

definida positiva arbitraria, tal que la derivada W_f a lo largo de las trayectorias de (5.93) satisface (5.44) con $\bar{v}_1 = \tilde{c}_3 = \lambda_{\min}(Q_f)$ y $\bar{v}_2 = 0$, es decir, ahí no la necesitamos para verificar (5.46). Finalmente, el control compuesto resulta

$$u(x, \eta, x_{sd}) = u_s(x, x_{sd}) + u_f(\eta) \quad (5.95)$$

Cuando este control compuesto es substituido dentro de la representación singularmente perturbada del modelo del robot con articulación flexible, uno obtiene el siguiente sistema singularmente perturbado en lazo cerrado

$$\dot{x} = f_c(x, \eta, x_{sd}) + g_\epsilon(x, x_{sd}) \quad \dot{x}_{sd} = A_s x + F_1(x)\eta + p_s(0, x_{sd}, \dot{x}_{sd}) \quad (5.96)$$

$$\dot{\eta} = g_c(x, \eta, 0) - \epsilon \frac{\partial h}{\partial x} [A_s x - F_1(x)\eta + p_s(0, x_{sd}, \dot{x}_{sd})] \quad (5.97)$$

5.6.2 Estimación de las velocidades angulares del motor y el eslabón

El controlador diseñado en la subsección 5.1 requiere de una medición completa de las posiciones angulares del eslabón y del eje del motor. Sin embargo, usualmente en la práctica, únicamente se puede disponer de las posiciones angulares. A fin de estimar las velocidades angulares del eslabón y del eje del motor, un observador de alta ganancia es diseñado en esta subsección; otra clase de observadores han sido diseñados para estos manipuladores (Busawon *et al* 1998). Haciendo el siguiente cambio de coordenadas.

$$\varsigma_1 = x_1, \quad \varsigma_2 = x_2, \quad \varsigma_3 = z_1, \quad \varsigma_4 = z_2. \quad (5.98)$$

se puede escribir el sistema singularmente perturbado asociado para el manipulador en la forma siguiente:

$$\left\{ \begin{array}{l} \dot{\zeta}_1 = \zeta_2 \\ \dot{\zeta}_2 = -\frac{mgl}{I} \sin(\zeta_1) - \frac{\zeta_3}{I} + m_1 + m_3 \cos(\zeta_1) \sin(\zeta_1) \\ \dot{\zeta}_3 = \frac{1}{\varepsilon} \zeta_4 \\ \dot{\zeta}_4 = -\frac{mgl}{\varepsilon I} \sin(\zeta_1) + \frac{\alpha B}{\varepsilon J} \zeta_2 - \frac{\alpha}{\varepsilon} \left(\frac{1}{I} + \frac{1}{J} \right) \zeta_3 - \frac{B}{J} \zeta_4 - \frac{\alpha}{\varepsilon J} u \\ y = \zeta_1 \end{array} \right. \quad (5.99)$$

con $\zeta = \text{col}(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ y

$$F(y) = F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{I} & 0 \\ 0 & 0 & 0 & \frac{1}{\varepsilon} \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$G(\zeta, u) = \begin{pmatrix} 0 \\ -\frac{mgl}{I} \sin(\zeta_1) \\ 0 \\ -\frac{mgl}{\varepsilon I} \sin(\zeta_1) + \frac{\alpha B}{\varepsilon J} \zeta_2 - \frac{\alpha}{\varepsilon} \left(\frac{1}{I} + \frac{1}{J} \right) \zeta_3 - \frac{B}{J} \zeta_4 - \frac{\alpha}{\varepsilon J} u \end{pmatrix}$$

Un observador para la variable de estado ζ es de la forma

$$\dot{\hat{\zeta}}(t) = F\hat{\zeta}(t) + G(\zeta, u) - \bar{S}_\theta^{-1}(y)C^T[\hat{\zeta}_1 - \zeta_1] \quad (5.100)$$

donde $\bar{S}_\theta = \Omega(y)S_\theta\Omega(y)$, siendo S_θ la solución única de (56), y

$$\Omega(y) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{I} & 0 \\ 0 & 0 & 0 & -\frac{1}{\varepsilon I} \end{pmatrix}.$$

La ganancia del observador $\tilde{S}_\theta^{-1}(y)C^T = (\Omega(y)S_\theta\Omega(y))^{-1}C^T$ tiene la forma.

$$\tilde{S}_\theta^{-1}(y)C^T = \begin{pmatrix} 4\theta^4 \\ 6\theta^3 \\ -4I\theta^2 \\ -\varepsilon I\theta \end{pmatrix}.$$

De acuerdo al teorema 1. el origen $e = 0$ de la estimación de la dinámica del error es un punto de equilibrio exponencialmente estable. mas precisamente. la función candidata de Lyapunov $V_o(\tilde{e}(t)) = \tilde{e}^T(t)S_1\tilde{e}(t)$. con $\tilde{e}(t) = \Omega(y)\Delta_\theta e(t)$. satisface

$$\dot{V}_o(e) \leq -\mu N \|e\|^2. \quad (5.101)$$

donde $\mu = \theta - \varrho[\tilde{\delta}_0\tilde{\delta}_1\tilde{k}_2 + \tilde{\delta}_2]$. $\sqrt{N} = \frac{\tilde{k}_3}{\theta^{n-m-1}}$. $n = 2$. $m = 2$.

Por supuesto. θ se selecciona de tal forma que $\mu > 0$.

5.6.3 Estabilidad en lazo cerrado

En esta aplicación el sistema aumentado en lazo-cerrado singularmente perturbado (5.65) resulta:

$$\begin{aligned} \dot{x} &= f_c(x, \eta, x_{sd}) + g_c(x, x_{sd}) \dot{x}_{sd} - g(x)\Delta u_s(x, \dot{x}, x_{sd}, \dot{x}_{sd}). \quad y = x_1, \\ \varepsilon \dot{\eta} &= g_c(x, \eta, 0) + g_2(x)\Delta u_f(x, \eta, \dot{x}, \hat{\eta}, 0, 0) \\ &\quad - \varepsilon \left[\left(\frac{\partial h}{\partial x} \right) + \left(\frac{\partial h}{\partial \zeta_x} \right) \left(\frac{\partial \Phi}{\partial x} \right) \right] \dot{x} - \varepsilon \left(\frac{\partial h}{\partial \zeta_x} \right) M_x \dot{e} \\ \dot{e}(t) &= \dot{e}(t) = \Omega^{-1}(y)\{A - S_\theta^{-1}C^TC\}\Omega(y)e(t) + G(u(t), \hat{\zeta}(t)) - G(u(t), \zeta(t)) \end{aligned} \quad (5.102)$$

donde $\Delta u_s(x, \hat{x}, x_{sd}, \dot{x}_{sd}) = u_s(\dot{x}, x_{sd}, \dot{x}_{sd}) - u_s(x, x_{sd}, \dot{x}_{sd})$ y $\Delta u_f(x, \eta, \hat{x}, \hat{\eta}, 0, 0) = u_f(\hat{x}, \hat{\eta}, 0, 0) - u_f(x, \eta, 0, 0)$.

De estas últimas expresiones uno puede verificar (despues de largos cálculos). las condiciones de Lipschitz (5.67) y las desigualdades (5.68).

Así las ecuaciones (5.69)-(5.72) se satisfacen. También, la diferenciabilidad continua de h con respecto a sus argumentos y del mapeo Φ , cuyas componentes son dadas por (5.87) y (5.98), nos permiten sostener las desigualdades (5.73) (también a través de largas operaciones matemáticas). Las constantes (5.74) son dadas por:

$$\begin{aligned}
\alpha_1 &= (c_3 - c_4 b_2), \\
\alpha_2 &= \left(\frac{\tilde{v}_1}{\epsilon} - \bar{c}_4 (l_{h_{x_1}} + l_{h_{x_2}} l_\Phi) (l_{f\eta} + \delta_2) \right), \\
\alpha_3 &= \mu N, \\
\beta_1 &= c_4 l_{f\eta} + \bar{c}_4 (l_{h_{x_1}} + l_{h_{x_2}} l_\Phi) l_{fx_1}, \\
\beta_2 &= c_4 m_0 m_s, \\
\beta_3 &= \bar{c}_4 \{ (l_{h_{x_1}} + l_{h_{x_2}} l_\Phi) - l_{h_{x_2}} \} (m_0 m_s + l_{fx_1} b_1) + \frac{1}{\epsilon} \bar{c}_4 m_2 m_f + \bar{c}_4 l_{h_{x_2}} (\tilde{\delta}_0 \tilde{\delta}_1 \tilde{k}_1 + \tilde{k}_2), \\
\beta_4 &= \bar{c}_4 (l_{h_{x_1}} + l_{h_{x_2}} l_\Phi) l_{fx_1} b_1, \\
\gamma &= v_2.
\end{aligned}$$

Los valores nominales de los parámetros del robot con articulación flexible fueron escogidos como [16]

$$mgl = 0.8; \quad \alpha = 7.13 \times 10^{-8}; \quad I = 0.031$$

$$J = 0.004; \quad B = 0.007; \quad \varepsilon = 10^{-4}$$

Con estos valores y la selección $s_1 = 5$, $s_2 = 10$, $l_s = 200$, $\bar{s}_1 = 0.1$, $\bar{s}_2 = 1.0$, $l_f = 0.010$, $\theta = 2$, la ecuación (5.75) se satisface, y el sistema en lazo cerrado (5.102) es finalmente acotado.

5.6.4 Resultados de Simulación

Nosotros mostramos ahora algunos resultados de simulación cuando el esquema controlador-observador es aplicado para el modelo del brazo robot con articulación flexible. La señal

de referencia a seguir fue puesta como $x_{sd_1} = 0.5 \sin(t)$. Las condiciones iniciales de las variables del robot, así como para los estimados fueron fijadas a los valores siguientes: $\varsigma_1(0) = 0$, $\varsigma_2(0) = 0$, $\varsigma_3(0) = 0$, $\varsigma_4(0) = 0$, $\hat{\varsigma}_1(0) = 0.2$, $\hat{\varsigma}_2(0) = 0.02$, $\hat{\varsigma}_3(0) = 0.002$, $\hat{\varsigma}_4(0) = 0.003$.

Las graficas de tiempo del sistema en lazo-cerrado muestran el comportamiento dinámico de la posición angular del eslabón, la señal de referencia, la velocidad angular del eslabón, la derivada de tiempo de la señal de referencia son graficadas en las figuras 5.1 y 5.2 . mientras que los errores de estimación e_1 y e_2 son dados en las figuras 5.3 y 5.4. Desde estas gráficas, uno puede observar que un buen desempeño en el seguimiento de la trayectoria del brazo robot es obtenida. También la variable de control es conservada dentro de ciertos límites de operación, tal como se muestra en la figura 5.5 .

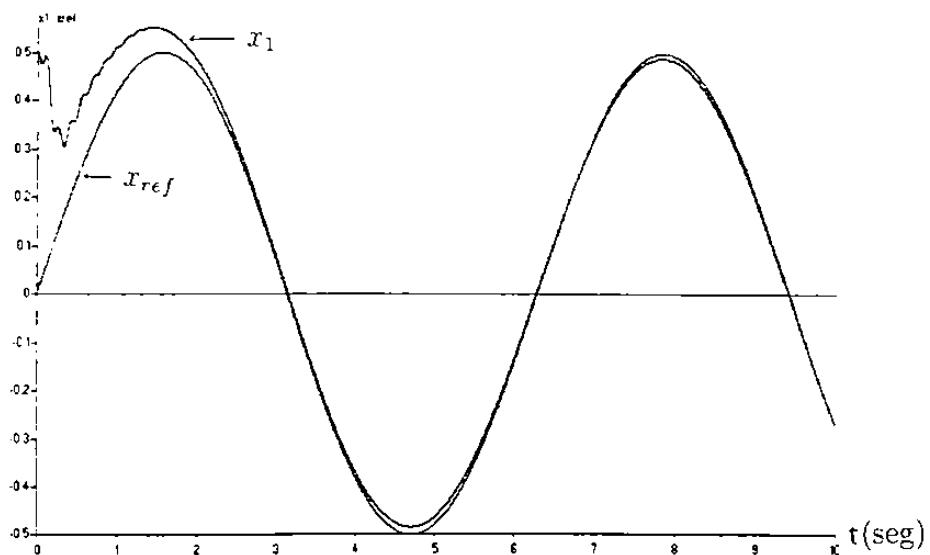


Figura No.5.1 Gráfica del estado x_1 y la señal de referencia x_{ref}

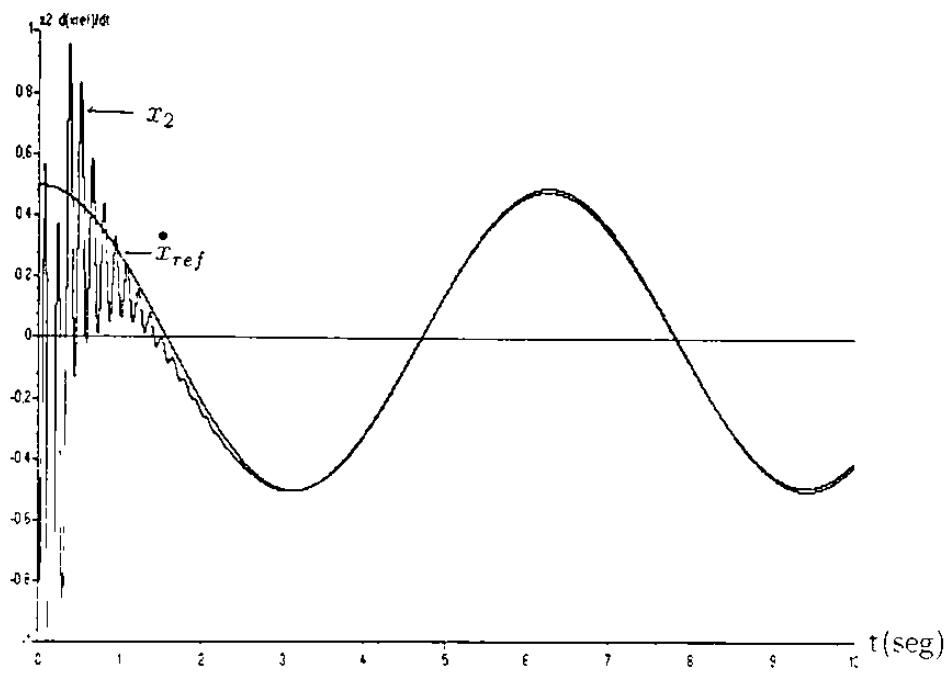


Figura No. 5.2 Gráfica del estado x_2 y la señal $d(x_{ref})/dt$

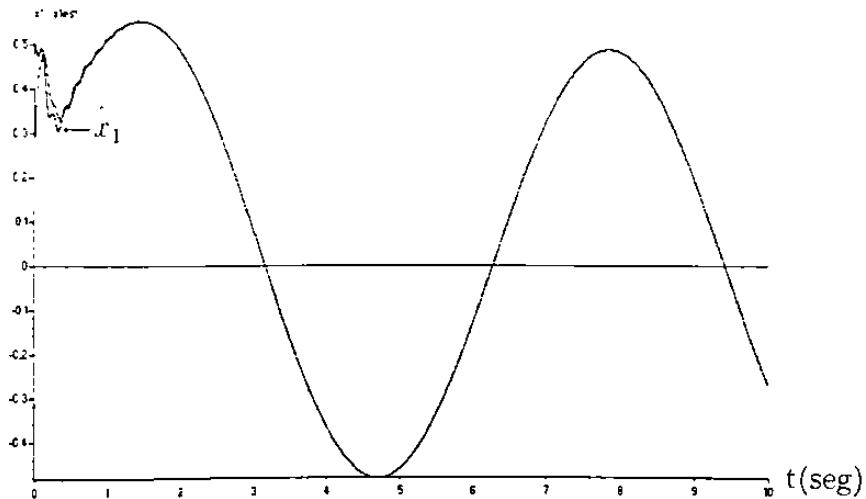


Figura 5.3 Grafica del estado x_1 y su estimado

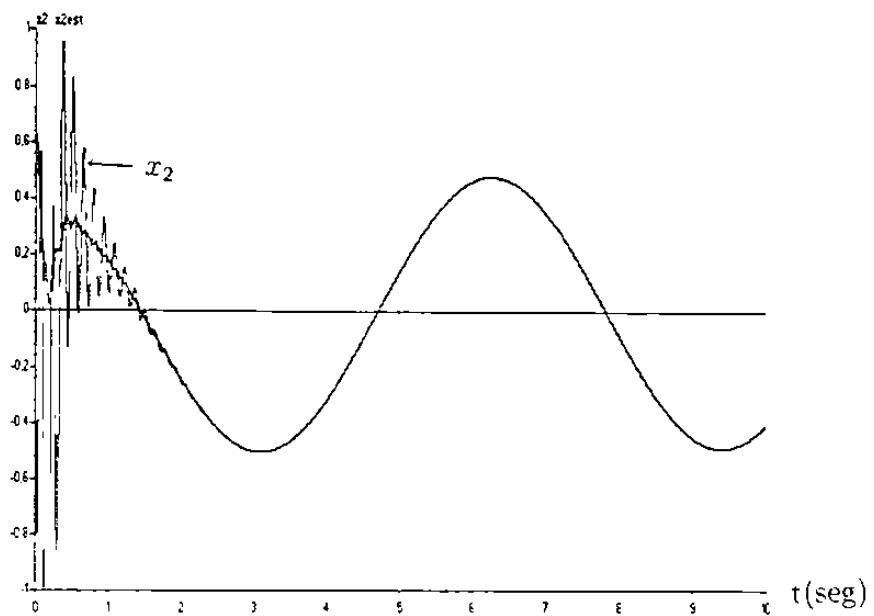


Figura No. 5.4 Gráfica del estado x_2 y su estimado

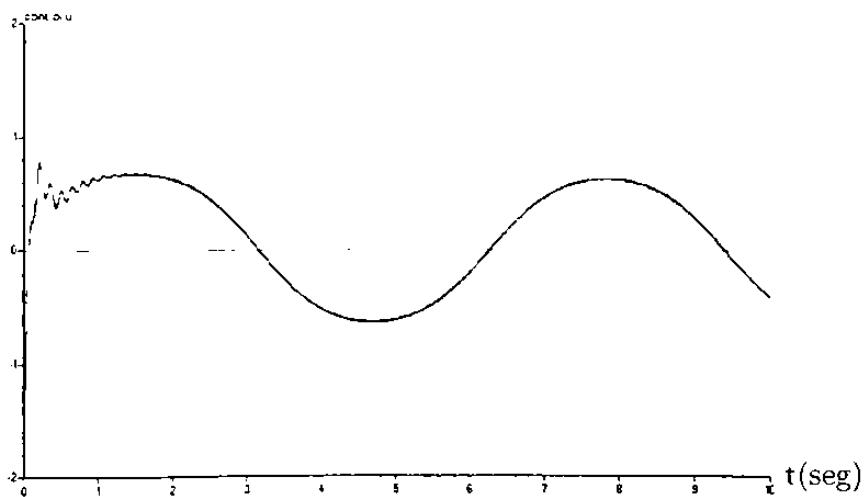


Figura 5.5 Gráfica del control u

5.7 Conclusiones

Una estructura no-lineal de control-observador, basada en una clase de sistemas no-lineales singularmente perturbados, nos permitió descomponer el sistema original, en dos subsistemas de menor dimensión, ambos descritos en diferentes escalas de tiempo. Esta técnica ha sido presentada y aplicada para el caso del modelo de un brazo robot de un simple eslabón, el cual tiene flexibilidad en la articulación. Un buen desempeño del seguimiento de la trayectoria se puede apreciar en este método para tales dispositivos mecánicos.

Capítulo 6

Estudio Comparativo de las distintas técnicas de Control empleadas.

6.0.1 Introducción

En este capítulo presentaremos un estudio comparativo, mediante resultados de simulación, de las técnicas de control para la estabilización global de sistemas no-lineales con aplicación para el caso de robots manipuladores con flexibilidad en la articulación han sido desarrollado en capítulos anteriores.

Considerando el modelo del brazo robot manipulador dado por

$$\begin{aligned} I \ddot{\dot{q}} + B_i \dot{q} + mgl \sin(q) &= k(q_m - q) \\ J \ddot{q}_m + B \dot{q}_m + k(q_m - q) &= u \end{aligned} \tag{6.1}$$

donde la primera ecuación de (6.1) representa la *dinámica del eslabón* y la segunda ecuación *la dinámica del motor*.

En este estudio se aplicarán las técnicas control para la estabilización y seguimiento de la trayectoria, las cuales se resumen en lo siguiente:

- *Método Geométrico Diferencial*

El diseño de un controlador para una clase de sistemas no-lineales feedback linealizante

mediante un observador de alta ganancia.

- *Método Algebraico Diferencial*

Un controlador dinámico linealizante por retroalimentación de estado, basando en las formas canónicas de observabilidad generalizada (FCOG) y de controlabilidad generalizada (FCCG) de Fliess. Este controlador está basado en la linealización del error de seguimiento, la estabilidad del sistema, así como el seguimiento de la trayectoria deseada para el manipulador con articulación flexible. La dinámica del error de seguimiento es estimada por un observador con el sistema en lazo cerrado.

- *Método basado en Perturbaciones Singulares*

Un controlador basado en un observador para una cierta clase de sistemas no-lineales Singularmente Perturbados fue diseñado. Condiciones suficientes para garantizar la estabilidad del esquema de control basado en estimadores de estado con el sistema en lazo cerrado fueron establecidas.

6.0.2 Comparación entre las técnicas de Control empleadas.

Resultados en simulación

Ahora, mostramos una comparación entre los esquemas de estos controladores, implementados para el modelo de un robot de un simple eslabón, con articulación flexible.

La simulaciones se efectuaron como sigue: se consideró a la señal de referencia deseada $x_{ref} = \sin(t)$, como la señal a seguir por la posición angular del motor x_1 .

En cada una de las estrategias de control se seleccionaron los parámetros de diseño como sigue:

Para la ley de control basada en la teoría de perturbaciones singulares, así como las ganancias, fueron seleccionadas, como sigue:

$$s_1 = 20; s_2 = 25; l_s = 60; s_{f1} = 1; s_{f2} = 10; l_f = 0.1.$$

El parámetro θ en las ecuaciones del observador fue $\theta = 20$.

Las condiciones iniciales de las variables del robot, y los estimados, se seleccionaron como:

$x_1(0) = 0, x_2(0) = 0, z_1(0) = 0, z_2(0) = 0, \hat{x}_1(0) = 0.2, \hat{x}_2(0) = 0.02, \hat{z}_1(0) = 0.002, \hat{z}_2(0) = 0.003.$

Para la técnica basada en el enfoque geométrico diferencial, las condiciones iniciales del sistema, así como las del observador fueron las siguientes:

$$x_1(0) = 1, x_2(0) = 0.1, x_3(0) = 1, x_4(0) = 0.1,$$

$$\hat{x}_1(0) = 1, \hat{x}_2(0) = 0.3, \hat{x}_3(0) = 1, \hat{x}_4(0) = 1.$$

El parámetro θ en las ecuaciones del observador, así como sus ganancias, fueron seleccionadas como:

$$\theta = 3, k_1 = 24, k_2 = 10, k_3 = 15, k_4 = 10.$$

Por otra parte las ganancias del controlador se seleccionaron como: $a_1 = 16, a_2 = 32, a_3 = 24, a_4 = 8$.

En las Figura 6.1 se muestran la posición angular del eslabón, usando todos los esquemas de control, y la señal de referencia deseada. La velocidad angular del eslabón, así como la derivada de la señal de referencia se muestran en la Figura 6.2. A partir de estas graficas, podemos observar, el buen desempeño para el seguimiento de la trayectoria de cada una de las metodologías propuestas. Sin embargo, podemos notar que el esquema de control basado en perturbaciones singulares presenta menos oscilaciones que los otros esquemas de control. Ademas, la acción de control esta dentro de los límites físicos. Aunque el control basado en la técnica geométrico diferencial, su acción es aun menor que el basado en la técnica perturbaciones singulares, su desempeño no es muy adecuado. Más aún, el método basado en la técnica del algebra diferencial presentó una alta oscilación y una acción de control mas severa, estos son mostrados en la figura 6.3

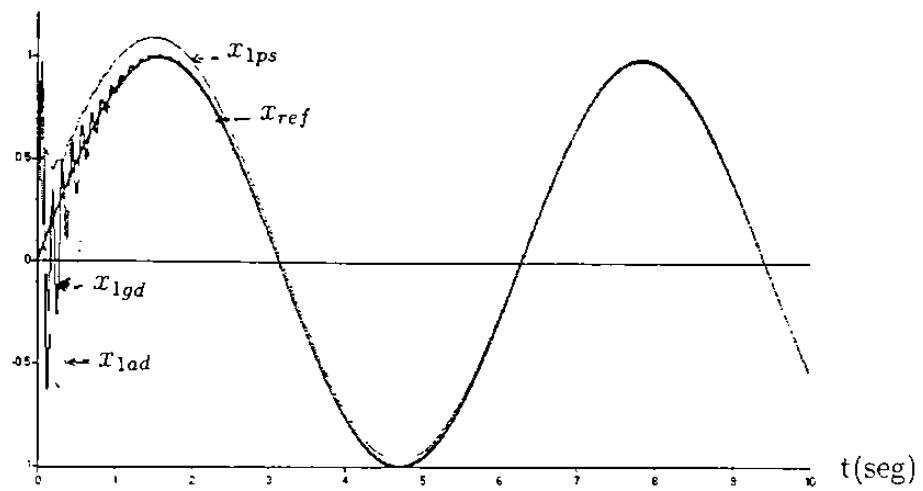


Figura 6.1 Gráfica del estado x_1 y la señal de referencia x_{ref}

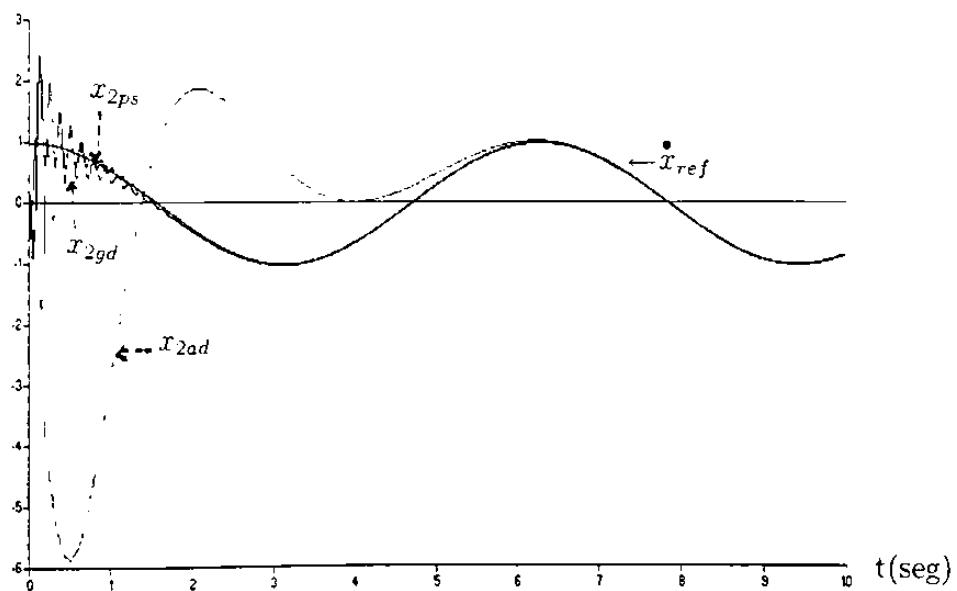


Figura 6.2 Gráfica del estado x_2 y la señal de referencia x_{ref}

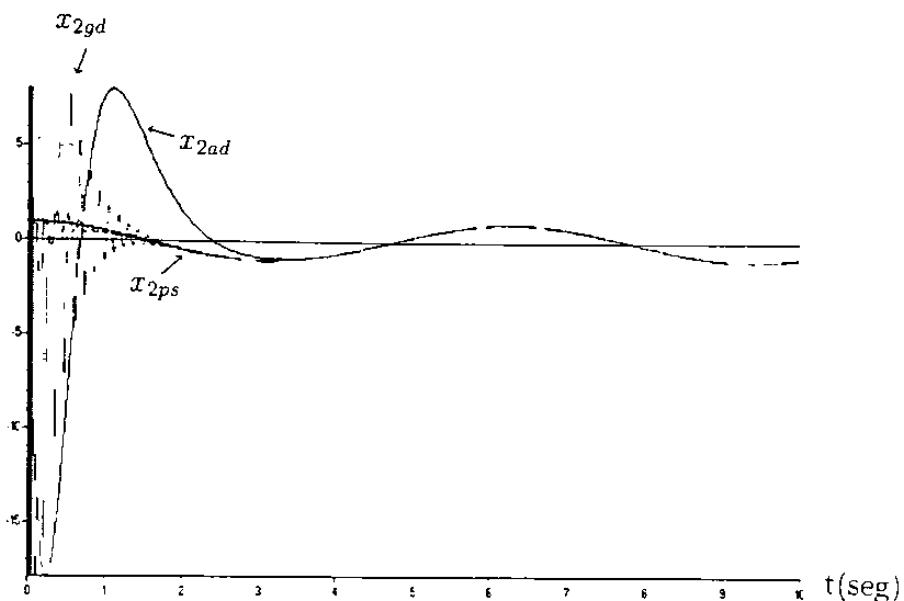


Figura 6.3 Gráficas del estado x_2 y x_{ref}

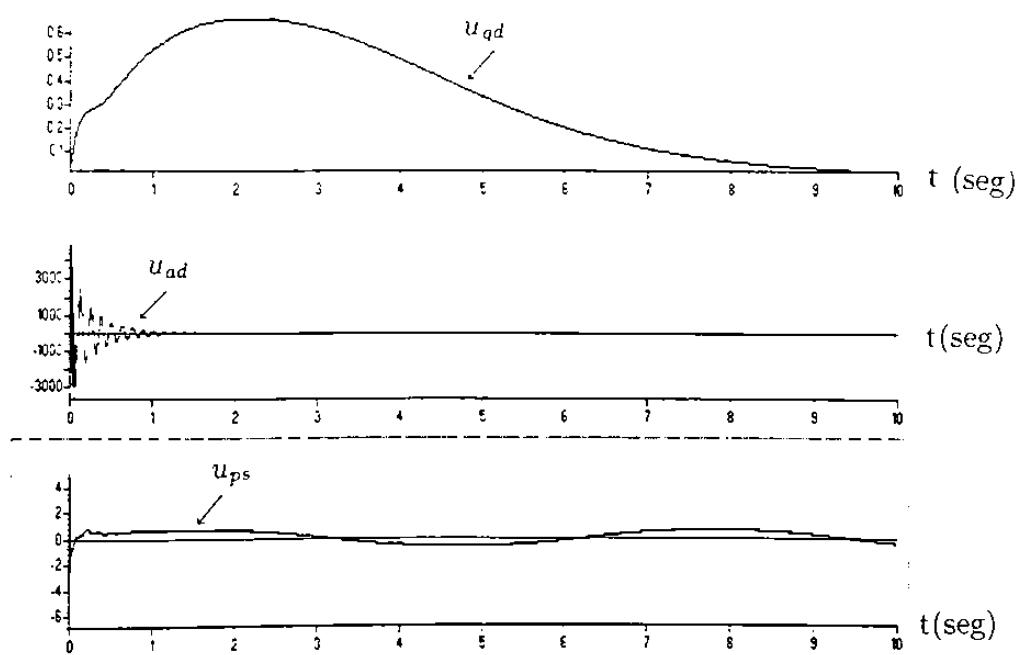


Figura 6.4 Gráfica de los controladores

6.1 Análisis ante perturbaciones o ruido

Un análisis de la técnicas de control desarrolladas, ante la presencia de perturbaciones fue implementada con una señal de ruido gaussiano $f(t) = 0.05\text{norm}(t)$. con media aritmética cero, desviación estandar 1, y magnitud de 5%. con respecto a la magnitud de la señal de salida. La figura 6.5 muestra el comportamiento del estado x_1, x_{ref} y la señal de ruido, mientras que la figura 6.6. nos muestra el comportamiento del estado x_2 , y la señal de ruido, mediante la técnica del método *Geométrico Diferencial*. La figura 6.7 muestra el comportamiento del estado x_1, x_{ref} , y la señal de ruido, la figura 6.8. nos muestra el comportamiento del estado x_2 , y la señal de ruido, mediante la técnica del método *Algebráico Diferencial*, y finalmente. Por otro lado, la figura 6.9 muestra el comportamiento del estado x_1, x_{ref} , y la señal de ruido, y finalmente la figura 6.10. nos muestra el comportamiento del estado x_2 , y la señal de ruido, mediante la técnica de *Perturbaciones Singulares*.

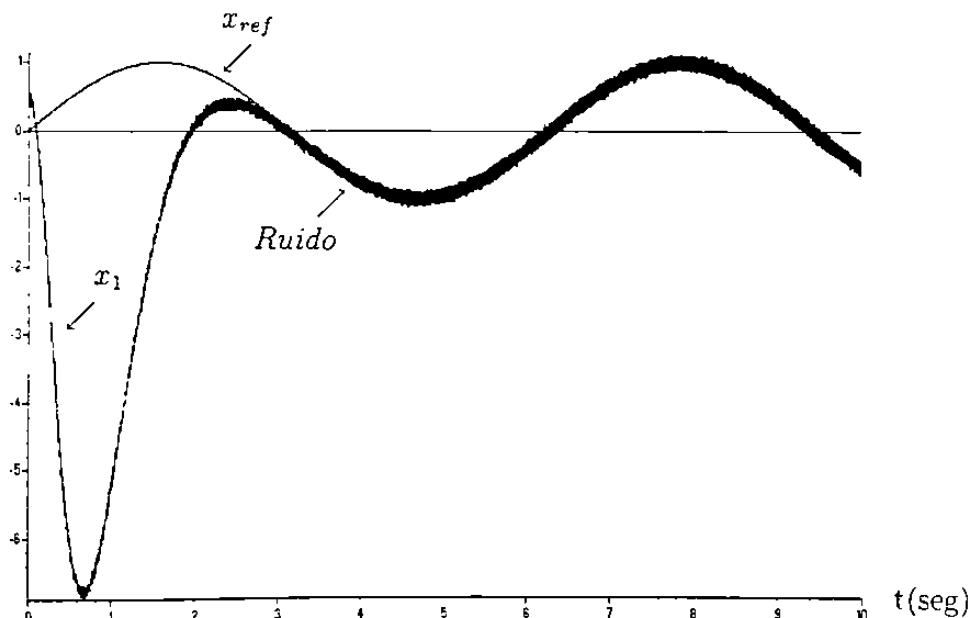


Figura 6.5 Estado x_1, x_{ref} y la señal de ruido

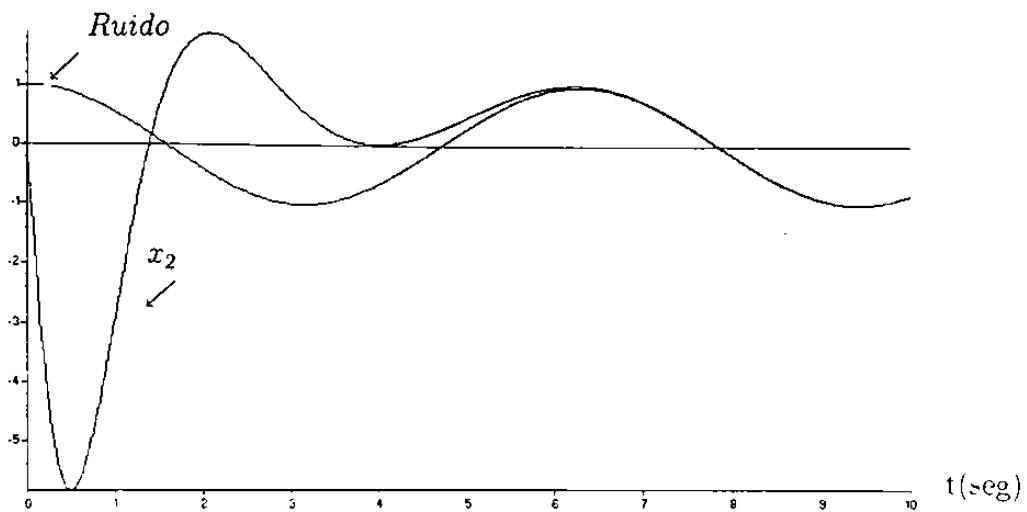


Figura 6.6 Estado x_2 y la señal con ruido. Geométrico Diferencial

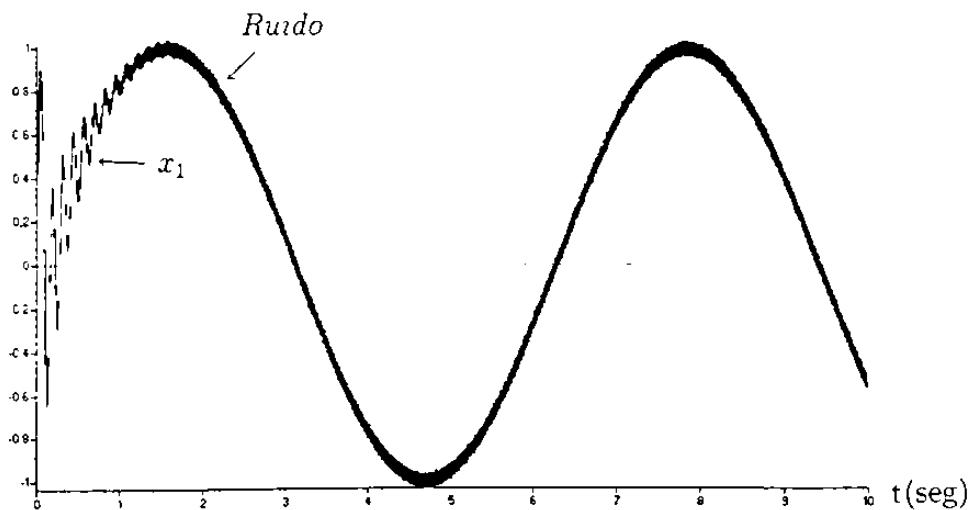


Figura 6.7 Estado x_1 y la señal de ruido. Algebráico Diferencial

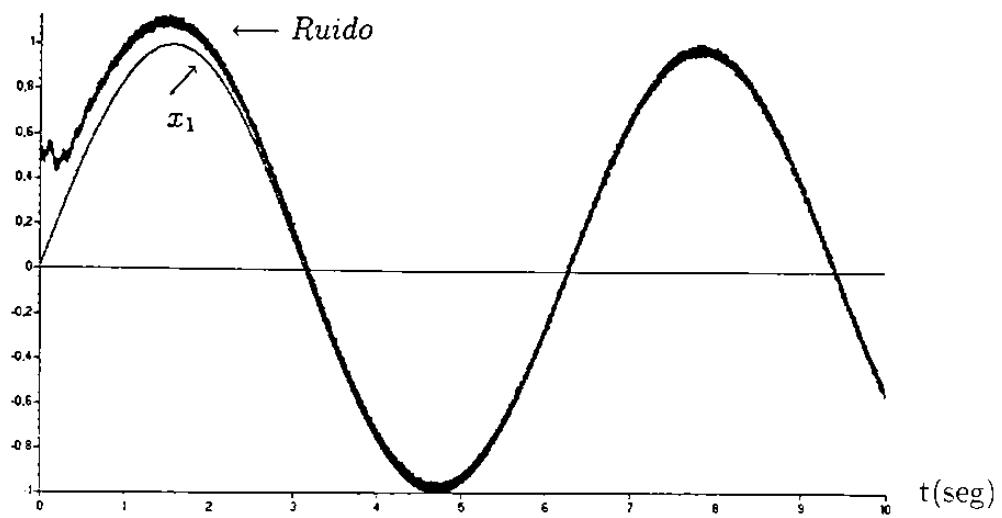


Figura 6.8 Estado x_1 y la señal de ruido. Peturbaciones Singulares

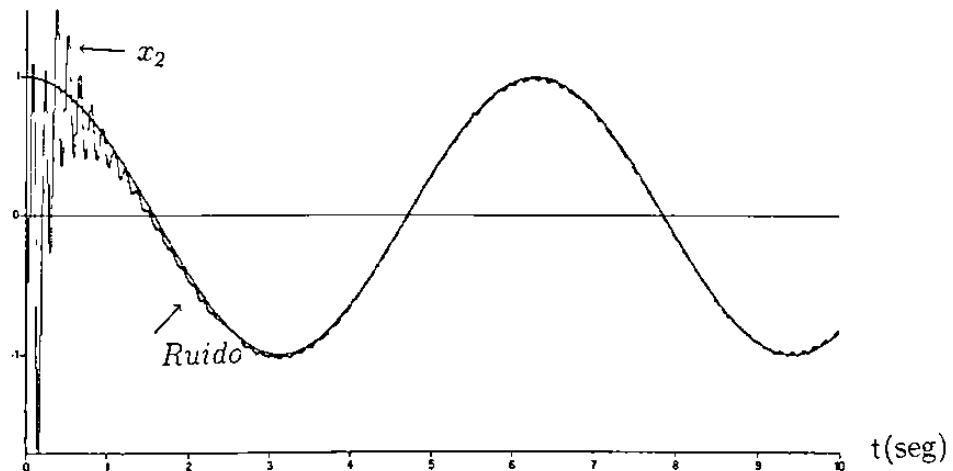


Figura 6.9 El estado x_1 y la señal de ruido. Perturbaciones singulares

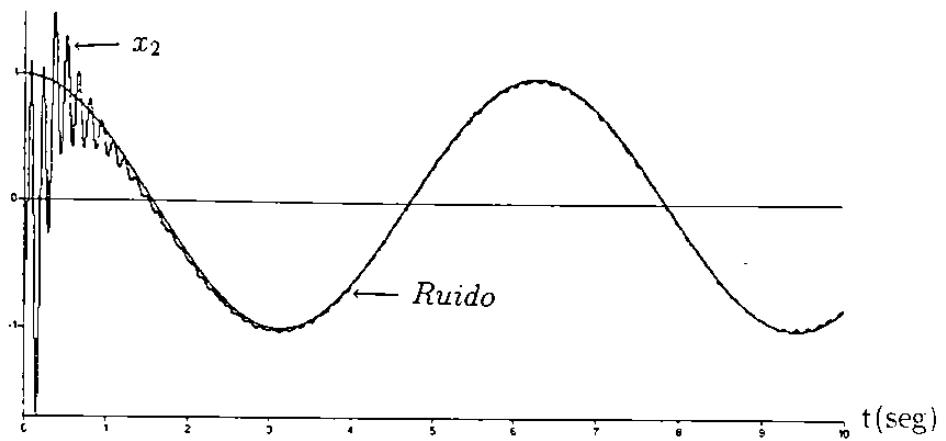


Figura 6.10 El estado x_2 y la señal de ruido. Perturbaciones singulares

Capítulo 7

Conclusiones

En el presente trabajo de tesis se presentaron tres técnicas de control basadas en un observador. para la estabilidad y el seguimiento de la trayectoria de un brazo robot de un simple eslabón con articulación flexible.

En cada una de las estrategias de control estudiadas. se hizo uso de un observador no lineal para estimar la parte del vector de estado no medible. permitiendo implementar estas estrategias.

En todas las técnicas presentadas. un estudio de estabilidad del sistema en lazo cerrado fue presentado. es decir en cada uno de los casos. condiciones necesarias para garantizar la estabilidad del sistema en lazo cerrado fueron presentadas. Ademas. resultados en simulacion fueron mostrados para ilustrar el desempeño de cada una de estas estrategias de control.

Un estudio comparativo de las técnicas estudiadas ha sido desarrollado para ilustrar el desempeño de estas. siendo la metodología basada en perturbaciones singulares la que mejor resultados mostró.

Por otra parte. extensión de estos resultados a el caso multivariable es un problema abierto. siendo actualmente tema de investigacion. Asi como las aplicaciones a modelos de robots con multiples eslabones es tambien un tema de investigacion. De hecho. aplicaciones a otro tipo de procesos puede ser realizada de manera similar.

En un análisis desarrollado en las tres técnicas de control para robots manipuladores con flexibilidad en la articulación, se sometió el sistema ante la presencia de ruido en la señal de salida (*perturbaciones en la salida*), obteniéndose los siguientes resultados:

1.-En cada una de las tres técnicas, se sigue conservando la convergencia del estado estimado, hacia la señal de referencia deseada.

2.-El método basado en la metodología Algebráico Diferencial, resulta demasiado lento en su respuesta

3.-La técnica basada en el enfoque Geométrico diferencial, resulta ser mejor comparado con el método Algebráico Diferencial, en cuanto a rapidez de convergencia.

4.- El método basado en la técnica de Perturbaciones Singulares, resultó ser el mejor en su respuesta de seguimiento de la trayectoria, ante la presencia de ruido en la señal de salida, presentando menos oscilaciones, con respecto a los otros dos métodos.

Por otra parte se efectuó un análisis, para determinar el desempeño del brazo robot manipulador con flexibilidad en la articulación, ante la presencia de incertidumbres parámetricas de la estructura del robot, teniendo como resultado lo siguiente:

El observador basado en la técnica de modos deslizantes, resultó ser el único que presenta mejor robustez, ante la presencia de incertidumbres parámetricas.

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APENDICE A

0.1 Conceptos y definiciones

En este apéndice, se daran algunas conceptos y definiciones necesarias para la comprensión de este trabajo. Para comenzar, daremos algunas definiciones de base sobre la teoría de la observación de sistemas no lineales en tiempo continuo. A continuación, recordaremos algunos resultados sobre los observadores y su síntesis. Posteriormente, se presentaran algunas definiciones y resultados útiles referentes a la estabilidad y estabilización de sistemas, y finalmente se anexan algunos resultados presentados en este trabajo de investigación, que fueron aceptados y presentados en Congresos Internacionales.

Primero, consideremos los sistemas no lineales de la forma:

$$\begin{cases} \dot{x} = f(u, x) \\ y = h(x) \end{cases} \quad (0.1)$$

donde el estado $x \in M$, una variedad analítica (C^ω); la entrada $u \in U \subset R^m$; la salida $y \in R^p$. Para toda $u \in U$, $f_u(x) = f(u, \cdot)$ denota el campo vectorial C^ω sobre M y la función $h = (h_1, \dots, h_p)$ es también C^ω .

Designaremos por $x_u(t, x_0)$ la solución de (0.1) a partir de la condición inicial $x_u(0, x_0) = x_0$.

0.2 Observabilidad de sistemas no lineales

La noción de observabilidad puede ser formulada de varia maneras en el contexto no lineal. En este trabajo adoptaremos la formulación dada por Hermann Krener que se

establece a partir de la noción de distinguibilidad de estados.

Definición 0.1 (Distinguibilidad-Indistinguibilidad): Dos estados iniciales $x_0 \in M$, $x_1 \in M$, $x_0 \neq x_1$, son indistinguibles si para todo $t \geq 0$ y para toda entrada admisible $u : [0, t] \rightarrow U$ las trayectorias correspondientes de (0.1) que resultan de x_0 , x_1 son tales que $h(x_u(t, x_0)) = h(x_u(t, x_1))$. Recíprocamente, decimos que dos estados iniciales $x_0 \in M$, $x_1 \in M$, $x_0 \neq x_1$, son distinguibles si existe $t \geq 0$ y una entrada admisible $u : [0, t] \rightarrow U$ tales que $h(x_u(t, x_0)) \neq h(x_u(t, x_1))$.

Definición 0.2 (Observabilidad): Un sistema es observable en $x_0 \in M$ si para otro estado $x_1 \neq x_0$ es distingible de x_0 . Un sistema es observable si es observable en todo $x_0 \in M$.

Para los sistemas lineales, la observabilidad está caracterizada por la famosa condición de rango:

Teorema 0.3: El sistema lineal estacionario :

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad x \in R^n, u \in R^m, y \in R^p \quad (0.2)$$

es observable si y sólo si la matriz $[C^T \ A^T C^T \ \dots \ A^{n-1} C^T]^T$ es de rango pleno.

La distinguibilidad es un concepto global. Sucede frecuentemente que para generar dos trayectorias a partir de x_0 y de x_1 , se tiene uno que alejar lo suficientemente de x_0 y x_1 . Las dos definiciones que siguen son de naturaleza local.

Definición 0.4 (Observabilidad local): Se dice que el sistema (0.1) es localmente observable en x_0 si para toda vecindad abierta V_{x_0} de x_0 el conjunto de puntos que son

indistinguibles de x_0 en V_{x_0} vía las trayectorias en V_{x_0} es el punto x_0 el mismo.

Cuando se trata de distinguir x_0 de sus vecinos, podemos avistar la condición de observabilidad local. Desde este punto de vista, se tiene:

Definición 0.5 (Observabilidad local débil): *Se dice que el sistema (0.1) es localmente débil observable en x_0 si existe una recindad abierta V_{x_0} de x_0 tal que para toda recindad abierta $V'_{x_0} \subset V_{x_0}$, el conjunto de puntos que son indistinguibles de x_0 en V'_{x_0} vía las trayectorias en V'_{x_0} es el punto x_0 el mismo.*

De manera general, un sistema es localmente débilmente observable si todo estado x_0 puede ser instantáneamente distinguido de sus vecinos al utilizar las trayectorias que permanecen en una vecindad de x_0 .

La propiedad de observabilidad local débil es importante, puesto que ella puede ser verificada por medio de una simple condición algebraica como en el caso lineal. Por lo anterior, nos lleva a definir el espacio de observación.

Definición 0.6 (Espacio de observación): *Se llama espacio de observación \mathcal{O} el mas pequeño subespacio vectorial de funciones reales de M que contiene h_1, \dots, h_p , y que sea cerrado por la derivación de Lie con respecto a todos los campos vectoriales del tipo f_u , $u \in U$, fijo.*

Teorema 0.7: *Sea $d\mathcal{O}$ el espacio de diferenciales de los elementos de \mathcal{O} . Designemos por $d\mathcal{O}(x_0)$ la evaluación de $d\mathcal{O}$ en x_0 . Si:*

$$\dim d\mathcal{O}(x_0) = n \quad (0.3)$$

entonces el sistema es localmente débilmente observable en x_0 .

La condición (0.3) se llama condición de rango. Si la condición (0.3) se verifica para todo $x_0 \in M$, se dice entonces que el sistema (2.1) es observable en el sentido del rango.

El teorema anterior da una condición suficiente de observabilidad local débil. El siguiente teorema da una condición necesaria:

Teorema 0.8: *Supongamos que el sistema (0.1) es localmente débilmente observable. Entonces la condición (0.3) se cumple casi en todas partes en M , es decir, $\dim d\mathcal{O}(x) = n$ en todo punto x que pertenece a un abierto denso M' en M .*

0.3 Entradas universales

Notemos que la definición de observabilidad dada anteriormente, no implica que toda función de entrada distinga parejas de punto de M . En la práctica, desearíamos centrarnos en el caso donde la entrada u que siendo fija, toda pareja de estados distintos den lugar a salidas diferentes. En efecto, este no es el caso en que se pueda reconocer el estado inicial del sistema a partir de la información anterior sobre la entrada y la salida. Esto no lleva a la noción de entrada universal.

Definición 0.9 (Entrada universal): *Una entrada admisible $u : [0, T] \rightarrow U$ es universal para el sistema (2.1) sobre $[0, T]$ si, para toda pareja de estados diferentes x_0 y x_1 , se tiene $h(x_u(t, x_0)) \neq h(x_u(t, x_1))$ en al menos en un tiempo $t \in [0, T]$. Una entrada universal sobre R^+ es universal.*

Definición 0.10 (Entrada singular): *Una entrada no universal es una entrada singular.*

El problema de la existencia de entradas universales ha sido estudiado por varios autores (Sontag , Sussmann). Ellos han demostrado que el conjunto de entradas universales es

denso en el espacio de entradas admisibles con una topología conveniente para los sistemas localmente débilmente observables.

0.4 Los sistemas uniformemente observables

Consideremos el sistema (0.1). En general, existen entradas singulares u tales que el sistema autónomo asociado:

$$\begin{cases} \dot{x} = f_u(x) \\ y = h(x) \end{cases} \quad (0.4)$$

no sea observable.

La noción de entrada universal permite definir una clase interesante de sistemas: los sistemas uniformemente observables.

Definición 0.11 (*Sistemas uniformemente observables localmente uniformemente observables*): *Un sistema en el cual todas las entradas son universales se llama uniformemente observable, o simplemente, observable para toda entrada. Un sistema es localmente uniformemente observable si y solo si para todo punto $x \in M$ existe una vecindad V_x de x tal que el sistema permanecerá en esta vecindad es uniformemente observable.*

Los sistemas lineales observables son uniformemente observables.

El problema de la caracterización de estos sistemas en el caso mono-salida ha sido abordado por Williamson (williamson) para los sistemas bilineales y por Gauthier et Bornard (Gauthier-Bornard) para los sistemas afines en el control. estos resultados se resumen en los dos teoremas siguientes:

Teorema 0.12 : *Consideremos el sistema bilineal:*

$$\begin{cases} \dot{x} = Ax + uBx + Fu \\ y = Cx \end{cases} \quad (0.5)$$

donde $x \in R^n$, $u \in R^m$, $y \in R$; A , B , F , C son matrices de dimensiones apropiadas.

Una condición necesaria y suficiente para que el sistema (0.5) sea uniformemente observable es que pueda trasformarse por un cambio de coordenadas $z = Tx$, en:

$$\begin{cases} \dot{z} = \bar{A}z + u\bar{B}z + \bar{F}u \\ y = \bar{C}z \end{cases} \quad (0.6)$$

donde:

$$\bar{A} = TAT^{-1} = \begin{pmatrix} 0 & 1 & \dots & 0 \\ \vdots & & & \vdots \\ 0 & \dots & 0 & 1 \\ a_1 & \dots & \dots & a_n \end{pmatrix}, \quad \bar{B} = TBT^{-1} = \begin{pmatrix} \bar{b}_{11} & & & 0 \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \bar{b}_{n1} & \dots & \dots & \bar{b}_{nn} \end{pmatrix}, \quad \bar{C} = CT^{-1} = [1, 0, \dots, 0].$$

Teorema 0.13 : Consideremos el sistema afín en el control:

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^m g_i(x)u_i \\ y = h(x) \end{cases} \quad x \in R^n, u \in R^m, y \in R. \quad (0.7)$$

Una condición necesaria y suficiente para que el sistema (0.7) sea localmente uniformemente observable es que exista un sistema de coordenadas locales sobre R^n tal que (0.7) sea casi por todos lados localmente de la forma:

$$\begin{cases} \dot{z} = \bar{A}z + \varphi(z) + \sum_{i=1}^m \bar{g}_i(z)u_i \\ y = \bar{C}z \end{cases} \quad (0.8)$$

donde $\bar{A} = \begin{pmatrix} 0 & 1 & 0 \\ & \ddots & \cdot \\ & & 1 \\ 0 & & 0 \end{pmatrix}$, $\varphi(z) = \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ \varphi_n(z) \end{pmatrix}$. $\bar{C} = [1, 0, \dots, 0]$
 $y \bar{g}_{i,j}(z) = \bar{g}_j(z_1, \dots, z_j)$ para $j = 1, \dots, n$; $i = 1, \dots, m$.

La caracterización de estos sistemas en el caso multivariable es todavía un problema abierto.

0.5 Observadores

Un observador o reconstructor de estado es un sistema dinámico que permite obtener una estimación del valor instantáneo del estado no medible de un sistema a partir de informaciones anteriores de las entradas del sistema original.

Definición 0.14 (Observador): Sea el sistema dinámico:

$$(\Sigma) \quad \begin{cases} \dot{x} = f(u, x) \\ y = h(x) \end{cases}$$

Sé llama observador de (Σ) un sistema dinámico auxiliar (O) cuyas entradas están constituidas por las entradas y las salidas del sistema (Σ) y cuyo vector de salida \hat{x} es el estado estimado :

$$(O) \quad \begin{cases} \dot{z} = \bar{f}(u, y, z) \\ \dot{\hat{x}} = \bar{h}(u, y, z) \end{cases}$$

tal que $\|e(t)\| = \|\hat{x}(t) - x(t)\| \rightarrow 0$ cuando $t \rightarrow \infty$.

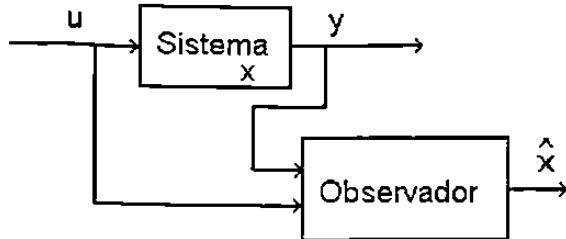


Figura 0-1: Esquema del observador

Cuando $\|e(t)\|$ tiende exponencialmente hacia cero cuando $t \rightarrow \infty$, se dice que el observador es exponencial. El esquema de un observador se muestra en la figura siguiente:

0.5.1 Síntesis de observadores

El estudio de la propiedad de observabilidad constituye una primera etapa en la construcción de un observador. En efecto, para garantizar el buen funcionamiento de un observador es necesario que el sistema permanezca "suficientemente observable" bajo todas las entradas aplicadas.

Es entonces evidente que uno de los obstáculos para la construcción de observadores es la presencia de entradas singulares. Este obstáculo no existe en los sistemas lineales estacionarios observables, puesto que estos sistemas no tienen entradas singulares. El observador más común para estos sistemas es el de Luenberger (luemberger).

- **Sistemas lineales estacionarios**

Sea el sistema lineal observable:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (0.9)$$

donde $x \in R^n$, $u \in R^m$, $y \in R^p$ y las matrices A , B , C son de dimensiones apropiadas.

El sistema gobernado por:

$$\dot{\hat{x}} = A\hat{x} + Bu - K(C\hat{x} - y) \quad (0.10)$$

donde la matriz K es seleccionada tal que los valores propios de $A - KC$ tienen parte real negativa. es un observador exponencial para el sistema (0.9).

• Sistema lineal variante en el tiempo

Para los sistemas lineales con parámetros variantes en el tiempo, existe el observador de Kalman bajo la hipótesis de la observabilidad completa uniforme:

\(\searrow\)

Definición 0.15 (Observabilidad completa uniforme): El sistema:

$$\begin{cases} \dot{x}(t) = A(t)x(t) \\ y(t) = C(t)x(t) \end{cases} \quad x \in R^n, u \in R^m, y \in R^p \quad (0.11)$$

es completamente uniformemente observable si existe $T > 0$, $\alpha > 0$ y $t_0 > 0$ tales que, para todo $t \geq t_0$, se tiene:

$$G(t, t+T) = \int_t^{t+T} \phi^T(\tau, t)C(\tau)^TC(\tau)\phi(\tau, t)d\tau > \alpha I$$

donde I es la matriz identidad y $\phi(\tau, t)$ es la matriz de transición de la parte autónoma de (0.11) definida por:

$$\begin{cases} \frac{d}{d\tau}\phi(\tau, t) = A(\tau)\phi(\tau, t) \\ \phi(t, t) = I \end{cases}$$

Es bien conocida la versión determinista del filtro de Kalman-Bucy (kalman-busy) que proporciona un observador no inicializado para los sistemas lineales variantes en el tiempo completamente observables. En efecto, se ha demostrado que si $A(t)$ y $C(t)$ son acotados y si el sistema (0.11) es uniformemente completamente observable, entonces el sistema:

$$\begin{cases} \dot{\hat{x}}(t) = A(t)\hat{x}(t) - S(t)^{-1}C(t)^TQ(C(t)\hat{x}(t) - y(t)) \\ \dot{S}(t) = -\theta S(t) - A(t)^TS(t) - S(t)A(t) + C(t)^TQC(t) \\ S(0) = S_0 \end{cases} \quad (0.12)$$

donde $\theta > 0$ y S_0 , Q que son matrices definidas positivas, es un observador exponencial para (0.11).

• Sistema afín en el estado

El resultado anterior puede ser extendido a los sistemas afines en el estado:

$$\begin{cases} \dot{x} = A(u)x + B(u) \\ y = C(u)x \end{cases} \quad x \in R^n, u \in R^m, y \in R^p \quad (0.13)$$

Se puede apreciar que toda función u fija genera un sistema lineal con parámetros variantes en el tiempo. Por consecuencia, se puede definir el Gramiano de observabilidad para estos sistemas, para u fija como sigue:

$$G(u, t, t+T) = \int_t^{t+T} \phi_u^T(\tau, t) C^T(u(\tau)) C(u(\tau)) \phi_u(\tau, t) d\tau$$

donde $\phi_u(\tau, t)$ es la matriz de transición definida por:

$$\begin{cases} \frac{d}{d\tau} \phi_u(\tau, t) = A(u(\tau)) \phi_u(\tau, t) \\ \phi_u(t, t) = I \end{cases}$$

El sistema (0.13) puede tener varias entradas singulares. En (hamouri-De León) se de-

muestra que las entradas universales no son suficientes para hacer funcionar un observador para los sistemas que poseen entradas singulares. Esto proviene del hecho que una entrada universal trasladada en el tiempo no puede ser universal. Es por esto que la noción de entradas persistentes ha sido introducida para garantizar el funcionamiento del observador, sobre todo en presencia de ruido o de perturbaciones. Ahora introduciremos la noción de entrada persistente.

Definición 0.16 (Entrada regularmente persistente): Una función de entrada u es regularmente persistente para el sistema (0.13) si existe $T > 0$, $\alpha > 0$ y $t_0 > 0$ tales que el valor propio más pequeño de la matriz $G(u, t, t + T)$ es superior a α por $t \geq t_0$.

El siguiente teorema permite garantizar la construcción de un observador tipo Kalman extendido para los sistemas afines en el estado.

Teorema 0.17 : Toda entrada regularmente persistente aplicada a un sistema afín al estado (0.13) genera un sistema lineal con parámetros variables en el tiempo completamente uniformemente observable.

Por consecuencia, se obtiene el siguiente resultado: consideremos el sistema (0.13) con matrices $A(u)$, $B(u)$ y $C(u)$ uniformemente acotadas sobre el dominio de las entradas admisibles. Entonces, para toda entrada regularmente persistente para (0.13), el sistema :

$$\begin{cases} \dot{\hat{x}} = A(u)\hat{x} + B(u) - S^{-1}C(u)^T Q(C(u)\hat{x} - y) \\ \dot{S} = -\theta S - A(u)^T S - S A(u) + C(u)^T Q C(u) \\ S(0) = S_0 \end{cases} \quad (0.14)$$

con $\theta > 0$ y S_0 matrices definidas positivas, es un observador exponencial para el sistema (0.13).

0.6 Estabilidad y estabilización

0.6.1 Estabilidad

El problema de estabilidad de un sistema corresponde al comportamiento de la trayectoria de este cuando su estado inicial está próximo del punto de equilibrio. Esta idea es intuitiva y resulta muy compleja para los sistemas no lineales. Por consecuencia una larga lista de definiciones han sido propuestos . El objetivo de la teoría de la estabilidad es de poder obtener conclusiones sobre el comportamiento de un sistema sin tener que calcular su solución. Aquí presentaremos algunas definiciones que son las más utilizadas en la literatura principalmente sobre la estabilidad en el sentido de Lyapunov, la estabilidad asintótica y la estabilidad asintótica uniforme.

Consideremos el sistema:

$$\dot{x}(t) = f(x(t), t) \quad (0.15)$$

donde $x \in R^n$ y $f : R^n \times R^+ \rightarrow R^n$ es continua. Supongase también que la ecuación (0.15) posee una solución única que corresponde a cada condición inicial.

Esto es en el caso en que f es globalmente Lipschitz. Designemos por $x(t, x_0, t_0)$ la solución al instante t de (0.15) surgida de la condición inicial $x(t_0, x_0, t_0) = x_0$.

Recordemos que un estado $x_e \in R^n$ es un punto de equilibrio para el sistema (0.15) si: $f(x_e, t) = 0, \forall t \geq 0$. Asumiendo que x_e es un punto de equilibrio de (0.15).

Definición 0.18 (Estabilidad en el sentido de Lyapunov): *El punto de equilibrio x_e de (0.15) es estable o estable en el sentido de Lyapunov si, para todo $\varepsilon > 0$ y para todo $t_0 \in R^+$, existe $\delta(\varepsilon, t_0) > 0$ tal que: $\|x_0 - x_e\| < \delta \Rightarrow \|x(t, x_0, t_0) - x_e\| < \varepsilon$ para todo $t \geq t_0$.*

Puesto que todas las normas son sobre R^n son topológicamente equivalentes.

Cuando δ depende solamente de ε , se habla de estabilidad uniforme:

Définition 0.19 (Estabilidad uniforme): El punto de equilibrio x_e de (2.15) es uniformemente estable si para todo $\varepsilon > 0$, existe $\delta(\varepsilon) > 0$ tal que $\|x_0 - x_e\| < \delta$. $\forall t_0 > 0 \Rightarrow \|x(t, x_0, t_0) - x_e\| < \varepsilon$. $\forall t \geq t_0$.

Para los sistemas autónomos $\dot{x}(t) = f(x(t))$, no hay distinción entre estabilidad y estabilidad uniforme.

El punto de equilibrio x_e es inestable si x_e no es estable.

La definición precedente no garantiza que las soluciones convergen hacia el punto de equilibrio. Definimos entonces la noción de atractividad de un punto de equilibrio y de estabilidad asintótica:

Definición 0.20 (Atractividad): El punto de equilibrio x_e de (2.15) es atractivo si, para todo $t_0 \in R^+$, existe $\eta(t_0) > 0$ tal que: $\|x_0 - x_e\| < \eta(t_0) \Rightarrow \|x(t_0 + t, x_0, t_0)\| \rightarrow x_e$ cuando $t \rightarrow \infty$.

Notemos que las soluciones inicializadas en estados diferentes en $B_{\eta(t_0)} = \{x: \|x - x_e\| < \eta(t_0)\}$ pueden aproximarse a x_e a velocidades diferentes.

La siguiente definición garantiza una convergencia uniforme hacia x_e .

Definición 0.21 (Atractividad uniforme): El punto de equilibrio x_e es uniformemente atractivo si existe $\eta > 0$ tal que: $\|x_0 - x_e\| < \eta$. $\forall t_0 \geq 0 \Rightarrow \|x(t_0 + t, x_0, t_0)\| \rightarrow x_e$ cuando $t \rightarrow \infty$ uniformemente en x_0, t_0 .

De manera equivalente: El punto de x_e es uniformemente atractivo si $\forall \varepsilon > 0$ existe un tiempo $T(\varepsilon)$ tal que : $\|x_0 - x_e\| < \eta$. $\forall t_0 \geq 0 \Rightarrow \|x(t_0 + t, x_0, t_0) - x_e\| < \varepsilon$. $\forall t \geq T(\varepsilon)$.

Definición 0.22 (Estabilidad aintótica y uniformemente asintótica): *El punto de equilibrio x_e de (2.15) es asintóticamente estable si es estable y atractivo. Es uniformemente asintóticamente estable si es uniformemente estable y uniformemente atractivo.*

Notemos que la atractividad y la estabilidad son dos propiedades independientes, es decir un punto de equilibrio puede ser atractivo sin ser estable. Sin embargo, no está todavía establecido si es posible para un punto de equilibrio ser uniformemente atractivo al ser inestable.

Los conceptos de estabilidad introducidos anteriormente son de naturaleza local en el sentido donde ellos describen el comportamiento de las trayectorias inicializadas próximas al punto de equilibrio. La definición que sigue describe el comportamiento global de las trayectorias.

Definición 0.23 (Estabilidad asintótica global): *El punto de equilibrio x_e de (0.15) es globalmente asintóticamente estable si este es uniformemente estable, y sí para un número par de números positivos M, ε con M arbitrariamente grande y ε arbitrariamente pequeño, existe un tiempo finito $T(M, \varepsilon)$ tal que : $\|x_0 - x_e\| < M$. $\forall t_0 \geq 0 \Rightarrow \|x(t_0 + t, x_0, t_0) - x_e\| < \varepsilon$. $\forall t \geq T(M, \varepsilon)$.*

Dentro de la práctica del estudio de la estabilidad se hace uso del segundo método de Lyapunov (ó método directo). Este consiste en definir una función de Lyapunov con las características apropiadas donde la existencia implica el tipo de estabilidad deseado. Notemos también, que podemos considerar que, $x_e = 0$, dado que siempre podemos realizar un cambio de variable, tal que nosotros caemos siempre en este caso. Para lo siguiente supongamos que es efectivamente el caso.

Definición 0.23a (*Estabilidad finalmente uniformemente acotada*): Una solución $x(t) : [t_0, \infty] \rightarrow \mathbb{R}^n$ del sistema $\dot{x} = f(x)$ con condición inicial $x(t_0) = x_0$ es llamada finalmente uniformemente acotada (*uniformly ultimately bounded*) con respecto a un conjunto S , si existe una constante no negativa $T(x_0, S)$ tal que :

$$x(t) \in S \text{ para toda } t \geq t_0 + T \quad (0.16)$$

Finalmente uniformemente acotada quiere decir que la trayectoria solución del sistema $\dot{x} = f(x)$ comienza en x_0 en un tiempo t_0 y deberá finalmente entrar y permanecer dentro del conjunto S .

Definición 0.24 (*Función de Lyapunov*): De manera general, se llama función de Lyapunov para el sistema (0.15) una función real $V(x, t)$ que posee las siguientes propiedades:

- i) $V(x, t)$ es de clase C^1 tal que $V(0, t) = 0$;
- ii) $V(x, t)$ es definida positiva, i.e. existe una función real continua no decreciente α tal que $\alpha(0) = 0$ y $0 < \alpha(\|x\|) \leq V(x, t), \forall t, \forall x \neq 0$ con $\alpha(\|x\|) \rightarrow \infty$ cuando $\|x\| \rightarrow \infty$.
- iii) Existe una función real γ tal que $\gamma(0) = 0$ y la derivada \dot{V} de V siguiendo las trayectorias de (0.15) es tal que: $\dot{V}(x, t) \leq -\gamma(\|x\|) < 0, \forall t, \forall x \neq 0$.
- iv) Existe una función real continua, no decreciente β tal que $\beta(0) = 0$ y $V(x, t) \leq \beta(\|x\|), \forall t$.

El siguiente teorema de Lyapunov muestra la existencia de tal función V es una condición necesaria y suficiente para la estabilidad asintótica uniforme.

Teorema 0.25: El origen del sistema (0.15), es un punto de equilibrio uniformemente asintóticamente estable si y solo si (0.15) admite una función de Lyapunov V .

Las propiedades requeridas de V pueden ser suavizadas según el tipo de estabilidad de-

seada. Entonces se tiene el siguiente corolario:

Corolario 0.26: *El origen del sistema (0.15), es un punto*

- a) *Estable si y solo si (0.15) admite una función de Lyapunov V que satisface las condiciones i). ii) y ii*): $\dot{V}(x, t) \leq 0$ para todo x y t .*
- b) *uniformemente estable si y solo si (0.15) admite una función de Lyapunov V que satisface las condiciones i). ii) iv) y ii*).*

Corolario 0.27: *Para un sistema dinámico continuo autónomo:*

$$\dot{x} = f(x) : f(0) = 0 \quad (0.17)$$

la estabilidad asintótica está asegurada para la existencia de una función real $V(x)$ de clase C^1 . tal que $V(0) = 0$ y

- $V(x) > 0$ para todo $x \neq 0$
- $\dot{V}(x) < 0$ para todo $x \neq 0$
- $V(x) \rightarrow \infty$ cuando $\|x\| \rightarrow \infty$.

En el caso de sistemas lineales . $\dot{x} = Ax$, la estabilidad asintótica esta caracterizada por la existencia de una forma cuadrática definida positiva $V(x) = x^T Px$ tal que : $A^T P + PA = -Q$ donde Q es una matriz definida positiva.

El siguiente teorema recíproco (inverso) es muy útil en el estudio de los sistemas dinámicos.

Teorema 0.28: *Sea el sistema autónomo (0.16). Si el punto de equilibrio 0 es asintóticamente estable entonces (0.16) admite una función de Lyapunov que es de clase C^∞ en una vecindad de x_0 .*

0.6.2 Estabilización

Uno de los problemas del control consiste en la estabilización de sistemas. Aquí recordaremos algunos resultados sobre la estabilización por retroalimentación de estado estático y dinámico. La estabilización de un sistema no lineal es un tema muy complicado. Existen pocos resultados generales que conciernen a este punto. Al mismo tiempo es difícil concluir sobre la estabilidad o la no estabilidad global asintótica de un sistema no lineal general.

Primeramente, recordemos que todo sistema lineal controlable es estabilizable por retroalimentación de estado estática lineal. Esto no es en general cierto, para los sistemas no lineales. En otros términos, existen sistemas no lineales controlables pero no estabilizables. De este hecho, un tema importante de estudio en la teoría del control es la caracterización de clases de sistemas que son estabilizables por retroalimentación de estado.

Definición 0.29 (Estabilización): *De manera general, se dice que el sistema : $\dot{x} = f(x) + g(x)u$ es estabilizable en el origen si existe una retroalimentación de estado $u = u(x)$, al menos continua, tal que el sistema en lazo-cerrado $\dot{x} = f(x) + g(x)u(x)$ admite el origen como punto de equilibrio asintóticamente estable.*

En lo que sigue, introduciremos algunas precisiones sobre las terminologías concernientes a los tipos de retroalimentación de estado utilizados en la práctica.

Definición 0.30 (Retroalimentación de estado estática) *Se llama retroalimentación de estado estática regular para el sistema no lineal:*

$$\dot{x} = f(x) + g(x)u \quad x \in R^n, u \in R^m \quad (0.18)$$

una función $u = \alpha(x) + \beta(x)v$ donde $\alpha : R^n \rightarrow R^m$ y $\beta : R^n \rightarrow R^{m \times m}$ son funciones regulares tales que la matriz $\beta(x)$ es invertible para todo $x \in R^n$ y $v = (v_1, \dots, v_m)$ representa el nuevo vector de control.

Una variante importante de esta retroalimentación regular es que solamente utiliza la salida.

Definición 0.31 (*Retroalimentación de estado estática de salida*): Se llama retroalimentación de estado estática de salida para el sistema no lineal con salida:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x), \quad x \in R^n, u \in R^m, y \in R^p \end{cases} \quad (0.19)$$

una función $u = \tilde{\alpha}(y) + \tilde{\beta}(y)v$ donde $\tilde{\alpha} : R^p \rightarrow R^m$ y $\tilde{\beta} : R^p \rightarrow R^{m \times m}$ son funciones regulares tales que la matriz $\tilde{\beta}(y)$ sea invertible para todo $y \in R^p$ y $v = (v_1, \dots, v_m)$ representa el nuevo vector de control.

Puesto que $y = h(x)$ la retroalimentación de estado estática de salida es un caso particular de la retroalimentación de estado estática anterior. Entonces, en general solo se puede esperar menos de las propiedades en el caso de una retroalimentación de salida.

Definición 0.32 (*Retroalimentación de estado dinámica*): Una retroalimentación de estado dinámica para el sistema (0.17) es definida por la relación:

$$\begin{cases} \dot{z} = \gamma(z, x) + \delta(z, x)v \\ u = \alpha(z, x) + \beta(z, x) \end{cases}$$

donde $z = (z_1, \dots, z_q) \in R^q$, y $\gamma : R^q \times R^n \rightarrow R^q$, $\delta : R^q \times R^n \rightarrow R^{q \times m}$, $\alpha : R^q \times R^n \rightarrow R^m$ y $\beta : R^q \times R^n \rightarrow R^{m \times m}$ son funciones regulares y $v = (v_1, \dots, v_m)$ representa el nuevo

vector de control.

De hecho, la retroalimentación de estado dinámica puedes ser considerada como la composición de un sistema (0.17) con el sistema:

$$\begin{cases} \dot{z} = \gamma(z, \tilde{x}) + \delta(z, \tilde{x})v \\ \tilde{u} = \alpha(z, \tilde{x}) + \beta(z, \tilde{x}) \end{cases} \quad (0.20)$$

donde \tilde{x} y \tilde{u} son respectivamente relacionados con x y con u . El sistema (0.19) es frecuentemente llamado compensador y esquemáticamente se puede representar como sigue :

Definición 0.33 (*Retroalimentación de estado dinámica de salida*): Una retroalimentación de estado dinámica de salida para el sistema (0.18) está definida por la relación:

$$\begin{cases} \dot{z} = \gamma(z, y) + \delta(z, y)v \\ u = \alpha(z, y) + \beta(z, y) \end{cases}$$

donde $z = (z_1, \dots, z_q) \in R^q$, $y : R^q \times R^p \rightarrow R^q$, $\gamma : R^q \times R^p \rightarrow R^q$, $\delta : R^q \times R^p \rightarrow R^{q \times m}$, $\alpha : R^q \times R^p \rightarrow R^m$ y $\beta : R^q \times R^p \rightarrow R^{m \times m}$ son funciones regulares y $v = (v_1, \dots, v_m)$ representan el nuevo vector de control

0.7 El principio de separación

La mayor parte de las leyes de control son funciones de estado del sistema. Cuando solo se dispone de una parte del vector de estado, no se puede aplicar los algoritmos de control diseñados. Una forma de evitar este problema es cerrar el lazo de control a través de un observador, es decir, una vez que se tiene un algoritmo de control para el sistema y que un observador ha sido diseñado para este último, se procede a reemplazar el estado

desconocido por el estado estimado en el algoritmo de control.

La pregunta que se plantea ahora es saber si la estabilidad del sistema en lazo cerrado se preserva. El estudio de la estabilidad del sistema en lazo cerrado mediante un observador se conoce como principio de separación. Esta terminología proviene del hecho de que la síntesis tanto de la ley de control, así como del observador se llevan al *finalmente uniformemente acotado* separadamente.

Este problema es de carácter muy especial. En general, se trata de encontrar un compromiso entre la velocidad de convergencia del observador y la velocidad de la acción de estabilización.

Para los sistemas lineales, el principio de separación se verifica de que el sistema sea controlable y observable. Esto se traduce mediante una descomposición del espectro del sistema en lazo cerrado con el estado estimado, el espectro del sistema con el del observador que permanece sin cambio, la estabilidad del sistema retroalimentado con el observador depende entonces solo de la estabilidad del observador y del sistema retroalimentado sin observador.

En el contexto no lineal, existen muy pocos resultados concernientes al principio de separación, siendo actualmente un problema abierto.

Notemos también de paso que la estrategia del principio de separación es equivalente a una estabilización dinámica por retroalimentación de salida.

APENDICE B

En esta parte del apéndice se anexan algunos de los resultados presentados en este trabajo y que fueron aceptados para su publicación en Congresos Internacionales

1.- A DYNAMICAL LINEARIZING FEEDBACK CONTROLLER FOR ROBOTS WITH FLEXIBLE JOINT

Presentado en el evento Conference on Control Application CCA ' 97 en Hartford, CT, U.S.A.

2.- SPEED AND POSITION CONTROL OF A FLEXIBLE JOINT ROBOT MANIPULATOR VIA A NONLINEAR CONTROL-OBSERVER SCHEME

Presentado en el evento Conference on Control Application CCA ' 97 en Hartford, CT, U.S.A.

3.- A COMPARATIVE STUDY OF SPEED AND POSITION CONTROL OF A FLEXIBLE JOINT ROBOT MANIPULATOR

Presentado en el evento 1998 IEEE RSJ INTERNATIONAL CONFERENCE Victoria Conference Centre, B.C., CANADA

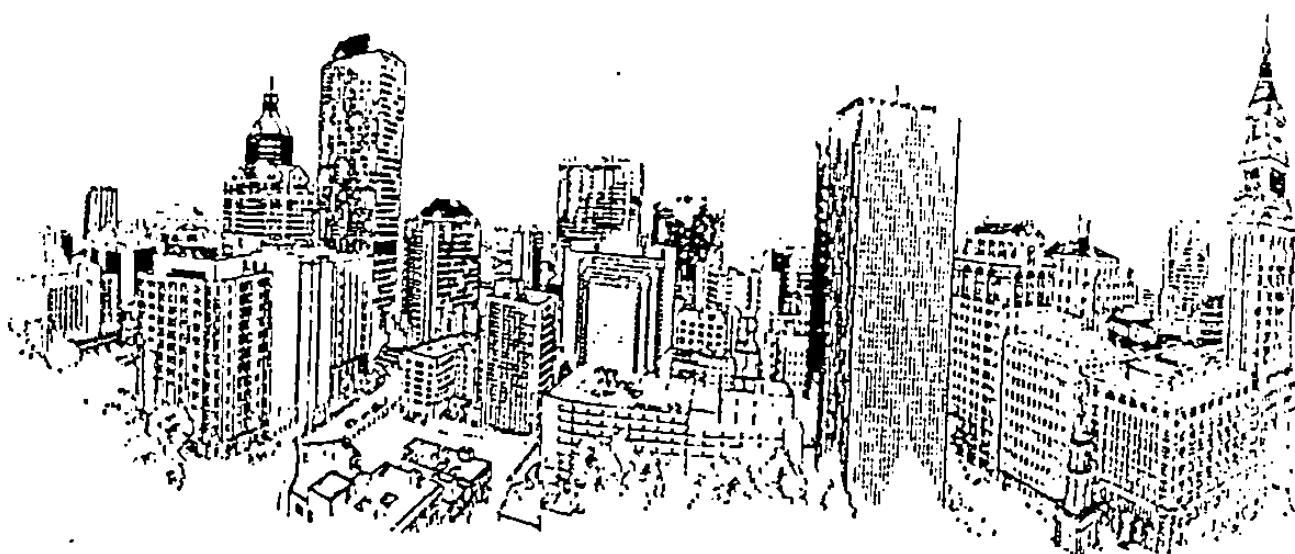
4.- OBSERVED-BASED CONTROLLER FOR A FLEXIBLE JOINT ROBOT MANIPULATOR

Presentado en el evento 37th IEEE CONFERENCE ON DECISION AND CONTROL • Tampa, Florida, U.S.A. December •1998



Proceedings of the

1997 IEEE International Conference on Control Applications



Sheraton Hartford Hotel
Hartford, Connecticut USA
October 5-7, 1997

Proceedings of the
1997 IEEE International
Conference on Control Applications

October 5-7, 1997

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Hartford, Connecticut USA



IEEE Control Systems Society
ASME Dynamic Systems & Control Division

97CH36055

A DYNAMICAL LINEARIZING FEEDBACK CONTROLLER FOR ROBOTS WITH FLEXIBLE JOINT.

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ABSTRACT

Techniques of differential algebra are used to derive an observer-based controller applied to a flexible manipulator. A dynamical linearizing feedback controller is derived on the basis of Fliess' Generalized Observability Canonical Form (GOCF). Our controller actually achieves both stability and tracking by linearizing the tracking error. The controller is then of error-driven type; the dynamics of the error is estimated by an observer in the closed-loop. An analysis of the stability of the closed-loop system is given. Simulation results are presented for illustrating the performance of this observer based controller scheme, when is applied to a mathematical model of a robot with flexible joints.

1.- INTRODUCTION

Several controller design techniques have been proposed, from different perspectives, for the stabilization of nonlinear systems. Recently, a considerable number of works have studied the problem of the control of robots manipulators with joint flexibility. This problem has a practical and theoretical interest. Because, from a practical point of view, we must be consider the effect of the elasticity in the robot to design the control laws; and from theoretical point of view, the number of degree of freedom is twice the number of control actions, and the matching property between nonlinearities and inputs is lost [8, 9].

In the literature, we can find different control schemes for nonlinear systems. Most of these schemes make use of the static feedback control schemes. The design of these static feedback are based on Adaptive control, Singular perturbations, Nonlinear control theory, and Energy-based Lyapunov control schemes, or more recently, Decoupling-based schemes, Backstepping-based schemes, or Passivity-based schemes [8, 9]. Further-

more, a considerable number of researchers have studied the stability and output tracking problems of dynamically systems using the commonest mathematical tool in nonlinear control theory of today: *differential geometry*. However, if the nonlinearities involved are all polynomial there are methods form differential algebra that can be used instead. It has been showed by M. Fliess that the differential algebra is a natural mathematical tool for dealing with polynomial systems.

We treat the stability and output tracking problems for nonlinear systems from the dynamically feedback error linearization strategy. The approach is based on Fliess' generalized observability canonical form (GOCF) and the generalized controller canonical form (GCCF) which are easy consequences of the differential primitive element theorem [2, 3, 5]. Recall that, from the external behavior of the system, the Fliess' GOCF is a generalized state description of the system where, in general, the state dynamics equations are control-dependent, including a finite number of the control time derivatives.

The controller we propose is obtained by means of exact linearization of the tracking error dynamics. This controller is a function of tracking error vector which is, in general, partially measurable, then it is necessary to estimate it. A variety of approaches have been proposed in the synthesis of nonlinear observers to overcome this difficulty. For that a high gain observer is used to estimate the tracking error in order to implement our controller. Finally, to guarantee the stability of the closed-loop system using the proposed observer-based controller, an analysis of the stability of the system in closed loop is given.

This note is organized as follows: In section 2, we introduce the generalized observability and controller canonical forms. The stabilization and output tracking

problems by means of an exponential observer, using the differential algebraic approach, are dealt in section 3. In section 4, we introduce a mathematical model which describes the behavior of a robot manipulator with joint flexibility. And simulation results are presented using our observer-based controller scheme. Some concluding remarks and proposals for further work will close the paper.

2.- GENERALIZED OBSERVABILITY AND CONTROLLER CANONICAL FORMS.

According to the theorem of the differential primitive element there exist an element $\bar{\xi} \in G$ such that $G = K\langle u, \bar{\xi} \rangle$. The Transcendence degree n of $G/K\langle u \rangle$ [2, 3], which is equal to the dimension of the system (1), and is the smallest integer such that $\frac{d^n \bar{\xi}}{dt^n}$ is $K\langle u \rangle$ -algebraically dependent on $\left\{ \bar{\xi}, \frac{d\bar{\xi}}{dt}, \dots, \frac{d^{n-1}\bar{\xi}}{dt^{n-1}} \right\}$ which is a transcendence basis of $G/K\langle u \rangle$ [2, 3]. Defining the following change of variable of the form $\xi_i = \frac{d^{i-1}\bar{\xi}}{dt^{i-1}}, 1 \leq i \leq n$. It is clear that $\{\xi_i\}_{i=1}^n$ is a transcendence basis of $G/K\langle u \rangle$ too. Hence, a nonlinear generalization of the controller canonical form is given by

$$(\Sigma_A) : \begin{cases} \frac{d\xi_i}{dt} = \xi_{i+1}, & 1 \leq i \leq n-1 \\ D\left(\frac{d\xi_n}{dt}, \xi, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n}\right) = 0 \end{cases} \quad (1)$$

where $D\left(\frac{d\xi_n}{dt}, \xi, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n}\right)$ is a polynomial with coefficients in K . If one can locally solve for $\frac{d\xi_n}{dt}$ in the second equation (1), one obtains an explicit system of first order differential equations, known as the Generalized Controller Canonical Form (GCCF).

$$(\Sigma_{GCCF}) : \begin{cases} \frac{d\xi_i}{dt} = \xi_{i+1}, & 1 \leq i \leq n-1 \\ \frac{d\xi_n}{dt} = L_c(\xi, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n}) \end{cases}$$

for γ a strictly positive integer. Now, let y be the scalar output and let n be the smallest integer such that $\frac{dy}{dt^n}$ is algebraically dependent on

$$\left\{ y, \frac{dy}{dt}, \dots, \frac{d^{n-1}y}{dt^{n-1}}, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n} \right\}$$

that means

$$\frac{d^n y}{dt^n} = -L_o\left(y, \frac{dy}{dt}, \dots, \frac{d^{n-1}y}{dt^{n-1}}, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n}\right)$$

Defining the following change of variable $\eta_i = \frac{d^i y}{dt^i}$; for $1 \leq i \leq n$, then one can write a local state space representation which has the special form of a Generalized Observability Canonical Form (GOCF):

$$(\Sigma_{GOCF}) : \begin{cases} \frac{d\eta_i}{dt} = \eta_{i+1}, & 1 \leq i \leq n-1 \\ \frac{d\eta_n}{dt} = -L_o(\eta, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n}) \\ y = \eta_1 \end{cases} \quad (2)$$

for ν a strictly positive integer.

3.- A DIFFERENTIAL ALGEBRAIC APPROACH TO ASYMPTOTIC STABILIZATION AND OUTPUT TRACKING.

Consider the following Nonlinear System

$$(\Sigma_{NL}) : \begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases} \quad (3)$$

where $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}$, f and h are assumed to be polynomial in their arguments. Systems (Σ_{NL}) are assumed to be universally observable (see [2, 3]) with external behavior described by equations of the form.

$$\frac{d^n y}{dt^n} = -L_o\left(y, \frac{dy}{dt}, \dots, \frac{d^{n-1}y}{dt^{n-1}}, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n}\right)$$

where L_o is a polynomial of its arguments.

By defining locally $\eta_i = \frac{d^{i-1}y}{dt^{i-1}}$, $1 \leq i \leq n$, we obtain an explicit GOCF of systems (Σ_{NL}) as follows $\dot{\eta}_i = \eta_{i+1}$, $1 \leq i \leq n$; $\eta_n = -L_o(\eta, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n})$ for some $\gamma > 0$.

Now, let $y_R(t)$ be a prescribed reference output function which is differentiable at least n times. The asymptotic output tracking problem consists in finding a dynamical controller described by a time-varying scalar ordinary differential equation, which is possible implicit, and which has as input:

- a) The output reference signal $y_R(t)$, together with a finit number of its time derivatives

$$\frac{d^i y_R}{dt^i}; 1 \leq i \leq n$$

- and b) The state coordinates η_i of the system.

The controller is supposed to produce a scalar function u , which locally forces y to asymptotically converge towards $y_R(t)$. Define an output tracking error function $e(t)$ as

$$e(t) = y(t) - y_R(t) \quad (4)$$

By definition, η_i is equal to the $(i-1)$ -th time derivative of $y(t)$, that is $\eta_i = \frac{d^{i-1}y}{dt^{i-1}}$; for $1 \leq i \leq n$. Then, we have

$$\frac{d^i e(t)}{dt^i} = \eta_{i+1} - \frac{d^i y_R(t)}{dt^i}; 1 \leq i \leq n-1 \quad (5)$$

$$\frac{d^n e(t)}{dt^n} = \frac{d\eta_n}{dt} - \frac{d^n y_R(t)}{dt^n} \quad (6)$$

Let $p(s) = s^n + \sum_{i=0}^{n-1} a_i s^i$ be Hurwitz polynomial. By requiring a linear time-invariant autonomous dynamics for the tracking error function:

$$\frac{d^n e(t)}{dt^n} + \sum_{i=0}^{n-1} a_i \frac{d^i e(t)}{dt^i} = 0 \quad (7)$$

it follows from (5) and (6), that (7) may be rewritten as

$$\frac{d\eta_n}{dt} - \frac{d^n y_R(t)}{dt^n} + \sum_{i=1}^n a_{i-1} \left(\eta_i - \frac{d^{i-1} y_R(t)}{dt^{i-1}} \right) = 0 \quad (8)$$

that is

$$-L_o \left(\eta, u, \dots, \frac{d^\nu u}{dt^\nu} \right) = \frac{d^n y_R(t)}{dt^n} - \sum_{i=1}^n a_{i-1} \left(\eta_i - \frac{d^i y_R(t)}{dt^i} \right) \quad (9)$$

Remark 1. The scalar time-varying differential equation (9) implicitly defines u , which accomplishes asymptotic stabilization to zero for the tracking error, in a manner entirely prescribed by the set of constant design coefficients $\{a_0, a_1, \dots, a_{n-1}\}$

Let $e_i = \frac{d^{i-1} e(t)}{dt^{i-1}}$, for $1 \leq i \leq n$, be the components of an error vector $e = \text{Col}(e_1, e_2, \dots, e_n)$. We obtain

$$\frac{de}{dt} = Fe \quad (10)$$

where

$$-L_o \left(\psi_R(t) + e, u, \dots, \frac{d^\nu u}{dt^\nu} \right) - \frac{d^n y_R(t)}{dt^n} = - \sum_{i=1}^n a_{i-1} e_i \quad (11)$$

with reference signals vector given by

$$\psi_R(t) = \text{Col} \left(y_R, \frac{dy_R}{dt}, \dots, \frac{d^{n-1} y_R}{dt^{n-1}} \right);$$

and

$$F = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{pmatrix}$$

The origin, $e = 0$, is an equilibrium point for the tracking error dynamics (10). We assume that the solution u of (11) is defined for all time, and is bounded for all bounded functions $y_R(t)$ which also exhibit bounded derivatives. Note that the dynamical feedback controller depends on the state vector of the tracking error dynamics, which should be estimated by means of an observer.

Now, writing the system (10) as follows:

$$\frac{de}{dt} = Ee + \varphi \left(e, y_R, \frac{dy_R}{dt}, \dots, \frac{d^n y_R}{dt^n}, u, \frac{du}{dt}, \dots, \frac{d^\nu u}{dt^\nu} \right)$$

where the elements of the matrix E are given by

$$E_{ij} = \delta_{i,j-1} = \begin{cases} 1 & \text{if } i = j - 1 \\ 0 & \text{everywhere} \end{cases}$$

and

$$\varphi \left(e, y_R, \dots, \frac{d^n y_R}{dt^n}, u, \dots, \frac{d^\nu u}{dt^\nu} \right) \\ \left(0 \dots 0 -L_o \left(\psi_R(t) + e, u, \dots, \frac{d^\nu u}{dt^\nu} \right) - \frac{d^n y_R(t)}{dt^n} \right)^T$$

Then, the estimation of the tracking error $e(t) = y(t) - y_R(t)$ is given by an exponential nonlinear observer (O) of the form:

$$(O) : \begin{cases} \frac{d\hat{e}}{dt} = E\hat{e} - \Delta_\theta K [C\hat{e}(t) - e_1(t)] \\ \quad + \varphi \left(\hat{e}(t), y_R, \dots, \frac{d^{n-1} y_R}{dt^{n-1}}, u, \dots, \frac{d^\nu u}{dt^\nu} \right) \end{cases}$$

where $\Delta_\theta = \text{diag}(\theta, \theta^2, \dots, \theta^n)$ for some $\theta > 0$ and $K = (K_1, K_2, \dots, K_n)^T$, K is chosen such that $\sigma(A - KC) \subset \mathbb{R}^-$ (See [6, 7] for more details).

Let

$$\sigma(u_\epsilon, y_R(t), \hat{e}) = -\frac{d^n y_R(t)}{dt^n} + \sum_{i=1}^n a_{i-1} \hat{e}_i \\ -L_o \left(\psi_R(t) + \hat{e}, u, \dots, \frac{d^\nu u}{dt^\nu} \right)$$

and u_ϵ be the observer-based control resulting from $\sigma(u_\epsilon, y_R(t), \hat{e}(t)) = 0$.

The dynamics of $\hat{e}(t)$ and $\epsilon_0(t) = \hat{e}(t) - e(t)$, (the estimate tracking error and the observation error, respectively), are given by:

$$\begin{cases} \frac{d\hat{e}}{dt} = E\hat{e}(t) - \Delta_\theta K [C\hat{e}(t) - e_1(t)] \\ \quad + \varphi \left(\hat{e}(t), y_R, \dots, \frac{d^{n-1} y_R}{dt^{n-1}}, u_\epsilon, \dots, \frac{d^\nu u_\epsilon}{dt^\nu} \right) \\ \frac{d\epsilon_0(t)}{dt} = (E - \Delta_\theta K C) \epsilon_0(t) + \delta\Phi(\epsilon_0(t), \hat{e}(t)) \end{cases}$$

which becomes

$$(\Sigma_{TOTAL}) : \begin{cases} \frac{d\hat{e}(t)}{dt} = F\hat{e} - \theta^{-1} \Delta_\theta K \tilde{\epsilon}_0 \\ \frac{d\epsilon_0(t)}{dt} = \theta(E - KC) \tilde{\epsilon}_0(t) \\ \quad + \Delta_\theta^{-1} \delta\Phi(\Delta_\theta \tilde{\epsilon}_0(t), \hat{e}(t)) \end{cases}$$

where

$$\tilde{\epsilon}_0 = \Delta_\theta^{-1} \epsilon_0, \quad \Delta_\theta^{-1} E \Delta_\theta = \theta E, \quad C \Delta_\theta = \theta C, \\ \Delta_\theta^{-1} = \text{diag}(\theta^{-1}, \theta^{-2}, \dots, \theta^{-n})$$

and

$$\delta\Phi(\epsilon_0, \hat{e}) = \varphi \left(\hat{e}(t), y_R, \dots, \frac{d^n y_R}{dt^n}, u_\epsilon, \dots, \frac{d^\nu u_\epsilon}{dt^\nu} \right) \\ - \varphi \left(e(t), y_R, \dots, \frac{d^n y_R}{dt^n}, u_\epsilon, \dots, \frac{d^\nu u_\epsilon}{dt^\nu} \right)$$

ASSUMPTIONS:

A1.- $\delta\Phi(\Delta_0 \tilde{\epsilon}_0(t), \dot{\epsilon}(t))$ is globally Lipschitzian in R^n with respect to $\Delta_0 \tilde{\epsilon}_0(t)$ and uniformly with respect to $\dot{\epsilon}(t)$.

A2.- The signals u_ϵ and its derivatives up to n at least are bounded.

THEOREM 1: Consider u_ϵ the linearizing dynamic state feedback which is solution of $\sigma(\dot{\epsilon}(t), u_\epsilon, y_R(t)) = 0$. Suppose that assumptions A1 and A2 are satisfied. Then the closed loop system (Σ_{TOTAL}) with control u_ϵ is globally asymptotically stable.

4.- OBSERVER-BASED CONTROLLER OF FLEXIBLE JOINT MANIPULATORS.

In this section, we apply the results obtained in the above sections to deal with the estimation and tracking problems of a single-link flexible joint manipulator [8].

MATHEMATICAL MODEL

Based on Lagrangian dynamics considerations, the dynamical system describing the behavior of a single-link flexible joint robot is obtained.

The dynamical system (Σ_M) describing the motion of the robot is given by

$$(\Sigma_M) : \begin{cases} \tau = D_m \ddot{q}_m + B_m \dot{q}_m + K_s(q_m - q) \\ 0 = D \ddot{q} + B \dot{q} + mgl \sin(q) - K_s(q_m - q) \end{cases} \quad (12)$$

where q denote the angular position of the link of length $\frac{l}{2}$ and mass m and let q_m be the angular position of the motor, D denotes the inertia of the link, D_m denotes the motor inertia, K_s is the flexible joint stiffness coefficient, B is the motor viscous damping, and B_m denotes the link viscous damping, and g is the gravitational acceleration; and τ is the vector of actuators torques.

Let us define $\bar{\beta}^2 = \frac{1}{K_s}$, which is not to be considered as a small constant related to the singular perturbation techniques. Defining the following change of coordinates: $x_1 = q_m$, the angular position of motor; $x_2 = \dot{q}_m$, the angular velocity of motor; $x_3 = K_s(q - q_m)$, the elastic force; $x_4 := \frac{(q - q_m)}{\bar{\beta}}$, the elastic velocity variation. Then, the dynamical model (Σ_M) represented in the new coordinates results as follows

$$(\Sigma_M) : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -k_5 x_2 + k_1 x_3 + k_1 u \\ \dot{x}_3 = \frac{x_4}{\bar{\beta}} \\ \dot{x}_4 = \{-k_2 k_3 \sin(\bar{\beta}^2 k_3 + x_1) - k_4 x_3 - k_7 x_2 \\ \quad - k_6 \bar{\beta} x_4 - k_1 u\}/\bar{\beta} \end{cases} \quad (13)$$

where $k_1 = \frac{1}{D_m}$; $k_2 = \frac{1}{D}$; $k_3 = mgl$; $k_4 = k_1 + k_2$; $k_5 = \frac{B_m}{D_m}$; $k_6 = \frac{B}{D}$; $k_7 = k_6 - k_5$; $u = \tau$

Remark 2: The system (Σ_M) has different relative degree, when the system output y is considered as the motor position x_1 instead of the link position $z = x_1 + \bar{\beta} x_3$.

On the other hand, an operation point of the system can be achieved when a torque input $u = u^0$ constant is applied, that is

$$\begin{aligned} u = u^0; x_1(u^0) &= \bar{\beta}^2 U + \sin^{-1}\left(\frac{u^0}{k_3}\right); x_2 = 0; \\ x_3(u^0) &= -u^0; x_4(u^0) = 0; z(u^0) = \sin^{-1}\left(\frac{u^0}{k_3}\right). \end{aligned} \quad (14)$$

When we consider the linearized system in the operating point, the zero dynamic follows to be minimum phase. However, this zero dynamic's property is a function of the parameter $\bar{\beta}$. The fact that $\bar{\beta} \neq 0$ is crucial for the flexible robot to be a minimum phase. If $\bar{\beta} = 0$, this technique can not to be applied, and the zero dynamic becomes oscillatory. In practice, this condition is verified for most mechanical manipulators. The following coordinate transformation depending on the input which allows us to obtain a generalized observability canonical form (GOCF). For that, we have:

$$\begin{aligned} y &= \eta_1 = x_1 \\ \dot{y} &= \eta_2 = x_2 \\ \ddot{y} &= \eta_3 = -k_5 x_2 + k_1 x_3 + k_1 u \\ \ddot{y} &= \eta_4 = (k_5)^2 x_2 - k_1 [k_5 x_3 + \bar{\beta}^{-1} x_4 + k_5 u + \frac{du}{dt}] \end{aligned} \quad (15)$$

$\{y, \dot{y}, \ddot{y}, \ddot{y}\}$ is a transcendence basis of $\mathcal{R}\langle u, y \rangle / \mathcal{R}\langle u \rangle$ which represents the nonlinear dynamics given in (13). The transcendence degree of $\mathcal{R}\langle u, y \rangle / \mathcal{R}\langle u \rangle$ is given by $d^0 tr(\mathcal{R}\langle u, y \rangle / \mathcal{R}\langle u \rangle) = 4$ and its corresponding inverse transformation.

$$\begin{aligned} x_1 &= \eta_1 \\ x_2 &= \eta_2 \\ x_3 &= (\eta_3 + k_5 \eta_2 - k_1 u)/k_1 \\ x_4 &= \bar{\beta}[\eta_4 - (k_5)^2 \eta_2 + k_1 k_5 \eta_3 - k_1 k_5 u - k_1 \frac{du}{dt}]/k_1 \end{aligned} \quad (16)$$

The Jacobian matrix of the state coordinate transformation (15) is given by

$$J = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -k_5 & k_1 & 0 \\ 0 & (k_5)^2 & -k_1 k_5 & k_1 \bar{\beta} \end{pmatrix}$$

which is clearly no singular if $\bar{\beta}$ is different to zero. Then the (GOCF) of (13) is given by

$$\begin{cases} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = \eta_3 \\ \dot{\eta}_3 = \eta_4 \\ \dot{\eta}_4 = -k_1 k_2 k_3 \bar{\beta}^{-2} \sin(\bar{\beta}^2 [k_5 \eta_2 - k_1 u + \eta_3]/k_1 + \eta_1) - \bar{\beta}^{-2} \{k_1 k_7 + k_4 k_5\} \eta_2 \\ \quad - \{\frac{k_4}{\bar{\beta}} + k_5 k_6\} \eta_3 - \{k_5 + k_6\} \eta_4 \\ \quad - \frac{k_4}{\bar{\beta}} \{k_4 - k_1\} u + k_1 k_6 \frac{du}{dt} + k_1 \frac{d^2 u}{dt^2} \end{cases} \quad (17)$$

And finally, the external behavior of the system is given by

$$\begin{aligned} & \frac{d^4 y}{dt^4} + \{k_5 + k_6\} \frac{d^3 y}{dt^3} + \left\{ \frac{k_4}{\beta} + k_5 k_6 \right\} \frac{d^2 y}{dt^2} \\ & + \bar{\beta}^{-2} \{k_1 k_7 + k_4 k_5\} \frac{dy}{dt} \\ & + k_1 k_2 k_3 \bar{\beta}^{-2} \sin(\bar{\beta}^2) [k_5 \frac{dy}{dt} + \frac{d^2 y}{dt^2} - k_1 u] / k_1 + y \\ & - \frac{k_1}{\beta} \{k_4 - k_1\} u - k_1 k_6 \frac{du}{dt} - k_1 \frac{d^2 u}{dt^2} = 0 \end{aligned} \quad (18)$$

Now, let $y_R(t)$ be a desired output reference trajectory of the angular position. Deriving the desired output reference $y_R(t)$ up to fourth time-derivative, we can apply the dynamical controller

$$-L_o \left(\eta, u, \dots, \frac{d^n u}{dt^n} \right) = \frac{d^n y_R(t)}{dt^n} - \sum_{i=1}^n a_{i-1} \left(\eta_i - \frac{d^{i-1} y_R(t)}{dt^{i-1}} \right)$$

with $n = 4$ and $\gamma = 2$.

Defining the tracking error as $e(t) = x_1 - y_R(t)$, one obtains the system of differential equations describing the tracking error dynamics as

$$\frac{de}{dt} = Ee + \varphi \left(e_1, \dots, e_4, y_R, \dots, \frac{d^4 y_R}{dt^4}, u, \frac{du}{dt}, \frac{d^2 u}{dt^2} \right) \quad (19)$$

with output $y_T = e_1$. Since system (19) is observable we propose the following nonlinear exponential observer for the estimation of the tracking error [6]:

$$\frac{d\hat{e}}{dt} = E\hat{e} - \Delta_e K [C\hat{e}(t) - e_1(t)] \quad (20)$$

$$+ \varphi \left(\hat{e}(t), y_R, \dots, \frac{d^4 y_R}{dt^4}, u_e, \frac{du_e}{dt}, \frac{d^2 u_e}{dt^2} \right) \quad (21)$$

Exact linearization of the tracking error dynamics can be now accomplished by equating the last differential of the system (19) to a linear time invariant expression in the error coordinates:

$$\begin{aligned} & -L_o \left(\psi_R(t) + e, u, \frac{du}{dt}, \frac{d^2 u}{dt^2} \right) = \frac{d^4 y_R(t)}{dt^4} - \sum_{i=1}^4 a_{i-1} e_i \\ & = -k_1 k_2 k_3 \bar{\beta}^{-2} \sin(\bar{\beta}^2) / k_1 [k_5 (e_2 + \frac{dy_R}{dt}) + \frac{d^2 y_R}{dt^2} \\ & - k_1 u + e_3] + y_R(t) \\ & - \bar{\beta}^{-2} (k_1 k_7 + k_4 k_5) (k_4 - k_1) u + k_1 k_6 \frac{du}{dt} \\ & + k_1 \frac{d^2 u}{dt^2} - (k_5 + k_6) \frac{d^2 y_R}{dt^2} - \left(\frac{k_4}{\beta} + k_5 \right) \frac{d^2 y_R}{dt^2} \\ & - \beta^{-2} (k_1 k_7 + k_4 k_5) \frac{dy}{dt} - \sum_{i=1}^4 a_{i-1} e_i \end{aligned} \quad (22)$$

Then, writing in the original coordinates the equation

of the dynamical controller, it follows that

$$\begin{aligned} & \frac{d^4 u}{dt^4} + (a_3 - k_5) \frac{du}{dt} + \left(k_5^2 - k_5 a_3 - k_1 \bar{\beta}^{-2} \right) u = \\ & - \frac{a_2}{k_1} x_1 - \left(\frac{k_7}{\beta} - \frac{a_1}{k_1} + \frac{k_4 a_2}{k_1} + \frac{k_5^2}{k_1} - \frac{k_5 a_3}{k_1} \right) x_2 \\ & + \left(\frac{k_5}{\beta} + \frac{k_6}{\beta} - \frac{a_2}{\beta} \right) x_4 + \frac{k_2 k_3}{\beta} \sin(x_1 + \bar{\beta}^2 x_3) \\ & + \left(\frac{d^4 y_R}{dt^4} + a_3 \frac{d^2 y_R}{dt^2} + a_2 \frac{d^2 y_R}{dt^2} + a_1 \frac{dy_R}{dt} + a_0 y_R \right) (k_1)^{-1} \end{aligned} \quad (23)$$

and the desired dynamic performance can be obtained by choosing suitable a_1, a_2, a_3, a_4 .

However, this controller depends on all state which is unmeasurable. To overcome this difficulty we will replace the state estimated by the observer. To achieve this goal we take into account that the inverse transformation is given by (ec.16), and written in terms of the tracking error, we obtain

$$\begin{aligned} x_1 &= e_1 + y_R \\ x_2 &= e_2 + \frac{dy_R}{dt} \\ x_3 &= \{e_3 + \frac{d^2 y_R}{dt^2} + k_5 (e_2 + \frac{dy_R}{dt}) - k_1 u\} / k_1 \\ x_4 &= \{e_4 + \frac{d^3 y_R}{dt^3} + k_5 (e_3 + \frac{d^2 y_R}{dt^2}) - k_1 \frac{du}{dt}\} / k_1 \bar{\beta} \end{aligned}$$

Now, replacing the states estimated given by the observer (20, 21) we get

$$\begin{aligned} \hat{x}_1 &= \hat{e}_1 + y_R \\ \hat{x}_2 &= \hat{e}_2 + \frac{dy_R}{dt} \\ \hat{x}_3 &= \{\hat{e}_3 + \frac{d^2 y_R}{dt^2} + k_5 (\hat{e}_2 + \frac{dy_R}{dt}) - k_1 u\} / k_1 \\ \hat{x}_4 &= \{\hat{e}_4 + \frac{d^3 y_R}{dt^3} + k_5 (\hat{e}_3 + \frac{d^2 y_R}{dt^2}) - k_1 \frac{du}{dt}\} / k_1 \bar{\beta} \end{aligned}$$

Then, the dynamical controller as a function of the estimates is given by

$$\begin{aligned} \frac{d^4 u_e}{dt^4} + k_6 \frac{du_e}{dt} - \frac{1}{\beta} \{k_4 - k_1\} u_e &= -k_2 k_3^{-2} \bar{\beta} \sin(\bar{\beta}^2 \hat{x}_3 + \hat{x}_1) \\ & - \bar{\beta}^{-2} \frac{1}{k_1} \{k_1 k_7 + k_4 k_5\} \hat{x}_2 - \frac{1}{k_1} \left\{ \frac{k_4}{\beta} + k_5 k_6 \right\} \hat{x}_3 \\ & - \{k_5 + k_6\} \hat{x}_4 - \sum_{i=1}^4 \frac{a_{i-1}}{k_1} \{\hat{x}_i + \frac{d^{i-1} y_R}{dt^{i-1}}\} + \frac{1}{k_1} \frac{d^4 y_R}{dt^4} \end{aligned}$$

Remark 3: The above dynamical controller does not exhibit any singularities if $\bar{\beta} \neq 0$. However, by considering a more general systems, the synthesis of a dynamical controller could be introduce some difficulties which are related to impasse points, nonminimum phase regions and others(further details can be found in [8]).

5.- SIMULATION RESULTS

Now, we will show how the observer-based dynamical controller scheme is implemented to a model of a flexible joint robot. The simulation were performed with the following parameters

$$\begin{aligned} k_1 &= 3.33 (m^2 Kg)^{-1}; k_2 = 1.0 (m^2 Kg)^{-1}; k_3 = 5.0 \\ Nm; k_4 &= 4.33 (m^2 Kg)^{-1} \\ k_5 &= 0.333 s^{-1}; k_6 = 0.1 s^{-1}; k_7 = -0.233 s^{-1}; K_s = 100 N/(m rad^{-1}). \end{aligned}$$

All initial conditions, for the system and the dynamical controller were chosen to be zero. The desired reference trajectory $y_R(t)$ for the motor position was set $y_R(t) = 0.5 \sin t$. The first task for the proposed observer-based controller was to track a desired reference. Figure 1 shows the state x_1 and the desired reference trajectories. We can see, in figure 1, that the trajectory x_1 converges towards the desired reference trajectory $y_R(t)$. In figure 2 shows the state x_2 and the desired reference $y'_R(t)$. Finally, the figure 3 depicts the obtained controller applied to the system using this technique observed-based controller.

6.- CONCLUDING REMARKS

An observer-based controller has been proposed for a class of nonlinear systems. In particular, sufficient conditions have been given to guarantee the stability of the closed-loop system including the observer. A connection between the gain of the observer and the gain of the controller in order have the origin globally asymptotically stable was obtained.

Concerning the multi-variable case, this technique can be extended to a class of multi-variable systems with a suitable canonical form.

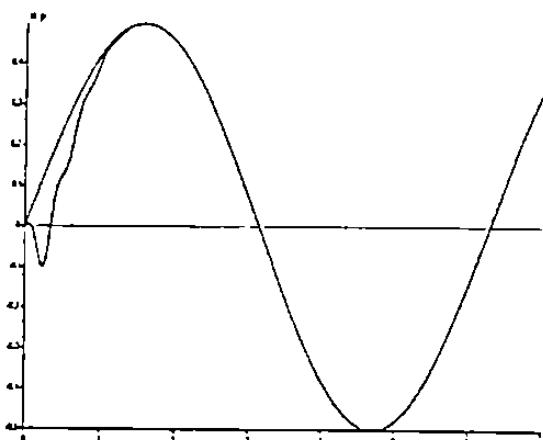


Figure 1

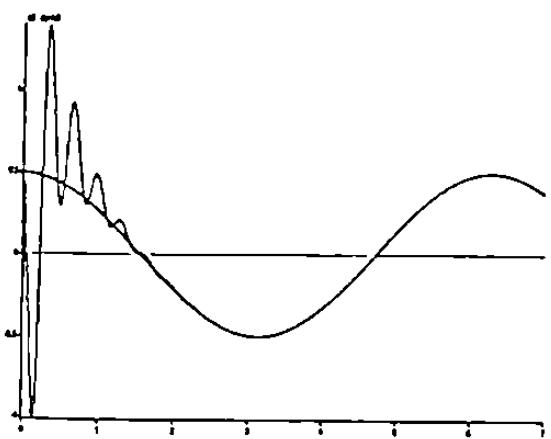


Figure 2

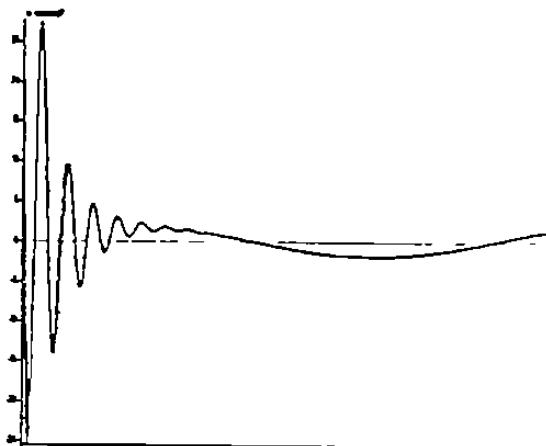


Figure 3

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Speed and Position Control of a Flexible Joint Robot Manipulator via a Nonlinear Control-Observer Scheme¹

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Abstract

A nonlinear control-observer structure for a class of nonlinear singularly perturbed systems, based on a two-time scale sliding-mode technique and a high gain estimator, is presented. The structure is applied to the model of a single-link flexible joint robot manipulator.

in [4], is designed for that class of systems and also applied to the robot manipulator model.

2. Model of the Manipulator.

1. Introduction.

During the last few years, considerable research efforts have been directed toward the control problem of flexible joint robots (see, for example, [3] and the references therein). On the other hand, the observer problem for flexible joint manipulators is an important one in robot control theory and of great practical importance. In fact, many control techniques for these robots require the knowledge of four variables for each joint, which may be either positions and velocities of the motors and of the links or positions, velocities, accelerations and jerks of the links. Some interesting results on the observer problem for these robots are reported in [8].

In this paper, a control law based on the sliding mode technique for a class of nonlinear singularly perturbed systems, recently reported in [1] and [5], is applied to the model of a single-link flexible joint robot manipulator. In addition, a nonlinear high gain observer, based on the one proposed

In this work, a single-link flexible joint robot manipulator directly actuated by a direct current electrical motor (hub motor) and whose rotor is elastically coupled to the link is considered (see Fig. 1). The mathematical model for the manipulator is given by the following equations (see [7] for a detailed derivation of the model)

$$\begin{aligned} I\ddot{q}_1 + mgl\sin(q_1) + k(q_1 - q_2) &= 0 \\ J\ddot{q}_2 + B\dot{q}_2 - k(q_1 - q_2) &= u \end{aligned} \quad (1)$$

where q_1 and q_2 are the angular positions of the link and the hub motor, respectively, while u is the input force from the actuator (motor torque). I is the inertia of the arm, J denotes the motor inertia, B is the motor viscous friction, mgl is the nominal load in the arm and k is the flexible joint stiffness coefficient. The model considered does not take into account the inertia of the actuator about the three independent axes. However, it has been shown that it adequately represents the manipulator dynamics and it is suitable for control design [7].

¹Work supported in part by CONACYT, Mexico, under Grant # 4417-A9406.

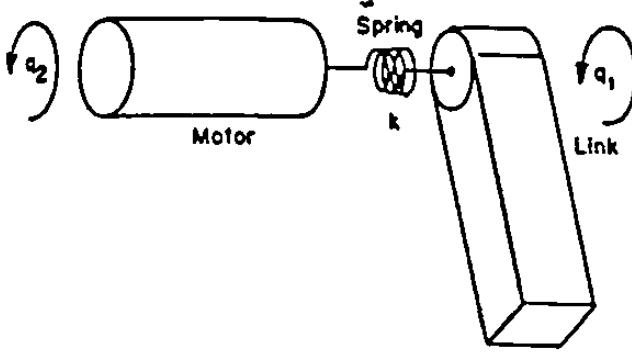


Figure 1: Single-link flexible joint robot manipulator.

3. Two-Time Scale Sliding Mode Control.

Let us consider the following class of nonlinear singularly perturbed systems

$$\dot{x} = f_1(x) + F_1(x)z + g_1(x)u, \quad x(t_0) = x_0, \quad (2)$$

$$\epsilon \dot{z} = f_2(x) + F_2(x)z + g_2(x)u, \quad z(t_0) = z_0 \quad (3)$$

where $t_0 \geq 0$, $x \in \mathbb{R}^n$ is the slow state, $z \in \mathbb{R}^m$ is the fast state, $u \in \mathbb{R}^r$ is the control input, ϵ is a small positive parameter such that $\epsilon \in [0, 1]$, and f_1, f_2 , the columns of the matrices F_1, F_2, g_1 and g_2 are assumed to be bounded analytical real vector fields. $F_2(x)$ is assumed to be nonsingular for all $x \in \mathbb{R}^n$. In addition, it is supposed that $f_1(0) = f_2(0) = 0$ and, for $u = 0$, the origin is an isolated equilibrium state.

3.1. The Singular Perturbation Methodology.

The singular perturbation methodology permits to decompose the original system in two subsystems of lower dimension, both described in different time scales. The slow reduced system is found by making $\epsilon = 0$ in (2)-(3), resulting in the following reduced $n - r$ order slow system

$$\dot{x}_s = f(x_s) + g(x_s)u_s, \quad x_s(t_0) = x_0, \quad (4)$$

$$z_s = h(x_s) := -F_2^{-1}(x_s)[f_2(x_s) + g_2(x_s)u_s] \quad (5)$$

where $x_s \in \mathbb{R}^n$, $z_s \in \mathbb{R}^m$ and $u_s \in \mathbb{R}^r$ denote the slow components of the original variables x , z and u , respectively, and

$$f(x_s) := f_1(x_s) - F_1(x_s)F_2^{-1}(x_s)f_2(x_s), \quad (6)$$

$$g(x_s) := g_1(x_s) - F_1(x_s)F_2^{-1}(x_s)g_2(x_s). \quad (7)$$

Then the slow manifold can be defined as $M_\epsilon := \{z \in \mathbb{R}^m : z = h(x)\}$, and the so-called manifold condition

$$\frac{\partial h}{\partial z}[f_1(x) + F_1(x)z + g_1(x)u] = f_2(x) + F_2(x)z + g_2(x)u$$

must be satisfied for M_ϵ to be an invariant manifold [6].

The fast dynamics (also named boundary layer system) is obtained by transforming the (slow) time scale t to the

(fast) time scale $\tau := (t - t_0)/\epsilon$ and making the change of variables $\eta := z - h(x, \epsilon)$. The original system (2)-(3) then becomes

$$\begin{aligned} \frac{d\tilde{x}}{d\tau} &= \epsilon[f_1(\tilde{x}) + F_1(\tilde{x})[\eta + h(\tilde{x})] + g_1(\tilde{x})u], \\ \frac{d\eta}{d\tau} &= f_2(\tilde{x}) + F_2(\tilde{x})[\eta + h(\tilde{x})] + g_2(\tilde{x})u - \frac{\partial h(\tilde{x})}{\partial \tilde{x}} \frac{d\tilde{x}}{d\tau} \end{aligned} \quad (8)$$

where $\tilde{x}(\tau) := z(\epsilon\tau + t_0)$, with $\tilde{x}(0) = z_0$, and $\tilde{x}(\tau) := x(\epsilon\tau + t_0)$, with $\tilde{x}(0) = x_0$. $u = u_s + u_f$ is the composite control for the full order system where u_s and u_f denotes the slow and fast controls, respectively. If ϵ is small enough, \tilde{x} remains relatively constant with respect to the fast time τ . Besides, if u_s and $\partial h(\tilde{x})/\partial \tilde{x}$ are bounded, then the term $[\partial h(\tilde{x})/\partial \tilde{x}] \cdot [d\tilde{x}/d\tau]$ can be neglected. Since the second equation in (8) defines the fast reduced subsystem, an $\mathcal{O}(\epsilon)$ approximation for this subsystem can be obtained from equation (5) and setting $\epsilon = 0$ in (8), namely $d\hat{\eta}/d\tau = F_2(\tilde{x})\hat{\eta} + g_2(\tilde{x})u_f$, with $\hat{\eta}(0) = z_0 - h(x_0)$, being $\hat{\eta}$ an $\mathcal{O}(\epsilon)$ approximation of η in the initial boundary layer.

3.2. Sliding-Mode Control Design.

The sliding-mode control for the complete system is designed in two stages. First, the slow control $u_s(x_s) \in \mathbb{R}^r$ is designed. To do this, let us consider a slow nonlinear switching surface defined by $\sigma_s(x_s) = \text{col}(\sigma_{s_1}(x_s), \dots, \sigma_{s_r}(x_s)) = 0$ where all the slow switching surfaces $\sigma_{s_i}(x_s)$, $i = 1, \dots, r$, are assumed to be locally Lipschitz-continuous functions for all x_s in \mathbb{R}^n . The equivalent control method [9] is used to determine the slow reduced system motion restricted to the slow switching surface $\sigma_s(x_s) = 0$, obtaining the so-called slow equivalent control

$$u_{s_e} = -([\partial \sigma_s / \partial x_s]g(x_s))^{-1}[\partial \sigma_s / \partial x_s]f(x_s) \quad (9)$$

where, of course, the matrix $[\partial \sigma_s / \partial x_s] \cdot g(x_s)$ is assumed to be nonsingular for all $x_s \in \mathbb{R}^n$. Substitution of the slow equivalent control in (4) yields the following slow sliding-mode equation

$$\dot{x}_s = \left[I_n - g(x_s) \left[\frac{\partial \sigma_s}{\partial x_s} g(x_s) \right]^{-1} \frac{\partial \sigma_s}{\partial x_s} \right] f(x_s) =: f_e(x_s) \quad (10)$$

where I_n denotes the $n \times n$ identity matrix. If one wants to guarantee the local stability of system (10), the nonlinear function $\sigma_s(x_s)$ should be chosen such that

$$\text{Re} \lambda \left\{ \frac{\partial f_e}{\partial x_s} \right\} \leq -c_s < 0, \quad \forall x_s \in \mathbb{R}^n \quad (11)$$

is satisfied, where $\lambda(\cdot)$ denotes the $(n - r)$ nonzero eigenvalues of the linear approximation of (10). Obviously, in order to satisfy the above condition, the stabilization of the linear approximation of the slow reduced system is necessary. The following control law [9] can be used to complete the design of the slow control

$$u_s = u_{s_e} + u_{s_N} \quad (12)$$

where u_{s_N} is the slow equivalent control given in (9), which acts when the slow reduced system is restricted to $\sigma_s(x_s) = 0$, while u_{s_N} is the discontinuous part of the slow control u_s acting when $\sigma_s(x_s) \neq 0$. In this work the control u_{s_N} is selected as a nonlinear feedback law of the type

$$u_{s_N}(x_s) = -([\partial\sigma_s/\partial x_s] \cdot g(x_s))^{-1} L_s(x_s) \sigma_s(x_s) \quad (13)$$

where $L_s(x_s)$ is a positive-definite matrix of dimension $r \times r$, whose elements are bounded nonlinear real functions of x_s . Hence, the following equation is obtained

$$\dot{\sigma}_s(x_s) = [\partial\sigma_s/\partial x_s] \dot{x}_s = -L_s(x_s) \sigma_s(x_s). \quad (14)$$

The stability of the slow reduced system in the sliding surface is assured if condition (11) is fulfilled.

The fast sliding-mode control can be obtained in a similar way than the one used for the slow sliding-mode control. This control law is given by

$$u_f = -\left[\frac{\partial\sigma_f}{\partial\hat{\eta}} g_2(\tilde{x})\right]^{-1} \left[\frac{\partial\sigma_f}{\partial\hat{\eta}} F_2(\tilde{x})\hat{\eta} + L_f(\hat{\eta})\sigma_f(\hat{\eta})\right] \quad (15)$$

where $\sigma_f(\hat{\eta}) = 0$ is the switching surface and $L_f(\hat{\eta})$ is a positive matrix of dimension $r \times r$. The fast sliding mode dynamics is given by

$$\frac{\partial\sigma_f}{\partial t} = -L_f(\hat{\eta})\sigma_f(\hat{\eta}), \quad (16)$$

and the same arguments used for the slow subsystem can be applied to the boundary layer system to carry out its stability analysis [1].

The composite control for the original nonlinear singularly perturbed system (2)-(3) can then be expressed by [6]

$$u = u_s + u_f. \quad (17)$$

The asymptotic stability properties of the closed-loop system nonlinear singularly perturbed system are studied in [1] and [5].

4. A High Gain Nonlinear Observer.

When all the inputs applied to a nonlinear system do not affect its observability, these are called *universal inputs* and the state estimators or observers designed are referred as *uniform observers*. In the literature, many observers have been proposed for nonlinear systems (see, for example, [2] and the references therein). In this paper, a uniform observer which allows to estimate the states of a nonlinear system is developed. Let us suppose that the singularly perturbed system (2)-(3) can be written, after

a (possible) state coordinate transformation $\zeta = \Phi(x, z)$, in the form [4]

$$\frac{d\zeta}{dt} = F(y)\zeta + G(u, \zeta) \quad (18)$$

together with a single output $y \in \mathbb{R}$

$$y = C\zeta \quad (19)$$

where $\zeta \in \mathbb{R}^{n+m}$, $u \in \mathbb{R}^r$ is the control input, $C = [1, 0, 0, \dots, 0, 0]$ and

$$F(y) = \begin{bmatrix} 0 & f_1(y) & 0 & \cdots & 0 \\ 0 & 0 & f_2(y) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & f_{n+m-1}(y) \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$G(u, \zeta) = \begin{bmatrix} g_1(u, \zeta_1) \\ g_2(u, \zeta_1, \zeta_2) \\ \vdots \\ g_{n+m-1}(u, \zeta_1, \zeta_2, \dots, \zeta_{n+m-1}) \\ g_{n+m}(u, \zeta_1, \zeta_2, \dots, \zeta_{n+m-1}, \zeta_{n+m}) \end{bmatrix}.$$

It is also assumed that the functions f_i , $i = 1, \dots, n+m-1$, are of class C^r , $r \geq 1$, with respect to y and that the functions g_i , $i = 1, \dots, n+m$, are globally Lipschitz with respect to ζ and uniformly with respect to u . In addition, one assumes there is a class of bounded controls $\mathcal{U} \subset \mathbb{R}^r$, a compact set $\mathcal{K} \subset \mathbb{R}^{n+m}$, and constants α and β such that, for each $u \in \mathcal{U}$ and each output y associated u and the initial condition $\zeta(0) \in \mathcal{K}$,

$$\alpha \leq f_i(y) \leq \beta, \quad i = 1, \dots, n+m-1. \quad (20)$$

Let us define the system

$$\frac{d\hat{\zeta}}{dt} = F(y)\hat{\zeta} + G(u, \hat{\zeta}) - \bar{S}_\theta^{-1}(y)C^T[C\hat{\zeta} - y] \quad (21)$$

where $\bar{S}_\theta = \Omega(y)S_\theta\Omega(y)$ with

$$\Omega = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & f_1(y) & 0 & \cdots & 0 \\ 0 & 0 & f_1(y)f_2(y) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \prod_{i=1}^{n+m-1} f_i(y) \end{bmatrix},$$

and S_θ being a symmetric positive definite matrix which is the unique solution of the algebraic equation

$$\theta S_\theta + A^T S_\theta + S_\theta A - C^T C = 0 \quad (22)$$

where the matrix A is in the Brunovsky canonical form. In addition, one also assumes that Ω is bounded.

Let $y(t)$ be the output of system (18) associated to the initial state $\zeta(0) \in \mathcal{K}$ and the input $u \in \mathcal{U}$. From (20), it

follows that $\Omega(y)$ is invertible and, when premultiplying both sides of (22) by Ω , one obtains

$$\theta \bar{S}_\theta + F^T(y) \bar{S}_\theta + \bar{S}_\theta F(y) - C^T C = 0$$

where one can verify that $F(y) = \Omega^{-1}(y) A \Omega(y)$ and $C \Omega(y) = C$. Let $e(t) = \zeta(t) - \hat{\zeta}(t)$ be the estimation error, then

$$\dot{e} = [F(y) - \bar{S}_\theta^{-1}(y) C^T C] e + [G(u, \hat{\zeta}) - G(u, \zeta)].$$

If one defines the diagonal matrix $\Delta_\theta := \text{diag}(1, 1/\theta, \dots, 1/\theta^{n+m-1})$, it is possible to show that $S_\theta = (1/\theta) \Delta_\theta S_1 \Delta_\theta$ where S_1 is the unique solution to equation (22) with $\theta = 1$. If the change of variable $\bar{e} = \Omega(y) \Delta_\theta e$ is now introduced, one has

$$\frac{d\bar{e}}{dt} = \theta [A - S_1^{-1} C^T C] \bar{e} + \Omega(y) \Delta G_\theta(u, \zeta, \hat{\zeta}) + \dot{\Omega}(y) \Omega^{-1}(y) \bar{e}$$

where $\Delta G_\theta(u, \zeta, \hat{\zeta}) = \Delta_\theta [G(u, \hat{\zeta}) - G(u, \zeta)]$, and the Lyapunov candidate function

$$V_\epsilon(\bar{e}) = \bar{e}^T S_1 \bar{e}.$$

can be used to prove the following result via stability Lyapunov methods.

Theorem 4.1 Consider the system (18). Then there exists $\theta_0 > 0$ such that for all $\theta \geq \theta_0$, for all $u \in \mathcal{U}$ and for all $\zeta(0)$ and $\hat{\zeta}(0)$ that belong to \mathcal{K} , the system (21) is an exponential observer for system (18) with arbitrarily decay exponential rate.

The stability of the closed-loop system when a composite control (17) is used and the state is replaced by its estimate (21) can be carried out following the same lines of [5].

5. Application to the Model of the Manipulator.

5.1. Control Law Design.

By means of the assignment $x = \text{col}(q_1, \dot{q}_1), z = \text{col}(k(q_1 - q_2), \epsilon k(\dot{q}_1 - \dot{q}_2))$ where $\epsilon \in [0, 1]$, it is possible to write the model (1) in the standard singularly perturbed form (2)-(3) with

$$f_1(x) = \begin{bmatrix} x_2 \\ -\frac{mgl}{J} \sin(x_1) \end{bmatrix},$$

$$F_1(x) = \begin{bmatrix} 0 & 0 \\ -1/I & 0 \end{bmatrix},$$

$$f_2(x) = \begin{bmatrix} 0 \\ -\frac{\alpha mgl}{J} \sin(x_1) + \frac{\alpha B}{J} x_2 \end{bmatrix},$$

$$F_2(x) = \begin{bmatrix} 0 & 1 \\ -\frac{\alpha(I+J)}{IJ} & -\frac{\epsilon B}{J} \end{bmatrix},$$

$$g_1(x) = 0, \quad g_2(x) = \begin{bmatrix} 0 \\ -\alpha/J \end{bmatrix}$$

where $\alpha = k\epsilon^2$, this is the flexible joint stiffness coefficient is assumed to be a $\mathcal{O}(1/\epsilon^2)$ for the parameter ϵ [7]. The slow reduced system then takes the form (4) with

$$f(x_s) = \begin{bmatrix} x_{s_2} \\ -\frac{mgl}{(I+J)} \sin(x_{s_1}) - \frac{B}{(I+J)} x_{s_2} \end{bmatrix},$$

$$g(x_s) = \begin{bmatrix} 0 \\ \frac{1}{(I+J)} \end{bmatrix}$$

while the $\mathcal{O}(\epsilon)$ approximation of the exact fast subsystem has the form

$$\frac{d\hat{\eta}}{dt} = A_2 \hat{\eta} + B_2 u_f \quad (23)$$

where

$$A_2 = \begin{bmatrix} 0 & 1 \\ -\frac{\alpha(I+J)}{IJ} & -\frac{\epsilon B}{J} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -\alpha/J \end{bmatrix}.$$

The approximation (23) is stable since the eigenvalues of the matrix A_2 have all negative real parts.

Since it is desired that the rotor position tracks a reference signal, the following slow switching function is chosen:

$$\sigma_s(x_s) = s_1(x_{s_1} - x_r) + s_2(x_{s_2} - \dot{x}_r) \quad (24)$$

where s_1 and s_2 are constant real coefficients, and x_r is the reference signal. It is also assumed that $x_r(t)$ and $\dot{x}_r(t)$ are bounded for all $t \geq 0$. The choice (24) leads, in accordance to section 3, to the slow control (12) with

$$u_{se} = mgl \sin(x_{s_1}) + (B - \frac{s_1(I+J)}{s_2}) x_{s_2}, \quad (25)$$

$$u_{sN} = -\frac{s_1(I+J)}{s_2} L_s(x_s)(x_{s_1} - x_r) - (I+J)L_s(x_s)(x_{s_2} - \dot{x}_r) \quad (26)$$

where $L_s(x_s)$ has been selected, for simplicity, as a strictly positive constant, i.e. $L_s(x_s) = \ell_s > 0$. After substitution of the slow control in the slow reduced system one can easily verify that the closed-loop slow sliding-mode equation is stable if the constants s_1, s_2 and ℓ_s are chosen in such a way that all the roots of the polynomial $s^2 + s(s_1/s_2 + \ell_s) + (s_1\ell_s/s_2) = 0$ are in the strictly left-half complex plane. On the other hand, from $\dot{\sigma}_s = -\ell_s \sigma_s$ one has that there exists a slow sliding manifold.

For the fast subsystem, one chooses the fast switching function

$$\sigma_f(\hat{\eta}) = \tilde{s}_1 \hat{\eta}_1 + \tilde{s}_2 \hat{\eta}_2 \quad (27)$$

where \bar{s}_1 and \bar{s}_2 are also constant real coefficients. In accordance to equation (15), one obtains the fast control

$$u_f = \left[\frac{L_f(\bar{\eta})J\bar{s}_1}{\alpha\bar{s}_2} - \left(\frac{J}{I} + 1 \right) \bar{\eta}_1 + \left[\frac{J\bar{s}_1}{\alpha\bar{s}_2} - \frac{\epsilon B}{\alpha} + \frac{L_f(\bar{\eta})J}{\alpha} \right] \bar{\eta}_2 \right] \quad (28)$$

where, again, for simplicity, $L_f(\bar{\eta})$ is chosen as a strictly positive constant, i.e. $L_f(\bar{\eta}) = \ell_f > 0$. In the same manner, and after substitution of this control into (23) one has that the closed-loop fast sliding-mode equation is asymptotically stable whenever the constants \bar{s}_1 , \bar{s}_2 , and ℓ_f are chosen such that the polynomial $s^2 + s(\bar{s}_1/\bar{s}_2 + \ell_f) + (\bar{s}_1\ell_f/\bar{s}_2) = 0$ has all roots strictly in the left-half complex plane. In the same way, from $\partial\sigma_f/\partial\tau = -\ell_f\sigma_f$, there also exists a fast sliding manifold.

5.2. Estimation of the Link and Hub Motor Angular Speeds.

The controller designed in subsection 5.1 requires a full measurement of the link and hub motor angular positions and speeds. However, only angular position measurements are usually available in practice. In order to estimate the angular speeds of the link and the hub motor, a high gain observer is designed in this subsection; other kinds of observers have been designed for this type of manipulators [8]. By making the coordinate transformation $\zeta_1 = x_1$, $\zeta_2 = x_2$, $\zeta_3 = z_1$ and $\zeta_4 = z_2$ one can write the singularly perturbed system associated to the manipulator in the form (18) with $\zeta = \text{col}(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ and

$$F(y) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1/I & 0 \\ 0 & 0 & 0 & 1/\epsilon \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$G(\zeta, u) = \begin{bmatrix} 0 \\ -\frac{mgl}{J} \sin(\zeta_1) \\ 0 \\ G_3(\zeta, u) \end{bmatrix}$$

where

$$G_3(\zeta, u) = -\frac{\alpha mgl}{\epsilon J} \sin(\zeta_1) + \frac{\alpha B}{\epsilon J} \zeta_2 - \frac{\alpha(I+J)}{\epsilon I J} \zeta_3 - \frac{B}{J} \zeta_4 - \frac{\alpha}{\epsilon J} u.$$

A high gain observer for the state variable ζ that has the form (21) is then proposed where S_θ is the unique solution of equation (22).

5.3. Simulation Results.

The single-link flexible joint robot manipulator described by equations (1) was simulated together with the controller and observer designed in subsections 5.1 and 5.2, and using the nominal values $I = 0.031$, $J = 0.004$, $B = 0.007$, $k = 7.13$ and $mgl = 0.8$, together with $\epsilon = 0.0001$ (i.e. $\alpha = 7.13 \times 10^{-8}$) [7]. The control law

gains were chosen as $s_1 = 20$, $s_2 = 25$, $\ell_s = 60$, $\bar{s}_1 = 1$, $\bar{s}_2 = 10$ and $\ell_f = 0.1$. Such a selection guarantees the stability of the slow and fast sliding-mode equations and the existence of the corresponding sliding modes. The parameter θ in the observer equations was chosen as $\theta = 20$. From this choice, a solution to equation (20) was found. The initial conditions of the manipulator variables and the estimates were fixed to $q_1(0) = 0$, $\dot{q}_1(0) = 0$, $q_2(0) = 0$, $\dot{q}_2(0) = 0$, $\zeta_1(0) = 0.2$, $\zeta_2(0) = 0.01$, $\zeta_3(0) = 0.002$ and $\zeta_4(0) = 0.003$. The reference signal considered was $x_r(t) = 0.5 \sin(t)$. The time closed-loop plots showing the dynamic behavior of the link angular position, the reference signal, the link angular speed, the time derivative of the reference signal, and the estimation errors e_1 and e_2 are given in figures 2 and 3. From these plots, one can notice that a good tracking performance is obtained. Also, the control variable is kept within practical limits of operation.

6. Conclusions.

A nonlinear control-observer structure based on a class of nonlinear singularly perturbed systems and canonical representations has been presented and applied to the model of a single-link flexible joint robot manipulator. A good trajectory tracking performance is obtained for the link angular position when the structure is used, thus making it a promising approach for the control of such electromechanical devices.

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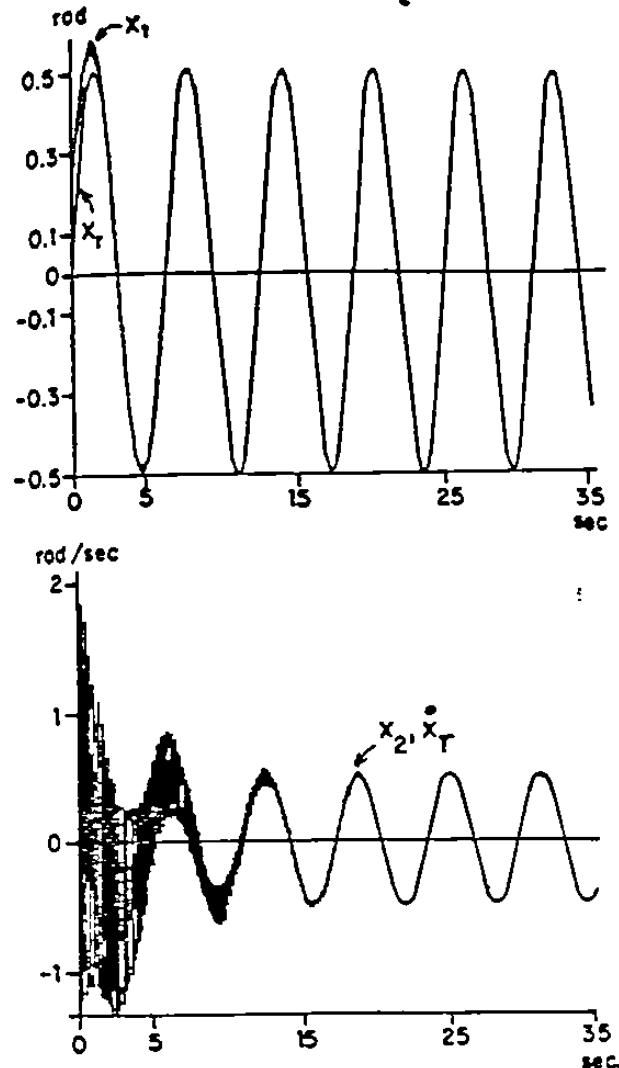


Figure 2: Time response of x_1 , x_r , x_2 and x_r .

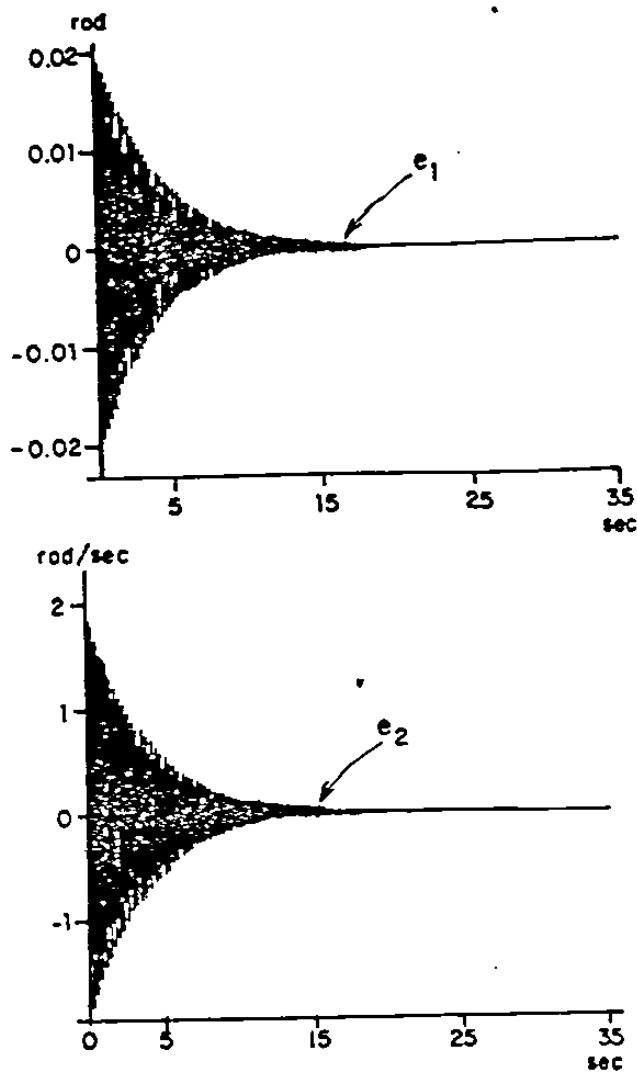


Figure 3: Time response of the estimation errors $e_1 = \dot{\zeta}_1 - \zeta_1$ and $e_2 = \dot{\zeta}_2 - \zeta_2$.

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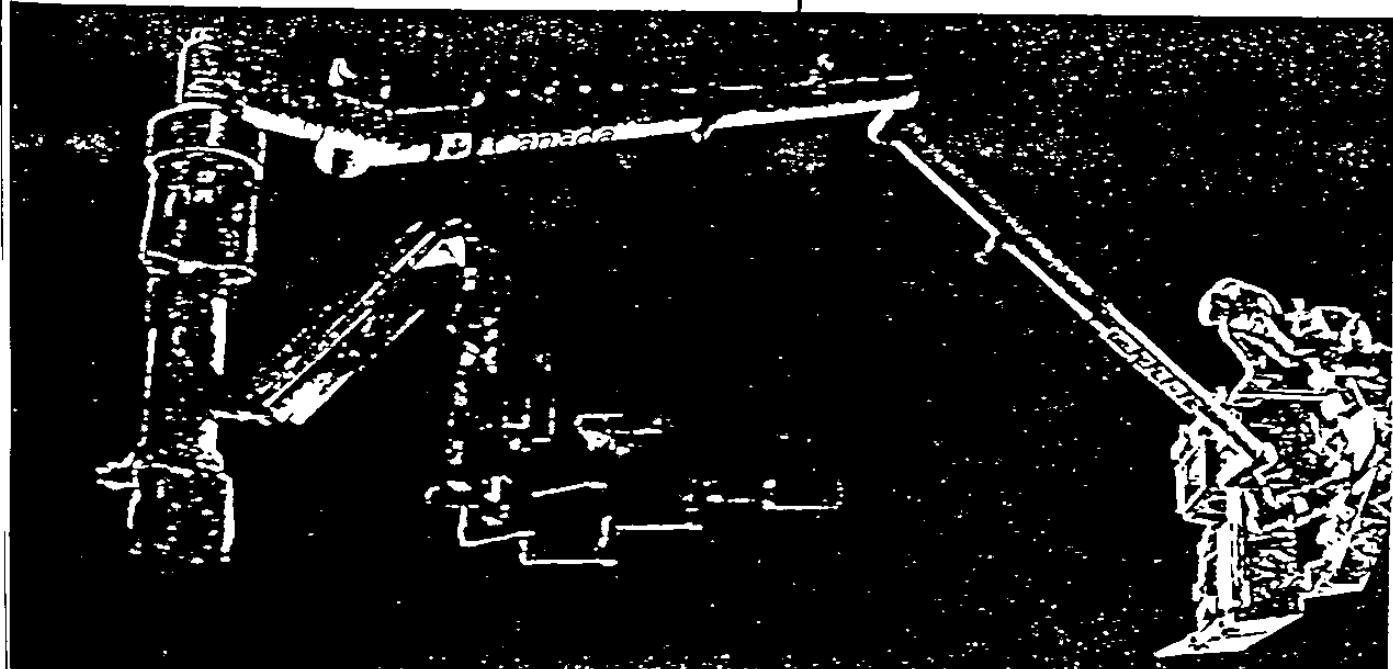
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0111	0010	1001	0110
1001	0110	1011	1101
0000	1011	0110	0000
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A Comparative Study of Speed and Position Control of a Flexible Joint Robot Manipulator.

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ABSTRACT

During the last few years, considerable research efforts have been directed toward the control problem of a flexible joint robots. In this paper we investigate two techniques for position and speed control of a single flexible joint robot manipulator. The connection between the two controllers are studied and compared using a flexible joint robot model.

Keywords: Flexible Joint Robots, Singular Perturbations, Differential Algebra, Observers.

I INTRODUCTION

During the last few years, considerable research efforts have been directed toward the control problem of flexible joint robots. In this paper we propose a controller based on an observer which is designed using two methodologies. Singular Perturbations Theory and Differential Algebra. On the other hand, the observer problem for flexible joint robots is an important one in robot control theory and the great practical importance. In fact many control techniques for these robots require the knowledge of four variables for each joint, which may be either positions and velocities of the motors and of the links or positions, velocities, accelerations and jerks of the links. Some interesting results on the observer problem for these robots are given for each controller.

Several controller design techniques have been proposed, from different perspectives, for the stabilization of nonlinear systems [1, 8, 12]. Recently, a considerable number of works have studied the robot control problem with joint flexibility. From a practical point of view, the effect of the elasticity in the robot must be considered in the control design. Moreover, the number of degree of freedom is twice the number of control actions, and the matching property between nonlinearities and inputs is lost [1].

Two-time scale sliding Mode control based on singularly perturbed theory for a class of nonlinear systems is proposed. Moreover, a dynamical feedback error linearization strategy based on Fliess' generalized observability canonical form (GOCF) and the generalized controller canonical form (GCCF) is given (see [4,6,10]).

On the other hand, the controllers designed here, require the knowledge of the whole vector state to be implemented. It is possible to substitute the unknown information by means of an auxiliary system called observer. In this paper, a high gain observers are used to estimate the state of the system for each methodology in order to implement ours controllers [2, 5].

This paper is organized as follows, a control law based on the sliding mode technique for a class of nonlinear singularly perturbed systems, as well as an observer is proposed in section 2. In section 3, using techniques of the differential algebra, a controller based on an observer is designed. Numerical simulations of these two schemes of control are given and compared. Finally, some conclusions close this paper.

2 TWO-TIME SCALE SLIDING MODE CONTROL

Let us consider the following class of nonlinear singularly perturbed systems

$$\begin{cases} \dot{x} = f_1(x) + F_1(x)z + g_1(x)u, & x(t_0) = x_0, t_0 \geq 0 \\ \epsilon \dot{z} = f_2(x) + F_2(x)z + g_2(x)u, & z(t_0) = z_0 \end{cases} \quad (1)$$

where $x \in R^n$ is the slow state, $z \in R^m$ is the fast state, $u \in R^r$ is the control input, ϵ is a small positive parameter such that $\epsilon \in [0, 1]$, and f_1, f_2 , the columns of the matrices F_1, F_2, g_1 and g_2 are assumed to be bounded analytical real vector fields. In addition, it is supposed that, $f_1(0) = f_2(0) = 0$ and, for $u = 0$, the origin is an isolated equilibrium state. Moreover $F_2(x)$ is assumed to be nonsingular for all $x \in B_z$, where B_z denote the

ball centered in $x = 0$.

2.1 THE SINGULAR PERTURBATION METHODOLOGY

The singular perturbation methodology permits to decompose the original system in two subsystems of lower dimension, both described in different time scales [7, 8]. The slow reduced system is found by making $\epsilon = 0$ in (1), resulting in the following reduced $n - th$ order slow system

$$\dot{x}_s = f(x_s) + g(x_s)u_s, \quad x_s(t_0) = x_0 \quad (2)$$

with $x_s = h(x_s) := -F_2^{-1}(x_s)[f_2(x_s) + g_2(x_s)u_s]$, and $x_s \in R^n$, $z_s \in R^m$ and $u_s \in R^r$ denote the slow components of the original variables x , z and u , respectively, and $f(x_s) := f_1(x_s) - F_1(x_s)F_2^{-1}(x_s)f_2(x_s)$; $g(x_s) := g_1(x_s) - F_1(x_s)F_2^{-1}(x_s)g_2(x_s)$. The fast dynamics is $\frac{d\hat{\eta}}{d\tau} = F_2(\tilde{x})\hat{\eta} + g_2(\tilde{x})u_f$; $\hat{\eta}(0) = z_0 - h(x_0, 0)$, being $\hat{\eta}$ an $O(\epsilon)$ approximation of η in the initial boundary layer and $\tilde{x}(\tau) = x(\epsilon\tau + t_0)$, $\tau = (t - t_0)$.

2.2 SLIDING-MODE CONTROL DESIGN

2.2.1 THE SLOW SUBSYSTEM

The equivalent control method [3] is used to determine the slow reduced system motion restricted to the slow switching surface $\sigma_s(x_s) = Col(\sigma_{s,1}(x_s), \dots, \sigma_{s,r}(x_s)) = 0$, where all the slow switching surfaces $\sigma_{s,i}(x_s)$, $i = 1, \dots, r$; are assumed to be locally Lipschitz-continuous functions for all $x_s \in R^n$, obtaining the so-called slow equivalent control

$$u_{se} = -[s_s(x_s)g(x_s)]^{-1}s_s(x_s)f(x_s); \quad (3)$$

where $s_s(x_s) = \frac{\partial \sigma_s}{\partial x_s}$, and the matrix $s_s(x_s)g(x_s)$ is assumed nonsingular $\forall x_s \in R^n$.

Obviously, in order to satisfy the above condition, the stabilization of the linear approximation of the slow reduced system is necessary. The following control law can be used to complete the design of the slow control

$$u_s = u_{se} + u_{sN} \quad (4)$$

where u_{se} is the slow equivalent control given by (3), which acts when the slow reduced system is restricted to $\sigma_s(x_s) = 0$, while u_{sN} is the discontinuous part of the control u_s acting when $\sigma_s(x_s) \neq 0$ (see [3]). In this work the control u_{sN} is selected as a nonlinear feedback law of the type

$$u_{sN} = -[s_s(x_s)g(x_s)]^{-1}L_s(x_s)\sigma_s(x_s) \quad (5)$$

where $L_s(x_s)$ is a positive definite matrix of dimension $r \times r$, $\forall x_s \in B_z$, whose elements are bounded nonlinear real functions of x_s .

The complete slow control law for the slow reduce subsystem is follows

$$u_s(x_s) = -[s_s(x_s)g(x_s)]^{-1}\{s_s(x_s)f(x_s) + L_s(x_s)\sigma_s(x_s)\} \quad (6)$$

2.2.2 THE FAST SUBSYSTEM

Consider the fast nonlinear switching surface defined by $\sigma_f(\hat{\eta}) = Col(\sigma_{f,1}(\hat{\eta}), \dots, \sigma_{f,r}(\hat{\eta})) = 0$, where all the slow switching surfaces $\sigma_{f,i}(\hat{\eta})$, $i = 1, \dots, r$; are assumed to be locally Lipschitz-continuous functions for all $\hat{\eta} \in R^n$. By using the equivalent control method for restricting the motion of the fast subsystem on the fast surface $\sigma_f(\hat{\eta}) = 0$, then the fast equivalent control is given by

$$u_{fe}(\hat{\eta}) = -(s_f(\hat{\eta})g_2(\tilde{x}))^{-1}s_f(\hat{\eta})F_2(\tilde{x})\hat{\eta}, \quad (7)$$

where $s_f(\hat{\eta}) = \frac{\partial \sigma_f}{\partial \hat{\eta}}$, and the matrix $s_f(\hat{\eta})g_2(\tilde{x})$ is nonsingular. In a similar way, the discontinuous fast control $u_{fN}(\hat{\eta})$ has the following form

$$u_{fN}(\hat{\eta}) = -\left\{s_f(\hat{\eta})g_2(\tilde{x})\right\}^{-1}L_f(\hat{\eta})\sigma_f(\hat{\eta}) \quad (8)$$

where L_f is a positive definite matrix and $s_f(\hat{\eta})g_2(\tilde{x})$ is nonsingular (see [13]). Then the complete fast control law for the fast reduce subsystem is follows

$$u_f(x_s) = -\{s_f(\hat{\eta})g_2(\tilde{x})\}^{-1}\{s_f(\hat{\eta})F_2(\tilde{x})\hat{\eta} + L_f(\hat{\eta})\sigma_f(\hat{\eta})\} \quad (9)$$

The composite control for the original nonlinear singularly perturbed system (1) can be expressed by

$$u = u_s + u_f. \quad (10)$$

However, the above control was obtained in function of the variables x_s and $\hat{\eta}$. In order to express this composite control in function of the original variables x and z , we replace x_s by x , and $\hat{\eta}$ by $z - h(x)$ in such a way the composite controller is given by

$$\begin{aligned} u &= -\{s_s(x)g(x)\}^{-1}\{s_s(x)f(x) + L_s(x)\sigma_s(x)\} \\ &\quad -\{s_f(x)g_2(x)\}^{-1}\{s_f(x)F_2(x)\eta + L_f(x)\sigma_f(x)\} \end{aligned} \quad (11)$$

2.3 A HIGH GAIN NONLINEAR OBSERVER

Let us suppose that the singularly perturbed system (1) can be written, after a state coordinate transformation $\zeta = \Phi(x, z)$, in the following form [2]:

$$\begin{cases} \dot{\zeta}(t) = F(y(t))\zeta(t) + G(u(t), \zeta(t)) \\ y = C\zeta(t) \end{cases} \quad (12)$$

where $\zeta \in R^n$, $u \in R^r$, $y \in R$, $C = (1 \ 0 \ \dots \ 0)$,
 $F(y(t))_{i,j} = \begin{cases} f_i(y(t)) & \text{if } i = j-1 \\ 0 & \text{everywhere} \end{cases}$, $G(u, \zeta) =$
 $\text{Col}(g_1(u, \zeta_1), g_2(u, \zeta_1, \zeta_2), \dots, g_n(u, \zeta_1, \dots, \zeta_n))$

It is also assumed that the functions f_i , $i = 1, \dots, n-1$, are of class C^r , $r \geq 1$, with respect to y and that the functions g_i , $i = 1, \dots, n$ are globally Lipschitz with respect to ζ and uniformly with respect to u . In addition, one assume there is a class of bounded controls $U \subset R^r$, and a compact set $K \subset R^n$, and constants α and β , such that for each $u \in U$ and each output $y(t)$ associated u and the initial condition $\zeta(0) \in K$, $0 \leq \alpha \leq f_i(y(t)) \leq \beta$, $i = 1, \dots, n-1$. Let us define the system

$$\begin{cases} \dot{\zeta}(t) = F(y(t))\zeta(t) + G(u(t), \zeta(t)) - \bar{S}_\theta^{-1}(y)C^T[C\zeta - y] \\ (13) \end{cases}$$

with $\Omega(y) = \text{Diag}\{1, f_1(y), f_1(y)f_2(y), \dots, \prod_{i=1}^{n-1} f_i(y)\}$, $\bar{S}_\theta = \Omega(y)S_\theta\Omega(y)$, and assume that the time derivative of $\Omega(y)$ is bounded. S_θ is a symmetric positive definite matrix which is the unique solution of the algebraic Lyapunov: $\theta S_\theta + A^T S_\theta + S_\theta A - C^T C = 0$; with $\theta > 0$, and $A_{i,j} = \delta_{i,j-1} = \begin{cases} 1 & \text{if } i = j-1 \\ 0 & \text{everywhere} \end{cases}$.

2.4 APPLICATION TO THE MODEL OF THE FLEXIBLE ROBOT

In this work, we consider a single-link flexible joint robot directly actuated by a direct current electrical motor (hub motor) whose rotor is elastically coupled to the link [1,10,11]. The mathematical model for the manipulator is given by

$$\begin{cases} I \ddot{q}_1 + B_v \dot{q}_1 + mgl \sin(q_1) + k(q_1 - q_2) = 0 \\ J \ddot{q}_2 + B \dot{q}_2 - k(q_1 - q_2) = u \end{cases} \quad (14)$$

where q_1 and q_2 are the angular positions of the link and the hub motor, respectively, while u is the input force from the actuator (motor torque). I is the inertia of the arm, J denotes the motor inertia, B is the motor viscous damping. B_v is the link viscous damping, mgl is the nominal load in the arm and k is the flexible joint stiffness coefficient.

By means of the assignment

$$x_1 = q_1, x_2 = \dot{q}_1, z_1 = k(q_1 - q_2), z_2 = \epsilon k(\dot{q}_1 - \dot{q}_2),$$

where $k = \frac{\alpha}{\epsilon^2}$ and $\epsilon \in [1, 0]$, it is possible to write the model (14) in the standard singularly perturbed form (1),

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -\frac{mgl}{I} \sin(x_1) - \frac{B}{I} x_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\frac{1}{I} & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$\begin{pmatrix} \epsilon \dot{z}_1 \\ \epsilon \dot{z}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\alpha B}{J} x_2 - \frac{mgl}{J} \sin(x_1) \end{pmatrix} + \begin{pmatrix} -\alpha \left(\frac{1}{I} + \frac{1}{J}\right) & 0 \\ 0 & -\frac{\epsilon B}{J} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{\alpha}{J} \end{pmatrix} u \quad (15)$$

where $\alpha = k\epsilon^2$, this is the flexible joint stiffness coefficient which is assumed to be a $O(\frac{1}{\epsilon^2})$ for the parameter ϵ .

Since it is desired that the rotor position tracks a reference signal, the following slow switching function is chosen

$$\sigma_s = s_1(x_1 - x_{ref}) + s_2(x_2 - \dot{x}_{ref}) \quad (16)$$

where s_1 and s_2 are constant real coefficients, and x_{ref} is the reference signal. It is also assumed that x_{ref} and \dot{x}_{ref} are bounded for all $t > 0$. Then, the slow control

$$u_s = \left(B - \frac{s_1}{s_2} (I + J) - l_s (I + J) \right) x_2 + mgl \sin(x_1) - (I + J) l_s \frac{s_1}{s_2} (x_1 - x_{ref})$$

where $L_s(x_s) = l_s > 0$.

For the fast subsystem, one chooses the fast switching function $\sigma_f = \sigma_{f_1} \hat{\eta}_1 + \sigma_{f_2} \hat{\eta}_2$ where are also constant real coefficients. In accordance to equation (9), one obtains the fast control

$$u_f = -\left(\frac{J}{I} + 1\right) \eta_1 + \frac{s_{f_1}}{s_{f_2}} \frac{J}{\alpha} \eta_2 + \frac{J}{\alpha} l_s \frac{s_{f_1}}{s_{f_2}} \eta_1 + \frac{J l_s}{\alpha} \eta_2$$

and $L_f(\hat{\eta}) = l_f > 0$ and $s_f(\eta) = (s_{f_1} \quad s_{f_2})$.

By making the following change of coordinates $\xi_1 = x_1$, $\xi_2 = x_2$, $\xi_3 = z_1$, $\xi_4 = z_2$, we obtain:

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = -\frac{mgl}{I} \sin(\xi_1) - \frac{B}{I} \xi_2 - \frac{1}{I} \xi_3 \\ \dot{\xi}_3 = \frac{1}{\epsilon} \xi_4 \\ \dot{\xi}_4 = -\frac{mgl}{\epsilon I} \sin(\xi_1) + \frac{\alpha B}{\epsilon I} \xi_2 - \frac{\alpha}{\epsilon} \left(\frac{1}{I} + \frac{1}{J}\right) \xi_3 - \frac{B}{\epsilon J} \xi_4 - \frac{\alpha}{\epsilon J} u \\ y = \xi_1 \end{cases}$$

A high gain observer for the state variable that has the form

$$\begin{cases} \dot{\widehat{\xi}}(t) = F(y(t))\widehat{\xi}(t) + G(u(t), \widehat{\xi}(t)) - \bar{S}_\theta^{-1}(y)C^T[C\widehat{\xi} - y] \\ \text{here } \bar{S}_\theta = \Omega(y)S_\theta\Omega(y), \end{cases}$$

$$F(y(t)) = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & -\frac{m}{I} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad G(x, u) = \begin{pmatrix} 0 \\ -\frac{m\omega}{I} \sin(\xi_1) - \frac{B}{I} \xi_2 \\ 0 \\ -\frac{m\omega}{I} \sin(\xi_1) + \frac{\alpha B}{I} \xi_2 - \frac{\alpha}{I} (\frac{1}{2} + \frac{1}{2}) \xi_3 - \frac{B}{I} \xi_4 - \frac{\alpha}{I} u \end{pmatrix}.$$

3 A DIFFERENTIAL ALGEBRAIC APPROACH

Consider the following Nonlinear System

$$(\Sigma_{NL}): \begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases} \quad (17)$$

where $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}$, f and h are assumed to be polynomial in their arguments. Systems (Σ_{NL}) are assumed to be universally observable (see [4, 6, 7]) with external behavior described by equations of the form,

$$\frac{d^n y}{dt^n} = -L_o \left(y, \frac{dy}{dt}, \dots, \frac{d^{n-1}y}{dt^{n-1}}, u, \frac{du}{dt}, \dots, \frac{d^\gamma u}{dt^\gamma} \right)$$

where L_o is a polynomial of its arguments. By defining locally $\eta_i = \frac{d^{i-1}y}{dt^{i-1}}$, $1 \leq i \leq n$, we obtain an explicit GOCF of systems (Σ_{NL}) as follows

$$\dot{\eta}_i = \eta_{i+1}, \quad 1 \leq i \leq n; \quad \dot{\eta}_n = -L_o \left(\eta, u, \frac{du}{dt}, \dots, \frac{d^\gamma u}{dt^\gamma} \right) \quad (18)$$

for some $\gamma > 0$. Now, let $y_R(t)$ be a prescribed reference output function which is differentiable at least n times, $\frac{d^i y_R}{dt^i}; 1 \leq i \leq n$. The controller is supposed to produce a scalar function u , which locally forces y to asymptotically converge towards $y_R(t)$. Define an output tracking error function $e(t)$ as $e(t) = y(t) - y_R(t)$. By definition, η_i is equal to the $(i-1)$ -the time derivative of $y(t)$, that is $\eta_i = \frac{d^{i-1}y}{dt^{i-1}}$; for $1 \leq i \leq n$. Then, we have

$$\frac{d^i e(t)}{dt^i} = \eta_{i+1} - \frac{d^i y_R(t)}{dt^i}; \quad 1 \leq i \leq n-1; \quad (19)$$

$$\frac{d^n e(t)}{dt^n} = \frac{d\eta_n}{dt} - \frac{d^n y_R(t)}{dt^n}.$$

Let $p(s) = s^n + \sum_{i=0}^{n-1} a_i s^i$ be Hurwitz polynomial and $\bar{u} = (u, \dots, \frac{d^{n-1}u}{dt^{n-1}})$. By requiring a linear time-invariant autonomous dynamics for the tracking error function:

$$\frac{d^n e(t)}{dt^n} + \sum_{i=0}^{n-1} a_i \frac{d^i e(t)}{dt^i} = 0.$$

Let $e_i = \frac{d^{i-1}e(t)}{dt^{i-1}}$, for $1 \leq i \leq n$, be the components of the error vector $e = Col(e_1, e_2, \dots, e_n)$, then we obtain

$$\frac{de}{dt} = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{pmatrix} e = Fe \quad (20)$$

where

$$-L_o \left(\psi_R(t) + e, \bar{u}, \frac{d^\nu u}{dt^\nu} \right) - \frac{d^n y_R(t)}{dt^n} = - \sum_{i=1}^n a_{i-1} e_i \quad (21)$$

with $\psi_R(t) = Col(y_R, \frac{dy_R}{dt}, \dots, \frac{d^{n-1}y_R}{dt^{n-1}})$, reference signals vector, and $\bar{y}_R = (y_R, \frac{dy_R}{dt}, \dots, \frac{d^{n-1}y_R}{dt^{n-1}})$. Now, writing the system (19) as follows:

$$\frac{de}{dt} = Ee + \varphi \left(e, \bar{y}_R, \frac{d^\nu u}{dt^\nu}, \bar{u}, \frac{d^\nu u}{dt^\nu} \right) \quad (22)$$

where the elements of the matrix E are given by $E_{ij} = \delta_{i,j-1} = \begin{cases} 1 & \text{if } i = j-1 \\ 0 & \text{everywhere} \end{cases}$ and $\varphi \left(e, \bar{y}_R, \frac{d^\nu u}{dt^\nu}, \bar{u}, \frac{d^\nu u}{dt^\nu} \right) = Col(0 \dots 0 -L_o(\psi_R(t) + e, \bar{u}, \frac{d^\nu u}{dt^\nu}) - \frac{d^n e(t)}{dt^n})$. Then, the estimation of the tracking error $\hat{e}(t) = y(t) - y_R(t)$ is given by an exponential nonlinear observer (O) of the form [9]:

$$(O): \begin{cases} \frac{d\hat{e}}{dt} = E\hat{e} - \Delta_0 K[C\hat{e}(t) - e_1(t)] \\ \quad + \varphi \left(\hat{e}(t), \bar{y}_R, \frac{d^\nu u}{dt^\nu}, \bar{u}, \frac{d^\nu u}{dt^\nu} \right) \end{cases} \quad (23)$$

where $\Delta_0 = diag(\theta, \theta^2, \dots, \theta^n)$ for some $\theta > 0$ and $K = Col(k_1, \dots, k_n)$, K is chosen such that $\sigma(A - KC) \subset \mathbb{R}^-$. Let $\sigma(u_\epsilon, y_R(t), \hat{e}) = -\frac{d^n y_R(t)}{dt^n} + \sum_{i=1}^n a_{i-1} \hat{e}_i - L_o(\psi_R(t) + \hat{e}, \bar{u}, \frac{d^\nu u}{dt^\nu})$ and u_ϵ be the observer-based control resulting from $\sigma(u_\epsilon, y_R(t), \hat{e}(t)) = 0$.

3.1 CONTROL BASED ON DIFFERENTIAL ALGEBRA

From the dynamical system (Σ_R) which describing the motion of the robot, the parameter B_v must be different to zero because if $B_v = 0$, this technique can not be applied, and the zero dynamics becomes oscillatory (see [10]). Now let us define $\bar{\beta}^2 = \frac{1}{k}$, which is not to be considered as a small constant related to the singular perturbation techniques. Defining the following change of coordinates:

$$x_1 = q_2, \quad x_2 = \dot{q}_2, \quad x_3 = k(q_1 - q_2); \quad x_4 = \frac{(q_1 - \dot{q}_2)}{\bar{\beta}}.$$

Then, the dynamical model (Σ_R) represented in the new coordinates results as follows

$$(\Sigma_M) : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -k_5 x_2 + k_1 x_3 + k_1 u \\ \dot{x}_3 = \frac{x_4}{\beta} \\ \dot{x}_4 = \{-k_2 k_3 \sin(\bar{\beta}^2 k_3 + x_1) - k_4 x_3 - k_7 x_2 - k_6 \beta x_4 - k_1 u\} / \bar{\beta} \end{cases} \quad (24)$$

where $k_1 = \frac{1}{J}$; $k_2 = \frac{1}{J}$; $k_3 = mgl$; $k_4 = k_1 + k_2$; $k_5 = \frac{B}{J}$; $k_6 = \frac{B}{J}$; $k_7 = k_6 - k_5$.

The following coordinate transformation depending on the input which allows us to obtain a generalized observability canonical form (GOCF). For that, we have:

$$y = \eta_1 = x_1, \quad \dot{y} = \eta_2 = x_2, \quad \ddot{y} = \eta_3 = -k_5 x_2 + k_1 x_3 + k_1 u,$$

$$\dddot{y} = \eta_4 = (k_5)^2 x_2 - k_1 [k_5 x_3 + \bar{\beta}^{-1} x_4 + k_5 u + \frac{du}{dt}]$$

and its corresponding inverse transformation is

$$x_1 = \eta_1, \quad x_2 = \eta_2, \quad x_3 = (\eta_3 + k_5 \eta_2 - k_1 u) / k_1,$$

$$x_4 = \bar{\beta} [\eta_4 - (k_5)^2 \eta_2 + k_1 k_5 \eta_3 - k_1 k_5 u - k_1 \frac{du}{dt}] / k_1.$$

Then the (GOCF) of (24) is given by

$$\begin{cases} \dot{\eta}_1 = \eta_2, \quad \dot{\eta}_2 = \eta_3, \quad \dot{\eta}_3 = \eta_4, \\ \dot{\eta}_4 = -k_1 k_2 k_3 \bar{\beta}^{-2} \sin(\bar{\beta}^2 [k_5 \eta_2 - k_1 u + +\eta_3]) / k_1 + \eta_1 \\ \quad - \bar{\beta}^{-2} \{k_1 k_7 + k_4 k_5\} \eta_2 - \{\frac{k_4}{\beta} + k_5 k_6\} \eta_3 \\ \quad - \{k_5 + k_6\} \eta_4 - \sum_{i=1}^4 \frac{a_{i-1}}{k_1} \{\hat{x}_i + \frac{d^{i-1} y}{dt^{i-1}}\} + \frac{1}{k_1} \frac{d^4 y}{dt^4} \end{cases} \quad (25)$$

Now, let $y_R(t)$ be a desired output reference trajectory of the angular position. Defining the tracking error as $e(t) = x_1 - y_R(t)$, then the tracking error dynamics is $\frac{de}{dt} = Ee + \varphi(e_1, \dots, e_4, y_R, \dots, \frac{d^4 y_R}{dt^4}, u, \frac{du}{dt}, \frac{d^2 u}{dt^2})$, with output $y_T = e_1$. Then, the following system is a nonlinear exponential observer for the estimation of the tracking error [9]: $\frac{de}{dt} = \widehat{Ee} - \Delta_\theta K [C\widehat{e}(t) - e_1(t)] + \varphi(\widehat{e}(t), y_R, \dots, \frac{d^4 y_R}{dt^4}, u, \frac{du}{dt}, \frac{d^2 u}{dt^2})$.

Then, the dynamical controller is given by

$$\begin{aligned} \frac{d^2 u}{dt^2} + (a_3 - k_5) \frac{du}{dt} + \left(k_5^2 - k_5 a_3 - k_1 \bar{\beta}^{-2} \right) u &= -\frac{a_2}{k_1} x_1 \\ - \left(\frac{k_7}{\beta} - \frac{a_1}{k_1} + \frac{k_4 a_2}{k_1} + \frac{k_5^2}{k_1} - \frac{k_4 a_3}{k_1} \right) x_2 + \left(\frac{k_4}{\beta} + \frac{k_5}{\beta} - \frac{a_3}{\beta} \right) x_4 \\ + \frac{k_1 k_3}{\beta} \sin(x_1 + \bar{\beta}^2 x_3) + \left(\frac{d^4 y_R}{dt^4} + \sum_{i=1}^4 a_{i-1} \frac{d^{i-1} y_R(t)}{dt^{i-1}} \right) \frac{1}{k_1} \end{aligned}$$

and the desired dynamic performance can be obtained by choosing suitable a_1, a_2, a_3, a_4 . However, this controller depends on all state which is unmeasurable, then we will

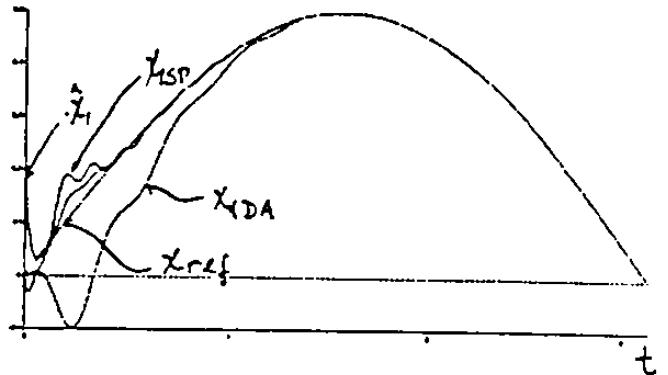


Figure 1 – Time response of the link angular position x_{1PS} and x_{1DA} , the estimate \hat{x}_1 , and the reference signal x_{ref}

replace the state estimated by the observer. Then, the dynamical controller as a function of the estimates is given by

$$\begin{aligned} \frac{d^2 u_e}{dt^2} + k_6 \frac{du_e}{dt} - \frac{1}{\beta^2} \{k_4 - k_1\} u_e &= -k_2 k_3^{-2} \bar{\beta} \sin(\bar{\beta}^2 \hat{x}_3 + \hat{x}_1) \\ - \bar{\beta}^{-2} \frac{1}{k_1} \{k_1 k_7 + k_4 k_5\} \hat{x}_2 - \frac{1}{k_1} \{\frac{k_4}{\beta} + k_5 k_6\} \hat{x}_3 \\ - \{k_5 + k_6\} \hat{x}_4 - \sum_{i=1}^4 \frac{a_{i-1}}{k_1} \{\hat{x}_i + \frac{d^{i-1} y}{dt^{i-1}}\} + \frac{1}{k_1} \frac{d^4 y}{dt^4} \end{aligned}$$

where $\hat{x}_1 = \hat{e}_1 + y_R$, $\hat{x}_2 = \hat{e}_2 + \frac{dy_R}{dt}$, $\hat{x}_3 = \{\hat{e}_3 + \frac{d^2 y_R}{dt^2} + k_5(\hat{e}_2 + \frac{dy_R}{dt}) - k_1 u\} / k_1$, $\hat{x}_4 = \{\hat{e}_4 + \frac{d^3 y_R}{dt^3} + k_5(\hat{e}_3 + \frac{d^2 y_R}{dt^2}) - k_1 \frac{du}{dt}\} \bar{\beta} / k_1$, is the inverse transformation in terms of the estimated tracking error.

4 SIMULATION RESULTS

Now, we will show how the controllers scheme are implemented to a model of a flexible joint robot. The simulation were performed with the following parameters $k_1 = 3.33(m^2 Kg)^{-1}$; $k_2 = 1.0(m^2 Kg)^{-1}$; $k_3 = 5.0Nm$; $k_4 = 4.33(m^2 Kg)^{-1}$, $k_5 = 0.333s^{-1}$; $k_6 = 0.1s^{-1}$; $k_7 = -0.233s^{-1}$; $k = 100N/(mrad^{-1})$. All initial conditions, for the system and the dynamical controller based on Differential Algebraic Approach were chosen to be zero.

The control law based on the Singularly Perturbation Theory, the gains were chosen as follows: $s_1 = 20$; $s_2 = 25$; $l_s = 60$; $s_{f1} = 1$; $s_{f2} = 10$; $l_f = 0.1$. Such a selection guarantees the stability of the slow and fast sliding modes systems and the existence of the corresponding sliding modes. The parameter θ in the observer equations was chosen as $\theta = 20$. From this initial conditions of the robot variables and the estimates were fixed to $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0$, $x_4(0) = 0$, $\hat{x}_1(0) = 0.2$, $\hat{x}_2(0) = 0.02$, $\hat{x}_3(0) = 0.002$, $\hat{x}_4(0) = 0.003$.

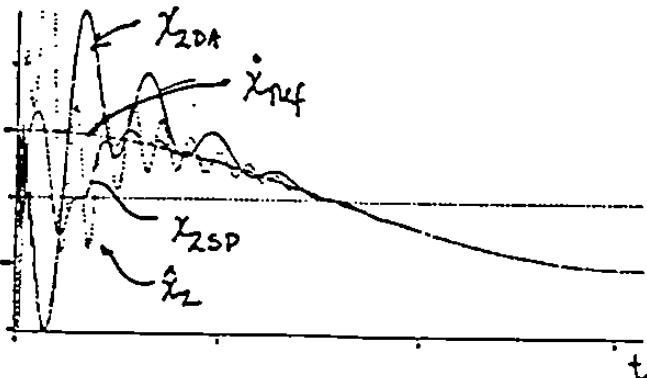


Figure 2 - Time response of the link angular speed x_{2SP} and x_{2DA} , the estimate \hat{x}_2 , and time derivative of the reference signal \dot{x}_{ref} .

The reference signal considered was $x_{ref} = 0.5 \sin(t)$. The time closed-loop plots showing the dynamic behavior of the link angular position using both schemes, the estimated and the desired reference signal, is plotted in figure 8.1. The link angular speed using Singular Perturbation Theory and Differential Algebra, the estimated and the time derivative of the reference signal are plotted in Figure 8.2. From these plots, one can notice that a good tracking performance is obtained.

Finally, the Figure 8.3 depicts the obtained controller applied to the system using these techniques. From figure 8.3, we can say that the control designed using singular perturbation theory is more robust and requires less energy to execute the control action than this designed by differential algebra. On the other hand, in both cases, the estimates converge to the state of the system.

5 CONCLUSIONS

Two controllers based on an observer were presented using the singular perturbation theory and based on differential algebra. A comparative study was done showing the performance of each controller, taking into account the design simplicity, and performance. High gain observers are given in order to estimate the unmeasurable state of the robot in each control scheme. Simulation results were given where these schemes are implemented to a mathematical model of a flexible robot.

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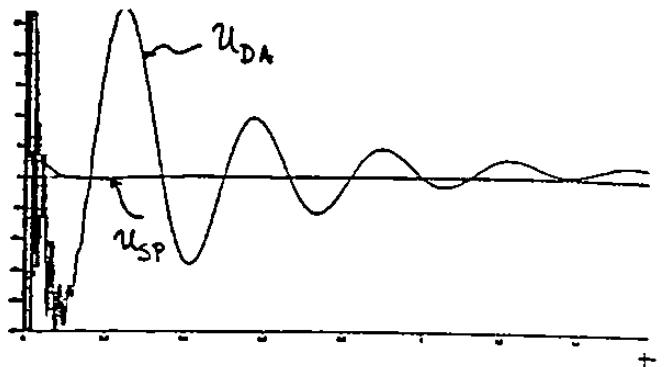


Figure 3 - Time response of the control input u_{SP} and u_{DA} .

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FINAL PROGRAM AND BOOK OF ABSTRACTS

37th IEEE CONFERENCE ON DECISION AND CONTROL



December 16-18, 1998

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IEEE CONTROL SYSTEMS SOCIETY



16:20			
<i>Slow control for global stabilization of feedforward systems with exponentially unstable Jacobian linearization</i>	<i>1452</i>		
Grognard, F.	Univ. Catholique de Louvain	Egerstedt, M.	Royal Inst. of Tech.
Sepulchre, Rodolphe J.	Univ. de Liege	Hu, Xiaoming	Royal Inst. of Tech.
Bastin, Georges	Univ. Catholique de Louvain	Stotsky, Alexander A.	Royal Inst. of Tech.
16:40			
<i>Global asymptotic stability for the averaged implies semi-global practical asymptotic stability for the actual</i>	<i>1458</i>		
Teel, Andrew R.	Univ. of California-Santa Barbara	Tian, Hua	Tongji University
Peuteman, Joan	Univ. Gent		
Aeyels, Dirk	Univ. Gent		
17:00			
<i>Control of unknown nonlinear systems using output feedback</i>	<i>1464</i>		
Kosmatopoulos, Elias B.	Univ. of Southern California	Laib, Abdelhamid	Ecole Superieure D'Electricite
17:20			
<i>An LMI approach to the control of a class of nonlinear systems</i>	<i>1470</i>		
Kiriakidis, Kiriakos	U.S. Naval Academy	Meressi, Tesfay	Univ. of Massachusetts-Dartmouth
17:40			
<i>Stabilization of orthogonal piecewise linear systems using piecewise linear Lyapunov-like functions</i>	<i>1476</i>		
Yfoulis, C. A.	UMIST		
Muir, A.	UMIST		
Pettit, N. B. O. L.	Danfoss A/S		
Wellstead, P. E.	UMIST		
18:00			
<i>Decentralized global robust stabilization of a class of large-scale interconnected minimum-phase nonlinear systems</i>	<i>1482</i>		
Xie, Shoulie	Nanyang Tech. Univ.		
Xie, Lihua	Nanyang Tech. Univ.		
	Wilson's Plover N&S		
WP14			
Advanced Methods for Robot Control II			
Chair: Spong, Mark W.	Univ. of Illinois at Urbana-Champaign		
Co-chair: Meressi, Tesfay	Univ. of Massachusetts Dartmouth		
16:00			
<i>Control of redundant manipulators using logic-based switching</i>	<i>1488</i>		
Bishop, Bradley E.	U.S. Naval Academy		
Spong, Mark W.	Univ. of Illinois at Urbana-Champaign		
16:20			
<i>Variable structure controller design for flexible-link robots under gravity</i>	<i>1494</i>		
Yang, H.	National Univ. of Singapore	Yokoyama, Makoto	Niigata Univ.
Krishnan, Hariharan	National Univ. of Singapore	Shimizu, H.	Niigata Univ.
Ang, Jr., M. H.	National Univ. of Singapore	Okamoto, N.	Unisia Jecs Co.
16:40			
<i>Observer-based controllers for a flexible joint robot manipulator</i>	<i>1500</i>		
De Leon-Morales, J.	FIME-UANL	Park, Kyihwan	Kwangju Inst. of Science & Tech.
Alvaro Leal, J. C.	Inst. Tecn. de Saltillo	Lee, Kap-Jin	Kwangju Inst. of Science & Tech.
17:00			
<i>Control of a car-like robot using a virtual vehicle approach</i>	<i>1502</i>		
Egerstedt, M.	Royal Inst. of Tech.		
Hu, Xiaoming	Royal Inst. of Tech.		
Stotsky, Alexander A.	Royal Inst. of Tech.		
17:20			
<i>Robotic action planning with the application of explanation-based learning</i>	<i>1508</i>		
Tian, Hua	Tongji University		
17:40			
<i>Output regulation of robot manipulators under actuator constraints</i>			
Laib, Abdelhamid	Ecole Superieure D'Electricite		
18:00			
<i>Modeling and control of a three dimensional gantry robot</i>	<i>1514</i>		
Meressi, Tesfay	Univ. of Massachusetts-Dartmouth		
	White Ibis N		
WP15			
Control of Automotive Systems			
Chair: Dunn, M. T.	Ford Research Lab.		
Co-chair: Larsen, Michael	Univ. of California-Santa Barbara		
16:00			
<i>Hub-coupled dynamometer control</i>	<i>1516</i>		
Sureshbabu, Natarajan	Ford Motor Company		
Dunn, M. T.	Ford Research Lab.		
16:20			
<i>Fundamental study of low fuel consumption control scheme based on combination of direct fuel injection engine and continuously variable transmission</i>	<i>1522</i>		
Takahashi, Shinsuke	Hitachi, Ltd.		
16:40			
<i>Adaptive control of diesel engine-dynamometer systems</i>	<i>1530</i>		
Yanakiev, Diana P.	Cummins Engine Company		
17:00			
<i>Passivation design for a turbocharged diesel engine model</i>	<i>1535</i>		
Larsen, Michael	Univ. of California-Santa Barbara		
Kokotovic, Petar V.	Univ. of California-Santa Barbara		
17:20			
<i>Application of sliding-mode servo controllers to electronic throttle control</i>	<i>1541</i>		
Yokoyama, Makoto	Niigata Univ.		
Shimizu, H.	Niigata Univ.		
Okamoto, N.	Unisia Jecs Co.		
17:40			
<i>Sliding mode control for an Eddy current brake system</i>			
Park, Kyihwan	Kwangju Inst. of Science & Tech.		
Lee, Kap-Jin	Kwangju Inst. of Science & Tech.		

Observer-based controllers for a flexible joint robot manipulator

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ABSTRACT

In this paper, observer-based controllers are designed using Singular Perturbations Theory and Differential Algebra, for position and speed control of a single flexible joint manipulator. Both schemes are compared in simulation.

1. Two-Time Scale Sliding Mode Control

Let us consider the following class of nonlinear singularly perturbed (SP) systems (see [2])

$$\begin{cases} \dot{x} = f_1(x) + F_1(x)z + g_1(x)u, & x(t_0) = x_0, t_0 \geq 0 \\ \dot{z} = f_2(x) + F_2(x)z + g_2(x)u, & z(t_0) = z_0 \end{cases} \quad (1)$$

where $x \in R^n$ is the slow state, $z \in R^m$ is the fast state, $u \in R^r$ is the input, ϵ is a small positive parameter such that $\epsilon \in [0, 1]$, and f_1, f_2 , the columns of the matrices F_1, F_2, g_1 and g_2 are assumed to be bounded analytical real vector fields. We assume that $f_1(0) = f_2(0) = 0$ and, for $u = 0$, the origin is an isolated equilibrium state. $F_2(x)$ is assumed to be nonsingular for all $x \in B_x$, where B_x denote the ball centered in $x = 0$. The slow reduced system is found by making $\epsilon = 0$ in (1), we obtain $\dot{x}_s = f(x_s) + g(x_s)u_s$, $x_s(t_0) = x_0$ with $z_s = h(x_s) := -F_2^{-1}(x_s)[f_2(x_s) - g_2(x_s)u_s]$, and $x_s \in R^n$, $z_s \in R^m$ and $u_s \in R^r$ denote the slow components of the original variables x , z and u , respectively, and $f(x_s) := f_1(x_s) - F_1(x_s)F_2^{-1}(x_s)f_2(x_s)$; $g(x_s) := g_1(x_s) - F_1(x_s)F_2^{-1}(x_s)g_2(x_s)$. The fast dynamics is $\dot{\eta} = F_2(\tilde{x})\hat{\eta} + g_2(\tilde{x})u_f$, $\hat{\eta}(0) = z_0 - h(x_0, 0)$, being $\hat{\eta}$ an $O(\epsilon)$ approximation of η in the initial boundary layer and $\tilde{x}(\tau) = x(\epsilon\tau + t_0)$, $\tau = (t - t_0)$.

2. Sliding-Mode Control Design

The equivalent control method is used to determine the slow reduced system motion restricted to the slow switching surface $\sigma_s(x_s) = \text{col}(\sigma_{s_1}(x_s), \dots, \sigma_{s_r}(x_s)) = 0$, where $\sigma_{s_i}(x_s)$, $i = 1, \dots, r$, are assumed to be locally Lipschitz-continuous functions for $\forall x_s \in R^n$.

R^n . Then the slow equivalent control is $u_{se} = -\{s_s(x_s)g(x_s)\}^{-1}s_s(x_s)f(x_s)$; where $s_s(x_s) = \frac{\partial \sigma_s}{\partial x_s}$, and the matrix $s_s(x_s)g(x_s)$ is assumed nonsingular $\forall x_s \in R^n$. The slow control is given by $u_s = u_{se} - u_{sN}$ where u_{se} is the slow equivalent control, which acts when the slow reduced system is restricted to $\sigma_s(x_s) = 0$, while u_{sN} is the discontinuous part of the control u_s acting when $\sigma_s(x_s) \neq 0$. Then, we get $u_{sN} = -\{s_s(x_s)g(x_s)\}^{-1}L_s(x_s)\sigma_s(x_s)$ where $L_s(x_s) \in R^{r \times r}$, $\forall x_s \in B_x$. The complete slow control law is follows

$$u_s = -\{s_s(x_s)g(x_s)\}^{-1}\{s_s(x_s)f(x_s) + L_s(x_s)\sigma_s(x_s)\}$$

The fast nonlinear switching surface defined by $\sigma_f(\hat{\eta}) = \text{col}(\sigma_{f_1}(\hat{\eta}), \dots, \sigma_{f_r}(\hat{\eta})) = 0$, where $\sigma_{f_i}(\hat{\eta})$, $i = 1, \dots, r$, are assumed to be locally Lipschitz-continuous functions for all $\hat{\eta} \in R^n$. The fast equivalent control is given by $u_{fe}(\hat{\eta}) = -\{s_f(\hat{\eta})g_2(\tilde{x})\}^{-1}s_f(\hat{\eta})F_2(\tilde{x})\hat{\eta}$, where $s_f(\hat{\eta}) = \partial \sigma_f / \partial \hat{\eta}$, and $s_f(\hat{\eta})g_2(\tilde{x})$ is nonsingular. The discontinuous fast control is given by $u_{ff}(\hat{\eta}) = -\{s_f(\hat{\eta})g_2(\tilde{x})\}^{-1}L_f(\hat{\eta})\sigma_f(\hat{\eta})$, where L_f is a positive definite matrix. Then the complete fast control law is

$$u_f = -\{s_f(\hat{\eta})g_2(\tilde{x})\}^{-1}\{s_f(\hat{\eta})F_2(\tilde{x})\hat{\eta} + L_f(\hat{\eta})\sigma_f(\hat{\eta})\}$$

The composite control for the original nonlinear singularly perturbed system (1) can be expressed by

$$u = u_s + u_f.$$

3. A High Gain Nonlinear Observer

In order to implement the above controller we design an observer. Let us suppose that the SP system (1) can be written in the following form (see [1]):

$$\dot{\zeta} = F(y)\zeta + G(u, \zeta), \quad y = C\zeta,$$

where $\zeta \in R^n$, $u \in R^r$, $y \in R$, $C = (1 \ 0 \ \dots \ 0)$, $F(y)_{ij} = f_i(y)$ if $i = j - 1$ else $F(y)_{ij} = 0$; $G(u, \zeta) = \text{col}(g_1(u, \zeta_1), \dots, g_n(u, \zeta_1, \dots, \zeta_n))$. We assume that the functions f_i , $i = 1, \dots, n - 1$, are of class C^r , $r \geq 1$, w. r. t. y and that the functions g_i , $i = 1, \dots, n$, are globally Lipschitz w.r. t. ζ and uniformly w. r. t. u . In addition, one assume there are constants α and β , such

that for each $u \in U$ and each output $y(t)$ associated u and the initial condition $\zeta(0) \in K$, $0 \leq \alpha \leq f_i(y(t)) \leq \beta$, $i = 1, \dots, n-1$. Let us define the system

$$\dot{\zeta} = F(y)\zeta + G(u, \zeta) - \tilde{S}_\theta^{-1}(y)C^T[C\zeta - y]$$

with $\Omega = \text{Diag}\{1, f_1(y), \dots, \prod_{i=1}^{n-1} f_i(y)\}$, $\tilde{S}_\theta = \Omega S_\theta \Omega$, and assume that the time derivative of Ω is bounded. $S_\theta = S_\theta^T > 0$ is the unique solution of $\theta S_\theta + A^T S_\theta + S_\theta A - C^T C = 0$; with $\theta > 0$, and $A_{ij} = 1$ if $i = j-1$ else $A_{ij} = 0$.

4. A Differential Algebraic Approach

Now, we consider the following nonlinear system

$$(\Sigma_{NL}): \quad \dot{x} = f(x, u), \quad y = h(x, u)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}$, f and h are assumed to be polynomial in their arguments (see [3,4]). Systems (Σ_{NL}) are assumed to be universally observable with external behavior described by $\frac{d^n y}{dt^n} = -L_0(y, \frac{dy}{dt}, \dots, \frac{d^{n-1} y}{dt^{n-1}}, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n})$ where L_0 is a polynomial of its arguments. By defining $\eta_i = \frac{d^{i-1} y}{dt^{i-1}}$, $1 \leq i \leq n$, we obtain an explicit GOCE of system (Σ_{NL}) as follows $\dot{\eta}_i = \eta_{i+1}$, $1 \leq i \leq n$; $\eta_n = -L_0(\eta, u, \frac{du}{dt}, \dots, \frac{d^n u}{dt^n})$; for some $\gamma > 0$. Now, let y_R be a prescribed reference output function which is differentiable at least n times. Define an output tracking error function e as $e = y - y_R$. Let $e_i = \frac{d^{i-1} e}{dt^{i-1}}$, for $1 \leq i \leq n$, be the components of the error vector $e = \text{col}(e_1, e_2, \dots, e_n)$, $\bar{y}_R = (y_R, \dots, \frac{d^{n-1} y_R}{dt^{n-1}})$, $\psi_R = \text{col } \bar{y}_R$ and $\bar{u} = (u, \dots, \frac{d^{n-1} u}{dt^{n-1}})$, then we obtain

$$\frac{de}{dt} = Ee + \varphi(e, \bar{y}_R, \frac{d^n y_R}{dt^n}, \bar{u}, \frac{d^n u}{dt^n})$$

where the elements of the matrix E are given by $E_{ij} = 1$ if $i = j-1$ else $E_{ij} = 0$, and $\varphi(e, \bar{y}_R, \frac{d^n y_R}{dt^n}, \bar{u}, \frac{d^n u}{dt^n}) = \text{col}(0, \dots, 0, -L_0(\bar{y}_R + e, \bar{u}, \frac{d^n u}{dt^n}))$. We assume that φ is Lipschitz w. r. t. e and uniformly w. r. t. \bar{y}_R , $\frac{d^n y_R}{dt^n}$, \bar{u} , $\frac{d^n u}{dt^n}$. Then, an exponential nonlinear observer of the above system is given by

$$\frac{d\hat{e}}{dt} = E\hat{e} - \Delta_0 K[C\hat{e} - e] - \varphi(\hat{e}, \bar{y}_R, \frac{d^n y_R}{dt^n}, \bar{u}, \frac{d^n u}{dt^n})$$

where $\Delta_0 = \text{diag}(\theta, \theta^2, \dots, \theta^n)$ for some $\theta > 0$ and $K = \text{col}(k_1, \dots, k_n)$. K is chosen such that $\sigma(A - KC) \subset \mathbb{R}^-$. Let

$$\pi(u_s, y_R, \hat{e}) = -\frac{d^n y_R}{dt^n} + \sum_{i=1}^n a_i \cdot i\hat{e}_i - L_0(\psi_R + \hat{e}, \bar{u}, \frac{d^n u}{dt^n})$$

and u_s be the observer-based control resulting from

$$\pi(u_s, y_R, \hat{e}) = 0.$$

5. Application to the Flexible Robot

In order to compare both schemes, we consider the following model of the flexible robot

$$I\ddot{q}_1 + B_v\dot{q}_1 + mgl\sin(q_1) + k(q_1 - q_2) = 0$$

$$J\ddot{q}_2 + B\dot{q}_2 - k(q_1 - q_2) = u$$

where q_1, q_2 : the angular positions of the link and the hub motor, u : the input force from the actuator. I : the arm inertia, J : the motor inertia, B : the motor viscous damping, B_v : the link viscous damping, mgl :

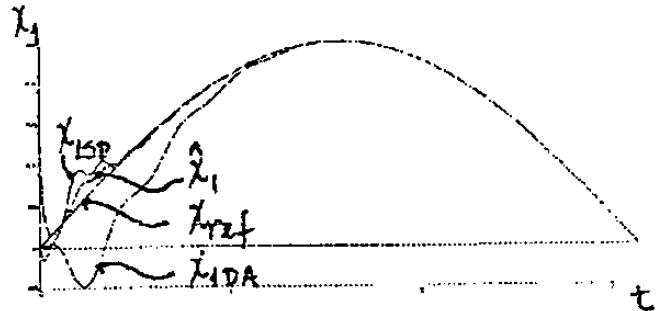


Figure 1 – Time response of the link angular position x_{1PS} , x_{1DA} , the estimate \hat{x}_1 and the reference x_{ref}

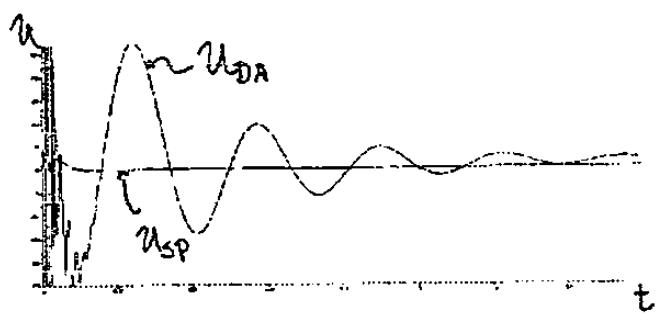


Figure 2 – Time response of the control u_{SP} and u_{DA}

the nominal load in the arm and k : the joint stiffness.

The reference signal considered was $x_{ref} = 0.5 \sin(t)$. The time closed-loop plots showing the dynamic behavior of the link angular position using both schemes, the estimated and the desired reference signal, is plotted in figure 1. From this plot, one can notice that a good tracking performance is obtained. Finally, the figure 2 depicts the obtained controller applied to the system using these techniques. From figure 2, we can say that the control designed using SP is more robust and requires less energy to execute the control action than this designed by Differential Algebra.

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