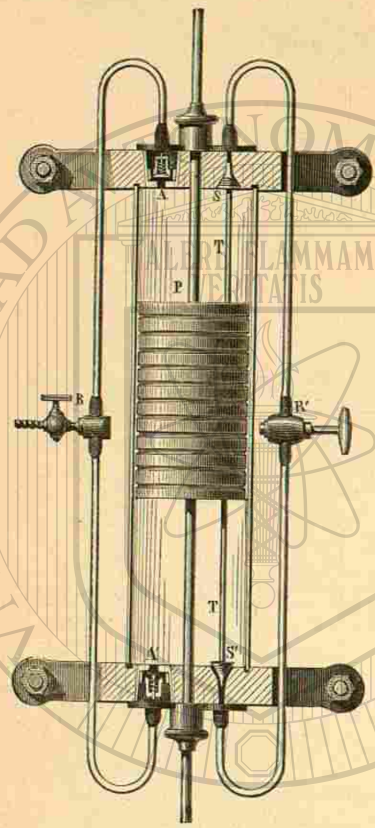


Fig. 146.
Piston and Barrel of Deleuil's Air-pump.

243. Compressing Pump.—It can easily be seen from the description of the air-pump, that if the expulsion-valves were connected with a tube communicating with a reservoir, the air removed by the pump would be forced into this reservoir. This communication is established in the instrument just described. If, therefore, R′ be made to communicate with the external air, this air will be continually drawn in at that point and forced out into the reservoir connected with R, so that the instrument will act as a compressing pump. The compressing-pump is thus seen to be the same instrument as the air-pump, the only difference being that the receiver is connected with the expulsion valves, instead of with the exhaustion-valves; it is thus, so to speak, the air-pump reversed. This fact can be very well seen in the structure of a small pump frequently employed in the laboratory, and represented in Fig. 147.

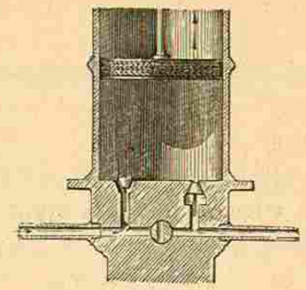


Fig. 147.—Barrel of Condensing Pump.

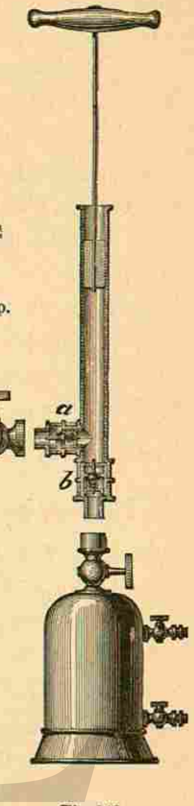


Fig. 148.
Condensing Pump.

At the bottom of the pump-barrel are two valves, communicating with two separate reservoirs, that on the left being an admission-valve, and that on the right an expulsion-valve.

When the piston is raised, rarefaction is produced in the reservoir to the left; and when it is pushed down, the air in the reservoir to the right is compressed.

In Fig. 148 is represented a compressing-pump often employed. At the bottom of the pump-barrel is a valve *b* opening downward; in a lateral tube is an admission-valve *a* opening inward. The position of these valves is shown in the figure. They are conical metal stoppers, fitted with a rod passing through a hole in a small plate behind, an arrangement which prevents the valve from overturning. The rod is surrounded by a small spiral spring, which keeps the valve pressed against the opening. If the lower part of the pump-barrel be screwed upon a reservoir, at each upward stroke of the piston the barrel will be filled with air through the valve *a*, and at every downward stroke this air will be forced into the reservoir.

If the lateral tube be made to communicate with a bladder or gas-holder filled with any gas, this gas will be forced into the reservoir, and compressed.

244. Calculation of the Effect of the Instrument.—The density of the compressed air after a given number of strokes of the piston may easily be calculated. If v be the volume of the pump-barrel, and V that of the reservoir; at each stroke of the piston there is forced into the reservoir a volume of air equal to that of the pump-barrel; which gives a volume nv at the end of n strokes. The air in the reservoir, accordingly, which when at atmospheric pressure had density D, and occupied a volume $V + nv$, will, when the volume is reduced to V, have the density $D \frac{V + nv}{V}$, and the pressure will, by Boyle's law, be $\frac{V + nv}{V}$ atmospheres.

If this formula were rigorously applicable in all cases, there would be no limits to the pressure attainable, except those depending on the strength of the reservoir and the motive power available. But, in fact, the untraversed space left below the piston, when at the end of its downward stroke, sets a limit to the action of the instrument, just as in the common air-pump. For when the air in the barrel is reduced from the volume of the barrel v to that of the untraversed space v', its tension becomes $H\frac{v}{v'}$; and this air cannot pass into the reservoir unless the tension of the air in the reservoir is less than this quantity. This is accordingly the utmost limit of compression that can be attained.

We must, however, carefully distinguish between the effects of untraversed space in the air-pump and in the compression-pump. In the first of these instruments the object aimed at is to rarefy the air to as great a degree as possible, and untraversed space must consequently be regarded as a defect of the most serious importance.

The object of the condensing-pump, on the contrary, is to compress the air, not indefinitely, but up to a certain point. Thus, for instance, one pump is intended to give a compression of five atmospheres, another of ten, &c. In each of these cases the maker

provides that this limit shall be reached, and the untraversed space has no injurious effect beyond increasing the number of strokes required to produce the desired amount of condensation.

245. Various Contrivances for producing Compression.—In order to expedite the process of compression, several pumps such as we have

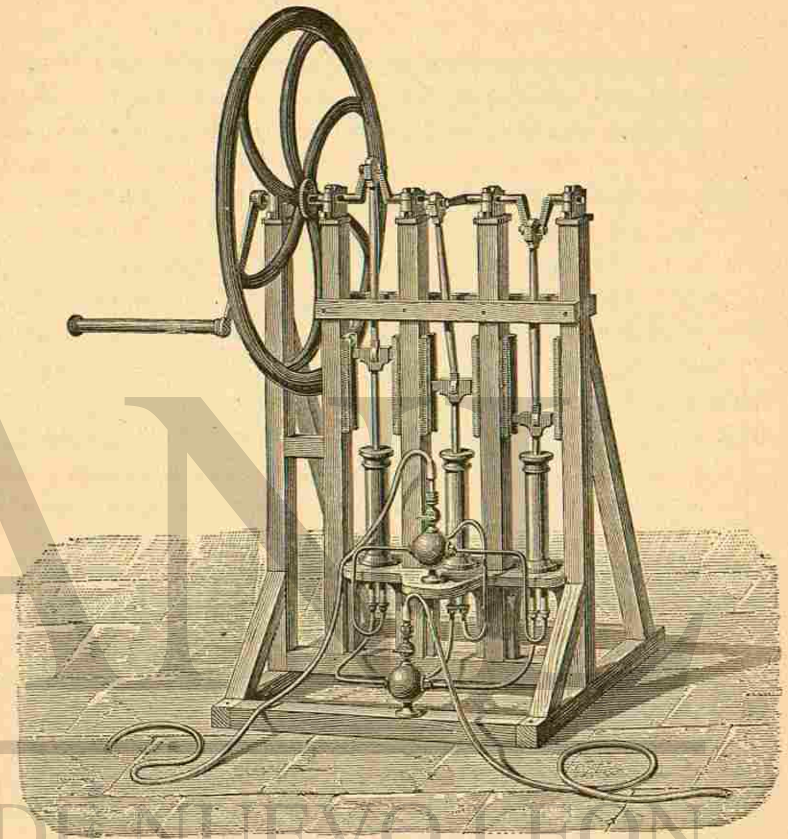


Fig. 149.—Connected Pumps.

described are combined, which may be done in various ways. Fig. 149 represents the system employed by Regnault in his investigations connected with Boyle's law and the elastic force of vapour. It consists of three pumps, the piston-rods of which are jointed to three cranks on a horizontal axle, by means of three connecting-rods. This axle, which carries a fly-wheel, is turned by means of one or two handles. The different admission-valves are in communication with a single reservoir in connection with the external air, and the compressed gas is forced into another reservoir which is in communication with the experimental apparatus.

A serious obstacle to the working of these instruments is the heat generated by the compression of the air, which expands the different parts of the instrument unequally, and often renders the piston so tight that it can scarcely be driven. In some of these instruments which are employed in the arts, this inconvenience is lessened by keeping the lower valves covered with water, which has the additional advantage of getting rid of "untraversed space." In this way a pressure of forty atmospheres may easily be obtained with air. Air may also be compressed directly, without the intervention of pumps, when a sufficient height of water can be obtained. It is only necessary to lead the liquid in a tube to the bottom of a reservoir containing air. This air will be compressed until its pressure exceeds that of the atmosphere by the amount due to the height of the summit of the tube. It is by a contrivance of this kind that compressed air has been obtained for driving the boring-machines employed in the great Alpine tunnels.

246. Practical Applications of the Air-pump and of Compressed Air.—Besides the use made of the air-pump and the compression-pump in the laboratory, these instruments are variously employed in the arts.

The air-pump is employed by sugar-refiners to lower the boiling point of the syrup. Compression-pumps are used by soda-water manufacturers to force the carbonic acid into the reservoirs containing the water which is to be aerated. The small apparatus described above (Fig. 148) is sufficient for this purpose; it is only necessary to fill the side-vessel with carbonic acid, and to pour a certain quantity of water into the reservoir below. Compressed air has for several years been employed to assist in laying the foundations of bridges in rivers where the sandy nature of the soil requires very deep excavations. Large tubes called *caissons*, in connection with a condensing pump, are gradually let down into the river; the air by its pressure keeps out the water, and the workmen, who are admitted into the apparatus by a sort of lock, are thus enabled to walk on dry ground.

In pneumatic despatch tubes, which have recently been established in many places, a kind of train is employed, consisting of a piston preceded by boxes containing the despatches. By exhausting the air at the forward end of the tube, or forcing in compressed air at

the other end, the train is blown through the tube with great velocity.

The atmospheric railway, which was for a few years in existence, was worked upon the same principle: an air-tight piston travelled through a fixed tube, and was connected by an ingenious arrangement with a train above.

Excavating machines driven by compressed air are coming into extensive use in mining operations. They have the advantage of assisting ventilation, inasmuch as the compressed air, which at each stroke of the machine escapes into the air of the mine, cools as it expands.

In the air-gun, the bullet is projected by a portion of compressed air which, on pulling the trigger, escapes into the barrel from a reservoir in which it has been artificially compressed.

We may add that the large machines employed in iron-works for supplying air to the furnaces, are really compression-pumps.

CHAPTER XXI.

UPWARD PRESSURE OF THE AIR.

247. The Baroscope.—The principle of Archimedes, explained in Chap. XIII., applies to all fluids, whether liquid or gaseous. Hence the resultant of the whole pressure of the atmosphere on the surface of a body is equal to the weight of the air displaced. The force required to support a body in air, is less than the force required to support it in vacuo, by this amount. This principle is illustrated by the baroscope (Fig. 150).

This is a kind of balance, the beam of which supports two balls of very unequal sizes, which balance each other in the air. If the apparatus is placed under the receiver of an air-pump, after a few strokes of the piston the beam will be seen to incline towards the larger ball, and the inclination will increase as the exhaustion proceeds. The reason is that the air, before it was pumped out, produced an upward pressure, which was greater for the large than for the small ball, on account of its greater displacement; and this disturbing force is now removed.

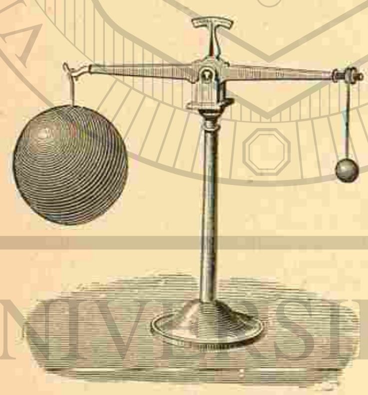


Fig. 150.—Baroscope.

If after exhausting the air, carbonic acid, which is heavier than air, were admitted at atmospheric pressure, the large ball would be subjected to a greater increase of upward pressure than the small one, and the beam would incline to the side of the latter.

248. Balloons.—Suppose a body to be lighter than an equal volume of air, then this body will rise in the atmosphere. For example, if

we fill soap-bubbles with hydrogen (Fig. 151), and shake them off from the end of the tube at which they are formed, they will be seen, if sufficiently large, to ascend in the air. This curious experiment is due to the philosopher Cavallo, who announced it in 1782.[1]

The same principle applies to balloons, which essentially consist of an envelope inclosing a gas lighter than air. In conse-

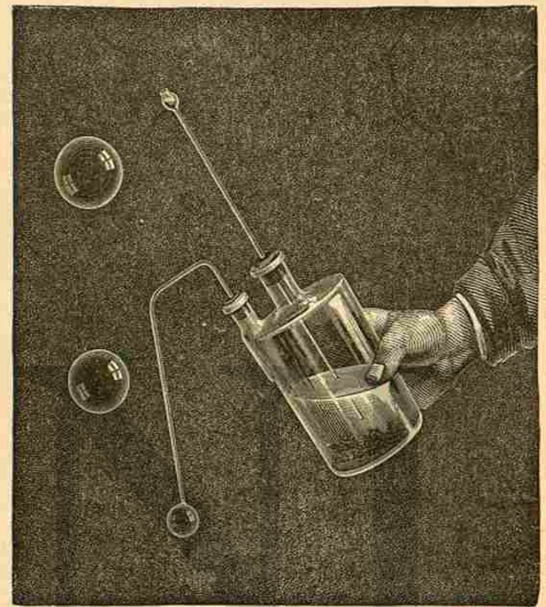


Fig. 151.—Ascent of Soap-bubbles filled with Hydrogen.

quence of this difference of density, we can always, by taking a sufficiently large volume, make the weight of the gas and containing envelope less than that of the air displaced. In this case the balloon will ascend.

The invention of balloons is due to the brothers Joseph and Stephen Montgolfier. The balloons made by them were globe-shaped, and constructed of paper, or of paper covered with cloth, the air inside being rarefied by the action of heat. It is curious to remark

[1] The first idea of a balloon must be attributed to Francisco de Lana, who, about 1670, proposed to exhaust the air in globes of copper of sufficient size and thinness to weigh less, under these conditions, than the air displaced. The experiment was not tried, and would certainly not have succeeded, for the pressure of the atmosphere would have caused the globes to collapse. The theory, however, was thoroughly understood by the author, who made an exact calculation of the amount of force tending to make the globes ascend. —*D.*

that in their first attempts they employed hydrogen gas, and showed that balloons filled with this gas could ascend. But as the hydrogen readily escaped through the paper, the flight of the balloons was short, and thus the use of hydrogen was abandoned, and hot air was alone employed.

The name *montgolfières* is still often applied to fire-balloons. They generally consist of a paper envelope with a wide opening below,

Fig. 152.—Fire balloon of Pilatre de Rozier.

in the centre of which is a sponge held in a wire frame. The sponge is dipped in spirit and ignited, when the balloon is to be sent up.

The first public experiment of the ascent of a balloon was performed at Annonay on the 5th June, 1783. On October 21st of the same year, Pilatre de Rozier and the Marquis d'Arlandes achieved the first aerial voyage in a fire-balloon, represented in our figure.

Charles proposed to reintroduce the use of hydrogen by employing an envelope less permeable to the gas. This is usually made of silk varnished on both sides, or of two sheets of silk with a sheet of india-rubber between. Instead of hydrogen, coal-gas is now generally employed, on account of its cheapness and of the facility with which it can be procured.

249.—The lifting power of a balloon is the difference between its weight and that of the air displaced. It is easy to compare the three modes of inflation in this respect.

A cubic metre of air weighs about	1·300	kilogramme.
A cubic metre of hydrogen	·089	„
A cubic metre of coal-gas about	·750	„
A cubic metre of air heated to 200° Cent.	·750	„

We thus see that the lifting power per cubic metre with hydrogen is 1·211, and with coal-gas or hot air about ·500 kilogramme. If, for instance, the total weight to be raised is estimated at 1500 kilogrammes, the volume of a balloon filled with hydrogen capable of raising the weight will be $\frac{1500}{1\cdot210}=1239$ cubic metres. If coal-gas were employed, the required volume would be $\frac{1500}{\cdot550}=2727$ cubic metres.

The car in which the aeronauts sit is usually made of wicker-work or whalebone. It is sustained by cords attached to a net-work (Fig. 153) covering the entire upper half of the balloon, so as to distribute the weight as evenly as possible. The balloon terminates below in a kind of neck opening freely into the air. At the top there is another opening in the inside, which is closed by a valve held to by a spring. Attached to the valve is a cord which passes through the interior of the balloon, and hangs above the car within reach of the hand of the aeronaut.

When the aeronaut wishes to descend, he opens the valve for a few moments and allows some of the gas to escape. An important part of the equipment consists of sand-bags for ballast, which are gradually emptied to check too rapid descent. In the figure is represented a contrivance called a parachute, by means of which the descent is sometimes effected. This is a kind of large umbrella with a hole at the top, from the circumference of which hang cords supporting a small car. When the parachute is left to itself, it opens out, and the resistance of the air, acting upon a large surface, moderates the rate of descent. The hole at the top is essential to safety, as it affords a regular passage for air which would otherwise escape from time to time from under the edge of the parachute, thus producing oscillations which might prove fatal to the aeronaut.

Balloons are not fully inflated at the commencement of the ascent; but the inclosed gas expands as the pressure diminishes outside. The lifting power thus remains nearly constant until the balloon has risen so high as to be fully inflated. Suppose, for instance, that the atmospheric pressure is reduced by one-half, the volume of the balloon will then be doubled; it will thus displace a volume of air twice as great as before, but of only half the density, so that the buoyancy will remain the same. This conclusion, however, is not quite exact, because the solid parts of the balloon do not expand like the gas, and the weight of air displaced by them accordingly diminishes as the balloon rises. If the balloon continues to ascend after it is completely inflated, its lifting power diminishes rapidly, becoming zero when a stratum of air is reached in which the weight of the volume displaced is equal to that of the balloon itself. It is carried past this stratum in the first instance in virtue of the velocity which it has acquired, and finally comes to rest in it after a number of oscillations.

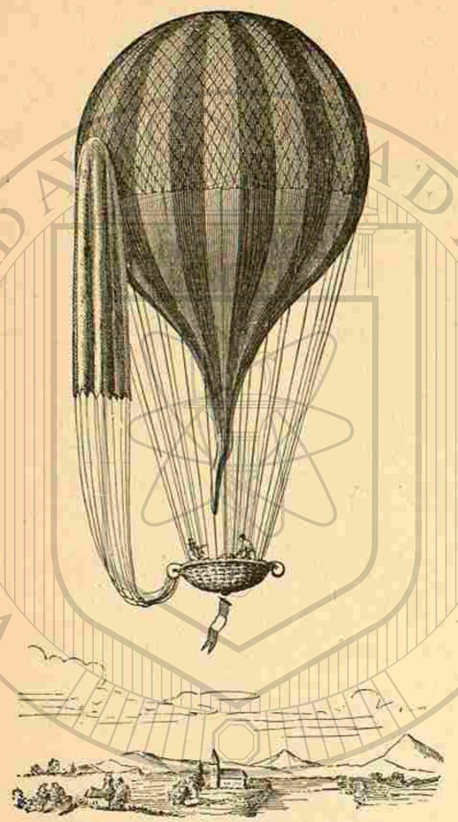


Fig. 153.—Balloon with Car and Parachute.

250. Height Attainable.—The pressure of the air in the stratum of equilibrium can be calculated as follows:

Let V be the volume of gas which the balloon can contain when fully inflated.

v the volume, and w the weight, of the solid parts, including the aeronauts themselves.

δ the density of the gas at the standard pressure and temperature, and D the density of air under the same conditions.

Then if P denote the standard pressure, and p the pressure in the stratum of equilibrium, the density of the gas when this stratum

has been reached will be $\frac{p}{P}\delta$, and the density of the air will be $\frac{p}{P}D$. Equating the weight of the air displaced to that of the floating body, we have

$$\frac{p}{P}(V+v)D = \frac{p}{P}V\delta + w,$$

whence p can be determined.

251. Effect of the Air upon the Weight of Bodies.—The upward pressure of the air impairs the exactness of weighings obtained even with a perfectly true balance, tending, by the principle of the baroscope, to make the denser of two equal masses preponderate. The stamped weights used in weighing are, strictly speaking, standards of mass, and will equilibrate any equal masses in vacuo; but in air the equilibrium will be destroyed by the greater upward pressure of the air upon the larger and less dense body. When the specific gravities of the weights and of the body weighed are known, it is easy from the apparent weight to deduce the true weight (that is to say, the mass) of the body.

Let x be the real weight (or mass) of a body which balances a standard weight of w grammes when the weighing is made in air. Let d be the density of the body, δ that of the standard weight, and α the density of the air. Then the weight of air displaced by the body is $\frac{\alpha}{d}x$, and the weight of air displaced by the standard weight is $\frac{\alpha}{\delta}w$. Hence we have

$$x - \frac{\alpha}{d}x = w - \frac{\alpha}{\delta}w,$$

$$x = w\frac{1-\frac{\alpha}{\delta}}{1-\frac{\alpha}{d}} = w\left\{1+\alpha\left(\frac{1}{d}-\frac{1}{\delta}\right)\right\} \text{ nearly.}$$

Let us take, for instance, a piece of sulphur whose weight has been found to be 100 grammes, the weights being of copper, the density of which is 8·8. The density of sulphur is 2.

We have, by applying the formula,

$$x = 100\left\{1+\frac{1}{770}\left(\frac{1}{2}-\frac{1}{8\cdot 8}\right)\right\} = 100\cdot 05 \text{ grammes.}$$

We see then that the difference is not altogether insensible. It varies in sign, as the formula shows, according as d or δ is the greater. When the density of the body to be weighed is less than that of the weights used, the real weight is greater than the apparent weight; if the contrary, the case is reversed. If the body to be weighed were of the same density as the weights used, the real and apparent weights would be equal. We may remark, that in determining the *ratio* of the weights of two bodies of the same density, by means of standard weights which are all of one material, we need not concern ourselves with the effect of the upward pressure of the air; as the correcting factor, which has the same value for both cases, will disappear in the quotient.

CHAPTER XXII.

PUMPS FOR LIQUIDS.

252. Machines for raising water have been known from very early ages, and the invention of the common pump is pretty generally ascribed to Ctesibius, teacher of the celebrated Hero of Alexandria; but the true theory of its action was not understood till the time of Galileo and Torricelli.

253. Reason of the Rising of Water in Pumps.—Suppose we take a tube with a piston at the bottom (Fig. 154), and immerse the lower end of it in water. The raising of the piston tends to produce a vacuum below it, and the atmospheric pressure, acting upon the external surface of the liquid, compels it to rise in the tube and follow the upward motion of the piston. This upward movement of the water would take place even if some air were interposed between the piston and the water; for on raising the piston, this air would be rarefied, and its pressure no longer balancing that of the atmosphere, this latter pressure would cause the liquid to ascend in a column whose weight, added to the pressure of the air below the piston, would be equal to the atmospheric pressure. This is the principle on which water rises in pumps. These instruments have a considerable variety of forms, of which we shall describe the most important types.

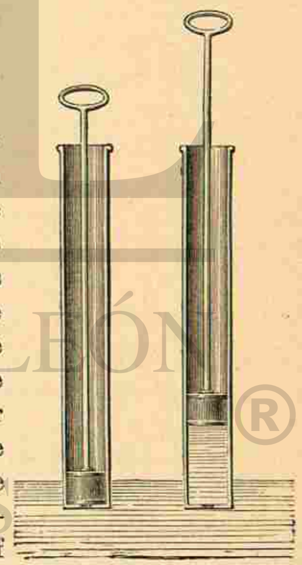


Fig. 154.—Principle of Suction-pump.

254. Suction-pump.—The suction-pump (Fig. 155) consists of a that of the weights used, the real weight is greater than the apparent weight; if the contrary, the case is reversed. If the body to be weighed were of the same density as the weights used, the real and apparent weights would be equal. We may remark, that in determining the *ratio* of the weights of two bodies of the same density, by means of standard weights which are all of one material, we need not concern ourselves with the effect of the upward pressure of the air; as the correcting factor, which has the same value for both cases, will disappear in the quotient.

CHAPTER XXII.

PUMPS FOR LIQUIDS.

252. Machines for raising water have been known from very early ages, and the invention of the common pump is pretty generally ascribed to Ctesibius, teacher of the celebrated Hero of Alexandria; but the true theory of its action was not understood till the time of Galileo and Torricelli.

253. **Reason of the Rising of Water in Pumps.**—Suppose we take a tube with a piston at the bottom (Fig. 154), and immerse the lower end of it in water. The raising of the piston tends to produce a vacuum below it, and the atmospheric pressure, acting upon the external surface of the liquid, compels it to rise in the tube and follow the upward motion of the piston. This upward movement of the water would take place even if some air were interposed between the piston and the water; for on raising the piston, this air would be rarefied, and its pressure no longer balancing that of the atmosphere, this latter pressure would cause the liquid to ascend in a column whose weight, added to the pressure of the air below the piston, would be equal to the atmospheric pressure. This is the principle on which water rises in pumps. These instruments have a considerable variety of forms, of which we shall describe the most important types.

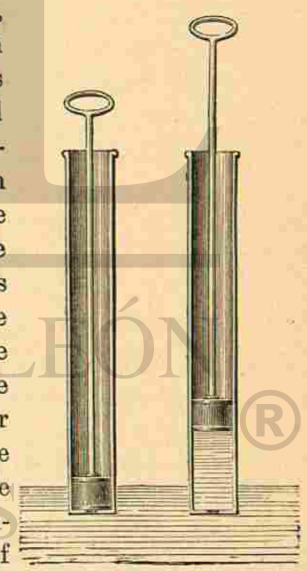


Fig. 154.—Principle of Suction-pump.

254. **Suction-pump.**—The suction-pump (Fig. 155) consists of a

cylindrical pump-barrel traversed by a piston, and communicating by means of a smaller tube, called the suction-tube, with the water in the pump-well. At the junction of the pump-barrel and the tube is a valve opening upward, called the suction-valve, and in the piston is an opening closed by another valve, also opening upward.

Suppose now the suction-tube to be filled with air at the atmospheric pressure, and the water consequently to be at the same level inside the tube and in the well. Suppose the piston to be at the end of its downward stroke, and to be now raised. This motion tends to produce a vacuum below the piston, hence the air contained in the suction-tube will open the suction-valve, and rush into the pump-barrel. The elastic force of this air being thus diminished, the atmospheric pressure will cause the water to rise in the tube to a height such that the pressure due to this height, increased by the pressure of the air inside, will exactly counterbalance the pressure of the atmosphere. If the piston now descends, the suction-valve closes, the water remains at the level to which it has been raised, and the air, being compressed in the barrel, opens the piston-valve and escapes. At the next stroke of the piston, the water will rise still further, and a fresh portion of air will escape.

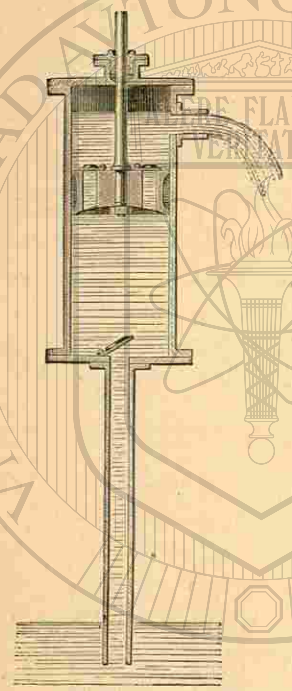


Fig. 155.—Suction-pump.

If, then, the length of the suction-tube is less than about 30 feet, the water will, after a certain number of strokes of the piston, be able to reach the suction-valve and rise into the pump-barrel. When this point has been reached the action changes. The piston in its downward stroke compresses the air, which escapes through it, but the water also passes through, so that the piston when at the bottom of the pump-barrel will have above it all the water which has previously risen into the barrel. If the piston be now raised, supposing the total height to which it is raised to be not more than 34 feet above the level of the water in the well, as should always be the case, the water will follow it in its upward movement, and will fill the

pump-barrel. In the downward stroke this water will pass up through the piston-valve, and in the following upward stroke it will be discharged at the spout. A fresh quantity of water will by this time have risen into the pump-barrel, and the same operations will be repeated.

We thus see that from the time when the water has entered the pump-barrel, at each upward stroke of the piston a volume of water is ejected equal to the contents of the pump-barrel.

In order that the water may be able to rise into the pump-barrel, the suction-valve must not be more than 34 feet above the level of the water in the well, otherwise the water would stop at a certain point of the tube, and could not be raised higher by any farther motion of the piston.

Moreover, in order that the working of the pump may be such as we have described, that is, that at each upward stroke of the piston a quantity of water may be removed equal to the volume of the pump-barrel, it is necessary that the piston when at the top of its stroke should not be more than 34 feet above the water in the well.

255. Effect of untraversed space.—If the piston does not descend to the bottom of the barrel, it is possible that the water may fall short of rising to the suction-valve, even though the total height reached by the piston be less than 34 feet. When the piston is at the end of its downward stroke, the air below it in the barrel is at atmospheric pressure; and when the limit of working has been reached, this air will expand during the upward stroke until it fills the barrel. Its pressure will now be the same as that of the air in the top of the suction-tube; and if this pressure be equivalent to h feet of water, the height to which water can be drawn up will be only $34-h$ feet.

Example. The suction-valve of a pump is at a height of 27 feet above the surface of the water, and the piston, the entire length of whose stroke is 7·8 inches, when at the lowest point is 3·1 inches from the fixed valve; find whether the water will be able to rise into the pump-barrel.

When the piston is at the end of its downward stroke, the air below it in the barrel is at the atmospheric pressure; when the piston is raised this air becomes rarefied, and its pressure, by Boyle's law, becomes $\frac{3 \cdot 1}{10 \cdot 9}$ that of the atmosphere; this pressure can therefore balance a column of water whose height is $34 \times \frac{3\cdot 1}{10\cdot 9}$ feet, or 9·67 feet. Hence, the maximum height to which the water can attain is $34 - 9\cdot 67$ feet $= 24\cdot 33$ feet; and consequently, as the suction-tube is 27 feet long, the water will not rise into the pump-barrel, even supposing the pump to be perfectly free from leakage.

Practically, the pump-barrel should not be more than about 25 feet above the surface of the water in the well; but the spout may be more than 34 feet above the barrel, as the water after rising above the piston is simply pushed up by the latter, an operation which is independent of atmospheric pressure. Pumps in which the spout is at a great height above the barrel are commonly called *lift-pumps*, but they are not essentially different from the suction-pump.

256. Force necessary to raise the Piston.—The force which must be expended in order to raise the piston, is equal to the weight of a column of water, whose base is the section of the piston, and whose height is that to which the water is raised. Let S be the section of the piston, P the atmospheric pressure upon this area, h the height of the column of water which is above the piston in its present position, and h' the height of the column of water below it; then the upper surface of the piston is subjected to a pressure equal to $P + Sh$; the lower face is subjected to a pressure in the opposite direction equal to $P - Sh'$, and the entire downward pressure is represented by the difference between these two, that is, by $S(h + h')$.

The same conclusion would be arrived at even if the water had not yet reached the piston. In this case, let l be the height of the column of water raised; then the pressure below the piston is $P - Sl$; the pressure above is simply the atmospheric pressure P, and, consequently, the difference of these pressures acts downward, and its value is Sl.

257. Efficiency of Pumps.—From the results of last section it follows that the force required to raise the piston, multiplied by the height through which it is raised, is equal to the weight of water discharged multiplied by the height of the spout above the water in the well. This is an illustration of the principle of work (§ 49). As this result has been obtained from merely statical considerations, and on the hypothesis of no friction, it presents too favourable a view of the actual efficiency of the pump.

Besides the friction of the solid parts of the mechanism, there is work wasted in generating the velocity with which the fluid, as a whole, is discharged at the spout, and also in producing eddies and other internal motions of the fluid. These eddies are especially produced at the sudden enlargements and contractions of the passages through which the fluid flows. To these drawbacks must be added loss from leakage of water, and at the commencement of the operation from leakage of air, through the valves and at the circumference of the piston. In common household pumps, which are generally roughly made, the *efficiency* may be as small as ·25 or ·3; that is to say, the product of the weight of water raised, and

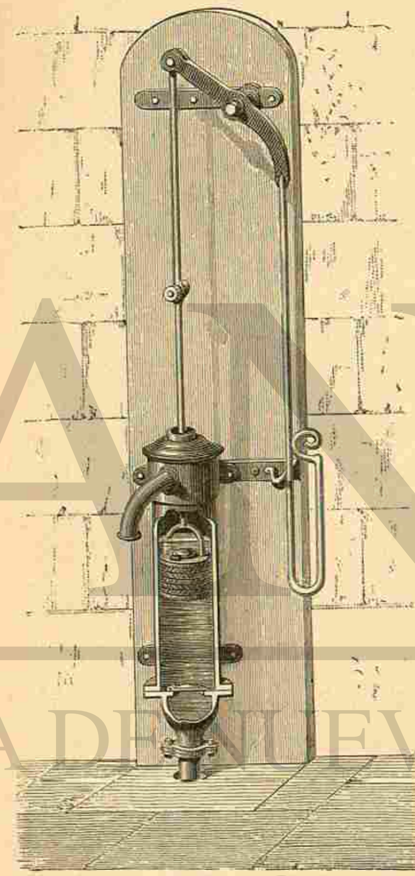


Fig. 156.

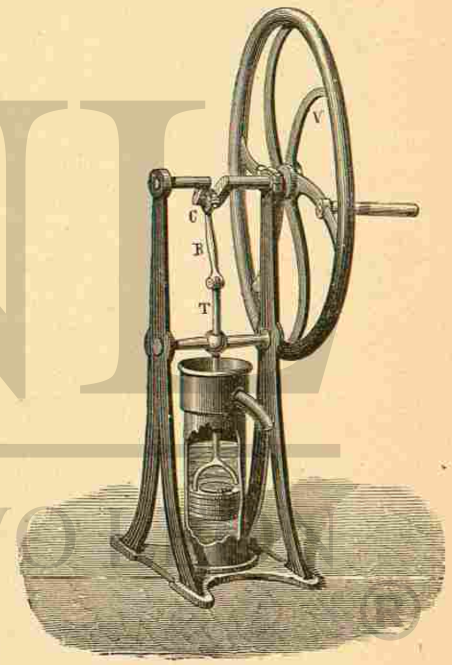


Fig. 157.

Suction-pump.

the height through which it is raised, may be only ·25 or ·3 of the work done in driving the pump.

In Figs. 156 and 157 are shown the means usually employed for working the piston. In the first figure the upward and downward movement of the piston is effected by means of a lever. The second figure represents an arrangement often employed, in which the alternate motion of the piston is effected by means of a rotatory motion. For this purpose the piston-rod T is joined by means of the connecting-rod B to the crank C of an axle turned by a handle attached to the fly-wheel V.

258. Forcing-pump.—The forcing-pump consists of a pump-barrel dipping into water, and having at the bottom a valve opening upward. In communication with the pump-barrel is a side-tube, with a valve at the point of junction, opening from the barrel into the tube. A solid piston moves up and down the pump-barrel, and it is evident that when this piston is raised, water enters the barrel by the lower valve, and that when the piston descends, this water is forced into the side-tube. The greater the height of this tube, the greater will be the force required to push the piston down, for the resistance to be overcome is the pressure due to the column of water raised.

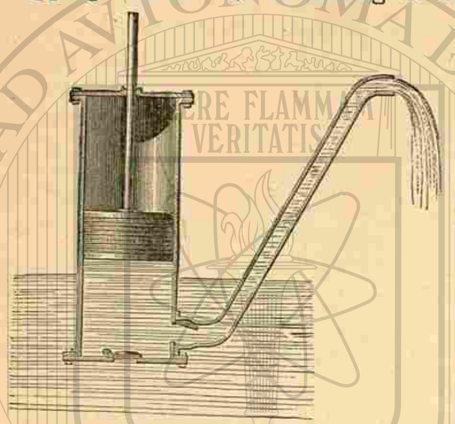


Fig. 158.—Forcing-pump.

The forcing-pump most frequently has a short suction-pipe leading from the reservoir, as represented in Fig. 159. In this case the water is raised from the reservoir into the barrel by atmospheric pressure during the up-stroke, and is forced from the barrel into the ascending pipe in the down-stroke.

259. Plunger.—When the height to which the water is to be forced is very considerable, the different parts of the pump must be very strongly made and fitted together, in order to resist the enormous pressure produced by the column of water, and to prevent leakage. In this case the ordinary piston stuffed with tow or leather washers cannot be used, but is replaced by a solid cylinder of metal called a *plunger*. Fig. 160 represents a section of a pump thus constructed. The plunger is of smaller section than the barrel, and passes through a stuffing-box in which it fits air-tight. The volume of water which enters the barrel at each up-stroke, and is expelled in the down-stroke, is the same as the volume of a length of the plunger equal

to the length of stroke; and the hydrostatic pressure to be overcome is proportional to the section of the plunger, not to that of the barrel. As the operation proceeds, air is set free from the water, and would eventually impede the working of the pump were it not permitted to escape. For this purpose the plunger is pierced with a narrow passage, which is opened from time to time to blow out the air.

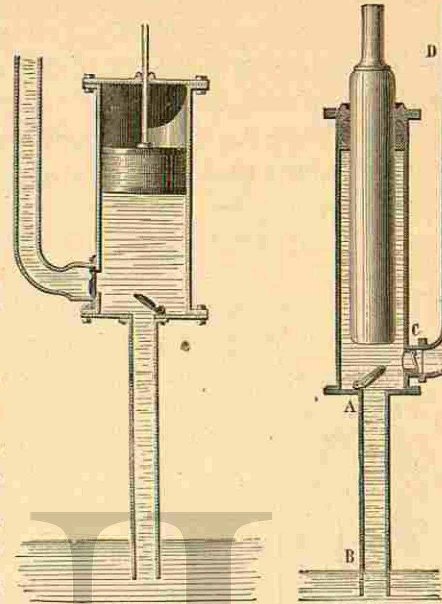


Fig. 159. Fig. 160.
Suction and Force Pump.

The drainage of deep mines is usually effected by a series of pumps. The water is first raised by one pump to a reservoir, into which dips the suction-tube of a second pump, which sends the water up to a second reservoir, and so on. The piston-rods of the different pumps are all joined to a single rod called the *spear*, which receives its motion from a steam-engine.

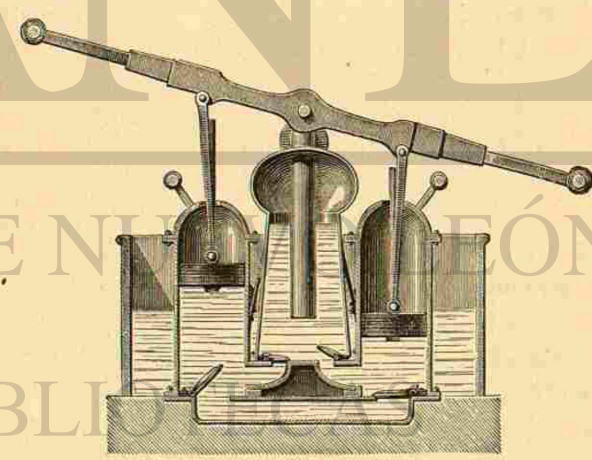


Fig. 161.—Fire-engine.

260. Fire-engine.—The ordinary fire-engine is formed by the union of two forcing-pumps which play into a common reservoir, containing in its upper portion (called the air-chamber) air compressed by the working of the engine. A tube dips into the water in this reservoir, and to the upper end of this tube is screwed the leather hose through which the water is discharged. The piston-rods are jointed to a lever, the ends of which are raised and depressed alternately, so that one piston is ascending while the other is descending. Water is thus continually being forced into the common reservoir except at the instant of reversing stroke, and as the compressed air in the air-chamber performs the part of a reservoir of work (nearly analogous to the fly-wheel), the discharge of water from the nozzle of the hose is very steady.

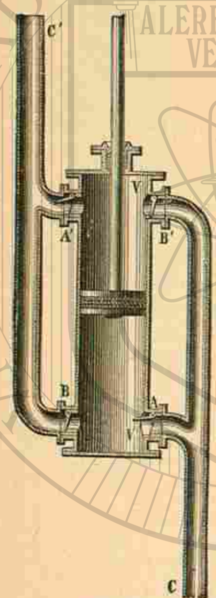


Fig. 162. Double-action Pump.

The engine is sometimes supplied with water by means of an attached cistern (as in Fig. 162) into which water is poured; but it is more usually furnished with a suction-pipe which renders it self-feeding.

261. Double-acting Pumps.—These pumps, the invention of which is due to Delahire, are often employed for household purposes. They consist of a pump-barrel VV (Fig. 162), with four openings in it, A, A′, B, B′. The openings A and B′ are in communication with the suction-tube C; A′ and B are in communication with the ejection-tube C′. The four openings are fitted with four valves opening all in the same direction, that is, from right to left, whence it follows that A and B′ act as suction-valves, and A′ and B as ejection-valves, and, consequently, in whichever direction the piston may be moving, the suction and ejection of water are taking place at the same time.

262. Centrifugal Pumps.—Centrifugal pumps, which have long been used as blowers for air, and have recently come into extensive use for purposes of drainage and irrigation, consist mainly of a flat casing or box of approximately circular outline, in which the fluid is made to revolve by a rotating propeller furnished with fans or blades. These extend from near the centre outwards to the circumference of the propeller, and are usually curved backwards. The

fluid between them, in virtue of the centrifugal force generated by its rotation, tends to move outwards, and is allowed to pass off through a large conduit which leaves the case tangentially.

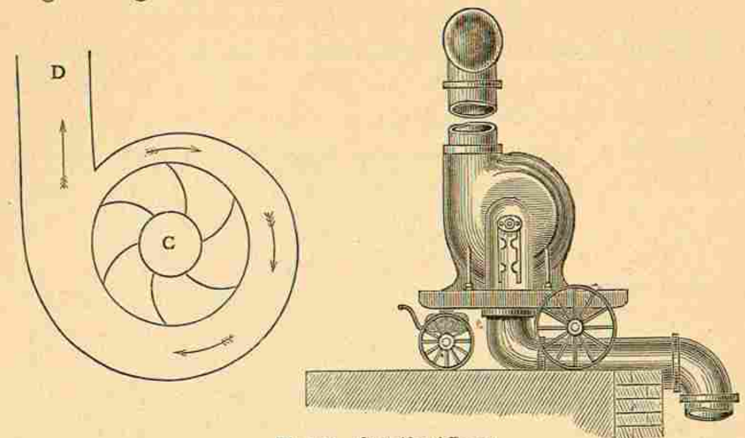


Fig. 163.—Centrifugal Pump.

The first part of Fig. 163 is a section of the propeller and casing, C being a central opening at which the fluid enters, and D the conduit through which it escapes. The second part of the figure represents a small pump as mounted for use. The largest class of

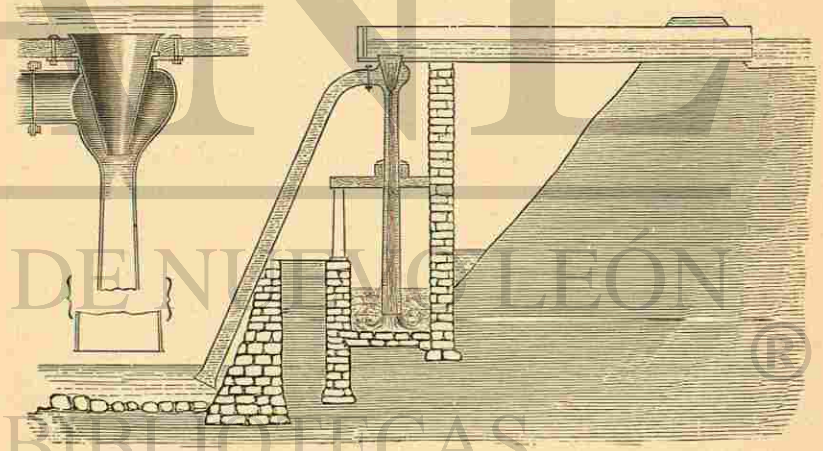


Fig. 164.—Jet Pump.

centrifugal pumps are usually immersed in the water to be pumped, and revolve horizontally.

263. Jet-pump.—The jet-pump is a contrivance by Professor

James Thomson for raising water by means of the descent of other water from above, the common outfall being at an intermediate level. Its action somewhat resembles that of the blast-pipe of the locomotive. The pipe corresponding to the locomotive chimney must have a narrow throat at the place where the jet enters, and must thence widen very gradually towards its outlet, which is immersed in the outfall water so as to prevent any admission of air during the pumping. The water is drawn up from the low level through a suction-pipe, terminating in a chamber surrounding the jet-nozzle.

Fig. 164 represents the pump in position, the jet-nozzle with its surroundings being also shown separately on a larger scale.

The action of the jet-pump is explained by the following considerations.

Suppose we have a horizontal pipe varying gradually in sectional area from one point to another, and completely filled by a liquid flowing steadily through it. Since the same quantity of liquid passes all cross-sections of the pipe, the velocity will vary inversely as the sectional area. Those portions of the liquid which are passing at any moment from the larger to the smaller parts of the pipe are being accelerated, and are therefore more strongly pushed behind than in front; while the opposite is the case with those which are passing from smaller to larger. Places of large sectional area are therefore places of small velocity and high pressure, and on the other hand, places of small area have high velocity and low pressure. Pressure, in such discussions as this, is most conveniently expressed by *pressure-height*, that is, by the height of an equivalent column of the liquid. Neglecting friction, it can be shown that if v_1, v_2 be the velocities at two points in the pipe, and h_1, h_2 the pressure-heights at these points,

$$v_2^2 - v_1^2 = 2g\,(h_1 - h_2),$$

g denoting the intensity of gravity. The change in pressure-height is therefore equal and opposite to the change in $\frac{v^2}{2g}$. This is for a horizontal pipe.

In an ascending or descending pipe, there is a further change of pressure-height, equal and opposite to the change of actual height.

Let H be the pressure-height at the free surfaces, that is, the height of a column of water which would balance atmospheric pressure;

k the difference of level between the jet-nozzle and the free surface above it.

l the difference of level between the jet-nozzle and the free surface of the water which is to be raised.

v the velocity with which the liquid rushes through the jet-nozzle,

then the pressure-height at the jet-nozzle may be taken as $H + k - \frac{v^2}{2g}$; and if this be less than $H - l$ the water will be sucked up. The condition of working is therefore that

$$H - l \text{ be greater than } H + k - \frac{v^2}{2g}, \text{ or}$$

$$\frac{v^2}{2g} \text{ greater than } k + l,$$

where it will be observed that $k + l$ is the difference of levels of the highest and lowest free surfaces.

264. Hydraulic Press.—The hydraulic press (Fig. 165) consists of a suction and force pump *aa* worked by means of a lever turning about an axis O. The water drawn from the reservoir BB is forced along

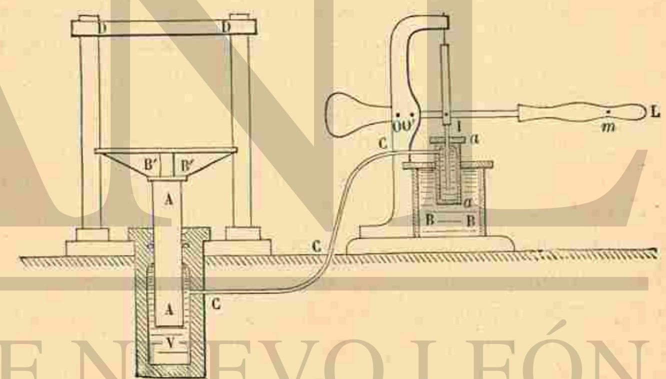


Fig. 165.—Bramah Press.

the tube CC into the cistern V. In the top of the cistern is an opening through which moves a heavy metal plunger AA. This carries on its upper end a large plate B′B′, upon which are placed the objects to be pressed. Suppose the plunger A to be in its lowest position when the pump begins to work. The cistern first begins to fill with water; then the pressure exerted by the plunger of the pump is transmitted, according to the principles laid down in § 141, to the bottom of the plunger A; which accordingly rises, and the objects to be pressed, being intercepted between the plate and the top of a fixed frame, are subjected to the transmitted pressure. The amount of this pressure depends both on the ratio of the sections of the pistons, and on the length of the lever used to work the force-pump. Suppose, for instance, that the distance of the point *m*, where the hand is applied, from the point O, is equal to twelve times the distance IO, and suppose the force exerted to be equal to fifty pounds. By the principle of the lever this is equivalent to a force of 50×12 at the point I; and if the section of the piston A be at the same time 100 times that of the piston of the pump, the pressure transmitted to A will be $50 \times 12 \times 100 = 60{,}000$ pounds. These are the ordinary conditions of the press usually employed in workshops. By drawing out the pin which serves as an axis at O, and introducing it at O′, we can increase the mechanical advantage of the lever.

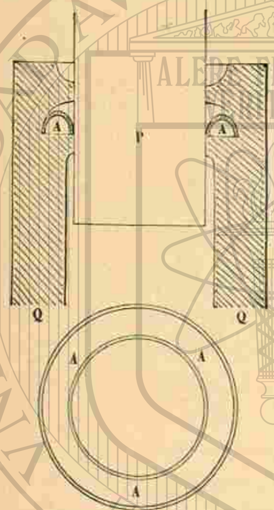


Fig. 166.—Cup-leather.

Two parts essential to the working of the hydraulic press are not represented in the figure. These are a safety-valve, which opens when the pressure attains the limit which is not to be exceeded; and, secondly, a tap in the tube C, which is opened when we wish to put an end to the action of the press. The water then runs off, and the piston A descends again to the bottom of the cistern.

The hydraulic press was clearly described by Pascal, and at a still earlier date by Stevinus, but for a long time remained practically useless; because as soon as the pressure began to be at all strong, the water escaped at the surface of the piston A. Bramah invented the *cupped leather collar*, which prevents the liquid from escaping, and thus enables us to utilize all the power of the machine. It consists of a leather ring AA (Fig. 166), bent so as to have a semicircular section. This is fitted into a hollow in the interior of the sides of the cistern, so that water passing between the piston and cylinder will fill the concavity of the cupped leather collar, and by pressing on it will produce a packing which fits more tightly as the pressure on the piston increases.

The hydraulic press is very extensively employed in the arts.

It is of great power, and may be constructed to give pressures of two or three hundred tons. It is the instrument generally employed in cases where very great force is required, as in testing anchors or raising very heavy weights. It was used for raising the sections of the Britannia tubular bridge, and for launching the *Great Eastern*.

CHAPTER XXIII.

EFFLUX OF LIQUIDS.—TORRICELLI'S THEOREM.

265. If an opening is made in the side of a vessel containing water, the liquid escapes with a velocity which is greater as the surface of the liquid in the vessel is higher above the orifice, or to employ the usual phrase, as the *head* of liquid is greater. This point in the dynamics of liquids was made the subject of experiments by Torricelli, and the result arrived at by him was that the velocity of efflux is equal to that which would be acquired by a body falling freely from the upper surface of the liquid to the centre of the orifice. If h be this height, the velocity of efflux is given by the formula

$$v = \sqrt{2gh}.$$

This is called Torricelli's theorem. It supposes the orifice to be small compared with the horizontal section of the vessel, and to be exposed to the same atmospheric pressure as the upper surface of the liquid in the vessel.

It may be deduced from the principle of conservation of energy; for the escape of a mass m of liquid involves a loss mgh of energy of position, and must involve an equal gain of energy of motion. But the gain of energy of motion is $\frac{1}{2}mv^2$; hence we have

$$\tfrac{1}{2}mv^2 = mgh,\ v^2 = 2gh.$$

The form of the issuing jet will depend, to some extent, on the form of the orifice. If the orifice be a round hole with sharp edges, in a thin plate, the flow through it will not be in parallel lines, but the outer portions will converge towards the axis, producing a rapid narrowing of the jet. The section of the jet at which this convergence ceases and the flow becomes sensibly parallel, is called the *contracted vein* or *vena contracta*. The pressure within the jet at this part is atmospheric, whereas in the converging part it is greater

than atmospheric; and it is to the contracted vein that Torricelli's formula properly applies, v denoting the velocity at the contracted vein, and h the depth of its central point below the free surface of the liquid in the vessel.

266. Area of Contracted Vein. Froude's Case.—A force is equal to the momentum which it generates in the unit of time. Let A denote the area of an orifice through which a liquid issues horizontally, and a the area of the contracted vein. From the equality of action and reaction it follows that the resultant force which ejects the issuing stream is equal and opposite to the resultant horizontal force exerted on the vessel. The latter may be taken as a first approximation to be equal to the pressure which would be exerted on a plug closing the orifice, that is to ghA if the density of the liquid be taken as unity.

The horizontal momentum generated in the water in one second is the product of the velocity v and the mass ejected in one second. The volume ejected in one second is va. This is equal to the mass, since the density is unity, and hence the momentum is v^2a, that is, $2gha$. Equating this last expression for the momentum to the foregoing expression for the force, we have

$$2gha = gh\text{A}$$
$$a = \tfrac{1}{2}\text{A},$$

that is, the area of the contracted vein is half the area of the orifice.

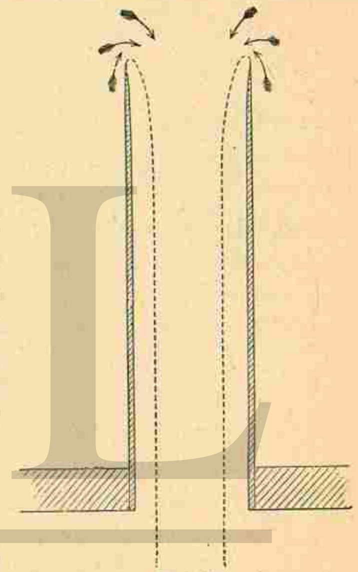


Fig. 167.

Mr. Froude has pointed out that this reasoning is strictly correct when the liquid is discharged through a cylindrical pipe projecting inwards into the vessel and terminating with a sharp edge (Fig. 167); and he has verified the result by accurate experiments in which the jet was discharged vertically downwards. The direction of flow in different parts of the jet is approximately indicated by the arrows and dotted lines in the figure; and, on a larger, scale by those in Fig. 168, in which the sections of the orifice and of the contracted vein are also indicated by the lines marked D and d. We may remark that since liquids press equally in all directions, there can

15

be no material difference between the velocities of a vertical and of a horizontal jet at the same depth below the free surface.

267. Contracted Vein for Orifice in Thin Plate.—When the liquid is simply discharged through a hole cut in the side of the vessel and bounded by a sharp edge, the direction of flow in different parts of the stream is shown by the arrows and dotted lines in Fig. 169. The pressure on the sides, in the neighbourhood of the orifice, is less than that due to the depth, because the curved form of the lines of flow implies (on the principles of centrifugal force) a smaller pressure on their concave than on their convex side. The pressure around the orifice is therefore less than it would be if the hole were plugged. The unbalanced horizontal pressure on the vessel (if we suppose the side containing the jet to be vertical) will therefore exceed the statical pressure on the plug ghA, since the removal of the plug not only removes the pressure on the plug but also a portion of the pressure on neighbouring parts. This unbalanced force, which is greater than ghA, is necessarily equal to the momentum generated per second in the liquid, which is still represented by the expression v^2a or $2gha$; hence $2gha$ is greater than ghA, or a is greater than $\frac{1}{2}A$. Reasoning similar to this applies to all ordinary forms of orifice. The peculiarity of the case investigated by Mr. Froude consists in the circumstance that the pressure on the parts of the vessel in the neighbourhood of the orifice is normal to the direction of the jet, and any changes in its amount which may be produced by unplugging the orifice have therefore no influence upon the pressures on the vessel in or opposite to the direction of the jet.[1]

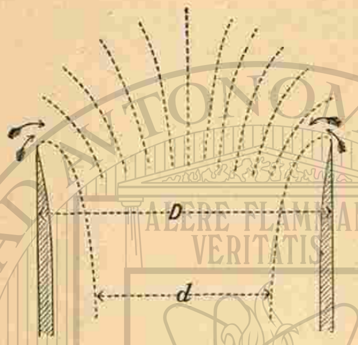


Fig. 168.

Fig. 169.

268. Apparatus for Illustration.—In the preceding investigations, no account is taken of friction. When experiments are conducted on too small a scale, friction may materially diminish the velocity; and further, if the velocity be tested by the height or distance to which the jet will spout, the resistance of the air will diminish this height or distance, and thus make the velocity appear less than it really is.

[1] This section and the preceding one are based on two communications read before the Philosophical Society of Glasgow, February 23d and March 31st, 1876; one being an extract from a letter from Mr. Froude to Sir William Thomson, and the other a communication from Professor James Thomson, to whom we are indebted for the accompanying illustrations.

Fig. 170 represents an apparatus frequently employed for illustrating some of the consequences of Torricelli's theorem. An upright cylindrical vessel is pierced on one side with a number of orifices in the same vertical line, which can be opened or closed at pleasure. A tap placed above the vessel supplies it with water, and, with the help of an overflow pipe, maintains the surface at a constant level, which is as much above the highest orifice as each orifice is above that next below it. The liquid which escapes is received in a trough, the edge of which is graduated. A travelling piece with an index

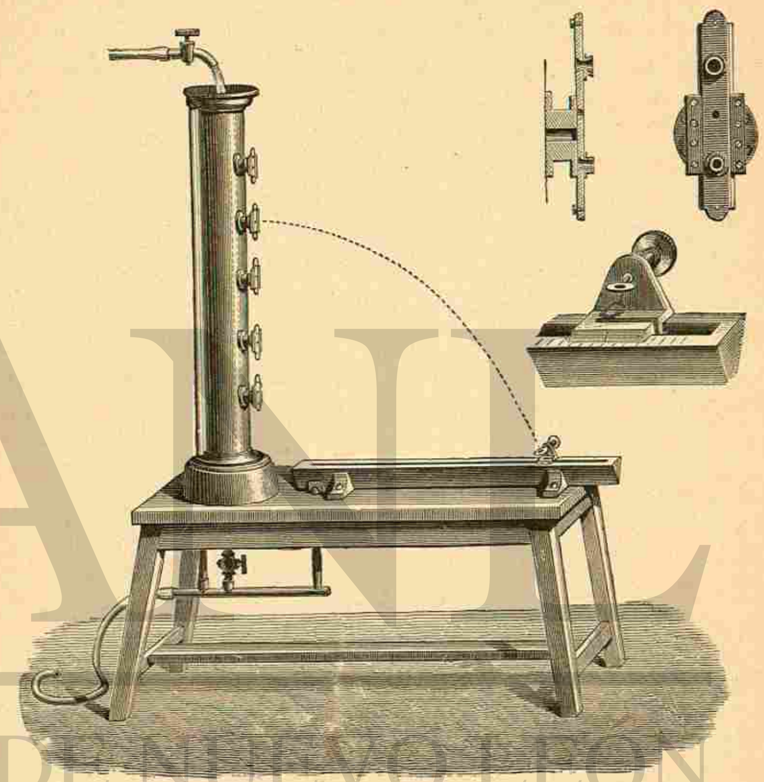


Fig. 170.—Apparatus for verifying Torricelli's Theorem.

line engraved on it slides along the trough; it carries, as shown in one of the separate figures, a disc pierced with a circular hole, and capable of being turned in any direction about a horizontal axis passing through its centre. In this way the disc can always be placed in such a position that its plane shall be at right angles to the liquid jet, and that the jet shall pass freely and exactly through its centre. The index line then indicates the range of the jet with considerable precision. This range is reckoned from the vertical plane containing the orifices, and is measured on the horizontal plane passing through the centre of the disc. The distance of this latter plane below the lowest orifice is equal to that between any two consecutive orifices.

The jet, consisting as it does of a series of projectiles travelling in the same path, has the form of a parabola.

Let a be the range of the jet, b the height of the orifice above the centre of the ring, and v the velocity of discharge, which we assume to be horizontal. Then if t be the time occupied by a particle of the liquid in passing from the orifice to the ring, we have to express that a is the distance due to the horizontal velocity v in the time t, and that b is the vertical distance due to gravity acting for the same time. We have therefore

$$a = vt$$
$$b = \tfrac{1}{2}gt^2$$
$$\text{whence } t^2 = \frac{a^2}{v^2} = \frac{2b}{g}, \quad v^2 = \frac{ga^2}{2b}.$$

But according to Torricelli's theorem, if h be the height of the surface of the water above the orifice, we have $v^2 = 2gh$; and comparing this with the above value of v^2 we deduce

$$\frac{a^2}{2b} = 2h, \quad a^2 = 4bh.$$

One consequence of this last formula is, that if the values of b and h be interchanged, the value of a will remain unaltered. This amounts to saying that the highest orifice will give the same range as the lowest, the highest but one the same as the lowest but one, and so on; a result which can be very accurately verified.

If we describe a semicircle on the line $b+h$, the length of an ordinate erected at the point of junction of b and h is $\sqrt{bh}$, and since $a = \sqrt{4bh} = 2\sqrt{bh}$, it follows that the range is double of this ordinate. This is on the hypothesis of no friction. Practically it is less than double. The greatest ordinate of the semicircle is the central one, and accordingly the greatest range is given by the central orifice.

269. Efflux from Air-tight Space.—When the air at the free surface of the liquid in a vessel is at a different pressure from the air into which the liquid is discharged, we must express this difference of pressures by an equivalent column of the liquid, and the velocity of efflux will be that due to the height of the surface above the orifice increased or diminished by this column. Efflux will cease altogether when the pressure on the free surface, together with that due to the height of the free surface above the orifice, is equal to the pressure outside the orifice; or if efflux continue under such circumstances it can only do so by the admission of bubbles of air. This explains the action of vent-pegs.

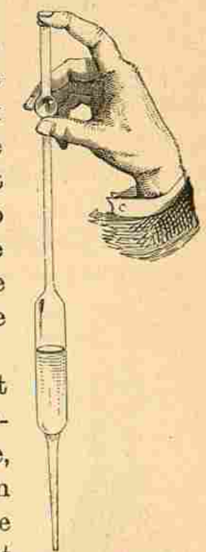


Fig 171.—Pipette.

Pipette.—This is a glass tube (Fig. 171) open at both ends, and terminating below in a small tapering spout. If water be introduced into the tube, either by aspiration or by direct immersion in water, and if the upper end be closed with the finger, the efflux of the liquid will cease almost instantly. On admitting the air above, the efflux will begin again, and can again be stopped at pleasure.

The Magic Funnel.—This funnel is double, as is shown in Fig. 172. Near the handle is a small opening by which the space between the two funnels communicates with the external air. Another opening connects this same space with the tube of the inner funnel. If the interval between the two funnels be filled with any liquid, this liquid will run out or will cease to flow according as the upper hole is open or closed. The opening and closing of the hole can be easily effected with the thumb of the hand holding the funnel without the knowledge of the spectator. This device has been known from very early times.

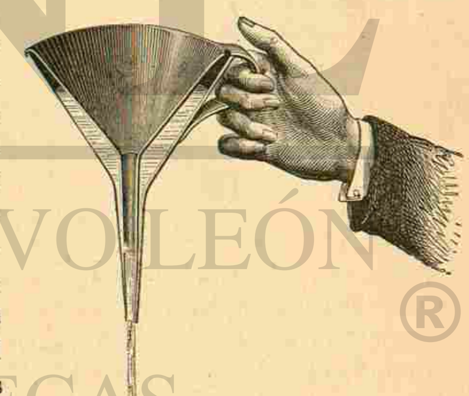


Fig. 172.—Magic Funnel.

The instrument may be used in a still more curious manner. For this purpose the space inside is secretly filled with highly-coloured wine, which is prevented from escaping by closing the opening above.

Water is then poured into the central funnel, and escapes either by itself or mixed with wine, according as the thumb closes or opens the orifice for the admission of air. In the second case, the water being coloured with the wine, it will appear that wine alone is issuing from the funnel; thus the operator will appear to have the power of making either water or wine flow from the vessel at his pleasure.

The Inexhaustible Bottle.—The inexhaustible bottle (Fig. 173) is a toy of the same kind. It is an opaque bottle of sheet-iron or

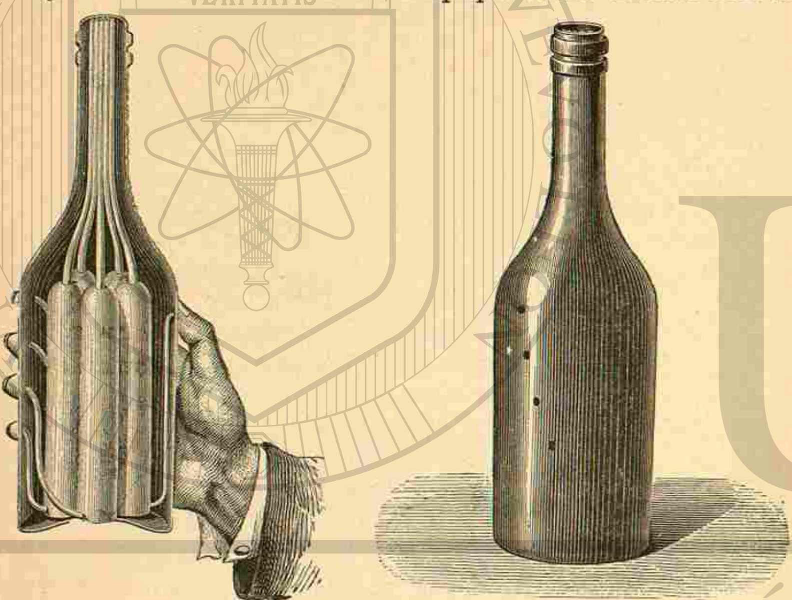


Fig. 173.—Inexhaustible Bottle.

gutta-percha, containing within it five small vials. These communicate with the exterior by five small holes, which can be closed by the five fingers of the hand. Each vial has also a small neck which passes up the large neck of the bottle. The five vials are filled with five different liquids, any one of which can be poured out at pleasure by uncovering the corresponding hole.

270. Intermittent Fountain.—The intermittent fountain is an apparatus analogous to the preceding, except that the interruptions in the efflux are produced automatically by the action of the instru-

ment, without the intervention of the operator. It consists of a globe V (Fig. 174), which can be closed air-tight by means of a stopper, and is in communication with efflux tubes *a*, which discharge into a basin B, having a small hole *o* in its bottom for permitting the water to escape into a lower basin C. A central tube *t*, open at both ends, extends nearly to the top of the globe, and nearly to the bottom of the basin B.

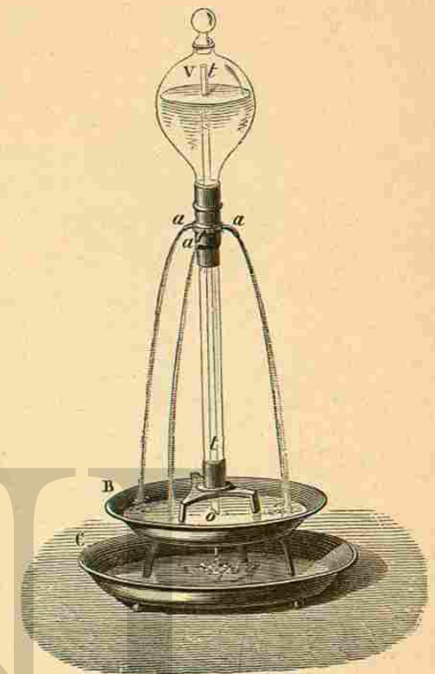


Fig. 174.—Intermittent Fountain.

Suppose the globe to be filled with water, the basins being empty. Then the water will flow from the efflux tubes *a*, while air will pass up through the central tube. As the water issues from the efflux tubes much faster than it escapes through the opening *o*, the level rises in the basin B till the lower end of the tube *t* is covered. The pressure of the air in the upper part of the globe then rapidly diminishes, and the efflux from the tubes *a* is stopped. But as the water continues to escape from the basin B through the opening *o*, the bottom of the tube *t* is again uncovered, the liquid again issues from the efflux tubes, and the same changes are repeated.

271. Siphon.—The siphon is an instrument in which a liquid, under the combined action of its own weight and atmospheric pressure, flows first up-hill and then down-hill, but always in such a way as to bring about a lowering of the centre of gravity of the whole liquid mass.

In its simplest form, it consists of a bent tube, one end of which is immersed in the liquid to be removed, while the other end either discharges into the air, at a lower level than the surface of the liquid in the vessel, as in Fig. 175, or dips into the liquid of a receiving vessel, the surface of this liquid being lower than that of the liquid in the discharging vessel.

We shall discuss the latter case, and shall denote the difference of levels of the two surfaces by h, while the height of a column of the liquid equivalent to atmospheric pressure will be denoted by H.

Let the siphon be full of liquid, and imagine a diaphragm to be drawn across it at any point, so as to prevent flow. Let this diaphragm be at a height x above the higher of the two free surfaces, and at a height y above the lower, so that we have

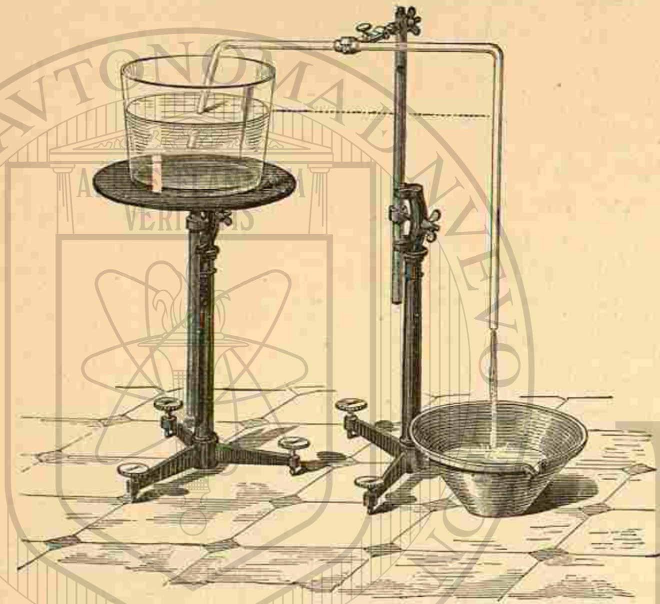


Fig. 175.—Siphon.

$$y - x = h.$$

The pressure on the side of the diaphragm next the higher free surface will be $H - x$, (pressure being expressed in terms of the equivalent liquid column,) and the pressure on the other side of the diaphragm will be $H - y$, which is less than the former by $y - x$, that is by h. The diaphragm therefore experiences a resultant force due to a depth h of the liquid, urging it from the higher to the lower free surface, and if the diaphragm be removed, the liquid will be propelled in this direction.

In practice, the two legs of the siphon are usually of unequal length, and the flow is from the shorter to the longer; but this is by no means essential, for by a sufficiently deep immersion of the long leg, the direction of flow may be reversed. The direction of flow depends not on the lengths of the legs, but on the levels of the two free surfaces.

If the liquid in the discharging vessel falls below the end of the siphon, or if the siphon is lifted out of it, air enters, and the siphon is immediately emptied of liquid. If the liquid in the receiving vessel is removed, so that the discharging end of the siphon is surrounded by air, as in the figure, the flow will continue, unless air bubbles up the tube and breaks the liquid column. This interruption is especially liable to occur in large tubes. It can be prevented by bending the end of the siphon round, so as to discharge the liquid in an ascending direction. To adapt the foregoing investigation to the case of a siphon discharging into air, we have only to substitute the level of the discharging end for the level of the lower free surface, so that y will denote the depth of the discharging end below the diaphragm, and h its depth below the surface of the liquid which is to be drawn off.

As the ascent of the liquid in the siphon is due to atmospheric pressure on the upper free surface, it is necessary that the highest point of the siphon (if intended for water) should not be more than about 33 feet above this surface.

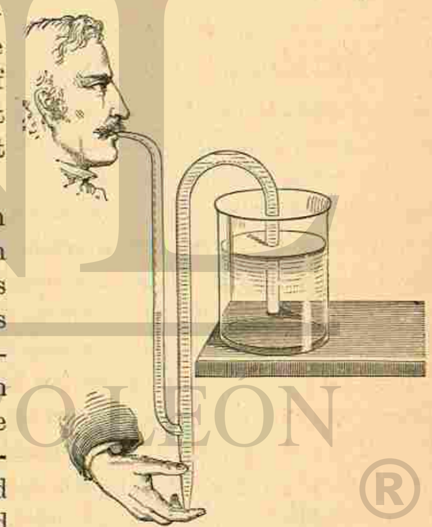


Fig. 176.—Starting the Siphon.

272. Starting the Siphon.—In order to make a siphon begin working, we must employ means to fill it with the liquid. This can sometimes be done by dipping it in the liquid, and then placing it in position while the ends are kept closed; or by inserting one end in the liquid which we wish to remove, and sucking at the other. It is usually more convenient to apply suction by means of a side tube, as in Fig. 176, this tube being sometimes provided with an enlargement to prevent the liquid from entering the mouth. One end of the siphon is inserted in the liquid which is to be removed, while the other end is stopped, and the operator applies suction at the side tube till the liquid flows over. In siphons for commercial purposes, the suction is usually produced by a pump.

273. Siphon for Sulphuric Acid.—Fig. 177 represents a siphon used for transferring sulphuric acid from one vessel to another. The long branch is first filled with sulphuric acid. This is effected by means of two funnels (which can be plugged at pleasure) at the bend of the tube. One of these admits the liquid, and the other suffers the air to escape. The two funnels are then closed, and the tap at the lower end of the tube is opened so as to allow the liquid to escape. The air in the short branch follows the acid, and becomes rarefied; the acid behind it rises, and if it passes the bend, the siphon will be started, for each portion of the liquid which issues from the tube will draw an equal portion from the short to the long branch.

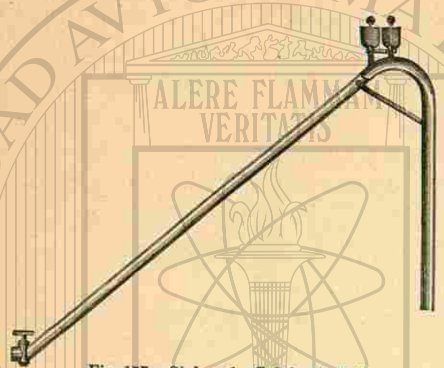


Fig. 177.—Siphon for Sulphuric Acid.

To insure the working of the sulphuric acid siphon, it is not sufficient to have the vertical height of the long branch greater than that of the short branch; it is farther necessary that it should exceed a certain limit, which depends upon the dimensions of the siphon in each particular case. In order to calculate this limit, we must remark that when the liquid begins to flow, its height diminishes in the long and increases in the short branch; if these two heights should become equal, there would be equilibrium. We see, then, that in order that the siphon may work, it is necessary that when the liquid rises to the bend of the tube, there should be in the long branch a column of liquid whose vertical height is at least equal to that of the short branch, which we shall denote by h, and the actual length of the short branch from the surface of the liquid in which it dips to the summit of the bend by h'. Then if α be the inclination of the long branch to the vertical, and L the length of the long branch, which we suppose barely sufficient, the length of the column of liquid remaining in the long branch will be $h \sec \alpha$. The air which at atmospheric pressure H occupied the length h', now under the pressure $H - h$ occupies a length $L - h \sec \alpha$; hence by Boyle's law, we have

$$Hh' = (H - h)(L - h \sec \alpha), \text{ whence } L = h \sec \alpha + \frac{Hh'}{H - h}.$$

In this formula H denotes the height of a column of sulphuric acid whose pressure equals that of the atmosphere.

274. Cup of Tantalus.—The siphon may be employed to produce the intermittent flow of a liquid. Suppose, for instance, that we have a cup (Fig. 178) in which is a bent tube rising to a height n, and with the short branch terminating near the bottom of the cup, while the long branch passes through the bottom. If liquid be poured into the cup, the level will gradually rise in the short branch of the bent tube, till it reaches the summit of the bend, when the siphon will begin to discharge the liquid. If the liquid then escapes by the siphon faster than it is poured into the vessel, the level of the liquid in the cup will gradually fall below the termination of the shorter branch. The siphon will then empty itself, and will not recommence its action till the liquid has again risen to the level of the bend.

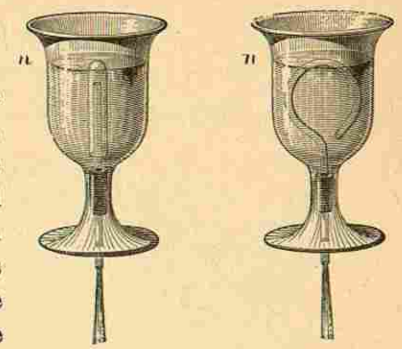


Fig. 178.—Vase of Tantalus.

The siphon may be concealed in the interior of the figure of a man whose mouth is just above the top of the siphon. If water be poured in very slowly, it will continually rise nearly to his lips and then descend again. Hence the name. Instead of a bent tube we may employ, as in the first figure, a straight tube covered by a bell-glass left open below; in this case the space between the tube and the bell takes the place of the shorter leg of the siphon.

It is to an action of this kind that natural intermittent springs are generally attributed. Suppose a reservoir (Fig. 179) to communicate with an outlet by a bent tube forming a siphon, and suppose it to be fed by a stream of water at a slower rate than the siphon is able to discharge it. When the water has reached the bend, the siphon will become charged, and the reservoir will be emptied; flow will then cease until it becomes charged again.

275. Mariotte's Bottle.—This is an apparatus often employed to obtain a uniform flow of water. Through the cork at the top of the bottle (Fig. 180) passes a straight vertical tube open at both ends, and in one side of the bottle near the bottom is a second opening furnished with a horizontal efflux tube *b* at a lower level than the lower end of the vertical tube. Suppose that both the bottle and the vertical tube are in the first instance full of water, and that the efflux tube is then opened. The liquid flows out, and the vertical tube is rapidly emptied. Air then enters the bottle through the vertical tube, and bubbles up from its lower end *a* through the liquid to the upper part of the bottle. As soon as this process begins, the velocity of efflux, which up to this point has been rapidly diminishing (as is shown by the diminished range of the

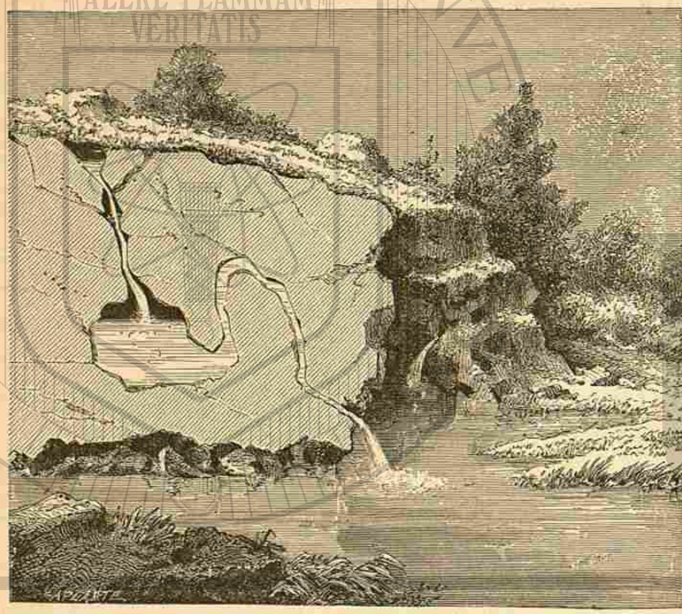


Fig. 179.—Intermittent Spring.

jet), becomes constant, and continues so till the level of the liquid has fallen to *a*, after which it again diminishes. During the time of constant flow, the velocity of efflux is that due to the height of *a* above *b*, and the air in the upper part of the bottle is at less than atmospheric pressure, the difference being measured by the height of the surface of the liquid above *a*. Strictly speaking, since the air enters not in a continuous stream but in bubbles, there must be slight oscillations of velocity, keeping time with the bubbles, but they are scarcely perceptible.

Instead of the vertical tube, we may have a second opening in the

side of the bottle, at a higher level than the first; as shown in Fig. 180. Air will enter through the pipe *a*, which is fitted in this upper opening, and the liquid will issue at the lower pipe *b*, with a constant velocity due to the height of *a* above *b*.

Mariotte's bottle is sometimes used in the laboratory to produce

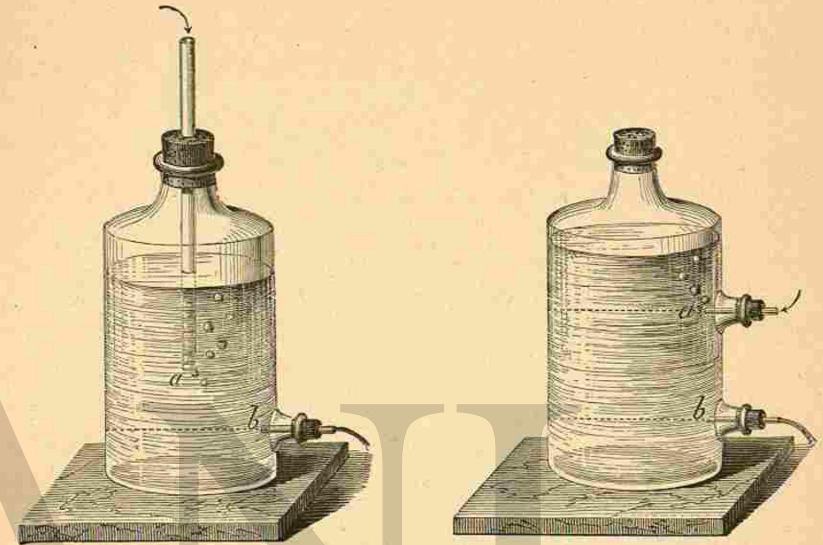


Fig. 180.—Mariotte's Bottle.

the uniform flow of a gas by employing the water which escapes to expel the gas. We may also draw in gas through the tube of Mariotte's bottle; in this case, the flow of the *water* is uniform, but the flow of the *gas* is continually accelerated, since the space occupied by it in the bottle increases uniformly, but the density of the gas in this space continually increases.

a U tube contains Hg to the height of 4 cu cms. in one leg and another liquid to the height of 54.4 cms in other Find density of other liquid Hg = 13.6

$4 \times 13.6 = 54.4 \times D.$

$D \quad D' \quad V \quad V'$

$p = wh$

$\frac{D}{D'} = \frac{V}{V'}$

$DV' = D'V$

4 = Volume of mercury
54.4 – Volume of liquid
13.6 = sp gr of mercury
x = sp " " of liquid

$4. \times 13.6 = 54.4. X$

$4.54.4) 54.4$

$x = 1$

water.

$pV = p_1(V+v)$

$p_1 = p\left(\frac{V}{V+v}\right)$

$p_2 = p_1\left(\frac{V}{V+v}\right) = p\left(\frac{V}{V+v}\right)^2$

$p_n = p\left(\frac{V}{V+v}\right)^n$

7.55
685
1.700) 7.38
5100 0.50

A vertical flood gate is 5 ft height & 3 feet wide Its upper edge is at the surface of the watter. Find pressure

5 x 3 = 15 sq feet

P = VD

5 | 1/2 P = 15. x 1

10, 2.5 P = 15 x 62.5

62.5
15
3125
625
937.5

62.5 weight of a cubic foot of watter.

P = 937.5 x 5/2 = 2,343.75 lbs.

P = height of centre of gravity times volume times weight of a cubic foot of watter

937.5
2.5
46875
18750
2343.75 ans

EXAMPLES.

Parallelogram of Velocities, and Parallelogram of Forces.

1. A ship sails through the water at the rate of 10 miles per hour, and a ball rolls across the deck in a direction perpendicular to the course, at the same rate. Find the velocity of the ball relative to the water.

2. The wind blows from a point intermediate between N. and E. The northerly component of its velocity is 5 miles per hour, and the easterly component is 12 miles per hour. Find the total velocity.

3. The wind is blowing due N.E. with a velocity of 10 miles an hour. Find the northerly and easterly components.

4. Two forces of 6 and 8 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

5. Two equal forces of 100 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

6. A force of 100 units acts at an inclination of 45° to the horizon. Resolve it into a horizontal and a vertical component.

7. Two equal forces act in lines which meet in a point, and the angle between their directions is 120°. Show that the resultant is equal to either of the forces.

8. A body is pulled north, south, east, and west by four strings whose directions meet in a point, and the forces of tension in the strings are equal to 10, 15, 20, and 32 lbs. weight respectively. Show that the resultant is equal to 13 lbs. weight.

9. Five equal forces act at a point, in one place. The angles between the first and second, between the second and third, between the third and fourth, and between the fourth and fifth, are each 60°. Find their resultant.

10. If θ be the angle between the directions of two forces P and Q acting at a point, and R be their resultant, show that

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta.$$

11. Show that the resultant of two equal forces P, acting at an angle θ, is $2P \cos \frac{1}{2}\theta$.

Parallel Forces, and Centre of Gravity.

10*. A straight rod 10 ft. long is supported at a point 3 ft. from one end. What weight hung from this end will be supported by 12 lbs. hung from the other, the weight of the rod being neglected?

11*. Weights of 15 and 20 lbs. are hung from the two ends of a straight rod 70 in. long. Find the point about which the rod will balance, its own weight being neglected.

A vertical flood gate is 5 ft height & 3 feet wide Its upper edge is at the surface of the water. Find pressure

5 x 3 = 15 sq feet

P = VD

5 | 2 10. 2.5

P = 15. x 1

P = 15 x 62.5

62.5
15
3125
625
937.5

62.5 weight of a cubic foot of water.

P = 937.5 x 5/2 = 2,343.75 lbs.

P = height of centre of gravity times volume times weight of a cubic foot of water

937.5
2.5
46875
18750
2343.75 ans

EXAMPLES.

Parallelogram of Velocities, and Parallelogram of Forces.

1. A ship sails through the water at the rate of 10 miles per hour, and a ball rolls across the deck in a direction perpendicular to the course, at the same rate. Find the velocity of the ball relative to the water.

2. The wind blows from a point intermediate between N. and E. The northerly component of its velocity is 5 miles per hour, and the easterly component is 12 miles per hour. Find the total velocity.

3. The wind is blowing due N.E. with a velocity of 10 miles an hour. Find the northerly and easterly components.

4. Two forces of 6 and 8 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

5. Two equal forces of 100 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

6. A force of 100 units acts at an inclination of 45° to the horizon. Resolve it into a horizontal and a vertical component.

7. Two equal forces act in lines which meet in a point, and the angle between their directions is 120°. Show that the resultant is equal to either of the forces.

8. A body is pulled north, south, east, and west by four strings whose directions meet in a point, and the forces of tension in the strings are equal to 10, 15, 20, and 32 lbs. weight respectively. Show that the resultant is equal to 13 lbs. weight.

9. Five equal forces act at a point, in one place. The angles between the first and second, between the second and third, between the third and fourth, and between the fourth and fifth, are each 60°. Find their resultant.

10. If θ be the angle between the directions of two forces P and Q acting at a point, and R be their resultant, show that

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta.$$

11. Show that the resultant of two equal forces P, acting at an angle θ, is $2P \cos \frac{1}{2}\theta$.

Parallel Forces, and Centre of Gravity.

10*. A straight rod 10 ft. long is supported at a point 3 ft. from one end. What weight hung from this end will be supported by 12 lbs. hung from the other, the weight of the rod being neglected?

11*. Weights of 15 and 20 lbs. are hung from the two ends of a straight rod 70 in. long. Find the point about which the rod will balance, its own weight being neglected.

12. A weight of 100 lbs. is slung from a pole which rests on the shoulders of two men, A and B. The distance between the points where the pole presses their shoulders is 10 ft., and the point where the weight is slung is 4 ft. from the point where the pole presses on A's shoulder. Find the weight borne by each, the weight of the pole being neglected.

13. A uniform straight lever 10 ft. long balances at a point 3 ft. from one end, when 12 lbs. are hung from this end and an unknown weight from the other. The lever itself weighs 8 lbs. Find the unknown weight.

14. A straight lever 6 ft. long weighs 10 lbs., and its centre of gravity is 4 ft. from one end. What weight at this end will support 20 lbs. at the other, when the lever is supported at 1 ft. distance from the latter?

15. Two equal weights of 10 lbs. each are hung one at each end of a straight lever 6 ft. long, which weighs 5 lbs.; and the lever, thus weighted, balances about a point 3 in. distant from the centre of its length. Find its centre of gravity.

16. A uniform lever 10 ft. long balances about a point 1 ft. from one end, when loaded at that end with 50 lbs. Find the weight of the lever.

17. A straight lever 10 ft. long, when unweighted, balances about a point 4 ft. from one end; but when loaded with 20 lbs. at this end and 4 lbs. at the other, it balances about a point 3 ft. from the end. Find the weight of the lever.

18. A lever is to be cut from a bar weighing 3 lbs. per ft. What must be its length that it may balance about a point 2 ft. from one end, when weighted at this end with 50 lbs.? (The solution of this question involves a quadratic equation.)

19. A lever is supported at its centre of gravity, which is nearer to one end than to the other. A weight P at the shorter arm is balanced by 2 lbs. at the longer; and the same weight P at the longer arm is balanced by 18 lbs. at the shorter. Find P.

20. Weights of 2, 3, 4 and 5 lbs. are hung at points distant respectively 1, 2, 3 and 4 ft. from one end of a lever whose weight may be neglected. Find the point about which the lever thus weighted will balance. (This and the following questions are best solved by taking moments round the end of the lever. The sum of the moments of the four weights is equal to the moment of their resultant.)

21. Solve the preceding question, supposing the lever to be 5 ft. long, uniform, and weighing 2 lbs.

22. Find, in position and magnitude, the resultant of two parallel and oppositely directed forces of 10 and 12 units, their lines of action being 1 yard apart.

23. A straight lever without weight is acted on by four parallel forces at the following distances from one end:—

At 1 ft.,	a force of	2 units,	acting	upwards.
At 2 ft.,	„	3	„	downwards.
At 3 ft.,	„	4	„	upwards.
At 4 ft.,	„	5	„	downwards.

Where must the fulcrum be placed that the lever may be in equilibrium, and what will be the pressure against the fulcrum?

24. A straight lever, turning freely about an axis at one end, is acted on by four parallel forces, namely—

A downward force of	3 lbs.	at 1 ft.	from axis.
A downward force of	5 „	3 ft.	„
An upward force of	4 „	2 ft.	„
An upward force of	6 „	4 ft.	„

What must be the weight of the lever that it may be in equilibrium, its centre of gravity being 3 ft. from the axis?

25. In a pair of nut-crackers, the nut is placed one inch from the hinge, and the hand is applied at a distance of six inches from the hinge. How much pressure must be applied by the hand, if the nut requires a pressure of 13 lbs. to break it, and what will be the amount of the pressure on the hinges?

26. In the steelyard, if the horizontal distance between the fulcrum and the knife-edge which supports the body weighed be 3 in., and the movable weight be 7 lbs., how far must the latter be shifted for a difference of 1 lb. in the body weighed?

27. The head of a hammer weighs 20 lbs. and the handle 2 lbs. The distance between their respective centres of gravity is 24 inches. Find the distance of the centre of gravity of the hammer from that of the head.

28. One of the four triangles into which a square is divided by its diagonals is removed. Find the distance of the centre of gravity of the remainder from the intersection of the diagonals.

29. A square is divided into four equal squares and one of these is removed. Find the distance of the centre of gravity of the remaining portion from the centre of the original square.

30. Find the centre of gravity of a sphere 1 decimetre in radius, having in its interior a spherical excavation whose centre is at a distance of 5 centimetres from the centre of the large sphere and whose radius is 4 centimetres.

31. Weights P, Q, R, S are hung from the corners A, B, C, D of a uniform square plate whose weight is W. Find the distances from the sides AB, AD of the point about which the plate will balance.

32. An isosceles triangle stands upon one side of a square as base, the altitude of the triangle being equal to a side of the square. Show that the distance of the centre of the whole figure from the opposite side of the square is $\frac{7}{9}$ of a side of the square.

33. A right cone stands upon one end of a right cylinder as base, the altitude of the cone being equal to the height of the cylinder. Show that the distance of the centre of the whole volume from the opposite end of the cylinder is $\frac{11}{16}$ of the height of the cylinder.

Work and Stability.

34. A body consists of three pieces, whose masses are as the numbers 1, 3, 9; and the centres of these masses are at heights of 2, 3, and 5 cm. above a certain level. Find the height of the centre of the whole mass above this level.

35. The body above-mentioned is moved into a new position, in which the heights of the centres of the three masses are 1, 3, and 7 cm. Find the new height of the centre of the whole mass.

36. Find the work done against gravity in moving the body from the first position into the second; employing as the unit of work the work done in raising the smallest of the three pieces through 1 cm.

37. Find the portions of this work done in moving each of the three pieces.

38. The dimensions of a rectangular block of stone of weight W are AB = a, AC = b, AD = c, and the edges AB, AC are initially horizontal. How much work is done against gravity in tilting the stone round the edge AB until it balances.

39. A chain of weight W and length l hangs freely by its upper end which is attached to a drum upon which the chain can be wound, the diameter of the drum being small compared with l. Compute the work done against gravity in winding up two-thirds of the chain.

40. Two equal and similar cylindrical vessels with their bases at the same level contain water to the respective heights h and H centimetres, the area of either base being a sq. cm. Find, in gramme-centimetres, the work done by gravity in equalizing the levels when the two vessels are connected.

41. Two forces acting at the ends of a rigid rod without weight equilibrate each other. Show that the equilibrium is stable if the forces are pulling outwards and unstable if they are pushing inwards.

42. Two equal weights hanging from the two ends of a string, which passes over a fixed pulley without friction, balance one another. Show that the equilibrium is neutral if the string is without weight, and is unstable if the string is heavy.

43. Show that a uniform hemisphere resting on a horizontal plane has two positions of stable equilibrium. Has it any positions of unstable equilibrium?

Inclined Plane, &c.

44. On an inclined plane whose height is $\frac{1}{8}$ of its length, what power acting parallel to the plane will sustain a weight of 112 lbs. resting on the plane without friction?

45. The height, base, and length of an inclined plane are as the numbers 3, 4, 5. What weight will be sustained on the plane without friction by a power of 100 lbs. acting (a) parallel to the base, (b) parallel to the plane?

46. Find the ratio of the power applied to the pressure produced in a screw-press without friction, the power being applied at the distance of 1 ft. from the axis of the screw, and the distance between the threads being $\frac{1}{8}$ in.

47. In the system of pulleys in which one cord passes round all the pulleys, its different portions being parallel, what power will sustain a weight of 2240 lbs. without friction, if the number of cords at the lower block be 6?

48. A balance has unequal arms, but the beam assumes the horizontal position when both scale-pans are empty. Show that if the two apparent weights of a body are observed when it is placed first in one pan and then in the other, the true weight will be found by multiplying these together and taking the square root.

Force, Mass, and Velocity.

The motion is supposed to be rectilinear.

49. A force of 1000 dynes acting on a certain mass for one second gives it a velocity of 20 cm. per sec. Find the mass in grammes.

50. A constant force acting on a mass of 12 gm. for one sec. gives it a velocity of 6 cm. per sec. Find the force in dynes.

51. A force of 490 dynes acts on a mass of 70 gm. for one sec. Find the velocity generated.

52. In the preceding example, if the time of action be increased to 5 sec., what will be the velocity generated?

In the following examples the unit of momentum referred to is the momentum of a gramme moving with a velocity of a centimetre per second.

53. What is the momentum of a mass of 15 gm. moving with a velocity of translation of 4 cm. per sec.?

54. What force, acting upon the mass for 1 sec., would produce this velocity?

55. What force, acting upon the mass for 10 sec., would produce the same velocity?

56. Find the force which, acting on an unknown mass for 12 sec., would produce a momentum of 84.

57. Two bodies initially at rest move towards each other in obedience to mutual attraction. Their masses are respectively 1 gm. and 100 gm. If the force of attraction be $\frac{1}{100}$ of a dyne, find the velocity acquired by each mass in 1 sec.

58. A gun is suspended by strings so that it can swing freely. Compare the velocity of discharge of the bullet with the velocity of recoil of the gun; the masses of the gun and bullet being given, and the mass of the powder being neglected.

59. A bullet fired vertically upwards, enters and becomes imbedded in a block of wood falling vertically overhead; and the block is brought to rest by the impact. If the velocities of the bullet and block immediately before collision were respectively 1500 and 100 ft. per sec., compare their masses.

Falling Bodies and Projectiles.

Assuming that a falling body acquires a velocity of 980 cm. per sec. by falling for 1 sec., find:—

60. The velocity acquired in $\frac{1}{10}$ of a second.

61. The distance passed over in $\frac{1}{10}$ sec.

62. The distance that a body must fall to acquire a velocity of 980 cm. per sec.

63. The time of rising to the highest point, when a body is thrown vertically upwards with a velocity of 6860 cm. per sec.

64. The height to which a body will rise, if thrown vertically upwards with a velocity of 490 cm. per sec.

65. The velocity with which a body must be thrown vertically upwards that it may rise to a height of 200 cm.

66. The velocity that a body will have after $\frac{3}{10}$ sec., if thrown vertically upwards with a velocity of 300 cm. per sec.

67. The point that the body in last question will have attained.

68. The velocity that a body will have after $2\frac{1}{2}$ secs., if thrown vertically upwards with a velocity of 800 cm. per sec.

69. The point that the body in last question will have reached.

Assuming that a falling body acquires a velocity of 32 ft. per sec. by falling for 1 sec., find:—

70. The velocity acquired in 12 sec.

71. The distance fallen in 12 sec.

72. The distance that a body must fall to acquire a velocity of 10 ft. per sec.

73. The time of rising to the highest point, when a body is thrown vertically upwards with a velocity of 160 ft. per sec.

74. The height to which a body will rise, if thrown vertically upwards with a velocity of 32 ft. per sec.

75. The velocity with which a body must be thrown vertically upwards that it may rise to a height of 25 ft.

76. The velocity that a body will have after 3 sec., if thrown vertically upwards with a velocity of 100 ft. per sec.

77. The height that the body in last question will have ascended.

78. The velocity that a body will have after $1\frac{1}{2}$ sec., if thrown vertically downwards with a velocity of 30 ft. per sec.

79. The distance that the body in last question will have described.

80. A body is thrown horizontally from the top of a tower 100 m. high with a velocity of 30 metres per sec. When and where will it strike the ground?

81. Two bodies are successively dropped from the same point, with an interval of $\frac{1}{3}$ of a second. When will the distance between them be one metre?

82. Show that if x and y are the horizontal and vertical co-ordinates of a projectile referred to the point of projection as origin, their values after time t are

$$x = Vt \cos a,\ y = Vt \sin a - \tfrac{1}{2} gt^2.$$

83. Show that the equation to the trajectory is

$$y = x \tan a - \frac{g x^2}{2V^2 \cos^2 a},$$

and that if V and a can be varied at pleasure, the projectile can in general be made to traverse any two given points in the same vertical plane with the point of projection.

Atwood's Machine.

Two weights are connected by a cord passing over a pulley as in Atwood's machine, friction being neglected, and also the masses of the pulley and cord; find:—

84. The acceleration when one weight is double of the other.

85. The acceleration when one weight is to the other as 20 to 21.

Taking g as 980, in terms of the cm. and sec., find:—

86. The velocity acquired in 10 sec., when one weight is to the other as 39 to 41.

87. The velocity acquired in moving through 50 cm., when the weights are as 19 to 21.

88. The distance through which the same weights must move that the velocity acquired may be double that in last question.

89. The distance through which two weights which are as 49 to 51 must move that they may acquire a velocity of 98 cm. per sec.

Energy and Work.

90. Express in ergs the kinetic energy of a mass of 50 gm. moving with a velocity of 60 cm. per sec.

91. Express in ergs the work done in raising a kilogram through a height of 1 metre, at a place where g is 981.

92. A mass of 123 gm. is at a height of 2000 cm. above a level floor. Find its energy of position estimated with respect to the floor as the standard level (g being 981).

93. A body is thrown vertically upwards at a place where g is 980. If the velocity of projection is 9800 cm. per sec. and the mass of the body is 22 gm., find the energy of the body's motion when it has ascended half way to its maximum height. Also find the work done against gravity in this part of the ascent.

94. The height of an inclined plane is 12 cm., and the length 24 cm. Find the work done by gravity upon a mass of 1 gm. in sliding down this plane (g being 980), and the velocity with which the body will reach the bottom if there be no friction.

95. If the plane in last question be not frictionless, and the velocity on reaching the bottom be 20 cm. per sec., find how much energy is consumed in friction.

96. Find the work expended in discharging a bullet whose mass is 30 gm. with a velocity of 40,000 cm. per sec.; and the number of such bullets that will be discharged with this velocity in a minute if the rate of working is 7460 million ergs per sec. (one horse-power).

97. One horse-power being defined as 550 foot-pounds per sec.; show that it is nearly equivalent to 8·8 cubic ft. of water lifted 1 ft. high per sec. (A cubic foot of water weighs $62\frac{1}{2}$ lbs. nearly. A foot-pound is the work done against gravity in lifting a pound through a height of 1 ft.)

98. How many cubic feet of water will be raised in one hour from a mine 200 ft. deep, if the rate of pumping be 15 horse-power?

Centrifugal Force.

99. What must be the radius of curvature, that the centrifugal force of a body travelling at 30 miles an hour may be one-tenth of the weight of the body; g being 981, and a mile an hour being 44·7 cm. per sec.?

100. A heavy particle moves freely along a frictionless tube which forms a vertical circle of radius a. Find the velocity which the particle will have at the lowest point, if it all but comes to rest at the highest. Also find its velocity at the lowest point if in passing the highest point it exerts no pressure against the tube. [Use the principle that what is lost in energy of position is gained in energy of motion.]

101. Show that the total intensity of centrifugal force due to the earth's rotation, at a place in latitude λ, is ω^2 R $\cos \lambda$, ω denoting $\frac{2\pi}{T}$, and R the earth's radius; that the vertical component (tending to diminish gravity) is ω^2 R $\cos^2 \lambda$, and that the horizontal component (directed from the pole towards the equator) is ω^2 R $\cos \lambda \sin \lambda$.

Pendulum, and Moment of Inertia.

101*. The length of the seconds pendulum at Greenwich is 99·413 cm.; find the length of a pendulum which makes a single vibration in $1\frac{1}{2}$ sec.

102. The weight of a fly-wheel is M grammes, and the distance of the inside of the rim from the axis of revolution is R centims. Supposing this distance to be identical with k (§ 117), find the moment of inertia.

If a force of F dynes acts steadily upon the wheel at an arm of a centims., what will be the value of the angular velocity $\frac{2\pi}{T}$ after the lapse of t seconds from the commencement of motion?

103. For a uniform thin rod of length a, swinging about a point of suspension at one end, the moment of inertia is the mass of the rod multiplied by $\frac{1}{3}a^2$. Find the length of the equivalent simple pendulum; also the moment of inertia round a parallel axis through the centre of the rod.

104. At what point in its length must the rod in last question be suspended to give a minimum time of vibration; and at what point must it be suspended to give the same time of vibration as if suspended at one end?

105. Show that if P be the mass of the pulley in Atwood's machine, r its radius, and Pk^2 its moment of inertia, the value of C in § 100 will be $P\frac{k^2}{r^2}$ *plus* the mass of the string. [The mass of the friction-wheels is neglected.]

106. A body moves with constant velocity in a vertical circle, going once round per second; and its shadow is cast upon level ground by a vertical sun. Find the value of μ (§ 111) for the shadow, using the centimetre and second as units.

107. What is the value of μ for one of the prongs of a C tuning-fork which makes 512 complete vibrations per second?

Pressure of Liquids.

Find, in gravitation measure (grammes per sq. cm.), atmospheric pressure being neglected:—

108. The pressure at the depth of a kilometre in sea-water of density 1·025.

109. The pressure at the depth of 65 cm. in mercury of density 13·59.

110. The pressure at the depth of 2 cm. in mercury of density 13·59 surmounted by 3 cm. of water of unit density, and this again by $1\frac{1}{2}$ cm. of oil of density ·9.

Find, in centimetres of mercury of density 13·6, atmospheric pressure being included, and the barometer being supposed to stand at 76 cm.:—

111. The pressure at the depth of 10 metres in water of unit density.

112. The pressure at the depth of a mile in sea-water of density 1·026, a mile being 160933 cm.

Find, in dynes per square centimetre, taking g as 981:—

113. The pressure due to 1 cm. of mercury of density 13·596.

114. The pressure due to a foot of water of unit density, a foot being 30·48 cm.

115. The pressure due to the weight of a layer a metre thick, of air of density ·00129.

116. At what depth, in brine of density 1·1, is the pressure the same as at a depth of 33 feet in water of unit density?

117. At what depth, in oil of density ·9, is the pressure the same as at the depth of 10 inches in mercury of density 13·596?

118. With what value of g will the pressure of 3 cm. of mercury of density 13·596 be 4×10^4?

Find, in grammes weight, the amount of pressure (atmospheric pressure being neglected):—

119. On a triangular area of 9 sq. cm. immersed in naphtha of density ·848; the centre of gravity of the triangle being at the depth of 6 cm.

120. On a rectangular area 12 cm. long, and 9 cm. broad, immersed in mercury of density 13·596; its highest and lowest corners being at depths of 3 cm. and 7 cm. respectively.

121. On a circular area of 10 cm. radius, immersed in alcohol of density ·791, the centre of the circle being at the depth of 4 cm.

122. On a triangle whose base is 5 cm. and altitude 6 cm., the base being at the uniform depth of 9 cm., and the vertex at the depth of 7 cm., in water of unit density.

123. On a sphere of radius r centimetres, completely immersed in a liquid of density d; the centre of the sphere being at the depth of h centimetres. [The amount of pressure in this case is not the resultant pressure.]

Density, and Principle of Archimedes.

Densities are to be expressed in grammes per cubic centimetre.

124. A rectangular block of stone measures 86 × 37 × 16 cm., and weighs 120 kilogrammes. Find its density.

125. A specific-gravity bottle holds 100 gm. of water, and 180 gm. of sulphuric acid. Find the density of the acid.

126. A certain volume of mercury of density 13·6 weighs 216 gm., and the same volume of another liquid weighs 14·8 gm. Find the density of this liquid.

127. Find the mean section of a tube 16 cm. long, which holds 1 gm. of mercury of density 13·6.

128. A bottle filled with water, weighs 212 gm. Fifty grammes of filings are thrown in, and the water which flows over is removed, still leaving the bottle just filled. The bottle then weighs 254 gm. Find the density of the filings.

129. Find the density of a body which weighs 58 gm. in air, and 46 gm. in water of unit density.

130. Find the density of a body which weighs 63 gm. in air, and 35 gm. in a liquid of density ·85.

131. A glass ball loses 33 gm. when weighed in water, and loses 6 gm. more when weighed in a saline solution. Find the density of the solution.

132. A body, lighter than water, weighs 102 gm. in air; and when it is immersed in water by the aid of a sinker, the joint weight is 23 gm. The sinker alone weighs 50 gm. in water. Find the density of the body.

133. A piece of iron, when plunged in a vessel full of water, makes 10 grammes run over. When placed in a vessel full of mercury it floats, displacing 78 grammes of mercury. Required the weight, volume, and specific gravity of the iron.

134. Find the volume of a solid which weighs 357 gm. in air, and 253 gm. in water of unit density.

135. Find the volume of a solid which weighs 458 gm. in air, and 409 gm. in brine of density 1·2.

136. How much weight will a body whose volume is 47 cubic cm. lose, by weighing in a liquid whose density is 2·5?

137. Find the weights in air, in water, and in mercury, of a cubic cm. of gold of density 19·3.

138. A wire 1293 cm. long loses 508 gm. by weighing in water. Find its mean section, and mean radius.

139. A copper wire 2156 cm. long weighs 158 gm. in air, and 140 gm. in water. Find its volume, density, mean section, and mean radius.

140. What will be the weights, in air and in water, of an iron wire 1000 cm. long and a millimetre in diameter, its density being 7·7?

141. How much water will be displaced by 1000 c.c. of oak of density ·9, floating in equilibrium?

142. A ball, of density 20 and volume 3 c.c., is surmounted by a cylindrical stem, of density 2·5, of length 12 cm., and of cross section $\frac{1}{3}$ sq. cm. What length of the stem will be in air when the body floats in equilibrium in mercury of density 13·6?

143. A hollow closed cylinder, of mean density ·4 (including the hollow space), is weighted with a ball of volume 5, and mean density 2. What must be the volume of the cylinder, that exactly half of it may be immersed, when the body is left to itself in water?

144. A long cylindrical tube, constructed of flint glass of density 3, is closed at both ends, and is found to have the property of remaining at whatever depth it is placed in water. If the mass of the ends can be neglected, show that the ratio of the internal to the external radius is $\sqrt{\frac{2}{3}}$.

145. A glass bottle provided with a stopper of the same material weighs 120 gm. when empty. When it is immersed in water, its apparent weight is 10 gm., but when the stopper is loosened and the water let in, its apparent weight is 80 gm. Find the density of the glass and the capacity of the bottle.

146. A hydrometer sinks to a certain depth in a fluid of density ·8; and if 100 gm. be placed upon it, it sinks to the same depth in water. Find the weight of the hydrometer.

147. Find the mean density of a combination of 8 parts by volume of a substance of density 7, with 19 of a substance of density 3.

148. Find the mean density of a combination of 8 parts by weight of a substance of density 7, with 19 of a substance of density 3.

149. What volume of fir, of density ·5, must be joined to 3 c.c. of iron, of density 7·1, that the mean density of the whole may be unity?

150. What mass of fir, of density ·5, must be joined to 300 gm. of iron, of density 7·1, that the mean density of the whole may be unity?

151. Two parts by volume of a liquid of density ·8, are mixed with 7 of water, and the mixture shrinks in the ratio of 21 to 20. Find its density.

152. A piece of iron of density 7·5 floats in mercury of density 13·5, and is completely covered with water which rests on the top of the mercury. How much of the iron is immersed in the mercury?

153. Two liquids are mixed. The total volume is 3 litres, with a sp. gr. of 0·9. The sp. gr. of the first liquid is 1·3, of the second 0·7. Find their volumes.

154. What volume of platinum of density 21·5 must be attached to a litre of iron of density 7·5 that the system may float freely at all depths in mercury of density 13·5?

155. What must be the thickness of a hollow sphere of platinum with an external radius of 1 decim., that it may barely float in water?

156. A sphere of cork of density ·24, 3 cm. in radius, is weighted with a sphere of gold of density 19·3. What must be the radius of the latter that the system may barely float in alcohol of density ·8?

157. An alloy of gold and silver has density D. The density of gold is d, that of silver d'. Find the proportions by weight of the two metals in the alloy, supposing that neither expansion nor contraction occurs in its formation.

158. A mixture of gold, of density 19·3, with silver, of density 10·5, has the density 18. Assuming that the volume of the alloy is the sum of the volumes of its components, find how many parts of gold it contains for one of silver—(a) by volume; (b) by weight.

159. A body weighs gM dynes in air of density A, gm in water, and gx in vacuo. Find x in terms of M, m, and A.

Capillarity.

160. A horizontal disc of glass is held up by means of a film of water between it and a similar disc of the same or a larger size above it.

If R denote the radius of the lower disc,

d the distance between the discs, which is very small compared with R,

T the surface tension of water,

show that the weight of the lower disc together with that of the water between the discs is approximately equal to $\frac{2T\pi R^2}{d}$.

[The disc of water will be concave at the edge, and the radius of curvature of the concavity may be taken as $\frac{1}{2}d$.]

161. The surface-tension of water at 20° C. is 81 dynes per linear centim. How high will water be elevated by capillary action in a wetted tube whose diameter is half a millimetre?

162. How much will mercury be depressed by capillary action in a glass tube of half a millimetre diameter, the surface-tension of mercury at 20° C. being 418 dynes per cm., its density 13·54, and the cosine of the angle of contact ·703?

163. Show by the method of § 186 that the capillary elevation or depression will be the same in a square tube as in a circular tube whose diameter is equal to a side of the square.

164. Two equal discs in a vertical position have a film of water between them sustained by capillary action. Show that if the water at the lowest point is at atmospheric pressure, the water at the centre of the discs is at a pressure less than atmospheric by rg dynes per sq. cm., r being the common radius of the discs in cm.; and that the discs are pressed together with a force of $\pi r^3 g$ dynes.

Barometer, and Boyle's Law.

165. A bent tube, having one end open and the other closed, contains mercury which stands 20 cm. higher in the open than in the closed branch. Compare the pressure of the air in the closed branch with that of the external air; the barometer at the time standing at 75 cm.

166. The cross sections of the open and closed branches of a siphon barometer are as 6 to 1. What distance will the mercury move in the closed branch, when a normal barometer alters its reading by 1 inch?

167. If the section of the closed limb of a siphon barometer is to that of the open limb as a to b, show that a rise of 1 cm. in the mercury in the closed limb corresponds to a rise of $\frac{a+b}{b}$ cm. of the theoretical barometer.

168. Compute, in dynes per sq. cm., the pressure due to the weight of a column of mercury 76 cm. high at the equator, where g is 978, and at the pole, where g is 983.

169. The volumes of a given quantity of mercury at 0° C. and 100° C. are as 1 to 1·0182. Compute the height of a column of mercury at 100°, which will produce the same pressure as 76 cm. of mercury at 0°.

170. The volumes of a given mass of mercury, at 0° and 20°, are as 1 to 1·0036. Find the height reduced to 0°, when the actual height (in true centimetres), at a temperature of 20°, is 76·2.

171. In performing the Torricellian experiment a little air is left above the mercury. If this air expands a thousandfold, what difference will it make in the height of the column of mercury sustained when a normal barometer reads 76 cm.?

172. In performing the Torricellian experiment, an inch in length of the tube is occupied with air at atmospheric pressure, before the tube is inverted. After the inversion, this air expands till it occupies 15 inches, while a column of mercury 28 inches high is sustained below it. Find the true barometric height.

173. The mercury stands at the same level in the open and in the closed branch of a bent tube of uniform section, when the air confined at the closed end is at the pressure of 30 inches of mercury, which is the same as the pressure of the external air. Express, in atmospheres, the pressure which, acting on the surface of the mercury in the open branch, compresses the confined air to half its original volume, and at the same time maintains a difference of 5 inches in the levels of the two mercurial columns.

174. At what pressure (expressed in atmospheres) will common air have the same density which hydrogen has at one atmosphere; their densities when compared at the same pressure being as 1276 to 88·4?

175. Two volumes of oxygen, of density ·00141, are mixed with three of nitrogen, of density ·00124. Find the density of the mixture—(a) if it occupies five volumes; (b) if it is reduced to four volumes.

176. The mass of a cub. cm. of air, at the temperature 0° C., and at the pressure of a million dynes to the square cm., is ·0012759 gramme. Find the mass of a cubic cm. of air at 0° C., under the pressure of 76 cm. of mercury—(a) at the pole, where g is 983·1; (b) at the equator, where g is 978·1; (c) at a place where g is 981.

177. Show that the density of air at a given temperature, and under the pressure of a given column of mercury, is greater at the pole than at the equator by about 1 part in 196; and that the gravitating force of a given volume of it is greater at the pole than at the equator by about 1 part in 98.

178. A cylindrical test-tube, 1 decim. long, is plunged, mouth downwards, into mercury. How deep must it be plunged that the volume of the inclosed air may be diminished by one-half?

179. The pressure indicated by a siphon barometer whose vacuum is defective is 750 mm., and when mercury is poured into the open branch till the barometric chamber is reduced to half its former volume, the pressure indicated is 740 mm. Deduce the true pressure.

180. An open manometer, formed of a bent tube of iron whose two branches are parallel and vertical, and of a glass tube of larger size, contains mercury at the same level in both branches, this level being higher than the junction of the iron with the glass tube. What must be the ratio of the sections of the two tubes, that the mercury may ascend half a metre in the glass tube when a pressure of 6 atmospheres is exerted in the opposite branch?

181. A curved tube has two vertical legs, one having a section of 1 sq. cm., the other of 10 sq. cm. Water is poured in, and stands at the same height in both legs. A piston, weighing 5 kilogrammes, is then allowed to descend, and press with its own weight upon the surface of the liquid in the larger leg. Find the elevation thus produced in the surface of the liquid in the smaller leg.

Pumps, &c.

182. The sectional area of the small plunger in a Bramah press is 1 sq. cm., and that of the larger 100 sq. cm. The lever handle gives a mechanical advantage of 6. What weight will the large plunger sustain when 1 cwt. is hung from the handle?

183. The diameter of the small plunger is half an inch; that of the larger 1 foot. The arms of the lever handle are 3 in. and 2 ft. Find the total mechanical advantage.

184. Find, in grammes weight, the force required to sustain the piston of a suction-pump without friction, if the radius of the piston be 15 cm., the depth

from it to the surface of the water in the well 600 cm., and the height of the column of water above it 50 cm. Show that the answer does not depend on the size of the pipe which leads down to the well.

185. Two vessels of water are connected by a siphon. A certain point P in its interior is 10 cm. and 30 cm. respectively above the levels of the liquid in the two vessels. The pressure of the atmosphere is 1000 grammes weight per sq. cm. Find the pressure which will exist at P—(*a*) if the end which dips in the upper vessel be plugged; (*b*) if the end which dips in the lower vessel be plugged.

186. If the receiver has double the volume of the barrel, find the density of the air remaining after 10 strokes, neglecting leakage, &c.

187. Air is forced into a vessel by a compression pump whose barrel has $\frac{1}{10}$th of the volume of the vessel. Compute the density of the air in the vessel after 20 strokes.

188. In the pump of Fig. 136 show that the excess of the pressure on the upper above that on the lower side of the piston, at the end of the first up-stroke, is $\frac{V}{V+V'}$ of an atmosphere [in the notation of § 230]; and hence that the first stroke is more laborious with a small than with a large receiver.

189. In Tate's pump show that the pressure to be overcome in the first stroke is nearly equal to an atmosphere during the greater part of the stroke; and that, when half the air has been expelled from the receiver, the pressure to be overcome varies, in different parts of the stroke, from half an atmosphere to an atmosphere.

ANSWERS TO EXAMPLES.

Ex. 1. 14·14. Ex. 2. 13. Ex. 3. 7·07 each. Ex. 4. 10. Ex. 5. 141·4. Ex. 6. 70·7 each. Ex. 7. Introduce a force equal and opposite to the resultant. Then we have three forces making angles of 120° with each other. Ex. 9. Equal to one of the forces.

Ex. 10*. 28. Ex. 11*. 40 in. from smaller weight. Ex. 12. 60 lbs. by A, 40 lbs. by B. Ex. 13. $2\frac{6}{7}$ lbs. Ex. 14. 2 lbs. Ex. 15. 15 in. from centre. Ex. 16. $12\frac{1}{2}$ lbs. Ex. 17. 32 lbs. Ex. 18. 10·4 ft. nearly. Ex. 19. 6 lbs. Ex. 20. $2\frac{6}{7}$ ft. from end. Ex. 21. $2\frac{13}{16}$. Ex. 22. 2 units acting at distance of 5 yards from the greater force. Ex. 23. 6 ft. from the end; pressure 2 units. Ex. 24. $4\frac{2}{3}$ lbs. Ex. 25. $2\frac{1}{6}$ lbs., $10\frac{5}{6}$ lbs. Ex. 26. $\frac{3}{7}$ in. Ex. 27. $2\frac{2}{11}$ in. Ex. 28. $\frac{1}{9}$ of side of square. Ex. 29. $\frac{1}{12}$ of diagonal of large square. Ex. 30. $\frac{40}{117}$ cm. from centre of large sphere. Ex. 31. Denoting side of square by a, distance from AB is $\frac{\frac{1}{2}W+R+S}{W+P+Q+R+S}a$, distance from AD is $\frac{\frac{1}{2}W+Q+R}{W+P+Q+R}a$.

Ex. 34. $4\frac{4}{13}$ cm. Ex. 35. $5\frac{8}{13}$ cm. Ex. 36. 17. Ex. 37. $-1, 0, +18$. Ex. 38. $\frac{1}{2}W(\sqrt{(b^2+c^2)}-c)$. Ex. 39. $\frac{4}{9}Wl$. Ex. 40. $\frac{a}{4}(H-h)^2$.

Ex. 44. 14 lbs. Ex. 45. (*a*) $133\frac{1}{3}$ lbs.; (*b*) $166\frac{2}{3}$ lbs. Ex. 46. 1 to 603 nearly. Ex. 47. $373\frac{1}{3}$.

Ex. 49. 50. Ex. 50. 72. Ex. 51. 7 cm. per sec. Ex. 52. 35. Ex. 53. 60. Ex. 54. 60 dynes. Ex. 55. 6 dynes. Ex. 56. 7 dynes. Ex. 57. Smaller mass $\frac{1}{100}$, larger $\frac{1}{10000}$ cm. per sec. Ex. 58. Inversely as masses of bullet and gun. Ex. 59. Mass of bullet is $\frac{1}{15}$ of mass of block.

Ex. 60. 98 cm. per sec. Ex. 61. 4·9 cm. Ex. 62. 490 cm. Ex. 63. 7 sec. Ex. 64. $122\frac{1}{2}$ cm. Ex. 65. 626 cm. per sec. Ex. 66. 6 cm. per sec. upwards. Ex. 67. 45·9 cm. above point of projection. Ex. 68. 1650 cm. per sec. downwards. Ex. 69. $1062\frac{1}{2}$ cm. below starting point. Ex. 70. 384 ft. per sec. Ex. 71. 2304 ft. Ex. 72. $1\frac{9}{16}$ ft. Ex. 73. 5 sec. Ex. 74. 16 ft. Ex. 75. 40 ft. per sec. Ex. 76. 4 ft. per sec. upwards. Ex. 77. 156 ft. Ex. 78. 78 ft. per sec. Ex. 79. 81 ft. Ex. 80. After 4·52 sec. At 135·6 m. from tower. Ex. 81. After ·41 sec. from dropping of second body.

Ex. 84. $\frac{1}{3}g$. Ex. 85. $\frac{1}{41}g$. Ex. 86. 245 cm. per sec. Ex. 87. 70 cm. per sec. Ex. 88. 200 cm. Ex. 89. 245 cm.

Ex. 90. 90,000 ergs. Ex. 91. 98,100,000 ergs. Ex. 92. 241,326,000 ergs. Ex. 93. 528,220,000 ergs each. Ex. 94. 11,760 ergs; $\sqrt{23520}=153{\cdot}4$ cm. per sec. Ex. 95. 11,560 ergs. Ex. 96. 24×10^9 ergs in each discharge. Not quite 19 discharges per min. Ex. 98. 2376 nearly.

Ex. 99. 18330 cm. or about 600 ft. Ex. 100. $2\sqrt{ga}$, $\sqrt{5ga}$.

Ex. 101*. 223·679 cm. Ex. 102. MR^2, $\frac{Fat}{MR^2}$. Ex. 103. $\frac{2}{3}a$; mass of rod multiplied by $\frac{1}{12}a^2$. Ex. 104. At either of the two points distant $\frac{a}{2\sqrt{3}}$ from centre; at either of the two points distant $\frac{a}{6}$ from centre. Ex. 106. $(2\pi)^2=39{\cdot}48$. Ex. 107. $(1024\pi)^2=10350000$.

Ex. 108. 102500. Ex. 109. 883·35. Ex. 110. 31·53. Ex. 111. 149·5. Ex. 112. 12217. Ex. 113. 13338. Ex. 114. 29901. Ex. 115. 126·5. Ex. 116. 30. Ex. 117. 12 ft. 7 in. Ex. 118. 980·68. Ex. 119. 45·79. Ex. 120. 7342. Ex. 121. 994. Ex. 122. 125. Ex. 123. $4\pi r^2hd$.

Ex. 124. 2·357. Ex. 125. 1·8. Ex. 126. ·932. Ex. 127. ·0046 sq. cm. Ex. 128. 6·25. Ex. 129. $4\frac{5}{8}$. Ex. 130. 1·9125. Ex. 131. $1\frac{2}{11}$. Ex. 132. $\frac{34}{43}$. Ex. 133. 10 cub. cm., 78 gm., 7·8. Ex. 134. 104. Ex. 135. $40{\cdot}8\dot{3}$. Ex. 136. 117·5. Ex. 137. 19·3, 18·3, 5·7. Ex. 138. ·393 sq. cm., ·354 cm. Ex. 139. 18, $8{\cdot}77\dot{7}$, ·00835 sq. cm., ·0516 cm. Ex. 140. 60·48, 52·62. Ex. 141. 900 c.c. Ex. 142. 5·56 cm. Ex. 143. 50 c.c. Ex. 145. 3, 70 c.c. Ex. 146. 400 gm. Ex. 147. $4\frac{5}{27}=4{\cdot}185$. Ex. 148. $3\frac{96}{157}=3{\cdot}6115$. Ex. 149. 36·6 c.c. Ex. 150. 257·7 gm. Ex. 151. 1·0033. Ex. 152. $\frac{13}{25}$ of the iron. Ex. 153. 1 lit. of first, 2 lit. of second. Ex. 154. $\frac{3}{4}$ of a litre. Ex. 155. $1-\sqrt[3]{\frac{41}{43}}$ decim. $=$ ·158 cm. Ex. 156. $\sqrt[3]{\frac{15{\cdot}12}{18{\cdot}5}}=$ ·935 cm. Ex. 157. Gold : silver $::\frac{1}{d'}-\frac{1}{D}:\frac{1}{D}-\frac{1}{d}$. Ex. 158. (*a*) 5·77, (*b*) 10·6. Ex. 159. $\frac{M-mA}{1-A}$.

Ex. 161. 6·6 cm. nearly. Ex. 162. 1·77 cm.

Ex. 165. $\frac{12}{13}$. Ex. 166. $\frac{6}{7}$ in. Ex. 168. 1010564, 1015730. Ex. 169. 77·3832. Ex. 170. 75·93. Ex. 171. ·076. Ex. 172. 30 in. Ex. 173. $2\frac{1}{6}$. Ex. 174. ·0693. Ex. 175. (*a*) ·001308, (*b*) ·001635. Ex. 176. (*a*) ·0012961, (*b*) ·0012895, (*c*) ·0012933. Ex. 177. d varies as g, and therefore gd varies as g^2. Ex. 178. Its top must be $76-5=71$ cm. deep. Ex. 179. 760 m. Ex. 180. 33 to 5. Ex. 181. $454\frac{6}{11}$ cm.

Ex. 182. 30 tons. Ex. 183. 4608. Ex. 184. 459500 nearly. Ex. 185. (*a*) 970. (*b*) 990 gm. wt. per sq. cm. Ex. 186. $\frac{1}{58}$ of an atmosphere, nearly. Ex. 187. 3 atmospheres.

INDEX TO PART I.