taron resultados vía simulación digital para estos casos de estudio, mostrando así el buen desempeño del observador.

5.2 Trabajos Futuros

Los temas a desarollar como trabajos de investigación futuros, a los que consideramos que sería adecuado dar seguimiento son los siguientes:

Con relación al estudio de sistemas electromecánicos, tales como el motor de inducción y los generadores síncronos, consideramos que un aspecto importante a tomar en cuenta es el uso de modelos más completos que tomen en cuenta efectos tales como la saturación, las dinámicas del regulador automático de voltaje (AVR), entre otros.

Además, sería deseable que las leyes de control desarrolladas para sistemas eléctricos de potencia sean decentralizadas, es decir se puedan implementar sólo con mediciones locales. O en los casos en el que no se dispone de todas las variables de estado necesarias para implementación de una ley de control, emplear un observador de estado.

En lo referente a los sistemas no lineales discretizados, en este trabajo de investigación se retomarón los trabajos reportados en [1, 19] y se efectuó un análisis de estabilidad para el sistema en lazo cerrado. El estudio se efectuó para la clase de sistemas no lineales linealizables por retroalimentación de estado, discretizados mediante el método de Euler. Como tema de investigación futuro nos gustaría estudiar la estabilidad del sistema en lazo cerrado para sistemas discretos de orden superior.

Por otra parte, se puede considerar el caso de tener un sistema continuo y un esquema de control y observación discreto, de modo que un estudio de estabilidad para esta clase de sistemas híbridos sería un tema interesante a realizar.

Finalmente, para los observadores adaptables, una extensión de este trabajo consistiría en considerar una clase más general de sistemas afines en el estado, así como sistemas en cascada. Además, la versión en discreto de este observador adaptable para una clase de sistemas no lineales puede ser también un tópico interesante a estudiar.

Anexo A

Sistemas de Potencia Multimáquinas

A.1 Nomenclatura del Sistema de Potencia Multi-Máquinas

$\delta_i(t)$ ángulo de potencia (del generador i), en p.u.

$\omega_i(t)$ velocidad relativa, en p.u.

$\omega_0 = 2\pi f_0$, velocidad síncrona

P_{m_i} potencia mecánica, en p.u.

$P_{e_i}(t)$ potencia activa, en p.u.

D_i constante de amortiguamiento, en p.u.

H_i constante de inercia, en segundos

$E'_{q_i}(t)$ voltaje transitorio en el eje de cuadratura, en p.u.

$E_{q_i}(t)$ voltaje en el eje de cuadratura, en p.u.

$E_{f_i}(t)$ voltaje de excitación, en p.u.

V_{t_i} voltaje en terminales, en p.u.

T'_{d_i} constante de tiempo transitoria de corto circuito del eje directo, en segundos

X_{d_i} reactancia del eje directo, en p.u.

X'_{d_i} reactancia transitoria del eje directo, en p.u.

B_{ij} elemento ij (i denota la fila y j la columna) de la matriz de susceptancia nodal (simétrica), en los nodos internos, después de eliminar todos los buses físicos, en p.u.

$Q_{ei}(t)$ potencia reactiva, in p.u.

$I_{qi}(t)$ corriente en el eje de cuadratura, en p.u.

$I_{di}(t)$ corriente en el eje directo, en p.u.

Anexo B

Sistemas Discretos

B.1 Equivalencia en Estabilidad Exponencial

La definición formal para la estabilidad exponencial[1] de sistemas discretos esta dada por

Definición B.1 *Decimos que el origen* [2] *del sistema* $\xi(k+1) = F_\tau(k, \xi(k))$ *sujeto a la condición inicial* $\xi(k_0) = \xi_0$ *es exponencialmente estable, si existen constantes* $a_1, a_2 > 0$ *y* $a_3 < 1$ *tal que la solución del sistema* $\xi(k)$ *satisface*

$$\|\xi_0\| \leq a_1 \quad \Rightarrow \quad \|\xi(k)\| \leq a_2 \|\xi_0\| a_3^k \quad \forall k \geq k_0 \geq 0 \tag{B.1}$$

si además (B.1) se satisface para toda $\xi \in \mathbb{R}^n$, *entonces decimos que el origen del sistema es globalmente exponencialmente estable.*

Si comparamos la definición anterior (B.1) con la dada en la sección 3.2 del capítulo 3 (ver (3.2)), notamos que la principal diferencia se presenta en los términos

$$\begin{aligned} \|\xi(k)\| &\leq a_2 \|\xi_0\| e^{-\lambda_\tau (k-k_0)} \\ \|\xi(k)\| &\leq a_2 \|\xi_0\| a_3^k \end{aligned}$$

pero, para la primera expresión tenemos

$$\|\xi(k)\| \leq a_2 \|\xi_0\| e^{-\lambda_\tau (k-k_0)} = a_2 \|\xi_0\| e^{-\lambda_\tau k} e^{\lambda_\tau k_0} = \widetilde{a}_2 \|\xi_0\| e^{-\lambda_\tau k}$$

donde $\widetilde{a}_2 = a_2 e^{\lambda_\tau k_0} =$ constante, por lo tanto

$$a_3^k \approx e^{-\lambda_\tau k} = \left(e^{-\lambda_\tau}\right)^k$$

es decir

$$a_3 \approx e^{-\lambda_\tau} = \text{constante} \tag{B.2}$$

Sin embargo, de la definición A.1 para garantizar estabilidad tenemos $a_3 < 1$, hecho que aplicado a (B.2) resulta en

$$\begin{aligned} e^{-\lambda_\tau} &< 1 \\ \ln\left(e^{-\lambda_\tau}\right) &< \ln(1) \\ -\lambda_\tau &< 0 \end{aligned}$$

[1] véase [49] página 266

[2] Se asumé sin perdida de generalidad que el equilibrio está en el origen

es decir, $\lambda_\tau > 0$, lo que concuerda con el requerimiento para λ_τ dado en la definición 3.1.

B.2 Propiedades de las Matrices de Control y Estimación

La estructura de las matrices utilizadas en el capítulo 3 para el control y la estimación están dadas por (3.4,3.10)

$$\begin{aligned}
\Omega_\rho &= \mathbf{diag}(\rho^n, \cdots, \rho) \quad \text{para} \quad \rho \geq 1 \\
F &= \begin{pmatrix} C_n^0 & \cdots & C_n^{n-1} \end{pmatrix} \quad \text{con} \quad C_n^p = \frac{n!}{(n-p)!p!} \\
\Delta_\theta &= \mathbf{diag}\left(\frac{1}{\theta} \cdots \frac{1}{\theta^n}\right) \quad \text{para} \quad \theta \geq 1 \\
K &= \mathbf{col}\left(C_n^1 \cdots C_n^n\right) \quad \text{con} \quad C_n^p = \frac{n!}{(n-p)!p!}
\end{aligned}$$

Entonces

$$\begin{aligned}
\Omega_\rho A_\tau \Omega_\rho^{-1} &= \begin{pmatrix} \rho^n & 0 & \cdots & 0 \\ 0 & \rho^{n-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \rho \end{pmatrix} \begin{pmatrix} 1 & \tau & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \tau \\ 0 & 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\rho^n} & 0 & \cdots & 0 \\ 0 & \frac{1}{\rho^{n-1}} & \ddots & \vdots \\ \vdots & \vdots & \vdots & 0 \\ 0 & 0 & \cdots & \frac{1}{\rho} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 1 \end{pmatrix} + \tau\rho \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix} = I + \tau\rho A
\end{aligned}$$

$$\Omega_\rho B = \begin{pmatrix} \rho^n & 0 & \cdots & 0 \\ 0 & \rho^{n-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \rho \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \rho \end{pmatrix} = \rho \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = \rho B$$

$$
\begin{aligned}
\Delta_\theta A_\tau \Delta_\theta^{-1} &= \begin{pmatrix} \frac{1}{\theta} & 0 & \cdots & 0 \\ 0 & \frac{1}{\theta^2} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \frac{1}{\theta^n} \end{pmatrix} \begin{pmatrix} 1 & \tau & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \tau \\ 0 & 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} \theta & 0 & \cdots & 0 \\ 0 & \theta^2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \theta^n \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 1 \end{pmatrix} + \tau\theta \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix} = I + \tau\theta A
\end{aligned}
$$

$$
\begin{aligned}
C\Delta_\theta^{-1} &= \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \theta & 0 & \cdots & 0 \\ 0 & \theta^2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \theta^n \end{pmatrix} = \begin{pmatrix} \theta & 0 & \cdots & 0 \end{pmatrix} \\
&= \theta \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix} = \theta C
\end{aligned}
$$

$$
\Delta_\theta B = \begin{pmatrix} \frac{1}{\theta} & 0 & \cdots & 0 \\ 0 & \frac{1}{\theta^2} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \frac{1}{\theta^n} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \frac{1}{\theta^n} \end{pmatrix} = \frac{1}{\theta^n} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = \frac{1}{\theta^n} B
$$

Por lo tanto

$$
\begin{cases} \Omega_\rho A_\tau \Omega_\rho^{-1} = I + \tau\rho A \\ \Omega_\rho B = \rho B \end{cases} \tag{B.3}
$$

$$
\begin{cases} \Delta_\theta A_\tau \Delta_\theta^{-1} = I + \tau\theta A \\ C\Delta_\theta^{-1} = \theta C \\ \Delta_\theta B = \dfrac{1}{\theta^n} B \end{cases} \tag{B.4}
$$

B.3 Resultados Aplicados al Análisis de Estabilidad para Sistemas Discretos

Declaración B.1 [14] *Sea $A_c = I + \gamma_c(A - BF)$, donde las matrices A y B tienen la estructura dada por la forma canonica controlable de Brunovsky y F y C_n^p son definidas como en (3.4). Entonces, para todo $\gamma_c \in (0,1)$, la única matriz simétrica definida positiva P_c que satisface la siguiente ecuación matricial algebraica*

$$A_c^T P_c A_c - P_c = -\gamma_c P_c - \gamma_c(1-\gamma_c)^n F^T F \tag{B.5}$$

está dada por $P_c = N^T N$, donde $N = \Lambda_c E_c$, $\Lambda_c = \textbf{diag}\left(1, (1-\gamma_c)^{\frac{1}{2}}, \cdots, (1-\gamma_c)^{\frac{n-1}{2}}\right)$, además si i y j denotan las filas y las columnas de E_c respectivamente, los elementos de E_c son $E_c(i,j) = C_{n-i}^{j-i}$ para $j \geq i$ y $E_c(i,j) = 0$ en caso contrario.

Declaración B.2 [14] *Sea $A_o = I + \gamma_o(A - KC)$, donde las matrices A y C tienen la estructura dada por la forma canonica controlable de Brunovsky y K y C_n^p son definidas como en (3.10). Entonces, para todo $\gamma_o \in (0,1)$, la única matriz simétrica definida positiva P_o que satisface la siguiente ecuación matricial algebraica*

$$A_o^T P_o A_o - P_o = -\gamma_o P_o - \gamma_o(1-\gamma_o)^n C^T C \tag{B.6}$$

está dada por $P_o = M^T M$, donde $M = \Lambda_o E_o$, $\Lambda_o = \textbf{diag}\left(1, (1-\gamma_o)^{\frac{1}{2}}, \cdots, (1-\gamma_o)^{\frac{n-1}{2}}\right)$, además si i y j denotan las filas y las columnas de E_c respectivamente, los elementos de E_o son $E_o(i,j) = (-1)^{i+j} C_{n-i}^{j-i}$ para $j \geq i$ y $E_o(i,j) = 0$ en caso contrario. [3]

Lema B.1 [30] *Si para un sistema $\xi(k+1) = F_\tau(k, \xi(k))$ existe un $p > 0$ (entero) , $\tau_{\max} > 0, \nu > 0$ y $c_\tau > 0$ proporcional a $\tau^{-1/p}$ tal que para todo $k \geq k_0$, toda $\xi(k_0) = \xi_0$ y todo $\tau \in (0, \tau_{\max})$*

$$\begin{aligned} \max \|\xi(k)\| &\leq \nu\|\xi_0\| \\ \left(\sum_{k=k_0}^{\infty} \|\xi(k)\|^p\right)^{1/p} &\leq c_\tau \|\xi_0\| \end{aligned} \tag{B.7}$$

entonces, existen κ y $\lambda_\tau > 0$ proporcional a τ tal que (3.2) se cumple para todo ξ_0.

[3] Para un análisis más detallado de ambas declaraciones véase [14]

B.4 Función de Lyapunov Discreta

Consideremos la expresión

$$\begin{aligned}\Delta V_2 &= V\left(x\left(k+1\right)\right) - V\left(x_{eq}\left(k+1\right)\right) \\ &= V\left(f_r\left(x\left(k\right),0\right) + p_r\left(x\left(k\right), x_{ref}\left(k\right)\right)\right) - V\left(f_r\left(x\left(k\right),0\right)\right)\end{aligned}$$

que en lo sucesivo por simplicidad escribiremos como

$$\Delta V_2 = V\left(f+p\right) - V\left(f\right) \tag{B.8}$$

Ahora, consideremos la siguiente figura para la expresión (B.8)

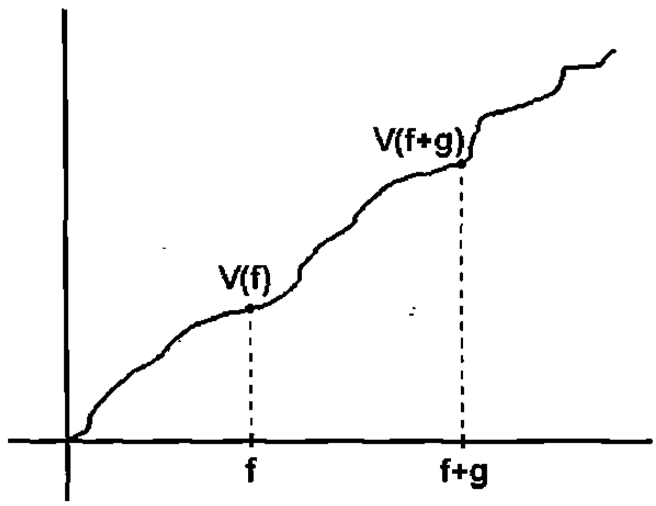

Figura B.1: Teorema de valor medio aplicado a la función de Lyapunov

Por lo tanto, por el teorema de valor medio

$$V'\left(h\right) = \frac{V\left(f+p\right) - V\left(f\right)}{\left(f+p\right) - f}, \qquad h \in \left(f, f+p\right)$$

Entonces ΔV_2 (B.8) podemos expresarla como

$$\begin{aligned}\Delta V_2 &= V\left(f+p\right) - V\left(f\right) = \dot{V}'\left(h\right) p \\ &= \frac{\partial V}{\partial x\left(k\right)}\left(h\right) p\end{aligned}$$

Finalmente, tomando la norma en ambos lados de la expresión resulta

$$
\begin{aligned}
\|\Delta V_2\| &\leq \left\| \frac{\partial V}{\partial x(k)}(h) \right\| \|p\| \\
&\leq l_V \|p\|
\end{aligned}
$$

donde $l_V = \max\limits_{k \geq k_0} \frac{\partial V}{\partial x(k)}(h)$

Anexo C

Observador Adaptable

C.1 Derivada de una Matriz Inversa

$$\begin{aligned} 0 &= \frac{d}{dt}(I) \\ &= \frac{d}{dt}(AA^{-1}) \\ &= \left[\frac{d}{dt}(A)\right]A^{-1} + A\frac{d}{dt}(A^{-1}) \end{aligned}$$

por lo tanto

$$\begin{aligned} A\frac{d}{dt}(A^{-1}) &= -\left[\frac{d}{dt}(A)\right]A^{-1} \\ \frac{d}{dt}(A^{-1}) &= -A^{-1}\left[\frac{d}{dt}(A)\right]A^{-1} \end{aligned} \tag{C.1}$$

C.2 Matriz Inversa para el Sistema Extendido

A continuación procedemos a la deducción de la inversa para la matriz del sistema aumentado (4.20), mediante operaciones de renglón[1]

$$\left(\begin{array}{cc|cc} S_1 & S_2 & I & O \\ S_2^T & S_3 & O & I \end{array}\right) \xrightarrow{S_1^{-1}R_1} \left(\begin{array}{cc|cc} I & S_1^{-1}S_2 & S_1^{-1} & O \\ S_2^T & S_3 & O & I \end{array}\right) \xrightarrow{-S_2^TR_1 + R_2}$$

$$\left(\begin{array}{cc|cc} I & S_1^{-1}S_2 & S_1^{-1} & O \\ O & -S_2^TS_1^{-1}S_2 + S_3 & -S_2^TS_1^{-1} & I \end{array}\right) \xrightarrow{\left(-S_2^TS_1^{-1}S_2 + S_3\right)^{-1}R_2}$$

[1]En lo sucesivo R_i $(i = 1, 2)$ denota el número de reglón

$$\left(\begin{array}{cc|cc} I & S_1^{-1}S_2 & S_1^{-1} & O \\ O & I & -\left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1}S_2^TS_1^{-1} & \left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1} \end{array}\right) \xrightarrow{-S_1^{-1}S_2R_2+R_1}$$

$$\left(\begin{array}{cc|cc} I & O & S_1^{-1}S_2\left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1}S_2^TS_1^{-1}+S_1^{-1} & -S_1^{-1}S_2\left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1} \\ O & I & -\left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1}S_2^TS_1^{-1} & \left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1} \end{array}\right)$$

por lo tanto

$$\begin{aligned} S_{11}^{-1} &= S_1^{-1}S_2\left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1}S_2^TS_1^{-1}+S_1^{-1} \\ &= S_1^{-1}+S_1^{-1}S_2\left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1}S_2^TS_1^{-1} \\ &= S_1^{-1}-S_1^{-1}S_2\left(S_2^TS_1^{-1}S_2-S_3\right)^{-1}S_2^TS_1^{-1} \\ &= S_1^{-1}\left(I-S_2\left(S_2^TS_1^{-1}S_2-S_3\right)^{-1}S_2^TS_1^{-1}\right) \end{aligned} \tag{C.2}$$

y además

$$\begin{aligned} S_{21}^{-1} &= -\left(-S_2^TS_1^{-1}S_2+S_3\right)^{-1}S_2^TS_1^{-1} \\ &= \left(S_2^TS_1^{-1}S_2-S_3\right)^{-1}S_2^TS_1^{-1} \end{aligned} \tag{C.3}$$

Anexo D

Publicaciones

1. Antonio Loria, Jesús de León Morales and Oscar Huerta-Guevara, "On Discrete-time Output-feedback control of Feedback Linearizable Systems", in Proc. of ACC, Denver Colorado, USA, June 2003.

2. Jesús de León Morales, Oscar Huerta-Guevara, Luc Dugard and J. M. Dion, "Discrete-time Nonlinear Control Scheme for Synchronous Generator", in Proc. of 42^{th} IEEE CDC, Maui Hawaii, USA, Dec. 2003.

3. Gildas Besançon, Jesús de León Morales and Oscar Huerta-Guevara, "On Adaptive Observers for State Affine Systems and Application to Synchronous Machines", in Proc. of 42^{th} IEEE CDC, Maui Hawaii, USA, Dec. 2003.

4. Jesús de León Morales, J. M. Dion and Oscar Huerta-Guevara, "Control Design for Multi-machine Power Systems using Continuous Sliding Modes Approach", XI Congreso Latinoamericano de Control Automático, Habana, Cuba, 2004.

On discrete-time output-feedback control of feedback linearizable systems

Antonio Loría[1], Jesús De León Morales[2]*, Oscar Huerta Guevara[2]
[1]C.N.R.S, Laboratoire des Signaux et Systèmes, Supélec,
91192 Gif sur Yvette, France
loria@lss.supelec.fr
[2]Universidad Autónoma de Nuevo León, Facultad de Ingeniería Mecánica y Eléctrica
P. O. Box 148-F, 66450, San Nicolás de Los Garza; Nuevo León, MEXICO.
drjleon@hotmail.com, ohuertag@infosel.com

Abstract

In this paper, we present a control-observer scheme for discrete-time nonlinear systems. A controller and an observer are proposed for a class of discrete-time nonlinear systems. The results obtained are applied to a flexible robot in order to illustrate the proposed scheme.

Keywords: Discrete controller, Euler discretization, Nonlinear Observer, Flexible robot.

1. Introduction

Motivated by the recent advances in digital technology, discrete-time nonlinear systems control theory is receiving an increasing attention in different aspects of control and dynamic systems theory originally developed for continuous-time systems. Such is the case of feedback linearization (see e.g. [1, 10, 11]), passivity-based (cf. [3]), backstepping (cf. [7]). See also [4].

The present paper deals with the problem of observer-based output feedback stabilization of Euler approximate discrete-time systems under the standing assumption that the continuous-time system is feedback linearizable. In particular, we will propose a control scheme which relies on the ability to make that the closed loop system has a cascaded structure. Earlier contributions in this direction include [2].

Our main contribution is an observer-based controller which ensures a form of exponential stability which has a uniform bound on the overshoot of the systems response and a convergence rate which is linear in the sampling period. This specific form of stability is important since only then, one can guarantee that the exact discrete-time and in its turn, the sampled-data systems have certain stability properties. See [8, 9].

*Corresponding Author

2. Problem statement

Notation. Given any symmetric positive definite matrices P, Q we will denote by $\|x\|_P^2 := x^\top P x$ for any $x \in R^n$ and use the constants c_1, c_2 in the relation $c_1\|x\|_P \le \|x\|_Q \le c_2\|x\|_P$. We will use c for a generic positive constant, i.e., we will write with an abuse of notation, $c + c = c^2 = c$. We denote by $\xi(k)$ the solution of the difference equation $\xi(k+1) = F_\tau(k, \xi(k))$ with initial conditions $k_0 \ge 0$ and $\xi_0 = \xi(k_0)$.

We consider feedback linearizable (in continuous time) nonlinear affine systems. We are concerned by the output feedback problem of the Euler discretization of nonlinear systems in the normal form, i.e., we are interested in designing an observer and an output-feedback controller for the Euler-based system

$$\begin{cases} x(k+1) = A_\tau x(k) + \tau B\left\{\alpha(x(k)) + \beta(x(k))u(k)\right\} \\ y = Cx(k) = x_1(k) \end{cases} \tag{2.1}$$

where $A_\tau = (I_n + \tau A) = \begin{pmatrix} 1 & \tau & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \tau \\ 0 & 0 & \cdots & 1 \end{pmatrix}$,

$$A = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}; \qquad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}.$$

We assume that $\beta(x) \neq 0$ for all x.

We address this control problem by designing an exponentially stabilizing observer-based controller for the approximated linearizable system (2.1). More precisely, we will design an observer-based controller which guarantees the following property for the closed-loop system:

Definition 1. (Uniform Exponential Stability) The origin of the system $\xi(k+1) = F_\tau(k, \xi(k))$ is said to be uniformly exponentially stable if there exist r, $\tau_{\max}$, $\kappa > 0$ and for each $\tau \in (0, \tau_{\max}]$, $\lambda_\tau > 0$ such that,

$$\|\xi(k_0)\| \le r \quad \Rightarrow \quad \|\xi(k)\| \le \kappa \|\xi_o\| e^{-\lambda_\tau (k-k_o)} \tag{2.2}$$

$\forall k \ge k_o$. If furthermore (2.2) holds for all $\xi(k_0) \in R^n$ then, the origin is said to be uniformly globally exponentially stable.

The property defined above is probably the most useful for discrete-time systems since it imposes a bound on the overshoots which is uniform in the initial conditions and the sampling time. Moreover, in the particular case when λ_τ is proportional to τ, this property guarantees that the *exact* discrete-time model is (globally) asymptotically practically stable. Roughly speaking, this means that the solutions tend to an arbitrarily small ball whose size is independent of τ and can be made smaller as $\tau_{\max}$ becomes smaller. See [8] for precise definitions and the only formal framework we are aware of, which establishes asymptotic practical stability of exact discrete-time nonlinear systems based on uniform (practical) asymptotic stability of approximate discrete-time systems.

3. Observer-based control

3.1. Control design

Consider the system (2.1) under the action of the static feedback-linearizing control law,

$$u = \beta^{-1}(x)\left[v(x) - \alpha(x)\right], \tag{3.1}$$

where $\alpha(x)$ and $\beta(x)$ are assumed to be known, $\beta(x) \neq 0$ for all $x \in \mathbb{R}^n$, and the external control input $v(x)$ is defined as

$$v(x) = -F\Omega_\rho x \tag{3.2}$$

where the matrices $F \in \mathbb{R}^{1\times n}$ and $\Omega_\rho \in \mathbb{R}^{n\times n}$ are given by

$$\begin{array}{rcl} \Omega_\rho & = & diag\,(\rho^n, \ldots, \rho), \\ F & = & \left(\begin{array}{ccc} C_n^0 & \cdots & C_n^{n-1} \end{array} \right), \end{array} \qquad \rho \ge 1, \tag{3.3}$$

with $C_n^p = \frac{n!}{(n-p)!p!}$. Then, the resulting closed-loop system is

$$x(k+1) = (A_\tau - \tau BF\Omega_\rho)\, x(k). \tag{3.4}$$

The following result is useful to establish our main result.

Lemma 1. *There exists $\tau_{\max} > 0$ sufficiently small such that the system (2.1) in closed-loop with (3.1),(3.2), (3.3) is uniformly globally exponentially stable with λ_τ proportional to $\tau \in (0, \tau_{\max})$ and for all $\rho > 0$ such that $\rho\tau_{\max} \in (0,1)$.*

The proof of this Lemma is based on the following statement and is omitted here for lack of space.

Claim 1 ([6]). *Let $A_c = I + \gamma_c(A - BF)$ where the matrices A and B are in the usual Brunovsky controllable form, and F and C_n^p are defined as (3.3). Then, for every $\gamma_c \in (0,1)$, the unique symmetric positive definite matrix P_c satisfying the algebraic equation*

$$A_c^T P_c A_c - P_c = -\gamma_c P_c - \gamma_c (1-\gamma_c)^n F^T F$$

is given by $P_c = N^T N$, where $N = \Lambda_c E_c$, $\Lambda_c = diag(1, (1-\gamma_c)^{\frac{1}{2}}, ..., (1-\gamma_c)^{\frac{n-1}{2}})$ and, letting i and j denote the rows and columns of E_c respectively, the elements of E_c are $E_c(i,j) = C_{n-i}^{j-i}$ for $j \ge i$ and $E_c(i,j) = 0$ otherwise.

3.2. Observer design

In this section we introduce an observer for the class of systems *(2.1)* which belongs to the class of systems with a triangular structure. This property of the nonlinearity is important because it ensures the uniform observability of the system.

An observer for the transformed system (2.1) is given by

$$\begin{array}{rcl} z(k+1) & = & A_\tau z(k) + \tau B\left[\alpha(z(k)) + \beta(z(k))\,u(k)\right] \\ & & + \tau\Delta_\theta^{-1} K\left[y(k) - \widehat{y}(k)\right] \end{array} \tag{3.5}$$

where

$$\begin{array}{rcl} \Delta_\theta & = & diag\left(\begin{array}{ccc} \frac{1}{\theta} & \cdots & \frac{1}{\theta^n} \end{array} \right) \quad \text{for} \quad \theta \ge 1, \qquad (3.6) \\ K & = & col\left(\begin{array}{ccc} C_n^1 & \cdots & C_n^n \end{array} \right) \quad \text{with} \quad C_n^p = \dfrac{n!}{(n-p)!p!}. \end{array}$$

The term $\tau\Delta_\theta^{-1}K$ represents the observer gain.

Defining the estimation error as $e = z - x$, it follows that the dynamics of the estimation error is of the form

$$\begin{array}{rcl} e(k+1) & = & \left\{A_\tau - \tau\Delta_\theta^{-1}KC\right\} e(k) \\ & & + \tau B\Psi_o^\tau(e(k), x(k), u(k)) \end{array} \tag{3.7}$$

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where

$$\Psi_o^\tau(e,x,u) := [\alpha(e+x)-\alpha(x)+(\beta(e+x)-\beta(x))\,u].$$

In order to make a statement on the stability of the observer we need the following hypothesis.

Assumption A. The function Ψ_o along the trajectories of (2.1) and (3.7), driven by any admissible control input $u(k)$ satisfy

$$\|B\Psi_o^\tau(e(k),x(k),u(k))\| \le l_1\,\|e(k)\|\,,$$

$\forall k \ge k_0 \ge 0\,, \quad \forall \tau \in (0,\tau_{\max})\,.$

Remark 1. Notice that this assumption holds for instance if, for each compact $\mathcal{X}$, and defining $\mathcal{U}_\tau := \{u \in R^n : u = \beta^{-1}(x)\,[v(x)-\alpha(x)]\,,\, x \in \mathcal{X}\}$ there exists $l_1 > 0$ such that $\|B\Psi_o^\tau(e,x,u)\| \le l_1\,\|e\|$, $x(k) \in \mathcal{X}$ and $u(k) \in \mathcal{U}_\tau$ for all $\tau \in (0,\tau_{\max})$ and all $k \ge k_0 \ge 0$.

Lemma 2. *Assume that the system (2.1) satisfies assumption* **A**. *Then, there exist $\tau_{\max} > 0$ sufficiently small and $\theta_{\min} > 0$ sufficiently large such that the estimation error dynamics (3.7) is uniformly globally exponentially stable with λ_τ proportional to $\tau \in (0,\tau_{\max})$, for all $\theta > \theta_{\min}$ such that $\theta_{\min}\tau_{\max} \in (0,1)$.*

The proof of this Lemma is based on the following claim which is the dual of Claim 1.

Claim 2 ([6]). *Let $A_o = I + \gamma_o(A-KC)$ where K is defined as in (3.6). Then for every $\gamma_o \in (0,1)$, the unique symmetric positive definite matrix P_o satisfying the algebraic equation,*

$$A_o^T P_o A_o - P_o = -\gamma_o P_o - \gamma_o(1-\gamma_o)^n C^T C,$$

is given by $P_o = M^T M$ where $M = \Lambda_o E_o$, $\Lambda_o = diag(1,(1-\gamma_o)^{\frac{1}{2}},\ldots,(1-\gamma_o)^{\frac{n-1}{2}})$ and, letting i and j denote the rows and columns of E_o respectively, the elements of E_o, are $E_o(i,j) = (-1)^{i+j}C_{j-1}^{i-1}$ for $i \le j \le n$ and $E_o(i,j) = 0$ otherwise.

3.3. Main result

We can now establish the following result.

Theorem 1. *Consider the discretized nonlinear system*

$$\begin{aligned} x(k+1) &= A_\tau x(k) + \tau B\,\{\alpha(x(k)) + \beta(x(k))u(k)\} \\ y(k) &= Cz(k) \end{aligned}$$

under Assumption A. Then the observer-based output feedback control law,

$$\begin{aligned} z(k+1) &= A_\tau z(k) + \tau B\,[\alpha(z(k)) + \beta(z(k))\,u(k)] \\ &\quad + \tau\Delta_\theta^{-1}K\,[y(k) - \hat{y}(k)] \\ u(k) &= \beta^{-1}(z(k))\,[-F\Omega_\rho z(k) - \alpha(z(k))]\,, \end{aligned}$$

renders the equilibrium $(x,z) = (0,0)$ of the closed-loop system (2.1), (3.1)-(3.3), (3.5)-(3.6) uniformly exponentially stable.

Proof: The result follows if and only if the origin of the estimation error and the observe dynamics, $(e,z) = (0,0)$, is exponentially stable. In view of Lemma 2, we only need to prove that the origin of the observer dynamics under the control action,

$$\begin{aligned} z(k+1) &= A_\tau z(k) + \tau B\,[\alpha(z(k)) + \beta(z(k))\,u(k)] \\ &\quad + \tau\Delta_\theta^{-1}K\,[y(k) - \hat{y}(k)]\,, \end{aligned} \tag{3.8}$$

is uniformly globally exponentially stable.

To prove this, we will invoke the following result.

Lemma 3. If for a system $\xi(k+1) = f_\tau(k,\xi(k))$ there exist $p > 0$, $\tau_{\max} > 0$, $\nu > 0$ and c_τ proportional to $\tau^{-1/p}$ such that for all $k \ge k_o$, all $\xi(k_o) = \xi_o$ and all $T \in (0,\tau_{\max})$,

$$\max_{k \ge k_o} \|\xi(k)\| \le \nu\|\xi_o\| \tag{3.9}$$

$$\left(\sum_{k=k_o}^{\infty} \|\xi(k)\|^p\right)^{1/p} \le c_\tau\|\xi_o\| \tag{3.10}$$

then, there exist κ and $\lambda_\tau > 0$ proportional to τ such that (2.2) holds for all $\xi_0 \in R^m$. □

Hence, we proceed to compute the bounds (3.9), (3.10) with $\xi := \mathrm{col}[e,z]$. We start with the bounds for $\|e(k)\|$. From Lemma 2, it follows that $\|\varepsilon(k)\|_{P_o} \le \|\varepsilon(k_0)\|_{P_o} e^{-\delta\tau(k-k_0)}$, *i.e.* $\|\varepsilon(k)\|_{P_o} \le \|\varepsilon(k_0)\|_{P_o}$ and therefore, there exists $c > 0$ such that

$$\|e(k)\| \le c\|e(k_0)\| \qquad \forall\, k \ge k_0\,. \tag{3.11}$$

Also from Lemma 2, we obtain $\Delta V_{\varepsilon_k} \le -\tau 2\delta V_{\varepsilon_k}$, then evaluating the sum from k_0 to ∞ on both sides of $\Delta V_{\varepsilon_k} \le -\tau 2\delta V_{\varepsilon_k}$, it follows that

$$V_{\varepsilon_{k_0}} \ge -\sum_{k=k_0}^{\infty} \Delta V_{\varepsilon_k} \ge \sum_{k=k_0}^{\infty} \tau\delta\,\|\varepsilon(k)\|_{P_o}^2$$

so using the equivalence of the norms $\|\cdot\|$ and $\|\cdot\|_{P_o}$ we conclude that there exists $c > 0$ such that

$$\left(\sum_{k=k_0}^{\infty} \|e(k)\|^2\right)^{1/2} \le \frac{c}{\sqrt{\tau\delta}}\,\|e(k_0)\|\,. \tag{3.12}$$

Next, we proceed to compute similar bounds for $z(k)$. To this end, reconsider the observer dynamics under the control action, and under the coordinate transformation $\eta = \Omega_\rho z$, *i.e.*,

$$\begin{aligned} \eta(k+1) &= (I_n + \tau\rho(A-BF))\,\eta(k) \\ &\quad + \tau\Omega_\rho\Delta_\theta^{-1}KC\Delta_\theta^{-1}\varepsilon(k) \\ &= A_c\eta(k) + \tau\Omega_\rho\Delta_\theta^{-1}KC\Delta_\theta^{-1}\varepsilon(k) \end{aligned} \tag{3.13}$$

where A_c is defined in Claim 1 with $\gamma_c = \tau\rho$. Define $V_\sigma = \sigma^T P_c \sigma$ then, we get that the difference equation $\Delta V_{\sigma_k} = V_{\sigma_{k+1}} - V_{\sigma_k}$ along the trajectories of $\sigma(k+1) = A_c\sigma(k)$ yields

$$\begin{aligned}\Delta V_{\sigma_k} &= V_{\sigma_{k+1}} - V_{\sigma_k}\\ &= \sigma^T(k)\left[A_c^T P_c A_c - P_c\right]\sigma(k).\end{aligned}$$

It is easy to see from Claim 1 with $\gamma_c = \tau\rho$, that

$$\begin{aligned}\Delta V_{\sigma_k} &= -\tau\rho\sigma^T(k)P_c\sigma(k)\\ &\quad -\tau\rho(1-\tau\rho)^n\sigma^T(k)F^TF\sigma(k)\\ \Delta V_{\sigma_k} &\leq -\tau\rho\,\|\sigma(k)\|_{P_c}^2\end{aligned}$$

Using this bound we now evaluate the difference equation $\Delta V_{\eta_k} = V_{\eta_{k-1}} - V_{\eta_k}$ where $V_{\eta_k} = \eta(k)^T P_c \eta(k)$ along the trajectories of (3.13) to obtain

$$\begin{aligned}\Delta V_{\eta_k} &= V_{\eta_{k-1}} - V_{\eta_k}\\ &\leq -\tau\rho\,\|\eta(k)\|_{P_c}^2 + \tau^2N^2\,\|\varepsilon(k)\|_{P_c}^2\\ &\quad +2\tau N\,\|\varepsilon(k)\|_{P_c}\,\|\eta(k)\|_{P_c}\\ &\leq -\tau(\rho-1)\,\|\eta(k)\|_{P_c}^2 + \tau N^2\,\|\varepsilon(k)\|_{P_c}^2\end{aligned}$$

where we defined $N := \|\Omega_\rho\Delta_\theta^{-1}KC\Delta_\theta^{-1}\|$. Evaluating the sum from k_0 to ∞ on both sides of the inequality above, and using (3.12) we obtain that

$$\begin{aligned}\sum_{k=k_0}^{\infty}\Delta V_{\eta_k} &\geq \tau\sum_{k=k_0}^{\infty}\left\{(\rho-1)\,\|\eta(k)\|_{P_c}^2 - cN^2\,\|\varepsilon(k)\|^2\right\}\\ &\geq \tau(\rho-1)\sum_{k=k_0}^{\infty}\|\eta(k)\|_{P_c}^2 - cN^2\frac{\|\varepsilon(k_0)\|^2}{\delta}\end{aligned}$$

which implies that

$$\sum_{k=k_0}^{\infty}\|\eta(k)\|^2 \leq \frac{c}{\tau(\rho-1)}\left(\|\eta(k_0)\|^2 + \frac{N^2}{\delta}\,\|\varepsilon(k_0)\|^2\right)$$

hence, setting $\rho_{\min} > 1$ and since $\eta = \Omega_\rho z$, we finally obtain that

$$\left(\sum_{k=k_0}^{\infty}\|z(k)\|^2\right)^{1/2} \leq \frac{c}{\sqrt{\tau}}\,\|\xi(k_0)\| \tag{3.14}$$

where it is clear that c is independent of τ. To determine the last bound, we recall that

$$\Delta V_{\eta_k} \leq -\tau(\rho-1)\,V_{\eta_k} + \tau N^2\,\|\varepsilon(k)\|_{P_c}^2\,.$$

Then, using $\|\varepsilon(k)\|_{P_o} \leq \|\varepsilon(k_0)\|_{P_o} e^{-\delta\tau(k-k_0)}$, we obtain that

$$\Delta V_{\eta_k} \leq -\tau(\rho-1)\,V_{\eta_k} + \tau cN^2\,\|\varepsilon(k_0)\|^2\,e^{-2\delta\tau(k-k_0)}.$$

To show contradiction, assume that $V_{\eta_k} \to \infty$ as $k \to \infty$. From the above we see that there exists $k^* > 0$, such that $\Delta V_{\eta_k} \leq 0$, which implies that $\|\eta(k)\|^2 \leq c\,\|\eta(k_0)\|^2$ for all $k \geq k^*$. On the other hand, $\|\eta(k)\|^2 \leq c\,\|\eta(k_0)\|^2 + k^*N\tau_{\max}\,\|\varepsilon(k_0)\|^2$ for all $k < k^*$. Therefore, $\|\eta(k)\|^2 \leq c\,\|\eta(k_0)\|^2 + ck^*N\tau_{\max}\,\|\varepsilon(k_0)\|^2$ for all $k \geq k_0$. We conclude that there exists $c > 0$ independent of τ such that

$$\|z(k)\| \leq c\|\xi(k_0)\| \qquad \forall k \geq 0\,. \tag{3.15}$$

From the bounds (3.11), (3.12), (3.14), (3.15), and invoking Lemma 3 with $\nu = c$, $p = 2$ and $c_\tau := c\left(\max\{\frac{1}{\tau\delta}\,\frac{1}{\tau}\}\right)^{1/2}$ (which is obviously proportional to $\tau^{-1/2}$), we conclude that there exist $\kappa > 0$ and λ_τ, proportional to τ, such that (2.2) holds. ■

4. Application to a flexible-joint robot

We apply the results developed above to the control of the flexible-joint robot. The dynamic equations of a single link robot arm with a revolute elastic joint rotating in a vertical plane are given by

$$\begin{aligned}J_l\ddot{q}_1 + F_l\dot{q}_1 + k(q_1 - q_2) + mgl\sin(q_1) &= 0\\ J_m\ddot{q}_2 + F_m\dot{q}_2 - k(q_1 - q_2) &= u\\ y &= q_1\end{aligned}$$

in which q_1 and q_2 are the link displacement and the rotor displacement, respectively. The link inertia J_l, the motor rotor inertia J_m, the elastic constant k, the link mass m, the gravity constant g, the center of mass l and the viscous friction coefficients F_l and F_m are positive constant parameters. The control u is the torque delivered by the motor. Assuming that only q_1 is measured, u is to be designed so that q_1 tracks a desired reference $q_{r1}(t)$ where the parameters are assumed to be known. Defining the state variables,

$$\xi_1 = q_1, \quad \xi_2 = \dot{q}_1, \quad \xi_3 = q_2, \quad \xi_4 = \dot{q}_2,$$

the model in state-space form is

$$\begin{aligned}\dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= -\frac{F_l}{J_l}\xi_2 - \frac{mgl}{J_l}\sin(\xi_1) - \frac{k}{J_l}(\xi_1 - \xi_3)\\ \dot{\xi}_3 &= \xi_4\\ \dot{\xi}_4 &= -\frac{F_m}{J_m}\xi_4 - \frac{k}{J_m}(\xi_1 - \xi_3) + \frac{1}{J_m}u.\end{aligned} \tag{4.1}$$

4.1. Control design

The system (4.1) is state-feedback linearizable by means of the change of coordinates (cf. [5])

$$x_1 = \xi_1,$$

$$
\begin{aligned}
x_2 &= \xi_2 \\
x_3 &= -\frac{F_l}{J_l}\xi_2 - \frac{mgl}{J_l}\sin(\xi_1) - \frac{k}{J_l}(\xi_1 - \xi_3) \\
x_4 &= \frac{F_l^2}{J_l^2}\xi_2 + \frac{F_l mgl}{J_l^2}\sin(\xi_1) \\
&\quad + \frac{F_l k}{J_l^2}(\xi_1 - \xi_3) - \frac{k}{J_l}(\xi_2 - \xi_4) \\
&\quad - \frac{mgl}{J_l \tau}\sin(\xi_1) + \frac{mgl}{J_l \tau}\sin \xi_1
\end{aligned}
$$

and feedback

$$u = \beta^{-1}[v(x) - \alpha(x)]$$

where $\beta(x) = \tau \frac{k}{J_l J_m}$ and

$$
\begin{aligned}
\alpha(x) &= \left(\frac{mgl}{J_l}\sin x_1 + \frac{F_l mgl}{J_l^2}\cos x_1 + \frac{kF_l{}^2}{J_l}\right) x_2 \\
&\quad + \frac{mgl}{J_l}\cos x_1 + \left(\frac{k}{J_l} - \frac{F_l^2}{J_l^2}\right) \times \\
&\quad \times \left[\frac{F_l}{J_l}x_2 + \frac{mgl}{J_l}\sin x_1 + \frac{x}{J_l}(x_1 - x_3)\right] \\
&\quad - \frac{kF_l}{J_l^2}x_4 + \frac{k}{J_l}\left[\frac{k}{J_m}(x_1 - x_3) - \frac{F_m}{J_m}x_4\right].
\end{aligned}
$$

The external control is given by $v(x) = -F\Omega_\rho x$, where the matrices F and Ω_ρ are defined as in (3.3), *i.e.* $F = \begin{pmatrix} C_4^0 & C_4^1 & C_4^2 & C_4^3 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 6 & 4 \end{pmatrix}$ and $\Omega_\rho = diag(\rho^4, \rho^3, \rho^2, \rho)$.

Then the external control is given by

$$
\begin{aligned}
v(x) &= -F\Omega_\rho x \\
&= -(\rho^4 x_1 + 4\rho^3 x_2 + -6\rho^2 x_3 + 4\rho x_4).
\end{aligned}
$$

4.2. Observer design

According to Section 3.2, the observer is given by

$$
\begin{aligned}
z(k+1) &= A_\tau z(k) + \tau B\{\alpha(z(k)) + \beta(z(k))u(k)\} \\
&\quad + \tau\Delta_\theta^{-1}K[y(k) - \hat{y}(k)]
\end{aligned}
$$

where the observer gain is

$$\tau\Delta_\theta^{-1}K = \tau \begin{pmatrix} 4\theta \\ 6\theta^2 \\ 4\theta^3 \\ \theta^4 \end{pmatrix}$$

with $\Delta_\theta^{-1} = diag(\theta, \theta^2, \theta^3, \theta^4)$;
$K = col\begin{pmatrix} C_4^1 & C_4^2 & C_4^3 & C_4^3 \end{pmatrix} = col\begin{pmatrix} 4 & 6 & 4 & 1 \end{pmatrix}$.

Therefore, the observer becomes

$$
\begin{aligned}
z_1(k+1) &= z_1(k) + \tau z_2(k) + 4\tau\theta(x_1(k) - z_1(k)) \\
z_2(k+1) &= z_2(k) + \tau z_3(k) + 6\tau\theta^2(x_1(k) - z_1(k)) \\
z_3(k+1) &= z_3(k) + \tau z_4(k) + 4\tau\theta^3(x_1(k) - z_1(k)) \\
z_3(k+1) &= z_3(k) + \tau\alpha(z(k)) + \tau\beta(z(k))u(k) \\
&\quad + \tau\theta^4(x_1(k) - z_1(k)).
\end{aligned}
$$

4.3. Simulation results

Numerical simulations were carried out to assess the closed loop responses of a flexible-joint robot using the above observer and controller algorithms was performed for the following numerical values: $m = 0.4$ Kg, $g = 9.81$ m/s^2, $l = 0.185$ m, $J_l = 0.002$ N-ms^2/rad, $J_m = 0.0059$ N-ms^2/rad, $k = 1.61$ N-m-s/rad

The initial conditions for the numerical simulation were selected as follows: $k_0 = 0$, $x(0) = col\begin{pmatrix} 0.1 & 0.2 & 0.03 & 0.04 \end{pmatrix}$ and $z(0) = col\begin{pmatrix} 0.2 & 0.3 & 0.15 & 0.25 \end{pmatrix}$. The sampling period was set to $\tau = 0.0001$. The parameter of the controller gain was set to $\rho = 30$, the parameter design of the observer was chosen as $\theta = 80$ and finally, the reference signal is $q_{r1}(t) = \frac{1}{2}\sin(4t)$. Figures 1-4 illustrate the performance of the proposed scheme.

5. Conclusions

An observer-based controller for feedback linearizable discrete-time nonlinear systems of Euler type was presented. Uniform exponential stability of the closed loop system was established. This allows to conclude on the practical asymptotic stability of the corresponding sampled-data system.

The usefulness and the performance of the proposed scheme was illustrated on the application to a flexible-joint robot. In particular, simulations show the fast convergence of the observer.

References

[1] C. Califano, S. Monaco, and D. Normand-Cyrot. "On the discrete-time normal form." *IEEE Trans. on Automat. Contr.*, 43(11):1654–1658, 1998.

[2] Z-P. Jiang and Y. Wang. "Input-to-state stability for discrete-time nonlinear systems". *Automatica*, 37(6):857–869, 2001.

[3] W. Lin and C. Byrnes. "Passivity and absolute stabilization of a class of discrete-time nonlinear systems". *Automatica*, 31(2):263–267, 1995.

[4] I. M. Y. Mareels, H. B. Penfold, and R. J. Evans. "Controlling nonlinear time-varying systems via euler approximations". *Automatica*, pages 681–696, 1992.

[5] R. Marino and P. Tomei. "Global adaptive output feedback control of nonlinear systems. Part I : Linear parameterization". *IEEE Trans. on Automat. Contr.*, 38:17–32, 1993.

[6] J. De Leon Morales, K. Busawon, and G. Acosta-Villarreal. "Digital implementation of an observer-based controller for rigid robot". In *Proc. 37th. IEEE Conf. Decision Contr.*, Tampa, FL, USA, 1998.

[7] D. Nešić and A. Teel. "Backstepping on the Euler approximate model for stabilization of sampled-data nonlinear systems". In *Proc. 39th. IEEE Conf. Decision Contr.*, San Diego, Ca, 2001.

[8] D. Nešić, A. Teel, and P. Kokotović. "Sufficient conditions for stabilization of sampled-data nonlinear systems via discrete-time approximations". *Syst. & Contr. Letters*, 38:259–270, 1999.

[9] D. Nešić, A. Teel, and E. Sontag. "Formulas relating $\mathcal{KL}$ stability estimates of discrete-time and sampled-data nonlinear systems". *Syst. & Contr. Letters*, 38:49–60, 1999.

[10] H. Nijmeijer and A. J. van der Schaft. *Nonlinear Dynamical Control Systems.* Springer-Verlag, New York, 1990.

[11] C. Simoes, H. Nijmeijer, and J. Tsinias. "Non-smooth stabilizability and feedback linearization of discrete-time sytems". *Int. J. Rob. Nonl. Contr.*, 6:171–188, 1996.

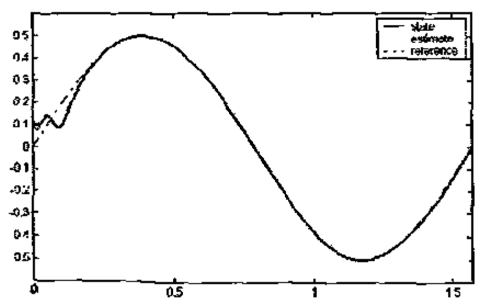

Figure 1. Link displacement.

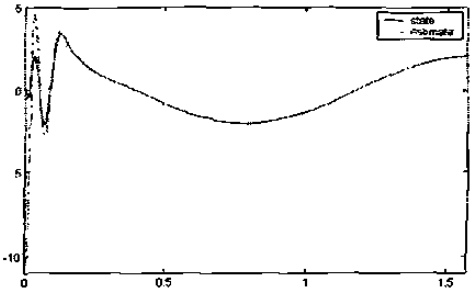

Figure2. Rotor displacement.

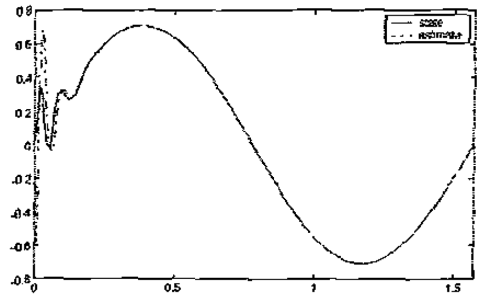

Figure 3. Link velocity.

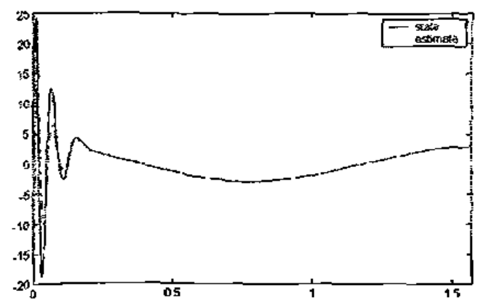

Figure 4. Rotor velocity.

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Discrete-time Nonlinear Control Scheme for Synchronous Generator

J. De León-Morales[1,2], O.Huerta-Guevara[1,2]*, L. Dugard[2] and J. M. Dion[2].
[1]Universidad Autónoma de Nuevo León
Programa Doctoral en Ingeniería
Apdo. Postal 148-F, San Nicolás de Los Garza; N. L 66451. MEXICO
[2]Laboratoire d'Automatique de Grenoble CNRS/INPG UMR 5528
ENSIEG-BP 46, 38402 Saint Martin d'Hères Cedex, FRANCE
Tel. (33) 4 76 82 62 36 - Fax: (33) 4 76 82 63 88
drjleon@hotmail.com, ohuertag@infosel.com, Luc.Dugard@inpg.fr, Jean-Michel.Dion@inpg.fr

Abstract

In this paper, we consider a class of nonlinear systems which are discretized via an Euler discretization procedure. A control design based on sliding-mode techniques is proposed. Furthermore, a discrete-time nonlinear observer is used. The proposed controller-observer scheme is applied to a synchronous generator connected to an infinite bus. Simulations are carried out to show the performance of the controller-observer scheme.

Keywords

Discrete-time systems, Sliding-mode, Nonlinear observer, Synchronous generator.

1. Introduction

The increasing complexity of electric power systems demands more efficient and powerful methods to ensure the control and operation of such systems. One of the strategies to improve the dynamic performance and large disturbance stability of synchronous generators consists in the design of excitation controllers. The main control function of excitation system is to regulate the generator terminal voltage.

Various techniques have been recently investigated to tackle the problem of transient stability by considering nonlinear models (see, for example [1, 3, 6]). Alternatively, the sliding-mode control technique has been extensively used when a robust control scheme is required [2, 8]. Usually these methods are developed for *continuous-time representation*.

However, these controllers are implemented via digital computers, then several different methods have been proposed to design digital controllers for continuos-time plants. One approach, which sometimes is refered to as the emulation method, considers a continuous-time plant model for which a continuous-time controller is designed, then the controller is discretized and implemented using sampler and hold devices. A second approach a discrete-time controller is designed using an exact disctre-time model of the plant. However, it is well-known that to obtain the exact discrete-time model is not evident. Instead, an approximated discrete-time model can be obtained using some numerical integration scheme. One of the simplest schemes is the Euler discretization. Furthermore, taking into account the new results which guarantee, under suitable conditions [7], that if a controller stabilizes an approximate (Euler) discrete-time model then for sufficiently small sampling periods the same controller will stabilize the exact discrete-time plant model in semiglobal and practical sense.

On the other hand, when all states of a control system are not available for feedback, an observer is necesary. In the nonlinear continuous-time case, several results have been proposed (see[5]). For discrete-time nonlinear systems this problem remains open, and some results have been proposed. In this paper, we present an observer design for a specific class of discrete-time nonlinear systems considered here.

In this paper, we propose a stabilizing control law based on sliding-mode methodology, which allows to track a rotor angle reference for a synchronous generator. The controller-observer scheme is then applied to the model of a synchronous generator and the overall stability is shown via simulation.

The paper is organized as follows. In Section 1, we introduce some basic notions on the structure of the class of nonlinear systems considered in this paper. The control and observer design proposed in this work are also introduced is given in Section 2 and 3 respectively. In Section 4, the controller-observer scheme is applied to the model of the synchronous generator and numerical simulations are presented. Finally, conclusions are drawn.

*Corresponding Author

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2. Problem setting and definitions

We denote by $\xi(k)$ the solution of the difference equation $\xi(k+1) = F_\tau(k, \xi(k))$ with initial conditions $k_0 \geq 0$ and $\xi_0 = \xi(k_0)$.

We consider the following class of continuous-time nonlinear systems

$$\Sigma_{NLC}: \begin{cases} \dot{\xi} = f(\xi) + g(\xi)u \\ y = h(\xi) \end{cases} \tag{2.1}$$

using the Euler approximation under the assumption of a sufficiently small sampling period,

$$\Sigma_{NLD}: \begin{cases} \xi(k+1) = \xi(k) + \tau\{f(\xi(k)) + g(\xi(k))u(k)\} \\ y(k) = h(\xi(k)) \end{cases} \tag{2.2}$$

where for simplicity we denote $\xi(k) = \xi(k\tau)$, for τ fixed.

In the sequel, the following definition can be used in order to design a controller and an observer.

Definition 1. Let $\Xi \subset R^n$ be a compact set. The system (2.2) is locally feedback linearizable if there exists a diffeomorphism $\Upsilon: \Xi \to \mathcal{X} \subset R^n$ such that $\mathcal{X} = \Upsilon(\Xi)$ contains the origin and defining $x = \Upsilon(\xi)$, the system (2.2) can be transformed into

$$\tilde{\Sigma}_{NLD}: \begin{cases} x(k+1) = A_\tau x(k) + \tau B\{\alpha(x(k)) + \beta(x(k))u(k)\} \\ y = Cx(k) = x_1(k) \end{cases} \tag{2.3}$$

where $A_\tau = (I_n + \tau A) = \begin{pmatrix} 1 & \tau & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \tau \\ 0 & 0 & \cdots & 1 \end{pmatrix}$,

$$A = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix},$$

$C = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}$ and τ is the sampling period, see [4].

We will address the above mentioned control problem by designing an observer-based controller scheme for the system (2.3). More precisely, we will design an observer for which the following property can be verified:

Definition 2. (Uniform exponential stability) The origin of the system $\xi(k+1) = F_\tau(k, \xi(k))$ is said to be uniformly exponentially stable if there exist r, $\tau_{\max}$, $\kappa > 0$ and for each $\tau \in (0, \tau_{\max})$, $\lambda_\tau > 0$ such that,

$$\|\xi(k_0)\| \leq r \quad \Rightarrow \quad \|\xi(k)\| \leq \kappa\|\xi_o\|e^{-\lambda_\tau(k-k_o)} \quad \forall k \geq k_o. \tag{2.4}$$

If furthermore (2.4) holds for all $\xi(k_0) \in R^n$ then, the origin is said to be *uniformly globally exponentially stable.*

The property defined above is probably the most useful for discrete-time systems since it imposes a bound on the overshoots which are uniform in the initial conditions and the sampling time. Moreover, in the particular case when λ_τ is proportional to τ, this property guarantees that the *exact* discrete-time model corresponding to (2.1) (hence with a discretized control input) is (globally) asymptotically practically stable. Roughly speaking, this means that the solutions tend to an arbitrarily small ball whose size is independent of τ and can be made smaller as $\tau_{\max}$ becomes smaller.

3. Sliding-Mode Control Design

In the sequel, a control design based on sliding mode techniques is proposed. The main idea is to design an asymptotically stabilizing feedback control law assuring the sliding motion on a (n-m) dimensional space $\mathcal{M} \subset R^n$. Consider the following nonlinear discrete-time dynamics

$$\tilde{\Sigma}: \begin{cases} x(k+1) = \mathcal{F}_\tau(x(k)) + \mathcal{G}_\tau(x(k))u(k) \\ y = Cx(k) = x_1(k) \end{cases} \tag{3.1}$$

The objective of the sliding mode control strategy is to steer the states of the system into a (n-m) dimensional manifold $\mathcal{M}$ and to maintain the subsequent motion of the trajectories on $\mathcal{M}$, such that as $k \to \infty$, $x(k) \to 0$.

For this system a sliding mode control is designed by considering the following switching surface

$$\sigma(k) = \mathcal{S}^T\left(x(k) - x_{ref}(k)\right) \tag{3.2}$$

where $\mathcal{S}$ is a vector: $\mathcal{S} = col\left(\mathcal{S}_1 \ ,..., \ \mathcal{S}_n\right)$ and $x_{ref}(k+1) = x_{ref}(k)$ is a constant reference signal. We assume that $\mathcal{S}^T\mathcal{G}(x(k))$ is invertible.

Remark 1:

i) From Definition 1, the system *(2.2)* can be transformed into *(2.3)*, which can be expressed as system *(3.1)* by taking $\mathcal{F}_\tau(x(k)) = A_\tau x(k) + \tau B\alpha(x(k))$ and $\mathcal{G}_\tau(x(k)) = \tau B\beta(x(k))$.

ii) It is clear that there exist others possibilities to define the switching surface. The choice depends on the control objective.

The proposed control is designed in two steps. Firstly, the *equivalent control* $u_e(k)$ is determined when the system motion is restricted to the switching surface $\sigma(k+1) = 0$, so that the control satisfying this sliding condition is given by

$$u_e(k) = \left[\mathcal{S}^T\mathcal{G}_\tau(x(k))\right]^{-1}\left[\mathcal{S}^T\mathcal{F}_\tau(x(k)) - \mathcal{S}^T x_{ref}(k+1)\right]$$

The next step is as follows. A *regulation control* Δu is added in order to satisfy the reaching condition. A necessary and sufficient condition for assuring both sliding motion and convergence onto $\mathcal{M}$ is the discrete-time reaching condition which can be stated as

$$|\sigma(k+1)| < |\sigma(k)|$$

which must be satisfied (see [2]). For that, the switching surface can be chosen as

$$\sigma(k+1) = \eta\mathcal{S}^T\left(x(k) - x_{ref}(k)\right) = \eta\sigma(k) \tag{3.3}$$

where $0 < \eta < 1$ is a scalar weighting value. It is clear that this choice satisfies the *reaching condition*, *i.e.*

$$\eta\,|\sigma(k)| < |\sigma(k)|.$$

Then, the regulation control Δu can be designed as follows

$$\Delta u(k) = \left[\mathcal{S}^T \mathcal{G}_\tau(x(k))\right]^{-1} \left[\eta \mathcal{S}^T \left(x(k) - x_{ref}(k)\right)\right].$$

Finally, the control law is given by

$$u(k) = u_e(k) + \Delta u(k) \tag{3.4}$$

The stability properties of $\sigma(k) = 0$ in (3.3) can be studied by means of the candidate Lyapunov function $V(\sigma(k)) = \sigma^T(k)\sigma(k)$. It follows that

$$\begin{aligned} V(\sigma(k+1)) - V(\sigma(k)) &= \sigma^T(k+1)\sigma(k+1) - \sigma^T(k)\sigma(k) \\ &= -(1-\eta^2)\sigma^T(k)\sigma(k) \end{aligned}$$

or equivalently $V(\sigma(k+1)) = \eta^2 V(\sigma(k))$

$$= \left(\eta^2\right)^k V(\sigma(0)).$$

Hence, $V(\sigma(k+1)) \to 0$ as $k \to \infty$.

To prove the stability of the closed-loop system under control action $u(k)$ it is necessary to introduce the notion of ultimate bound for the solutions of the unperturbed system

$$\xi(k+1) = F_\tau(\xi(k), k) \tag{3.5}$$

where $F_\tau(\xi(k), k) = \xi(k) + \tau f(\xi(k))$, which will be used to study the stability properties of a class of perturbed discrete nonlinear systems when the equilibrium point is affected by a small perturbation in some sense.

Definition 3. *The solutions of system (3.5) are said to be uniformly ultimately bounded if there exist positive constants β_1 and β_2 and for every $r \in (0, \beta_2)$ there is a constant $T = T(r)$, such that*

$$\|\xi(k_0)\| < r \Rightarrow \|\xi(k)\| < \beta_1, \quad \forall k > k_0 + T.$$

The constant β_1 is known as the ultimate bound.

Furthermore, we introduce a result of existence of *the ultimate bound* for the solution of system *(3.5)*.

Consider the following assumptions:

A1. *There exists $\mu > 0$ such that the equilibrium point $\xi = 0$ is uniformly stable on B_μ.*

A2. *There exists a continuous function $V : B_r \times Z_+ \to R$ such that*

$$c_1 \|\xi(k)\|^2 \le V(\xi, k) \le c_2 \|\xi(k)\|^2$$
$$\Delta V(\xi, k) \le -c_3 \|\xi(k)\|^2$$

for $0 < \mu < \sqrt{\frac{c_1}{c_2}}r$, for some positive constants c_1, c_2 and c_3, for all $k > 0$ and for all $\xi \in B_r$.

Theorem 1. *Consider the system (3.5). Assume that A1 and A2 hold. There exists a class $\mathcal{KL}$ function $\varphi(.,.) = \phi(.)\rho(.)$ such that ρ is a function of class $\mathcal{K}$, ρ is a decreasing function and a finite time k_1, depending on $\xi(k_0)$ and μ, such that the solution of (3.5) satisfies*

$$\|\xi(k)\| \le \phi(\|\xi(k_0)\|)\rho(k - k_0)$$

and

$$\|\xi(k)\| \le \sqrt{\tfrac{c_2}{c_1}}\mu, \qquad \forall k \ge k_1$$

for $\|\xi(k_0)\| < \sqrt{\frac{c_1}{c_2}}r$.

Now, the system *(3.1)* under the action of the control (3.4) yields the closed-loop system

$$x(k+1) = f_\tau(x(k), 0) + p_\tau(x(k), x_{ref}(k)) \tag{3.6}$$

where

$$f_\tau(x(k), 0) = \mathcal{F}_\tau(x(k)) + \mathcal{G}_\tau(x(k)) \left[\mathcal{S}^T \mathcal{G}_\tau(x(k))\right]^{-1} \left[\eta \mathcal{S}^T x(k) - \mathcal{S}^T \mathcal{F}_\tau(x(k))\right]$$

and

$$p_\tau(x(k), x_{ref}(k)) = \mathcal{G}_\tau(x(k)) \left[\mathcal{S}^T \mathcal{G}_\tau(x(k))\right]^{-1} \times \left[\mathcal{S}^T x_{ref}(k+1) - \eta \mathcal{S}^T x_{ref}(k)\right]$$

It is clear that the closed-loop system (3.6) can be seen as a system with a unperturbed part, represented by $f_\tau(x(k), 0)$ and a perturbed part given by $p_\tau(x(k), x_{ref}(k))$.

From the boundedness of the columns of $\mathcal{G}_\tau(x(k))$ and the non-singularity of $\mathcal{S}^T \mathcal{G}_\tau(x(k))$, it follows that the perturbed part satisfies the following inequality

$$\|p_\tau(x(k), x_{ref}(k))\| \le l_1 \|x(k)\|^2 + l_2 \|x_{ref}(k)\|^2 \tag{3.7}$$

for $x(k), x_{ref}(k) \in B_r$, where l_1 and l_2 are positive constants.

Now, we consider the following assumptions about the perturbed system:

A3. *The equilibrium point of $x(k+1) = f_\tau(x(k), 0)$, is locally exponentially stable.*

A4. *The reference signal $x_{ref}(k)$ is uniformly bounded and satisfy $\|x_{ref}(k)\| \le b$, for some positive constant b.*

By a converse theorem of Lyapunov, assumption A3 assures the existence of a Lyapunov function $V(x, k)$ which satisfies

$$c_1 \|x(k)\|^2 \le V(x, k) \le c_2 \|x(k)\|^2 \tag{3.8}$$

$$\Delta V_1(x, k) = V(x, k+1) - V(x, k) \le -c_3 \|x(k)\|^2 \tag{3.9}$$

for some positive constants c_1, c_2 and c_3.

Then, the forward difference function $\Delta V(x, k)$ along the trajectories of the closed-loop system is given by

$$\Delta V(x, k) = \Delta\ V_1(x, k) + \Delta V_2(x, k)$$

where

$$\Delta V_1(x, k) = V(f_\tau(x(k), 0), k+1) - V(x, k),$$

and

$$\Delta V_2(x, k) = V(f_\tau(x(k), 0) + p_\tau(x(k), x_{ref}(k)), k+1) - V(f_\tau(x(k), 0), k+1).$$

Furthermore, from assumption A4 and (3.7), the function $\Delta V_2(x, k)$ satisfies the following inequality

$$\begin{aligned} |\Delta V_2(x, k)| &\le l_p \|\pm f_\tau(x(k), 0) + p_\tau(x(k), x_{ref}(k))\| \\ &\le l_p l_1 \|x(k)\|^2 + l_p l_2 \|x_{ref}(k)\|^2 \\ &\le l_p l_1 \|x(k)\|^2 + l_p l_2 b_1^2 \end{aligned}$$

Using the condition (3.9) and the above inequality, we have

$$\Delta V(x, k) \le -(c_3 - l_p l_1) \|x(k)\|^2 + l_p l_2 b^2.$$

If l_1 is sufficiently small such that $l_1 < \tilde{l}_1 < \frac{c_3}{l_p}$ is satisfied. It follows that

$$\Delta V(x,k) \le -a\|x(k)\|^2 + l_p l_2 b^2$$

where $a = (c_3 - l_p \tilde{l}_1)$.

Then, the forward difference function $\Delta V(x,k)$ satisfies

$$\begin{aligned}\Delta V(x,k) &\le -(1-\gamma)a\|x(k)\|^2 - \gamma a\|x(k)\|^2 + l_p l_2 b^2 \\ &\le -(1-\gamma)a\|x(k)\|^2,\end{aligned}$$

for some γ such that $0 < \gamma < 1$ and for all $\|x(k)\| \ge \sqrt{\frac{l_p l_2 b^2}{\gamma a}}$.

It follows that $l_2 \le \frac{\gamma a}{l_p b^2}\|x(k)\|^2$ for $\|x(k)\| < \sqrt{\frac{c_1}{c_2}}r$, and a bound for l_2 is given by $l_2 \le \tilde{l}_2 < \frac{\gamma a}{l_p b^2}\frac{c_1}{c_2}r^2$. From Theorem 1, the ultimate bound of the solution of system (3.6) is given by $B = \sqrt{\frac{c_2}{c_1}}\sqrt{\frac{l_p \tilde{l}_2 b^2}{\gamma a}}$ where the solutions of the slow system satisfy

$$\|x(k)\| < \sqrt{\frac{c_2}{c_1}}\sqrt{\frac{l_p \tilde{l}_2 b^2}{\gamma a}}, \qquad \forall k \ge k_1,$$

for some finite time k_1.

To prove that the closed-loop system is locally ultimately bounded, we have the following lemma.

Lemma 2. *Consider the discrete-time nonlinear system (2.2) for which a control (3.4) is designed. Suppose that assumptions A3 and A4 hold. Then, there exist positive constants $\tilde{l}_1$ and $\tilde{l}_2$ such that, for any initial state $x(k_0)$, the solutions of the closed-loop system (3.6) are ultimately bounded.*

For $x_{ref}(k) = 0, \forall k > k_0$; the following result can be obtained.

Corollary 1: *Consider the discrete-time nonlinear system (2.2) for which a control (3.4) is designed. Suppose that assumption A3 holds. Then, there exists a positive constant $\tilde{l}_1$ such that, for any initial state $x(k_0)$, the solutions of the closed-loop system (3.6) are uniformly exponentially stable.*

4. Observer design

In this section we introduce an observer for the class of systems *(2.3)* which belongs to the class of systems with a triangular structure. This property of the nonlinearity is important because it ensures the uniform observability of the system.

An observer for the transformed system (2.3) is given by

$$\begin{aligned}z(k+1) &= A_\tau z(k) + \tau B[\alpha(z(k)) + \beta(z(k))u(k)] \\ &\quad + \tau\Delta_\theta^{-1}K[y(k) - \widehat{y}(k)] \qquad (4.1)\end{aligned}$$

where

$$\begin{aligned}\Delta_\theta &= diag\left(\ \tfrac{1}{\theta}\ ,\cdots,\ \tfrac{1}{\theta^n}\ \right) \quad \text{for} \quad \theta \ge 1, \qquad (4.2) \\ K &= col\left(\ C_n^1\ ,\cdots,\ C_n^n\ \right) \quad \text{with} \quad C_n^p = \frac{n!}{(n-p)!p!}.\end{aligned}$$

The term $\tau\Delta_\theta^{-1}K$ represents the observer gain.

Defining the estimation error as $e = z - x$, it follows that the dynamics of the estimation error is of the form

$$e(k+1) = \left(A_\tau - \tau\Delta_\theta^{-1}KC\right)e(k) + \tau B\Psi_o^\tau\left(e(k), x(k), u(k)\right) \qquad (4.3)$$

where $\Psi_o^\tau(e,x,u) := [\alpha(e+x) - \alpha(x) + (\beta(e+x) - \beta(x))\,u]$.

In order to make a statement on the stability of the observer we need the following hypothesis.

A5. The function Ψ_o along the trajectories of (2.3) and (4.3), driven by any admissible control input $u(k)$ satisfies

$\|B\Psi_o^\tau(e(k), x(k), u(k))\| \le l_3\|e(k)\|, \quad \forall k \ge k_0 \ge 0$, $\forall \tau \in (0, \tau_{\max})$.

Remark 2. Notice that this assumption holds for instance if, for each compact $\mathcal{X}$, and defining

$\mathcal{U}_\tau := \{u \in R^n : u = \left[S^T\mathcal{G}_\tau(x(k))\right]^{-1} \times \left[S^T x_{ref}(k+1) + \eta S^T\left(x(k) - x_{ref}(k)\right) - S^T\mathcal{F}_\tau(x(k))\right],$ $x \in \mathcal{X}\}$ there exists $l_3 > 0$ such that $\|B\Psi_o^\tau(e,x,u)\| \le l_3\|e\|$, $x(k) \in \mathcal{X}$ and $u(k) \in \mathcal{U}_\tau$ for all $\tau \in (0, \tau_{\max})$ and all $k \ge k_0 \ge 0$.

Lemma 3. *Assume that the system (2.3) satisfies assumption **A5**. Then, there exist $\tau_{\max} > 0$ sufficiently small and $\theta_{\min} > 0$ sufficiently large such that the estimation error dynamics (4.3) is uniformly globally exponentially stable with λ_τ proportional to $\tau \in (0, \tau_{\max})$, for all $\theta > \theta_{\min}$ such that $\theta_{\min}\tau_{\max} \in (0,1)$.*

The proof of this theorem is based on the following claim.

Claim 1. *Let $A_o = I + \gamma_o(A - KC)$ where K is defined as in (4.2). Then for every $\gamma_o \in (0,1)$, the unique symmetric positive definite matrix P_o satisfying the algebraic equation,*

$$A_o^T P_o A_o - P_o = -\gamma_o P_o - \gamma_o(1-\gamma_o)^n C^T C,$$

is given by $P_o = M^T M$ where $M = \Lambda_o E_o$, $\Lambda_o = diag(1, (1-\gamma_o)^{\frac{1}{2}}, \ldots, (1-\gamma_o)^{\frac{n-1}{2}})$ and, letting i and j denote the rows and columns of E_o respectively, the elements of E_o, are $E_o(i,j) = (-1)^{i+j}C_{j-1}^{i-1}$ for $i \le j \le n$ and $E_o(i,j) = 0$ otherwise.

5. Application to the Synchronous Generator

In this section, we apply the previous control and observer design techniques to a synchronous generator. We consider a synchronous generator connected through purely reactive transmission lines to the rest of the network which is represented by an infinite bus, *i.e.* a machine rotating at a synchronous speed ω_s and capable of absorbing or delivering any amount of energy [6]. Such a generator can be modelled as

$$\begin{aligned}&\frac{d\delta}{dt} = \omega - \omega_s \\ &M\frac{d\omega}{dt} = T_m - P_g \\ &T'_{do}\frac{dE'_q}{dt} = -\frac{X_d}{X'_d}E'_q - \left(\frac{X'_d - X_d}{X'_d}\right)V\cos(\delta) + E_{fd}\end{aligned} \qquad (5.1)$$

where $\delta = \angle E_q' - \angle V$ is the generator rotor angle referred to the infinite bus (also called power angle), $\omega = \dot{\delta}$ is the rotor angular speed and E_q' is the stator voltage which is proportional to flux linkages. M is the per unit inertia constant, T_m is the constant mechanical power supplied by the turbine, and T_{do}' is the transient open circuit time constant. $X_d = x_d + x_L$ is the augmented reactance, where x_d is the direct axis reactance and x_L is the line reactance, X_d' is the transient augmented reactance and V is the infinite bus voltage which is fixed. P_g is the generated power while E_{fd} is the stator equivalent voltage given by field voltage v_f.

$$P_g = \frac{1}{X_d'}E_q' V\sin(\delta) + \frac{1}{2}\left(\frac{1}{X_q} - \frac{1}{X_d'}\right)V^2\sin(2\delta),$$
$$E_{fd} = \frac{\omega_s M_f}{\sqrt{2}R_f}v_f$$

where v_f is the scaled field excitation voltage, x_d' is the transient direct axis reactance, x_q is the quadrature axis reactance, M_f is the mutual inductance between stator coils and R_f is the field resistance. We only consider the case where the dynamics of the damper windings are neglected, *i.e.* $D = 0$.

For a given constant field voltage $E_{fd} = E_{fd}^*$, the generator possesses two equilibrium points - one stable and one unstable. Throughout this work, the analysis and design are made around the stable equilibrium point eventhough similar analysis can be made around the unstable equilibrium point. Setting $(\delta^*, \omega^*, E_q'^*)$ as the stable equilibrium point of (5.1), then the system, represented in terms of the deviations variables $\Delta\delta = \delta - \delta^*, \Delta\omega = \omega - \omega^*, \Delta E_q' = E_q' - E_q'^*, u = E_{fd} - E_{fd}^*$ and of the following constants $m_1 = \frac{T_m}{M}$, $m_2 = \frac{-V}{MX_d'}$, $m_3 = \frac{V^2}{M}\left(\frac{1}{X_d'} - \frac{1}{X_q}\right)$, $m_4 = -\frac{X_d}{T_{do}'X_d'}$, $m_5 = -\left(\frac{X_d' - X_d}{T_{do}'X_d'}\right)V$, $m_6 = \frac{1}{T_{do}'}$, is given by

$$\begin{aligned}\frac{d\Delta\delta}{dt} &= \Delta\omega\\ \frac{d\Delta\omega}{dt} &= m_1 + \left\{m_2(\Delta E_q' + E_q'^*) + m_3\cos(\delta)\right\}\sin(\delta)\\ \frac{d\Delta E_q'}{dt} &= m_4(\Delta E_q' + E_q'^*) + m_5\cos(\delta) + m_6(u + E_{fd}^*)\end{aligned} \tag{5.2}$$

where $\delta = \Delta\delta + \delta^*$. Defining the following change of variable $x_1 = \Delta\delta$, $x_2 = \Delta\omega$, $x_3 = \Delta E_q'$, and applying the methodology given in section 2, it follows that the Euler approximate model of the synchronous generator is given by

$$x(k+1) = \mathcal{F}_\tau(x(k)) + \mathcal{G}_\tau(x(k))u(k) \tag{5.3}$$

where $\mathcal{F}_\tau(x(k)) = x(k) +$
$$+\tau\begin{pmatrix} x_2(k)\\ m_1 + \left\{m_2(x_3(k) + E_q'^*) + m_3\cos(\tilde{x}_1)\right\}\sin(\tilde{x}_1)\\ m_4(x_3(k) + E_q'^*) + m_5\cos(\tilde{x}_1(k)) + m_6E_{fd}^*\end{pmatrix}$$
$$\mathcal{G}_\tau(x(k)) = \tau\begin{pmatrix}0\\0\\m_6\end{pmatrix}, \qquad \tilde{x}_1 = x_1(k) + \delta^*.$$

A. Control law design.

In order to regulate the power angle of the generator (5.3), the following switching function was chosen

$$\begin{aligned}\sigma(k) &= \mathcal{S}^T(x(k) - x_{ref}(k)) &(5.4)\\ &= \mathcal{S}_1(x_1(k) - x_{1ref}(k)) + \mathcal{S}_2x_2(k) + \mathcal{S}_3x_3(k)\end{aligned}$$

where $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ are constants that are chosen to satisfy the sliding condition and $x_{1ref}(k)$ is a constant reference signal.

Then the control law is given by
$$u(k) = u_e(k) + \Delta u(k).$$

B. Observer design

Consider the following change of coordinates $x_1 = \Delta\delta$, $x_2 = \Delta\omega$, $x_3 = m_1 + \left\{m_2(\Delta E_q' + E_q'^*) + m_3\cos(\delta)\right\}\sin(\delta)$. Taking the Euler discretization , we obtain

$$\begin{aligned}x_1(k+1) &= x_1(k) + \tau x_2(k)\\ x_2(k+1) &= x_2(k) + \tau x_3(k)\\ x_3(k+1) &= x_3(k) + \tau\{m_4(\Delta E_q'(k) + E_q'^*)\\ &\quad + m_5\cos(\delta) + m_6(u(k) + E_{fd}^*)\}\end{aligned} \tag{5.5}$$

where $m_4E_q'^* + m_5\cos(\delta^*) + m_6E_{fd}^* = 0$.

The dynamic system described in the new coordinates has the following structure

$$\begin{aligned}x(k+1) &= A_\tau x(k) + \tau B\left\{\alpha(x(k)) + \beta(x(k))u(k)\right\}\\ y(k) &= Cx(k)\end{aligned} \tag{5.6}$$

where $\alpha(x(k))$ and $\beta(x(k))$, in the original coordinates, are given by

$$\begin{aligned}\alpha(x(k)) &= m_2\sin(\delta(k) + \tau\Delta\omega(k))E_q'^* + \Delta E_q'(k)\\ &\quad + \tau m_4(\Delta E_q'(k) + E_q'^*) + \tau m_5\cos(\delta(k))\\ &\quad + m_3\cos(\delta(k) + \tau\Delta\omega(k))\sin(\delta(k) + \tau\Delta\omega(k))\\ &\quad - \left(m_2(\Delta E_q'(k) + E_q'^*) + m_3\cos(\delta(k))\right)\sin(\delta(k))\\ \beta(x(k)) &= \tau m_2m_6\sin(\delta(k) + \tau\Delta\omega(k))\end{aligned}$$

Then, an observer for system (5.6) is of the form
$$\begin{aligned}z(k+1) &= A_\tau z(k) + \tau B\left\{\alpha(z(k)) + \beta(z(k))u(k)\right\}\\ &\quad + \tau\Delta_\theta^{-1}K\left[y(k) - \hat{y}(k)\right].\end{aligned}$$

where $K = col\left(\ C_3^1,\ \ C_3^2,\ \ C_3^3\ \right) = col\left(\ 3,\ \ 3,\ \ 1\ \right)$, and the observer gain is given by
$$\tau\Delta_\theta^{-1}K = col\left(\ 3\tau\theta,\ \ 3\tau\theta^2,\ \ \tau\theta^3\ \right).$$

C. Simulation results.

The simulations were done considering the following nominal values of the generator's parameters (per unit) $T_m = 1$; $M = 0.033$; $\omega_s = 1$; $T_{do}' = 0.033$; $X_q = X_d = 0.9$; $X_d' = 0.3$; $V = 1.0$. Furthermore, the stable equilibrium point was obtained from (5.1) for a stator equivalent field equivalent voltage $E_{fd}^* = 1.1773$: $\delta^* = 0.870204$, $\omega^* = 1$, $E_q'^* = 0.822213$. The control and observer parameters are chosen as $\mathcal{S}_1 = 5$, $\mathcal{S}_2 = 2, \mathcal{S}_3 = 2$, $\eta = 0.1$, $\theta = 0.8$ and τ was chosen as $\tau = 0.01$.

Simulations were performed with the proposed discrete-time controller-observer scheme. First, in order to illustrate the performance of the observer, we consider the open-loop case. For this set of simulations, the initial conditions for the generator variables and the estimates were fixed to $\delta(0) = 0.8$, $\omega(0) = 0.1$, $E_q'(0) = 0.8$, $\hat{\delta}(0) = 0.79$, $\hat{\omega}(0) = 0.0$ and $\hat{E}_q'(0) = 0.8$. Figures 1-3 show that the estimates given by the observer converge to the state of the system in open-loop.

In Figures 4, the performance of the observer-controller scheme is shown, where the initial conditions of the system were fixed as : $\delta(0) = 0.77$, $\omega(0) = 0.1$, $E_q'(0) = 0.85$. From this plot, we can see that the power angle converge to the desired reference.

6. Conclusions

In this paper a discrete-time nonlinear controller-observer scheme has been developed and applied to the continuous-time model of a synchronous generator. A tracking control was designed for the generator using the sliding-mode technique. Furthermore, an observer was designed to estimate the internal voltage and the angular speed, assuming that the power angle is available. Simulations results have shown the good performance of the observer-controller scheme.

Acknowledgments

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7. References

[1] A. S. Bazanella, A.S. Silva, P.V. Kokotovick, "Lyapunov design of excitation control for synchronous machines", Proceedings. 36th IEEE CDC, pp. 211-216, San Diego, California, USA, 1997.

[2] R. A. De Carlo, S. H. Zak and G.P. Matthews, "Variable structure control of nonlinear multivariable systems: A tutorial," Proceedings. of the IEEE, Vol. 76, No. 3, pp. 212-233, 1988.

[3] J. De Leon-Morales, K. Busawon, G. Acosta Villarreal, S. Acha Daza. " Nonlinear control for small synchronous generator", Electrical Power and Energy Systems 23 (2001), pp.1-11.

[4] S. Monaco and D. Normand-Cyrot, " Minimum-phase nonlinear discrete-time systems and feedback stabilization", Proceedings. of the 26th Conference on Decision and Control, 979-986, Los Angeles, CA. USA. 1987.

[5] H. Nijmeijer and T.I. Fossen, (Eds) "New Directions in Nonlinear Observer Design", Springer, 1999.

[6] P. W. Sauer and M. A. Pai, "Power System Dynamics and Stability", Prentice Hall, 1998.

[7] D. Nesic, A. R. Teel, P. V. Kokotovic, "Sufficient conditions for stabilization of sampled-data nonlinear systems via discrete-time approximations". Sustems and Control Letters, 38 pp. 259-270, 1999.

[8] V. I. Utkin, Sliding Modes in Control and Optimization, Springer-Verlag, Heidelberg, 1992.

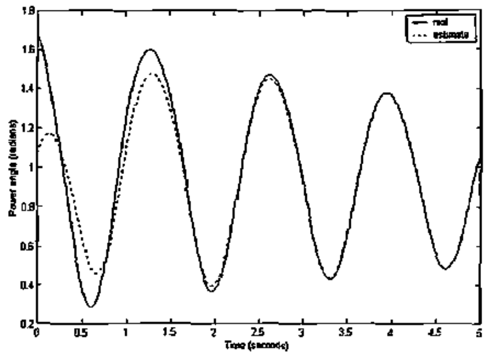

Figure 1. Power angle and its estimate.

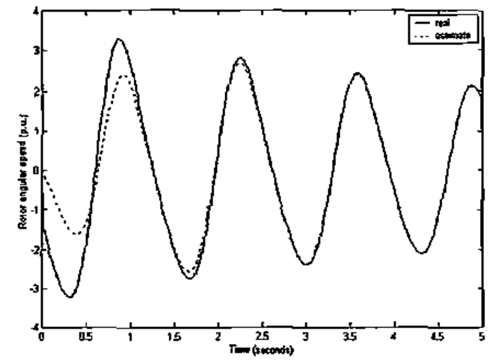

Figure 2. Rotor angular speed and its estimate.

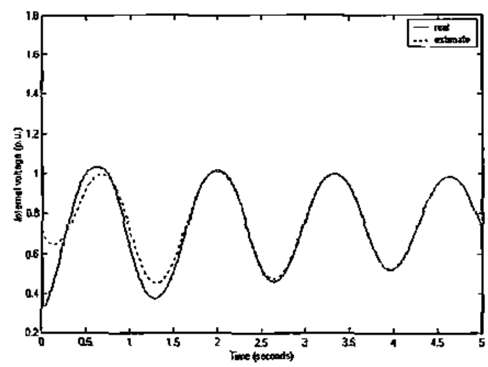

Figure 3. The internal voltage and its estimate.

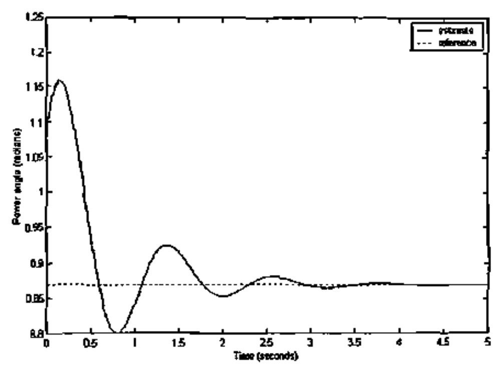

Figure 4. Response of the power angle.

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On adaptive observers for state affine systems and application to synchronous machines [1]

G. Besançon*
Gildas.Besancon@inpg.fr

J. de León-Morales*,†
drjleon@hotmail.com

O. Huerta-Guevara†
ohuertag@infosel.com

* Laboratoire d'Automatique de Grenoble
ENSIEG-BP 46, 38402 Saint Martin d'Hères
Cedex, France

† Universidad Autónoma de Nuevo León
Programa Doctoral en Ingeniería
Apdo. Postal 148-F, San Nicolás de Los Garza;
N. L 66451. Mexico

***Abstract*—A recently proposed adaptive observer for time-varying linear systems [21] is revisited on the basis of the well-known Kalman-like design for state affine systems [13], [4]. This approach in particular allows to emphasize the possible arbitrary rate of convergence in the design. The corresponding observer is applied to a problem of state and parameter estimation for a synchronous machine connected to an infinite bus, and its performances are illustrated in simulation.**

Keywords: State affine systems, adaptive observers, exponential convergence, synchronous machines.

I. INTRODUCTION

The problem of parameter identification has been extensively studied in many aspects during the last decades, including the problem of nonlinear systems, but generally without taking care of lack of state-space measurements. In the same time, the problem of state estimation for nonlinear systems has attracted a growing attention in the control community, and several results have been proposed to tackle this problem.

When dealing with the simultaneous estimation of state variables and constant parameters, the situation becomes more difficult, and the resulting problem of so-called *adaptive observer* has also attracted the attention of various control research groups. In short, an adaptive observer is a recursive algorithm allowing the joint estimation of the state and the unknown parameters in a dynamical system. Different approaches have already been proposed, in particular for linear systems (e.g. as in [7], [14] for early results, and [21] for a recent one), but also for nonlinear ones (see e.g. [2], [15], [5] and references therein).

Such adaptive observers are motivated by purposes of fault detection and isolation, signal transmission and adaptive control for instance. Here we are more particularly interested in such problems in the field of electrical power systems. There has indeed been a growing interest in this field during the last few years [10]. One of the problems in power systems is to preserve stability under changes in operating conditions, in particular due to network disturbances. Several control techniques are already available, but generally assuming that all components of the state vector are measurable and all the parameters are known.Such a situation is most likely not met in practice, and in turn hinders the possibility to apply the corresponding controllers.

The purpose here is thus to take advantage of recent developments in adaptive observers to discuss some algorithm for both state and parameter estimation for a class of nonlinear systems which can in particular be found in power systems: the considered class of systems is that of state-affine systems, and an illustration is given by the case of a synchronous generator connected to a infinite bus. The basic ingredients of our discussion are that of adaptive observer for linear time-varying systems as in [21] on the one hand, and that of state observers for state-affine systems as in [13], [4] on the other hand. Basically we show that by choosing a time-varying gain in the adaptive design of [21] (roughly as in [20]), we end up with an observer which actually corresponds to the well-known Kalman-like design for state-affine systems. This in turn yields a design with arbitrarily tunable rate of convergence. These results are illustrated in simulation for a synchronous machine.

The paper is thus organized as follows: in section II, previous results on adaptive observers for linear time-varying systems on the one hand, and state observers for state affine systems on the other hand, are recalled, highlighting the relationship between the two approaches. As an illustrative application, the case of a synchronous generator model is then considered in section III, where simulation results for the state estimation and simultaneously the identification of the mechanical power, are presented. Finally, some conclusions are given.

II. BACKGROUND RESULTS AND PROPOSED INTERPRETATION

A. Exponential adaptive observer for linear time-varying systems

Let us recall here the basic result of [21] on adaptive observer design for linear time-varying systems of the following form:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) + \Phi(t)\theta \\ y(t) &= C(t)x(t) \end{aligned} \tag{1}$$

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where x, u, y classically denote the state, the input and the measured output vectors respectively, and θ some vector of unknown parameters. A, B, C, Φ are assumed to be known matrices of appropriate dimensions, continuous and uniformly bounded in time.

The main result of [21] can be summarized as follows:

If the following assumptions hold,

(A1) There exists a bounded time-varying matrix $K(t)$ such that: $\dot{\eta}(t) = (A(t) - K(t)C(t))\eta(t)$ is exponentially stable.

(A2) The solution $\Lambda(t)$ of $\dot{\Lambda}(t) = [A(t) - K(t)C(t)]\Lambda(t) + \Phi(t)$ is persistently exciting in the sense that there exist α, β, T such that:

$$\alpha I \leq \int_t^{t+T} \Lambda(\tau)^T C^T \Sigma(\tau) C(\tau) \Lambda(\tau) d\tau \leq \beta I \quad (2)$$

for some bounded positive definite matrix Σ.

Then, the system (3) below is an exponential observer for system (1), in the sense that for any set of initial conditions, $\hat{x}(t) - x(t)$ and $\hat{\theta}(t) - \theta$ exponentially decay to zero:

$$\begin{aligned} \dot{\Lambda}(t) &= [A(t) - K(t)C(t)]\Lambda(t) + \Phi(t) \\ \dot{\hat{x}}(t) &= A(t)\hat{x}(t) + B(t)u(t) + \Phi(t)\hat{\theta}(t) \\ &+ [K(t) + \Lambda(t)\Gamma\Lambda^T(t)C^T(t)\Sigma(t)].[y(t) - C(t)\hat{x}(t)] \\ \dot{\hat{\theta}}(t) &= \Gamma\Lambda^T(t)C^T(t)\Sigma(t)[y(t) - C(t)\hat{x}(t)] \end{aligned} \quad (3)$$

Taking advantage of classical recursive least square algorithms, an adaptation law for the parameter gain Γ of the above observer can obviously be obtained as follows (*e.g.* as in [20]):

$$\dot{\Gamma}(t) = -\Gamma(t)\Lambda^T(t)C^T(t)\Sigma(t)C(t)\Lambda(t)\Gamma(t) + \lambda\Gamma(t), \quad (4)$$

for $\lambda > 0$.

Our purpose here is to discuss such a design at the light of available results on observers for state-affine systems [13], [4].

B. Kalman-like interpretation of the adaptive observer

Let us first recall the result on state observer design for so-called state-affine systems of the following form [13]:

$$\begin{aligned} \dot{x} &= A(u,y)x + \varphi(u,y) \\ y &= Cx \end{aligned} \quad (5)$$

where the components of matrix $A(u,y)$ and vector $\varphi(u,y)$ are continuous functions depending on u and y, uniformly bounded.

The result is as follows:

If the input is persistently exciting, in the sense that there exist α, β, T such that:

$$\alpha I \leq \int_t^{t+T} \Psi_u(t,\tau)^T C^T \Sigma(\tau) C \Psi_u(t,\tau) d\tau \leq \beta I, \quad (6)$$

where Ψ_u denotes the transition matrix for the system $\dot{x}= A(u,y)x$, $y = Cx$, and Σ some positive definite bounded matrix.

Then, an exponential observer for system (5) is given by:

$$\begin{aligned} \dot{\hat{x}} &= A(u,y)\hat{x} + \varphi(u,y) + S^{-1}C^T\Sigma(y - C\hat{x}) \quad (7) \\ \hat{y} &= C\hat{x} \end{aligned}$$

where S is the solution of the equation:

$$\dot{S}= -\rho S - A(u,y)^T S - SA(u,y) + C^T\Sigma C, \ S(0) > 0 \quad (8)$$

for some positive constant ρ sufficiently large.

Defining indeed the estimation error as $e = \hat{x} - x$, the error system is given by:

$$\dot{e}= \left\{A(u,y) - S^{-1}C^T\Sigma C\right\} e \quad (9)$$

and from (6), $V(e) = e^T Se$ is a Lyapunov function for this system satisfying $\dot{V}\leq -\rho V$ [13].

Now, in the case of a system affine in the state and depending on unknown parameters in an affine way, a model can be given as follows:

$$\begin{aligned} \dot{x} &= A(u,y)x + \varphi(u,y) + \Phi(u,y)\theta \quad (10) \\ y &= Cx \end{aligned}$$

where Φ satisfies the same properties as A, φ.

Assuming excitation condition (6) for state estimation on the one hand, and some additional one of the form (2) with $K = S^{-1}C^T$ and S as in (8) for parameter estimation on the other hand, an adaptive observer can be proposed as follows (where S_θ corresponds to Γ^{-1} of (4)):

$$\begin{aligned} \dot{\hat{x}} &= A(u,y)\hat{x} + \varphi(u,y) + \Phi(u,y)\hat{\theta} \quad (11) \\ &+ \left\{\Lambda S_\theta^{-1}\Lambda^T C^T + S_x^{-1}C^T\right\}\Sigma(y - C\hat{x}) \quad (12) \\ \dot{\hat{\theta}} &= S_\theta^{-1}\Lambda^T C^T\Sigma(y - C\hat{x}) \quad (13) \\ \dot{\Lambda} &= \left\{A(u,y) - S_x^{-1}C^T C\right\}\Lambda + \Phi(u,y) \quad (14) \\ \dot{S}_x &= -\rho_x S_x - A(u,y)^T S_x - S_x A(u,y) + C^T\Sigma C \quad (15) \\ \dot{S}_\theta &= -\rho_\theta S_\theta + \Lambda^T C^T\Sigma C\Lambda, \ S_x(0) > 0, S_\theta(0) > 0 \quad (16) \end{aligned}$$

where ρ_x and ρ_θ are sufficiently large positive constants (and Σ is as in (2)).

With $e_x := \hat{x} - x$ and $\epsilon_\theta := \hat{\theta} - \theta$, we indeed get:

$$\begin{aligned} \dot{e}_x &= \left\{A(u,y) - \Lambda S_\theta^{-1}\Lambda^T C^T\Sigma C - S_x^{-1}C^T\Sigma C\right\} e_x \\ &+ \Phi(u,y)\epsilon_\theta \\ \dot{\epsilon}_\theta &= -S_\theta^{-1}\Lambda^T C^T\Sigma C e_x \end{aligned}$$

and following the same idea as in [21], the transformation:

$$\epsilon_x = e_x - \Lambda\epsilon_\theta,$$

yields:

$$\begin{aligned}\dot{\epsilon}_x &= \left\{A(u,y) - \Lambda S_\theta^{-1}\Lambda^T C^T\Sigma C - S_x^{-1}C^T\Sigma C\right\} e_x \\ &+\Phi(u,y)\epsilon_\theta - \dot{\Lambda}\,\epsilon_\theta - \Lambda\,\dot{\epsilon}_\theta\,.\end{aligned}$$

Replacing the suitable expressions in the above equation, we finally get:

$$\dot{\epsilon}_x = \left\{A(u,y) - S_x^{-1}C^T\Sigma C\right\}\epsilon_x \qquad (17)$$

$$\dot{\epsilon}_\theta = -S_\theta^{-1}\Lambda^T C^T \Sigma C\,(\epsilon_x + \Lambda\epsilon_\theta) \qquad (18)$$

Now noting that under the considered excitation conditions, S_x and S_θ are positive definite matrices [4], one can choose:

$$V(\epsilon_x\,,\epsilon_\theta) = \epsilon_x^T S_x\epsilon_x + \epsilon_\theta^T S_\theta\epsilon_\theta$$

as a Lyapunov function. Then, the time derivative of V is given by:

$$\begin{aligned}\dot{V}(\epsilon_x\,,\epsilon_\theta) &= \epsilon_x^T\left\{A(u,y) - S_x^{-1}C^T\Sigma C\right\}^T S_x\epsilon_x \\ &+ \epsilon_x^T S_x\left\{A(u,y) - S_x^{-1}C^T\Sigma C\right\}\epsilon_x \\ &- (\epsilon_x + \Lambda\epsilon_\theta)^T\left\{S_\theta^{-1}\Lambda^T C^T\Sigma C\right\}^T S_\theta\epsilon_\theta \\ &- \epsilon_\theta^T S_\theta\left\{S_\theta^{-1}\Lambda^T C^T\Sigma C\right\}(\epsilon_x + \Lambda\epsilon_\theta) \\ &+ \epsilon_x^T\,\dot{S}_x\,\epsilon_x + \epsilon_\theta^T\,\dot{S}_\theta\,\epsilon_\theta\end{aligned}$$

and substituting the appropriate expressions, we obtain:

$$\begin{aligned}\dot{V}(\epsilon_x\,,\epsilon_\theta) &= -\rho_x\epsilon_x^T S_x\epsilon_x - \rho_\theta\epsilon_\theta^T S_\theta\epsilon_\theta - \epsilon_x^T C^T\Sigma C\epsilon_x \\ &-\epsilon_x^T C^T\Sigma C\Lambda\epsilon_\theta - \epsilon_\theta^T\Lambda^T C^T\Sigma C\epsilon_x \\ &-\epsilon_\theta^T\Lambda^T C^T\Sigma C\Lambda\epsilon_\theta\end{aligned}$$

Since $-\epsilon_x^T C^T\Sigma C\epsilon_x\ \ -\epsilon_x^T C^T\Sigma C\Lambda\epsilon_\theta\ \ -\epsilon_\theta^T\Lambda^T C^T\Sigma C\epsilon_x$ $-\epsilon_\theta^T\Lambda^T C^T\Sigma C\Lambda\epsilon_\theta$
$= -(\epsilon_x + \Lambda\epsilon_\theta)^T C^T\Sigma C\,(\epsilon_x + \Lambda\epsilon_\theta) <\leq 0$, it follows that:

$$\dot{V}(\epsilon_x\,,\epsilon_\theta) \leq -\rho_x\epsilon_x^T S_x\epsilon_x - \rho_\theta\epsilon_\theta^T S_\theta\epsilon_\theta$$

which finally gives:

$$\dot{V}(\epsilon_x\,,\epsilon_\theta) \leq -\rho V(\epsilon_x\,,\epsilon_\theta), \quad \text{for} \quad \rho = \min(\rho_x, \rho_\theta). \qquad (19)$$

As a conclusion, ϵ_x and ϵ_θ exponentially go to zero with a rate driven by ρ, and so does e_x.

Discussion on observer (12)-(16):
First of all, in view of the form of the considered system (10), it is clear that extending the state vector by the vector of constant parameters θ, into $X := \begin{pmatrix} x \\ \theta \end{pmatrix}$, the state affine structure is preserved:

$$\begin{aligned}\dot{X} &= \begin{pmatrix} A(u,y) & \Phi(u,y) \\ 0 & 0 \end{pmatrix} X + \begin{pmatrix} \varphi(u,y) \\ 0 \end{pmatrix} \\ &:= F(u,y)X + G(u,y) \\ y &= \begin{pmatrix} C & 0 \end{pmatrix} X = HX\end{aligned} \qquad (20)$$

Obviously if the condition (6) is satisfied for this extended system, an observer of the form (7) can be designed for X, providing an adaptive observer for the original system.
Now our point is that observer (12)-(16) is actually *the same* as observer (7) for (20):

Proposition 2.1: *The adaptive observer design (12)-(16) for system (10) coincides with observer (7) for system (20) when $\rho_x = \rho_\theta$.*

Proof. Let S denote the solution of Riccati equation (8) for extended system (20), and consider a partition $\begin{pmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{pmatrix}$ corresponding to the partition in x and θ of X (namely S_1 is of same dimensions as A).
Then we can show that for the corresponding initialization, S_x, S_θ, Λ of (12)-(16) are related to S through:

$$\begin{aligned}S_x &= S_1 \\ S_\theta &= S_3 - S_2^T S_1^{-1} S_2 \\ \Lambda &= -S_1^{-1}S_2\end{aligned} \qquad (21)$$

From (8) and (20) indeed, we first have:

$$\dot{S}_1 = -\rho S_1 - A^T(u,y)S_1 - S_1A(u,y) + C^T\Sigma C \qquad (22)$$

$$\dot{S}_2 = -\rho S_2 - A^T(u,y)S_2 - S_1\Phi(u,y) \qquad (23)$$

$$\dot{S}_3 = -\rho S_3 - \Phi^T(u,y)S_2 - S_2^T\Phi(u,y) \qquad (24)$$

and clearly from (22), S_1 satisfies the same equation (15) as S_x (for $\rho_x = \rho$).
By using (22), (23), one can check that:

$$\frac{d}{dt}(S_1^{-1}S_2) = (A(u,y) - S_1^{-1}C^T\Sigma C)(S_1^{-1}S_2) - \Phi(u,y)$$

and thus $-S_1^{-1}S_2$ satisfies the same equation (14) as Γ.
In the same way, durect computations show that from (22)-(24), we get:

$$\begin{aligned}\tfrac{d}{dt}(S_3 - S_2^T S_1^{-1}S_2) &= -\rho(S_3 - S_2^T S_1^{-1}S_2) \\ &\quad +S_2^T S_1^{-1}C^T\Sigma C S_1^{-1}S_2\end{aligned}$$

namely, with $\Lambda = -S_2^T S_1^{-1}$, $S_3 - S_2^T S_1^{-1}S_2$ satisfies the same equation (16) as S_θ (for $\rho_\theta = \rho$).
Finally, the gain in observer (7) is given by $S^{-1}H^T\Sigma$ (with H from (20)), and from matrix manipulation, one can check that S^{-1} takes the following form:

$$S^{-1} = \begin{pmatrix} (S_1 - S_2S_3^{-1}S_2^T)^{-1} & * \\ (S_2^T S_1^{-1}S_2 - S_3)^{-1}S_2^T S_1^{-1} & * \end{pmatrix}$$

i.e.

$$S^{-1}H^T\Sigma = \begin{pmatrix} (S_1 - S_2S_3^{-1}S_2^T)^{-1}C^T\Sigma \\ (S_2^T S_1^{-1}S_2 - S_3)^{-1}S_2^T S_1^{-1}C^T\Sigma \end{pmatrix}$$

By using again some matrix manipulations, one can check that:

$$\begin{aligned}&(S_1 - S_2S_3^{-1}S_2^T)^{-1}C^T\Sigma && (25)\\ =\ & S_1^{-1}(I - S_2S_3^{-1}S_2^T S_1^{-1})^{-1}C^T\Sigma && (26)\\ =\ & S_1^{-1}(I - S_2[S_2^T S_1^{-1}S_2 - S_3]^{-1}S_2^T S_1^{-1})C^T\Sigma && (27)\end{aligned}$$

and

$$\begin{aligned}&(S_2^T S_1^{-1} S_2 - S_3)^{-1} S_2^T S_1^{-1} C^T \Sigma \\ =\ &(S_3 - S_2^T S_1^{-1} S_2)^{-1}[-S_2^T S_1^{-1}] C^T \Sigma\end{aligned} \tag{28}$$

From (21), the term (27) reads as $(S_x^{-1}C^T + \Lambda S_\theta^{-1}\Lambda^T C^T)\Sigma$ which is the gain in the $\hat{x}$ equation (12) of observer (12)-(16), while the term (28) is $S_\theta^{-1}\Lambda^T C^T \Sigma$, namely the gain in the $\hat{\theta}$ equation (13) of observer (12)-(16), and the conclusion follows. □

At this point, we can conjecture that excitation conditions (6) and (2) for system (10) should be equivalent to some condition of the form (6) for the extended system (20).

From the discussion of this section, it results that choosing a time-varying gain as in (4) in the adaptive design (3) leads to the same design as the usual Kalman one for the system extended with the parameter vector. This increases the on-line computation burden, but as a counterpart, from the above computations (e.g. equation (19)), it allows to tune the rate of convergence of the observer for both state and parameter estimation, by simple tuning of ρ_x and ρ_θ (or only ρ).
An example of practical use of such an adaptive observer is presented in next section.

III. Application to synchronous machines

As an illustrative example of adaptive state affine observation, let us consider here the problem of monitoring synchronous machines.

A. Machine model and observer design

Let us consider a nominal flux decay model of a single machine connected to an infinite bus through purely reactive transmission lines to the rest of the network, which is represented by an infinite bus (*i.e.* a machine rotating at constant synchronous speed ω_0 and capable of absorbing or delivering any amount of energy). Such a generator can be modelled by the following differential equations:

Mechanical equation

$$M\, \ddot{\delta} + D\, \dot{\delta} + P_g = P_m$$

Electrical equation

$$T'_{do}\, \dot{E}'_q + \frac{X_d}{X'_d} E'_q = -\left(\frac{X'_d - X_d}{X'_d}\right) V \cos(\delta) + E_{fd}$$

where $\delta = \angle E'_q - \angle V$ is the generator rotor angle referred to the infinite bus (*power angle*), $\omega = \dot{\delta}$ is the *rotor angular speed* and E'_q is the transient voltage (*transient electromagnetic force*). M is the per unit inertia constant, D is the per unit damping constant, P_m is the constant mechanical power supplied by the turbine, and T'_{do} is the open circuit transient time constant. $X_d = x_d + x_L$ is the augmented reactance, where x_d is the direct axis reactance and x_L is the line reactance, X'_d is the transient augmented reactance and V is the infinite bus voltage which is fixed. P_g is the generated power while E_{fd} is the field excitation voltage respectively given by:

$$\begin{aligned} P_g &= \tfrac{1}{X'_d} E'_q V \sin(\delta_m) + \tfrac{1}{2}\left(\tfrac{1}{X_q} - \tfrac{1}{X'_d}\right) V^2 \sin(2\delta_m) \\ E_{fd} &= \tfrac{\omega_s M_f}{\sqrt{2} r_f} v_f, \end{aligned}$$

where v_f is the field excitation voltage, x'_d is the transient direct axis reactance, x_q is the quadrature axis reactance and X_q the quadrature axis augmented reactance, M_f is the mutual inductance between stator and rotor windings, from phase windings to the field winding and r_f is the field resistance. Finally, a state-space model reads as:

$$\dot{\delta} = \omega - \omega_0 \tag{29}$$

$$\dot{\omega} = \frac{\omega_0}{2H} P_m - \frac{\omega_0}{2H}\left(\frac{V}{x'_d}\right) \sin(\delta)\, \mathrm{E}'_q \tag{30}$$

$$-\frac{\omega_0}{2H} V^2 \left(\frac{1}{x_q} - \frac{1}{x'_d}\right) \cos(\delta) \sin(\delta) \tag{31}$$

$$-\frac{D}{2H}(\omega - \omega_0)$$

$$\dot{E}'_q = -\left(\frac{x_d}{T'_{d_0} x'_d}\right) E'_q + \left(\frac{x_d - x'_d}{T'_{d_0} x'_d}\right) V \cos(\delta) + \frac{1}{T'_{d_0}} E_f$$

The equilibrium points of the above system are solutions of:

$$\begin{aligned} \omega^* - \omega_0 &= 0 \\ m_1 - m_2 \sin(\delta^*)\, \mathrm{E}'^{*}_q - & \\ -m_3 \cos(\delta^*) \sin(\delta^*) - m_4(\omega^* - \omega_0) &= 0 \\ -m_5 E'^{*}_q + m_6 \cos(\delta^*) + m_7 E^*_{fd} &= 0 \end{aligned}$$

where the parameters m_i depend on the machine type, the transmission line parameters, the rotor inertia and the infinite bus constant voltage, and which are constant at only one operating point. These constants are *all positive* and are given by:

$$m_1 = \frac{P_m}{M}, m_2 = \frac{V}{M X'_d}, m_3 = \frac{V^2}{M}\left(\frac{1}{X_q} - \frac{1}{X'_d}\right), m_4 = \frac{D}{M},$$

$$m_5 = \frac{X_d}{T'_{do} X'_d},\ m_6 = \left(\frac{X_d - X'_d}{T'_{do} X'_d}\right) V,\ m_7 = \frac{1}{T'_{do}}.$$

For a given constant field voltage $E_{fd} = E^*_{fd}$, the generator possesses two equilibrium points - one stable and one unstable. In what follows, the analysis and design is made around the stable equilibrium point, which we denote by $col[\delta^*, \omega^*, E'^{*}_q]$. Then, the system equations in terms of the set point error variables $\tilde{\delta} = \delta - \delta^*$, $\tilde{\omega} = \omega - \omega^*$,

$\tilde{E}'_q = E'_q - E'^*_q$ and $u = E_{fd} - E^*_{fd}$ are:

$$\dot{\tilde{\delta}} = \tilde{\omega} \tag{32}$$

$$\dot{\tilde{\omega}} = m_1 - m_2 \sin\left(\tilde{\delta} + \delta^*\right)\left(\tilde{\mathrm{E}}'_q + \mathrm{E}'^*_q\right) - \frac{m_3}{2}\sin(2(\tilde{\delta} + \delta^*)) - \mathrm{m}_4\tilde{\omega} \tag{33}$$

$$\dot{\tilde{E}}'_q = -m_5\left(\tilde{E}'_q + E'^*_q\right) + m_6\cos\left(\tilde{\delta} + \delta^*\right) + m_7\left(u + E^*_{fd}\right) \tag{34}$$

Let us assume that the rotor angle is available for measurement, and that ω and E'_q are bounded, while all parameters are known except the mechanical power P_m;*i.e.* the parameter m_1 which represents the acceleration provided by the turbine is assumed to be unknown.
Then with the notations:

$$x_1 = \tilde{\delta}, \qquad x_2 = \tilde{\omega}, \qquad x_3 = \tilde{E}'_q, \qquad \theta = m_1$$

system (32) is of a state affine form (10) as follows:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -m_4 & -m_2\sin(\mathrm{x}_1 + \delta^*) \\ 0 & 0 & -m_5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -m_2\sin(\mathrm{x}_1 + \delta^*)\mathrm{E}'^*_q - \frac{m_3}{2}\sin(2(\mathrm{x}_1 + \delta^*)) \\ -m_5 E'^*_q + m_6\cos(\mathrm{x}_1 + \delta^*) + \mathrm{m}_7(\mathrm{u} + \mathrm{E}^*_{fd}) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\theta$$
$$y = x_1 \tag{35}$$

From this, an observer (12)-(16) can be designed to estimate x and θ, with:
$A(u, y) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -m_4 & -m_2\sin(\mathrm{x}_1 + \delta^*) \\ 0 & 0 & -m_5 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$; $\Phi(u, y) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\varphi(u, y) = \begin{pmatrix} 0 \\ -m_2\sin(\mathrm{x}_1 + \delta^*)\mathrm{E}'^*_q - \frac{m_3}{2}\sin(2(\mathrm{x}_1 + \delta^*)) \\ -m_5 E'^*_q + m_6\cos(\mathrm{x}_1 + \delta^*) + \mathrm{m}_7(\mathrm{u} + \mathrm{E}^*_{fd}) \end{pmatrix}$,
and here with $\Sigma = I$.

B. Simulation results

The purpose here is to illustrate the results obtained with the proposed adaptive observer via digital simulations. The numerical values for the generator parameters (per unit) here considered are:

$X'_d = 0.408$, $X_d = 1.07$, $H = 6.68$, $T'_{do} = 5.4$, $X_l = 0.415$, $E_f = 1.3$, $P_m = 1$ and $\omega_s = 377$.

Under this choice the stable equilibrium position of the generator is:

$\delta^* = 1.12$, $\omega^* = 0$, $E'^*_q = 0.91469$.

The initial value of state variables in all simulations are: $\delta(0) = 1.17$, $\omega(0) = 0.01$ and $E'_q(0) = 0.91$, while the initial conditions of the adaptive observer are $\hat{x}_1(0) = 1.0$, $\hat{x}_2(0) = 0.05$, $\hat{x}_3(0) = 1.0$. The observer gain is initialized at $S_x(0) = I$ for $\hat{x}$, and $S_\theta(0) = 10$ for $\hat{\theta}$, and simulations were performed for various choices of ρ_x and ρ_θ, so as to illustrate the effect of the tuning parameters on the observer performances.

The pictures of Figure 1 depict the dynamical behavior of both real and estimated state variables for $\rho_x = 5$ and $\rho_\theta = 5$, as well as the estimated value for m_1 versus its actual constant value. A good converging performance of the estimated variables can be observed. In the pictures of Figure 2 are shown the responses of each variable under parametric variation of parameter $m_1$in order to emphasize the performance of the adaptive law. From those pictures it can be seen that all state variables are still well estimated, while the parameter estimation tracksits actual value under step changes.

Finally, in order to illustrate how the convergence of the adaptive observer can be affected by parameters ρ_x and ρ_θ, simulation results are shown for different values: it can be seen how for larger values, the convergence speed increases (inducing in turn some overshoots). In figure 3 we show the performance of the adaptive observer for ρ_x and ρ_θ taking the following values: $\rho_x = \rho_\theta = 5, 10, 15$. It is clear that the corresponding time of convergence of the adaptive observer is improved.

IV. Conclusions

The problem of adaptive observer for the class of state affine systems has been discussed at the light of recent results on adaptive observers for time-varying linear systems and available state observers for state affine systems. In particular it has been shown how an adaptive design with a purpose of arbitrarily tunable rate of convergence is equivalent to a state affine observer for an extended system.
The practical interest of such observers has been illustrated in simulation by the example of a single synchronous machine connected to an infinite bus, for which the state vector and some parameter have been jointly estimated.

V. References

[1] A.S. Bazanelle, A.S. Silva, P.V. Kokotovick, "Lyapunov design of excitation control for synchronous machines", Proc. 36th IEEE CDC, pp. 211-216, San Diego, California, USA, 1997.

[2] G. Bastin and M. Gevers, "Stable adaptive observers for nonlinear time vary systems", IEEE Trans. Automatic Control, vol. 37, pp.650-658,1988.

[3] A. Bergen, Power System Analysis, Prentice-Hall, 1986.

[4] G. Besançon, H. Hammouri and G. Bornard "Observer synthesis for a class of nonlinear control systems", European Journal of Control, Vol. 2, No. 3, pp. 176-192, 1996.

[5] G. Besançon, "Remarks on nonlinear adaptive observer design", Systems and Control Letters, Vol. 41, pp. 271-280, 2000.

[6] K. Busawon, J. De Leon Morales and M. Aguilera-Hernandez, "A new high gain observer for uniformly observable systems", IASTED, Austria, Feb. 1999.

[7] R.L. Carol and D.P. Lindorff, "An adaptive observer for single-input single-output linear systems", IEEE Trans. on Automatic Control, Vol. 18 pp. 428-435, 1973.

[8] Chen, C.T., "Introduction to linear system theory", HRW series in Electrical Engineering, Electronics, and Systems, 1970.

[9] J.W. Chapman, M.D. Ilic, C.A. King, L. Eng, H. Kaufman, "Stabilizing a multimachine power system via decentralized feedback linearizing excitation control", IEEE Trans. on Power Systems, Vol. 8, No. 1, pp. 830-839, February 1993.

[10] J. H. Chow, P. Kokotovic, R. J. Thomas, Editors, Systems and Control Theory for Power Systems, IMA Volumes in Mathematics and its Applications, Vol. 64, Springer-Verlag, 1995.

[11] EPRI, "Power-Angle Instrument for large Synchronous Machines", EPRI EL-3667, Final Report, August, 1984.

[12] Hammouri, H. and De Leon J., "On systems equivalence and observer synthesis", New Trends in Systems Theory, 340-347, 1991.

[13] H. Hammouri and J. De León-Morales, 1990. "Observers synthesis for state affine systems", in Proc. of the 29th IEEE Conf. Dec. and Control, pp. 784-785, Honolulu, Hawaii.

[14] G. Kreisselmeier, "Adaptive observers with exponential rate of convergence", IEEE Trans. on Automatic Control, Vol. 22, No. 7, pp. 2-8, 1977.

[15] R. Marino and P. Tomei, "Adaptive observers with arbitrary exponential rate of convergence for nonlinear systems", IEEE Trans. on Automatic Control, Vol. 40, No. 7, pp. 1300-1304, July 1995.

[16] F. P. de Mello, C. Concordia, "Concepts of Synchronous Machine Stability as Affected by Excitation Control", IEEE Trans. on Power Apparatus and Systems, pp. 316-329, April, 1969.

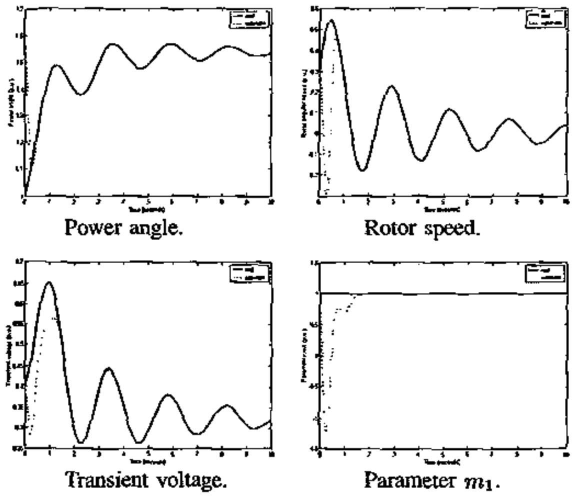
[17] W. Mielczarski, A. M. Zajaczkowzki, "Nonlinear field voltage control of a synchronous generator using feedback linearization", Automatica, Vol. 30, No. 10, pp. 11625-1630, 1994.

[18] M.A. Pai, Energy Function Analysis for Power System Stability, Kluwer Academic Publishers, 1989.

[19] P. W. Sauer, S. Ahmed-Zaid, P. V. Kokotovic, "An integral manifold approach to reduced order dynamic modeling of synchronous machines", IEEE Trans on Power Systems, Vol. 3, No. 1, pp. 17-23, Feb. 1988.

[20] Q. Zhang and A. Clavel, 2001. "Adaptive observer with exponential forgetting factor for linear time-varying systems", in Proc. of the 40th IEEE Conf. Dec. and Control, pp. 3886-3891, Orlando, USA.

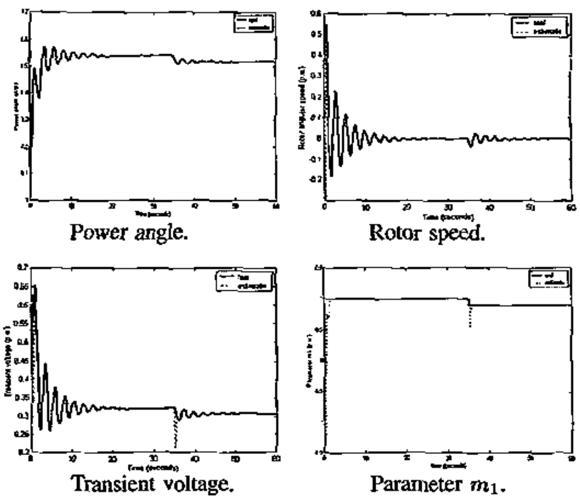
[21] Q. Zhang, "Adaptive observers for MIMO linear time-varying systems", IEEE Trans on Automatic Control, Vol. 47, No. 3, pp. 525-529, 2002.



Power angle. Rotor speed.

Transient voltage. Parameter m_1.

Fig. 1. State variables and parameter m_1 with their estimates.



Power angle. Rotor speed.

Transient voltage. Parameter m_1.

Fig. 2. Variables and estimates under parametric variation.

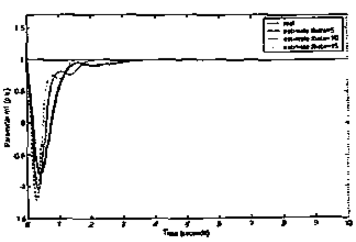


Fig. 3. Sensitivity of the adaptive law ($\rho_x = \rho_\theta = 5, 10, 15$).

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CONTROL DESIGN FOR MULTI-MACHINE POWER SYSTEMS USING A CONTINUOUS SLIDING MODE APPROACH

J. de León-Morales, D.Georges, O. Huerta-Guevara
Contacto: drjleon@hotmail.com.
Universidad Autónoma de Nuevo León, Laboratoire d'Automatique de Grenoble
México, Francia

Resumen:

This paper is concerned with the control of multimachine power systems. We propose a continuous slidingmode control design. The designed controller is smooth: in that sense, it di ers from classical sliding mode controllers subject to chattering phenomena. Two versions of the sliding-mode controller are then applied to the control of a multi-machine power system. The practical implementation of these two controllers leads to a fully decentralized control schemes. Simulations results demonstrate better performances of these two controllers compared to a Hamiltonian passive controller.

keywords
Multi-machine power systems, large-scale decentralized nonlinear control, continuous sliding mode control.

Control Design for Multi-machine Power Systems Using Continuous Sliding Mode Approach[1]

J. de León-Morales[+,2], Didier Goerges[*], Oscar Huerta-Guevara[+]

[+] *Universidad Autónoma de Nuevo León, Facultad de Ing. Mecánica y Eléctrica*
San Nicolás de los Garza, Nuevo León, MEXICO
[*]*Laboratoire d'Automatique de Grenoble CNRS/INPG UMR 5528*
ENSIEG-BP 46, 38402 Saint Martin d'Hères Cedex, FRANCE

Abstract
This paper is concerned with the control of multi-machine power systems. We propose a continuous *sliding*-mode control design. The designed controller is smooth: in that sense, it differs from classical sliding mode controllers subject to chattering phenomena. Two versions of the sliding-mode controller are then applied to the control of a multimachine power system. The practical implementation of these two controllers leads to a fully decentralized control schemes. Simulations results demonstrate better performances of these two controllers compared to a Hamiltonian passive controller.

Keywords: Multi-machine power systems, large-scale decentralized nonlinear control, continuous sliding mode control.

1. Introduction

The stability of an electrical power system (EPS) may be defined as the ability to remain in synchronous operation under normal operating conditions as well as after a disturbance (a default like a short-cut or a change of operating conditions for example).

Ensuring the transient stability under different operating conditions in order to maintain synchronism between generators is an important issue in power system control and we will focus our attention on this problem hereafter.

Excitation control, that is one of the possible actions to maintain transient stability of power systems under disturbed conditions, will be considered in this paper.

The use of advanced control techniques for power system control has been one of the more promising application areas of automatic control. To enhance transient stability of power systems, a great attention has been paid to the application of nonlinear control theory.

To improve the robustness of closed-loop power systems, different approaches based on nonlinear control theory have been proposed; for example, those based on variable structure, singular perturbation methods, control Lyapunov function (Bazanella, 1997), Hamiltonian function method (Masschke et al., 1998, Ortega et al., 1998) or adaptive control.

Recently, port-controlled Hamiltonian systems have been introduced in (Masschke et al., 1998, Ortega et al., 1998). For this class of systems the Hamiltonian function is considered as the total energy and play the role of Lyapunov function for the system. The key feature of this technique is to express the electrical power system dynamics under the form of a port-controlled Hamiltonian representation. This method has already been applied for improving the transient stability of a multi-machine power system by means of decentralized nonlinear excitation control (Xi, 2002).

Another technique for improving robustness under parameter uncertainties and external disturbances is sliding-mode control design which has attracted a number of researches (see De Carlo et al., 1988, Utkin, 1992). It can be viewed as a high-speed switching controller that provides a robust means of controlling nonlinear systems by forcing the trajectories to reach a sliding manifold in finite time and to stay on the manifold for all time.

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[2] corresponding author. e-mail: drjleon@hotmail.com

2. Dynamical model of a multi-machine power system

Now we consider a power system made of n generators. Under some standard assumptions, the motion of the interconnected generators can be described by the classical model with flux decay dynamics. The generator is modeled by the voltage behind direct axis transient reactance. The angle of the voltage coincides with the mechanical angle relative to the synchronous rotating frame. The network has been reduced to internal bus representation. The dynamical model of the i-th machine is represented by (Bergen, 1986, Pai et al., 1989):

$$\begin{aligned} \overset{\bullet}{\delta}_i &= \omega_i - \omega_0 \\ \overset{\bullet}{\omega}_i &= \frac{1}{2H_i}\left(-D_i\left(\omega_i - \omega_0\right) + \omega_0\left(P_{m_i} - P_{e_i}\right)\right) \\ \overset{\bullet}{E}{}'_{q_i} &= \frac{1}{T'_{d_i}}\left(E_{f_i} - E_{q_i}\right) \end{aligned} \tag{1}$$

where

$$\begin{aligned} P_{e_i} &= E'_{q_i} \sum_{j=1, i\neq j}^{n} E'_{q_j} B_{ij} \sin\left(\delta_i - \delta_j\right) \\ E_{q_i} &= E'_{q_i} - \left(X_{d_i} - X'_{d_i}\right) \sum_{j=1, i\neq j}^{n} E'_{q_j} B_{ij} \cos\left(\delta_i - \delta_j\right) \end{aligned}$$

and $\delta_i(t)$ is the power angle of the i-th generator, $\omega_i(t)$ represents the relative speed, $E'_{q_i}(t)$ is the transient EMF in the quadrature axis

We consider that the $E_{f_i}(t)$, $i = 1, ..., n$ are the control inputs and the $\delta_i(t)$ are measurable outputs, together with the P_{e_i} and V_{t_i}, where V_{t_i} represents the terminal voltage at generator i. The P_{m_i} are supposed to be constant (standard assumption).

Then, the state representation of a n-machine power system is given by

$$\begin{aligned} \overset{\bullet}{x}_{i1} &= x_{i2} \qquad i = 1, ..., n \\ \overset{\bullet}{x}_{i2} &= -a_i x_{i2} + b_i - c_i x_{i3} \sum_{j=1}^{n} x_{j3} B_{ij} \sin(x_{i1} - x_{j1}) \\ \overset{\bullet}{x}_{i3} &= -e_i x_{i3} + d_i \sum_{j=1}^{n} x_{j3} B_{ij} \cos(x_{i1} - x_{j1}) + u_i \end{aligned} \tag{2}$$

where $a_i = D_i/2H_i$, $b_i = (\omega_o/2H_i)P_{m_i}$, $c_i = (\omega_o/2H_i)$, $d_i = (X_{d_i} - X'_{d_i})/T'_{d_i}$, $e_i = 1/T'_{d_i}$, are the systems parameters, $[x_{i1}, x_{i2}, x_{i3}]^T = [\delta_i(t), \omega_i(t), E'_{qi}(t)]^T$ represents the state vector, and the control input is given by $u_i = (1/T'_{d_i})k_{ci}u_{fi(t)}$

$$f_i(x) = \begin{bmatrix} x_{i2} \\ -a_i x_{i2} + b_i - c_i x_{i3} \sum_{j=1}^{n} x_{j3} B_{ij} \sin(x_{i1} - x_{j1}) \\ -e_i x_{i3} + d_i \sum_{j=1}^{n} x_{j3} B_{ij} \cos(x_{i1} - x_{j1}) \end{bmatrix}$$

$$g_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

We will now present our controller design based on the idea of continuous sliding mode control. In the same time, we present a controller design based on passivity theory in order to compare the performances of these two methodologies.

3. A continuous sliding-mode controller design

We consider the class of affine nonlinear systems described in the state space by

$$\overset{\bullet}{x} = f(x) + g(x)u, \qquad x(t_0) = x_0, \tag{3}$$

where $t_0 \geq 0$, $x \in B_x \subset R^n$ is the *state vector*, $u \in R^r$ is the *control input vector* , f and g are assumed to be bounded with their components being smooth functions of x. B_x denotes a closed and bounded subset centered at the origin.

The continuous sliding-mode control for the system (3), is designed as follows. Consider the following $(n-r)$-dimensional nonlinear sliding surface defined by

$$\sigma(x - x^*) = (\sigma_1(x - x^*), ..., \sigma_r(x - x^*))^T = 0 \tag{4}$$

where x^* is equilibrium point and each function σ_i : $B_x \times B_x \to R$, $i = 1, ..., r$, is a C^1 function such that $\sigma_i(0) = 0$.

The so-called *equivalent control method* (see De Carlo, 1988, Utkin, 1992) is used to determine the system motion restricted to surface $\sigma(x - x^*) = 0$, leading to the *equivalent control*

$$u_e = -\left[\frac{\partial\sigma}{\partial x} g(x)\right]^{-1} \left[\frac{\partial\sigma}{\partial x} f(x)\right] \tag{5}$$

where the matrix $[\partial\sigma/\partial x]g(x)$ is assumed to be nonsingular for all $x, x^* \in B_x$.

In order to complete the control design an additional control term u_N is added to the control input:

$$u = u_e + u_N \tag{6}$$

where u_e is the equivalent control (5), which acts when the system is restricted to $\sigma(x - x^*) = 0$, while u_N acts when $\sigma(x - x^*) \neq 0$.

The control u_N is selected as

$$u_N = -[\frac{\partial\sigma}{\partial x} g(x)]^{-1} L\sigma(x - x^*) \tag{7}$$

where L is an $r \times r$ positive definite matrix.

We can easily check that the system trajectory $x(t)$ is such that the following stable ordinary differential equation

$$\overset{\bullet}{\sigma}(x - x^*) = -L\sigma(x - x^*) \tag{8}$$

is satisfied for all t. This means that the system trajectory reaches the sliding surface asymptotically, since $\sigma(x(t) - x^*) = e^{-L(t-t_0)}\sigma(x(t_0) - x^*)$, $\forall t_0 > 0$, then, $\sigma(x(t) - x^*) \to 0$, when $t \to +\infty$. In fact, the input-output behavior of the closed-loop system (with the output $y = \sigma(x(t) - x^*)$ is given by equation (8).

On the basis of the continuous sliding-mode control described above, the resulting the composite control is given by

$$u = -\left[\frac{\partial\sigma}{\partial x}g(x)\right]^{-1}\left[\frac{\partial\sigma}{\partial x}f(x) + L\sigma(x - x^*)\right]. \quad (9)$$

When the composite control (9) is applied to (3), one obtains the closed-loop nonlinear system

$$\overset{\bullet}{x} = f_e(x, x^*) + p(x, x^*) \quad (10)$$

where
$f_e(x, x^*) = \left\{I_{n\times n} - g(x)\left[\frac{\partial\sigma}{\partial x}g(x)\right]^{-1}\left(\frac{\partial\sigma}{\partial x}\right)\right\}f(x).$
and
$p(x, x^*) = -g(x)[\frac{\partial\sigma}{\partial x}g(x)]^{-1}L\sigma(x - x^*).$

Now, in order to study the stability properties of the closed-loop system, we introduce the following assumption.

Assumption 1. *The equilibrium point x^* of $\overset{\bullet}{x} = f_e(x, x^*)$ is locally exponentially stable.*

By use of Lyapunov's converse theorem (see Khalil, 1996), Assumption 1 ensures the existence of a Lyapunov function $V(e)$ with $e = x - x^*$ which satisfies the following inequalities

$$\left\|\frac{\partial V(e)}{\partial e}\right\| \leq \alpha_4\|e\|, \quad \alpha_1\|e\|^2 \leq V(e) \leq \alpha_2\|e\|^2$$

$$\frac{\partial V(e)}{\partial e}\{f_e(e + x^*, x^*) + p(e + x^*, x^*)\} \leq -\alpha_3\|e\|^2 \quad (11)$$

for some positive constants $\alpha_1, \alpha_2, \alpha_3$ and α_4.

Let consider $V(e)$ as a Lyapunov function candidate to investigate the stability of the origin $e = 0$ as an equilibrium point for the system (10). From both Assumption 1 and equation (11), the time derivative of V along the trajectories of (10) satisfies

$$\overset{\bullet}{V}(e) \leq -c_3\|e\|^2 \quad (12)$$

then the system (10) is exponentially stable.

The Lyapunov function candidate V is instrumental to investigate the stability properties of the closed-loop system obtained when the composite control u is used. Then the following proposition can be stated.

Proposition 1: *Consider the nonlinear system (3) for which a composite control (5), (6), (7) is designed such that Assumption 1 is satisfied. Then, the closed-loop nonlinear system (10) is locally exponentially stable.*

4. Hamiltonian controller design

Now we derive an excitation controller using the methodology based on the notions of energy function and port-controlled Hamiltonian systems (PCHS).

We onsider the following affine nonlinear system

$$\begin{aligned}\overset{\bullet}{x} &= f(x) + g(x)u \quad (13)\\ y &= h(x)\end{aligned}$$

where $x \in I\!R^n$ is the state vector of the system, $u \in I\!R^m$ is the control vector and $y \in I\!R^p$ is the output vector. In this paper we are interested in the class of systems that can be equivalently represented in a Hamiltonian form with dissipative terms in the following way

$$\begin{aligned}\overset{\bullet}{x} &= (\mathcal{J}(x) - \mathcal{R}(x))\frac{\partial H^T}{\partial x} + g(x)u \quad (14)\\ y &= g^T(x)\frac{\partial H^T}{\partial x}\end{aligned}$$

where x , u, y are the energy variables, $H(x_1, ..., x_n)$: $\mathbb{R}^n \rightarrow \mathbb{R}$ represents the total stored energy and the interconnection structure is captured in the $n \times n$ matrix $\mathcal{J}(x)$ and the $n \times m$ matrix $g(x)$. The matrix $\mathcal{J}(x)$ is skew-symmetric, *i.e.*

$$\mathcal{J}(x) = -\mathcal{J}^T(x), \quad \forall x \in I\!R^n$$

and $\mathcal{R}(x)$ is a non-negative symmetric matrix depending on x, *i.e.*

$$\mathcal{R}(x) = \mathcal{R}^T(x) \geq 0, \quad \forall x \in I\!R^n.$$

The main advantage of this kind of representation is that the total energy function can be considered as a Lyapunov function. Moreover, from (14), we obtain the power-balance equation

$$\frac{dH}{dt} = -\frac{\partial H}{\partial x}\mathcal{R}(x)\frac{\partial H^T}{\partial x} + u^T y$$

with $u^T y$ the power externally supplied to the system and $-\frac{\partial H}{\partial x}\mathcal{R}(x)\frac{\partial H^T}{\partial x}$ representing the energy-dissipation due to the resistive elements. As it is well known (see Maschke et al., 1998), the equality above establishes the passivity properties of the system in the following sense.

Definition 1: *System (13) is passive with respect the output $y = h(x)$ if there exists a smooth non-negative function $H(x)$, such that $H(0) = 0$ and the following inequality holds*

$$H(x(t)) - H(x(0)) \leq \int_0^t u(s)y(s)ds. \quad (15)$$

If in addition, the system satisfies the detectability properties stated in the next definition

Definition 2: *The system (13) is zero-state detectable if $u(t) = 0$, $y(t) = 0$ $\forall t \geq 0$, implies that $\lim_{t\rightarrow\infty} x(t) = 0$.*

Then it is possible to formulate the following result, that is fundamental concerning the stability properties of the considered class of systems.

Theorem 1: *Consider the class of systems defined by (14). Assume that the system is zero-state detectable and that the generalized Hamiltonian has a strict local minimum. Then it follows that x^* is a Lyapunov stable equilibrium point of the unforced dynamics. Moreover, the following output feedback*

$$u = -Fy = -Fg^T(x)\frac{\partial H^T}{\partial x} \qquad (16)$$

renders the equilibrium point asymptotically stable.

5. Application to a multi-machine power system

A three-machine power system is now introduced to demonstrate the effectiveness of the continuous sliding mode controller. In this system, generator 3 is considered as an infinite bus, then generator 3 is used as the reference, *i. e.* $\left(E'_{q3} = const = 1\angle 0°\right)$

The system has the following state-space representation

$$\begin{aligned}
\dot{x}_{11} &= x_{12},\ \dot{x}_{21} = x_{22} && (17)\\
\dot{x}_{12} &= \frac{1}{2H_1}\left[-D_1 x_{12} + \omega_0\left(P_{m_1} - P_{e_1}\right)\right]\\
\dot{x}_{22} &= \frac{1}{2H_2}\left[-D_2 x_{22} + \omega_0\left(P_{m_2} - P_{e_2}\right)\right]\\
\dot{x}_{13} &= \frac{1}{T'_{d_1}}\left(E_{f_1} - E_{q_1}\right),\ \dot{x}_{23} = \frac{1}{T'_{d_2}}\left(E_{f_2} - E_{q_2}\right)
\end{aligned}$$

where $x_{11} = \delta_1$, $x_{21} = \delta_2$, $x_{12} = \omega_1$, $x_{22} = \omega_2$, $x_{13} = E'_{q_1}$, $x_{23} = E'_{q_2}$

5.1 Sliding-mode control design

In this paper we introduce two continuous sliding mode controllers corresponding to two particular choices of the sliding surface:

Sliding-Mode Control 1

We consider the following nonlinear switching surface defined by

$$\sigma(x,x^*) = (\sigma_1(x,x^*), \sigma_2(x,x^*))^T = 0$$

where

$$\sigma_i(x,x^*) = s_{i1}(x_{i1} - x^*_{i1}) + s_{i2}(x_{i2} - x^*_{i2}) + s_{i3}(x_{i3} - x^*_{i3})$$

for $i = 1,2$ and $x^*_i = (x^*_{i1}, x^*_{i2}, x^*_{i3})$, for $i = 1,2$; is an equilibrium point of system (17).

Then, the *equivalent control* is given by

$$\begin{aligned}
u_{e_i} &= -\left[\frac{\partial\sigma_i}{\partial x_i}g_i(x)\right]^{-1}\left[\frac{\partial\sigma_i}{\partial x_i}f_i(x)\right]\\
&= -\frac{1}{s_{i3}}\left\{\begin{array}{c} s_{i1}x_{i2} + s_{i3}\left(\begin{array}{c} -e_i x_{i3} + d_i\sum\limits_{j=1}^{n} x_{j3} \\ B_{ij}\cos(x_{i1} - x_{j1})\end{array}\right) \\ +s_{i2}\left(\begin{array}{c} -a_i x_{i2} + b_i - c_i x_{i3} \\ \sum\limits_{j=1}^{n} x_{j3}B_{ij}\sin(x_{i1} - x_{j1})\end{array}\right)\end{array}\right\}
\end{aligned}$$

On the other hand, the control u_{N_i} is selected as

$$\begin{aligned}
u_{N_i} &= -\left[\frac{\partial\sigma_i}{\partial x_i}g_i(x)\right]^{-1}L_i\sigma_i(x,x^*)\\
&= -\frac{L_i}{s_{i3}}\left\{\begin{array}{c} s_{i1}(x_{i1} - x^*_{i1}) + s_{i2}(x_{i2} - x^*_{i2}) \\ +s_{i3}(x_{i3} - x^*_{i3})\end{array}\right\}
\end{aligned}$$

Sliding-Mode Control 2

Now, let us consider the following nonlinear switching surface given by

$$\sigma_i(x,x^*) = s_{i1}\tilde{x}_{i1} + s_{i2}\dot{\tilde{x}}_{i1} + s_{i3}\ddot{\tilde{x}}_{i1}$$

where $\tilde{x}_{i1} = x_{i1} - x^*_{i1}$.

This is equivalent to

$$\begin{aligned}
\sigma_i(x,x^*) &= s_{i1}(x_{i1} - x^*_{i1}) + s_{i2}x_{i2}\\
&\quad + s_{i3}\left(\begin{array}{c} -a_i x_{i2} + b_i - c_i x_{i3} \\ \sum\limits_{j=1}^{n=3} x_{j3}B_{ij}\sin(x_{i1} - x_{j1})\end{array}\right)
\end{aligned}$$

where

$$\frac{\partial\sigma_i}{\partial x_i}g_i(x) = -s_{i3}c_i\sum_{j=1}^{n=3} x_{j3}B_{ij}\sin(x_{i1} - x_{j1})$$

for all $x_i \in B_{x_i}$.

Remarks:

1. The coefficients s_{ij}, $j = 1,2,3$ of sliding mode controller 1 must be chosen in order Assumption 1 is verified.

2. We can notice that the sliding surface of controller 2 differs from the surface of controller 1 by only one term: $x_{i3} - x^*_{i3}$ is replaced by $\ddot{\tilde{x}}_{i1}$. In this case, when the s_{i1} are some positive constants, the sliding surfaces can be viewed as some stable second-order ordinary differential equations in the power angle δ_i, ensuring convergence of the power angles to their equilibrium values, when the system trajectory remains on the sliding surface.

3. Furthermore the equivalent control u_e can be viewed as an output linearizing controller rendering the system dynamics equivalent to the linear dynamics

$$\dot{\sigma}_i(x,x^*) = s_{i1}\dot{\tilde{x}}_{i1} + s_{i2}\ddot{\tilde{x}}_{i1} + s_{i3}\dddot{\tilde{x}}_{i1} = 0$$

The relative degree of each output (power angle) is equal to 3, thus the system has no zero dynamics in this case. Furthermore, stability can be stated by using stability analysis arguments (Lasalle theorem (Khalil, 1996)) apart from Lyapunov function candidate $V(x - x^*) = \frac{1}{2}\sigma^T(x - x^*)\sigma(x - x^*)$.

5.2 Hamiltonian Control design

Now we design a control law based on passivity theory and energy function. The system is described in a Hamiltonian representation providing that the stability of the system can be guaranteed.

Consider system (2) and the following energy function

$$H = \sum_{j=1}^{n=3} \left(\begin{array}{c} \frac{1}{2c_i}x_{i2}^2 - \frac{b_i}{c_i}x_{i1} + \frac{e_i}{2d_i}x_{i3}^2 \\ -\frac{1}{2}x_{i3}\sum_{j=1}^{n=3} x_{j3}B_{ij}\cos(x_{i1} - x_{j1}) \end{array} \right) \tag{18}$$

It follows that the system dynamics can be written as a generalized Hamiltonian control system with dissipation according to what follows

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix} \frac{\partial H}{\partial x_i} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_i \tag{19}$$

where

$$x_i = \mathrm{col}(x_{i1}, x_{i2}, x_{i3}),\ \mathcal{J}_i(x) = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{R}_i(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix}, g_i(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Let $(x_{i1}^*, x_{i2}^*, x_{i3}^*)$ be the equilibrium point of (2), obtained from the following equations

$$x_{i2}^* = 0$$

$$-a_i x_{i2}^* + b_i - c_i x_{i3}^* \sum_{j=1}^{n=3} x_{j3}^* B_{ij} \sin(x_{i1}^* - x_{j1}^*) = 0$$

$$-e_i x_{i3}^* + d_i \sum_{j=1}^{n} x_{j3}^* B_{ij} \cos(x_{i1}^* - x_{j1}^*) + \tilde{u}_i = 0 \tag{20}$$

Defining the constant excitation control $\tilde{u}_i$, it follows that

$$\tilde{u}_i = e_i x_{i3}^* - d_i \sum_{j=1}^{n=3} x_{j3}^* B_{ij} \cos(x_{i1}^* - x_{j1}^*). \tag{21}$$

Now, defining the energy function which includes the equilibrium point of the following form

$$\begin{aligned} H_e = & \sum_{j=1}^{n=3} \left(\frac{1}{2c_i}x_{i2}^2 - \frac{b_i}{c_i}(x_{i1} - x_{i1}^*) + \frac{e_i}{2d_i}(x_{i3} - x_{i3}^*)^2 \right) \\ & + \sum_{i=1}^{n=3} \left(\begin{array}{c} x_{i3}\sum_{j=1}^{n} x_{j3}B_{ij}\cos(x_{i1} - x_{j1}) \\ +x_{i3}\sum_{j=1}^{n} x_{j3}^* B_{ij}\cos(x_{i1}^* - x_{j1}^*) \end{array} \right) \end{aligned}$$

Then, system (19) can be represented by the Hamiltonian system with dissipation as

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix} \frac{\partial H_e}{\partial x_i} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v_i.$$

Since H_e is bounded from below, because of $x_{i1} \in [-\pi, \pi]$, and $\forall l > 0$ the set $\{x : H_e(x) \le l\}$ is compact. Thus $H_e(x)$ has a strict local minimum at $(x_{i1}^*, x_{i2}^*, x_{i3}^*)$.

Then, a control law which stabilizes the multi-machine power system is given by

$$u_i = \tilde{u}_i + v_i.$$

where

$$\begin{aligned} v_i &= -f_i g_i^T \frac{\partial H_e}{\partial x_i} \\ &= -f_i \left(\begin{array}{c} -\sum_{j=1}^{n=3} B_{ij} \left[\begin{array}{c} x_{j3}\cos(x_{i1} - x_{j1}) \\ -x_{j3}^*\cos(x_{i1}^* - x_{j1}^*) \end{array} \right] \\ +\frac{e_i}{d_i}(x_{i3} - x_{i3}^*) \end{array} \right) \\ &= -f_i \left\{ \begin{array}{c} I_{d_i} + \frac{e_i}{d_i}x_{i3} \\ +\frac{1}{d_i}\left(\begin{array}{c} d_i\sum_{j=1}^{n=3} B_{ij}x_{j3}^* \\ \cos(x_{i1}^* - x_{j1}^*) - e_i x_{i3}^* \end{array} \right) \end{array} \right\} \\ &= -f_i \left\{ I_{d_i} + \frac{e_i}{d_i}x_{i3} - \frac{1}{d_i}\tilde{u}_i \right\} \end{aligned}$$

where $\tilde{u}_i = e_i x_{i3}^* - d_i \sum_{j=1}^{n=3} x_{j3}^* B_{ij} \cos(x_{i1}^* - x_{j1}^*)$. Next, using $E_{q_i} = E'_{q_i} + (X_{d_i} - X'_{d_i}) I_{q_i}$, and $d_i = (X_{d_i} - X'_{d_i})/T'_{d_i}$, $e_i = 1/T'_{d_i}$, it follows that $\frac{e_i}{d_i} = \frac{1}{(X_{d_i} - X'_{d_i})}$. Finally, the controller can be expressed only in terms of local measurable signals:

$$\begin{aligned} u &= \tilde{u}_i - f_i \left\{ \frac{1}{(X_{d_i} - X'_{d_i})} E_{q_i} - \frac{1}{d_i}\tilde{u}_i \right\} \\ &= \tilde{u}_i + \frac{f_i}{d_i}\tilde{u}_i - \frac{f_i}{(X_{d_i} - X'_{d_i})}\left(V_{t_i} + \frac{Q_{e_i} X_{d_i}}{V_{t_i}} \right) \end{aligned}$$

where $E_{q_i} = V_{t_i} + \frac{Q_{e_i} X_{d_i}}{V_{t_i}}$. Consequently, the resulting controller is a decentralized static output feedback.

6. Simulation results

The effectiveness of the here-proposed sliding-mode controller design has been validated through computer simulations.

The numerical values of the generator parameters (in per unit) were $D_1 = 5$, $D_2 = 3$, $X'_{d_1} = 0.252$, $X'_{d_2} = 0.319$, $X_{d_1} = 1.863$, $X_{d_2} = 2.36$, $H_1 = 1$, $H_2 = 2$, $T'_{d_1} = 6.9$, $T'_{d_2} = 7.96$, $E_{f_1} = 1.3$, $P_{m_1} = 0.35$, $P_{m_2} = 0.35$ and $\omega_s = 377$, $B_{12} = 0.56$, $B_{13} = 0.53$, $B_{23} = 0.6$,

With this parameter choice, the stable equilibrium state of the generator is

$$\begin{array}{lll} x_{11}^* = 0.6654, & x_{12}^* = 0, & x_{13}^* = 1.03 \\ x_{12}^* = 0.6425, & x_{22}^* = 0, & x_{23}^* = 1.01 \end{array}$$

The initial value of the states variables are

$$\begin{array}{lll} x_{11}(0) = 0.8, & x_{12}(0) = 0.3, & x_{13}(0) = 1.5 \\ x_{12}(0) = 0.5, & x_{22}(0) = -0.3, & x_{23}(0) = 0.5 \end{array}$$

The controller parameters are chosen as follows

Control 1

$$s_{11} = 10, \quad s_{12} = 15, \quad s_{13} = 8, \quad L_1 = 25$$

Control 2

$$s_{21} = 10, \quad s_{22} = 15, \quad s_{23} = 8, \quad L_2 = 25$$

The system responses obtained for the rotor angle are shown in figures 1-3. From the different figures, we can see that the dynamic response of the rotor angle is such that their equilibrium position is reached.

From these figures, it can be also seen that the sliding mode controller 2 can provide better transient performances than the sliding mode controller 1. However, the transient response of the two continuous sliding mode controllers is significantly better than the one of the Hamiltonian controller. We suggest than an explanation can be found in the fact that the sliding controller 2 has no zero dynamics due to the particular choice of the sliding surface whose time-derivative corresponds to a equivalent linear system obtained via an input-output linearization technique, without zero dynamics in this case.

7. Conclusions

A nonlinear control strategy for a class of nonlinear systems has been developed and successfully applied to multi-machine power system control. Two new controllers have been designed using continuous sliding-mode techniques. This controller design has been successfully applied to a three-machine power system, where two different switching surfaces have been considered. The overall methodology can be obviously extended to a more general system made of n generators. Closed-loop performance of these two controllers appears to be better than the one obtained with a port-controlled Hamiltonian design.

References

Bazanella A.S, Silva A.S, Kokotovick P.V. (1997), "Lyapunov design of excitation control for synchronous machines", Proc. 36th IEEE CDC, pp. 211-216, San Diego, California, USA.

Bergen A. (1986), Power System Analysis, Prentice-Hall.

De Carlo R.A., Zak S.H. and Matthews G.P. (1988), "Variable structure control of nonlinear multivariable systems: A tutorial", Proc. of the IEEE, Vol. 76, No. 3, pp. 212-233.

Khalil H. (1996), "Nonlinear Systems", 2nd. Edition, Prentice Hall.

Pai M.A. (1989), Energy Function Analysis for Power System Stability, Kluwer Academic Publishers.

Maschke B.M.J., Ortega R., Van der Schaft A.J. (1998), "Energy-based Lyapunov functions for forced Hamiltonian systems with dissipation", Proc. of the 37th IEEE CDC, Tampa, Florida.

Ortega R., Loria A., Nicklasson P.J. and Sira-Ramirez H. (1998). "Passivity-based Control of Euler-La-grange Systems". Springer-Verlag.

Utkin V.I. (1992), "Sliding Modes in Control and Optimization", Springer-Verlag, Heidelberg.

Xi Z. (2002), "Nonlinear dentralized saturated controller design for multi-machine power systems", Int. J. of Control, Vol. 75, pp. 1002-1011.

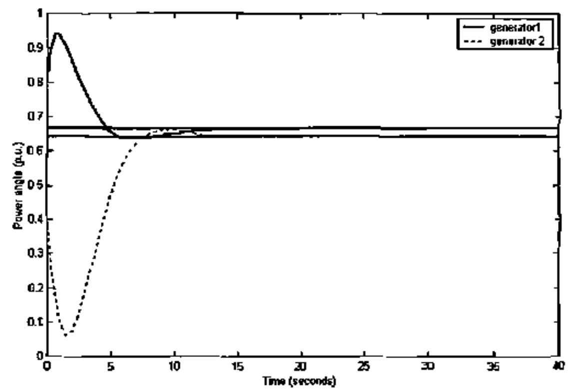

Fig. 1 Rotor angle with sliding modes 1 control

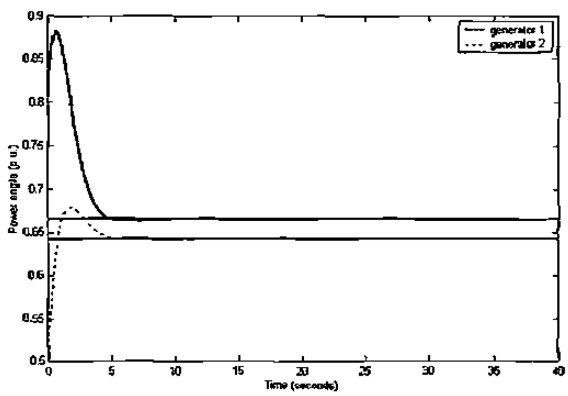

Fig. 2 Rototr angle with sliding modes 2 control

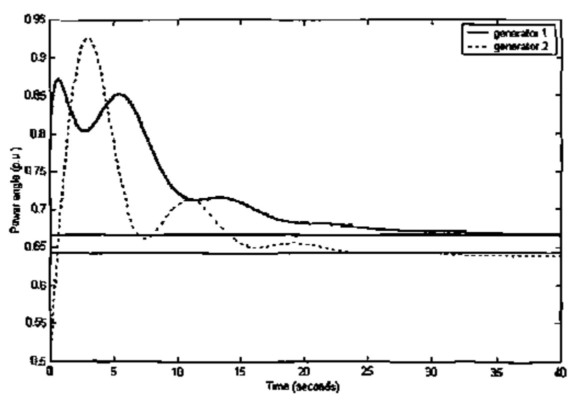

Fig. 3 Rotor angle with Hamiltonian control

Bibliografía

[1] Guadalupe Acosta , "Técnicas de control para sistemas no lineales", Tesis de Doctorado, Doctorado en Ingeniería Eléctrica, opción Control Automático, FIME-UANL, San Nicolás de los Garza, N.L, México, Junio 2002.

[2] J. H. Anderson, "The control of Synchronous Machine Using Optimal Control Theory" Proc. IEEE Vol. 59, pp. 25-35, Jan. 1971.

[3] A. S. Bazanella, A.S. Silva, P.V. Kokotovick, "Lyapunov design of excitation control for synchronous machines", Proceedings 36^{th} IEEE CDC, pp. 211-216, San Diego, California, USA, 1997.

[4] A. Bergen, "Power System Analysis", Prentice-Hall, 1986.

[5] G. Besançon, H. Hammouri and G. Bornard, "Observer synthesis for a class of nonlinear control systems", European Journal of Control, Vol. 2, No. 3, pp. 176-192, 1996.

[6] G. Besançon, "Remarks on nonlinear adaptive observer design", Systems and Control Letters, Vol. 41, pp. 271-280, 2000.

[7] K. Busawon, M. Saif and J. De León Morales, "Estimation and control for a class of Euler discretized nonlinear system", American Control Conference, pp. 3579-3583, San Diego Cal., USA, June 1999.

[8] K. Busawon, J. De León Morales and M. Aguilera-Hernandez, "A new high gain observer for uniformly observable systems", IASTED, Austria, Feb. 1999.

[9] K. Busawon, "Sur les observateurs des systémes nonlinéaires et le principe de separation", Ph. D. Thesis, Université Claude Bernard Lyon I, Lyon France, 1993

[10] J.W. Chapman, M.D. Ilic, C.A. King, L. Eng, H. Kaufman, "Stabilizing a multimachine power system via decentralized feedback linearizing excitation control", IEEE Transactions on Power Systems, Vol. 8, No. 1, pp. 830-839, February 1993.

[11] C. Califano, S. Monaco, and D. Normand-Cyrot. "On the discrete-time normal form", IEEE Transactions on Automatic Control, Vol. 43, No. 11, pp. 1654–1658, 1998.

[12] R.L. Carol and D.P. Lindorff, "An adaptive observer for single-input single-output linear systems", IEEE Transactions on Automatic Control, Vol. 18 pp. 428-435, 1973.

[13] R. A. De Carlo, S. H. Zak and G.P. Matthews, "Variable structure control of non-linear multivariable systems: A tutorial", Proc. of the IEEE, Vol. 76, No. 3, pp. 212-233, 1988.

[14] J. De Leon Morales, K. Busawon and G. Acosta-Villarreal, "Digital implementation of an observer-based controller for rigid robot", IEEE CDC, Tampa Florida, USA, 1998.

[15] Francisco Humberto Florenzano Lara, "Diseño de Algoritmos de Control para Sistemas Multimáquinas" Tesis de Maestría, Doctorado en Ingeniería Eléctrica, opción Control Automático, FIME-UANL, San Nicolás de los Garza, N.L, México, Enero 2002.

[16] J. P. Gauthier and I. Kupka,"Observability and Observers for Nonlinear Systems", SIAM Journal of Control Optimization, 32, pp. 975-994, 1994.

[17] D. Georges, "Nonlinear Robust Control of Power Systems", European Control Conference, Porto Portugal, 4-7 septembre 2001.

[18] A. Ghosh, O. P. Malik, G. S. Hope, "Power System Stabilizer Based On Adaptive Control Techniques", IEEE Trans. Vol. PAS-103, pp. 1983-1989, 1983.

[19] Cesar Guerra Torres, "Diseño de controladores y observadores para sistemas no lineales", Tesis de Maestría, Doctorado en Ingeniería Eléctrica, opción Control Automático, FIME-UANL, San Nicolás de los Garza, N.L, México, Nov. 2002.

[20] H. Hammouri and J. De León-Morales, "Observers synthesis for state affine systems", in Proc. of the 29th IEEE CDC, pp. 784-785, Honolulu Hawaii, USA, 1990.

[21] Y. Wang, G. Guo, D. J. Hill, "Robust Decentralized Control of a Class of Nonlinear Systems and Applications to Multimachine Power System Stabilization", in Proc. of the 36^{th} IEEE CDC, San Diego, USA.

[22] Y. Y. Hsu, W. C. Chan, "Stabilization of Power Systems Using Variable Structure", Journal of Electrical Engineering, Vol. 24, pp.31-41, April 1981.

[23] Oscar Francisco Huerta Guevara, "Observadores no lineales para generadores síncronos", Tesis de Maestría, Doctorado en Ingeniería Elé+ctrica, opción Control Automático, FIME-UANL, San Nicolás de los Garza, N.L, México, Mayo 2000.

[24] H. Khalil,1996, "Nonlinear Systems", 2nd. Edition, Prentice Hall.

[25] Gerhard Kreisselmeier, "On sampling without Loss of Observability/Controllability", IEEE Transactions on Automatic Control, vol 44, No. 5, May 1999.

[26] G. Kreisselmeier, "Adaptive observers with exponential rate of convergence", IEEE Transactions on Automatic Control, Vol. 22, No. 7, pp. 2-8, 1977.

[27] A. N. Kolmogorov and S. V. Fomin, "Elementos de la teoría de funciones y del análisis funcional", Editotial Mir, URSS, 1975, Traducción al español por C. Vega.

[28] P. Kundur, "Power System Stability and Control", McGraw-Hill, pp.45-136, USA, 1994.

[29] W. Lin and C. Byrnes, "Passivity and absolute stabilization of a class of discrete-time nonlinear systems", Automatica, 31 (2), pp. 263-267, 1995.

[30] A. Loria, "Summation-type conditions for uniform asymptotic stability of discrete-time systems", Technical report, LSS, 2004.

[31] Patricio Tomei and Ricardo Marino, "Nonlinear Control Design, Geometric, Adaptiva an Robust", Prentice Hall, 1995

[32] R. Marino and P. Tomei, "Adaptive observers with arbitrary exponential rate of convergence for nonlinear systems", IEEE Transactions on Automatic Control, Vol. 40, No. 7, pp. 1300-1304, July 1995.

[33] B. M.J. Maschke, R. Ortega and A. J. van der Shaft. "Energy-based Lyapunov functions for forced Hamiltonian systems with dissipation.", in Proc. of the 37^{th} IEEE Conference on Control and Decision, Tampa Florida, pp. 3599-3599, 1998.

[34] Juan Manuel Mendivil Avila, "Diseño de controladores para una clase de sistemas no lineales, aplicación al motor a pasos de magneto permanente", Tesis de Maestría, Doctorado en Ingeniería Electrica, opción Control Automático, FIME-UANL, San Nicolás de los Garza, N.L, México, Agosto 2002.

[35] W. Mielczarski, A. M. Zajaczkowski, "Nonlinear field voltage control of a synchronous generator using feedback linearization", Automatica, Vol. 30, No. 10, pp. 1625-1630, 1994

[36] S. Monaco and D. Normand-Cyrot, "Minimum-phase nonlinear discrete-time systems and feedback stabilization", in Proc. of the 26th IEEE, Conference on Decision and Control, pp. 979-986, Los Angeles, CA. USA. 1987.

[37] Dragan Nešić and Antonio Loria, "On uniform asymptotic stability of time-varying parametrized discrete-time cascades", arXiv:math.OC/0307167 v1, 11 Jul 2003

[38] D. Nešić and A. R. Teel, "Backstepping on Euler approximate model for stabilization of sampled-data nonlinear systems", in Proc. of 40^{th} IEEE, Conference on Decision and Control, Orlando Florida USA, Dec 2001

[39] D. Nešić , A.Teel, and P. Kokotović. "Sufficient conditions for stabilization of sampled-data nonlinear systems via discrete-time approximations". Systems & Control Letters, 38, pp. 259–270, 1999.

[40] D. Nešić , A. Teel, and E. Sontag. "Formulas relating $\mathcal{KL}$ stability estimates of discrete-time and sampled-data nonlinear systems". Systems & Control Letters, 38, pp. 49–60, 1999.

[41] Yuanzhang Sun, Tielong Shen, Romeo Ortega and Qianjin Liu, "Decentralized Controller Desing for Multimachine Power Systems based on the Hamiltonian Structure", Proceedings. of the 40th IEEE Conference on Decision and Control, pp. 3045-3050, Orlando Florida, USA. Dec.2001.

[42] M. A. Pai, "Energy Function Analysis for Power System Stability", Kluwer Publishers, 1989.

[43] Mohammed S. Santina, Allen R. Stubberud and Gene H. Hostetter, "Discrete-Time Equivalents to Continuous-Time Systems", The Control Handbook, CRC Press, Inc., pp. 265-277.

[44] P. W. Sauer, S. Ahmed-Zaid, P. V. Kokotovic, "An Integral Manifold Approach to Reduced Order Dynamic Modeling of Synchronous Machines", IEEE Transactions on Power Systems, Vol. 3, No. 1, February 1988.

[45] P. W. Sauer and M. A. Pai, "Power System Dynamics and Stability", Prentice Hall, 1998.

[46] R. Sepulchre, M. Janković and P. V. Kokotović, "Constructive Nonlinear Control", Springer-Verlag, London 1997.

[47] C. Simoes, H. Nijmeijer and J. Tsinias, "Nonsmooth stabilizability and feedback linearization of discrete-time systems", International Journal Robust Nonlinear Control, (6), pp. 171-188, 1996.

[48] V. I. Utkin, "Sliding Modes in Control and Optimization", Springer-Verlag, Heidelberg, 1992.

[49] M. Vidyasagar, Nonlinear Systems Analysis, second edition, *Prentice Hall*, New Jersey, 1993

[50] Jacques L. Willems, "Direct Methods for Transient Stability Studies in Power System Analisys", IEEE Transactions on Automatic Control, Vol. AC-16, No. 4, August 1971.

[51] Z. Xi, "Nonlinear decentralized controller design for multimachine power systems using Hamiltonian function method", Automatica, 38, pp. 527–5324, 2002.

[52] Zairong Xi and Daizhan Cheng, "Pasivity-based stabilization and H_∞ control of Hamiltonian control systems with dissipation and its applicatios to power systems", Int. J. of control, Vol. 73, No. 18, pp. 1686-1691, 2000.

[53] MingQing Xiao, Nikolaos Kazantzis, Costas Kravaris and Arthur J. Krener, "Nonlinear Discrete-Time Observer Design with Linearizable Error Dynamics", IEEE Transactions on Automatic Control, Vol. 48, No. 4, April 2003.

[54] Q. Zhang and A. Clavel, 2001. "Adaptive observer with exponential forgetting factor for linear time-varying systems", in Proc. of the 40th IEEE CDC, pp. 3886-3891, Orlando Florida, USA.

[55] Q. Zhang, "Adaptive observers for MIMO linear time-varying systems", IEEE Transactions on Automatic Control, Vol. 47, No. 3, pp. 525-529, 2002.