de las propiedades de Δ_θ (B.4), se tiene

$$\begin{aligned}
\overline{e}_{es}(k+1) &= (I+\tau\theta A)\overline{e}_{es}(k) - \tau K\theta C\overline{e}_{es}(k) + \tau\frac{1}{\theta^n}B\Psi_o\left(\Delta_\theta^{-1}\overline{e}_{es}(k), u(k)\right) \\
&\quad -\Delta_\theta e_{ap}(k+1) \\
&= \{I+\gamma_o(A-KC)\}\,\overline{e}_{es}(k) + \frac{\tau}{\theta^n}B\Psi_o\left(\Delta_\theta^{-1}\overline{e}_{es}(k), u(k)\right) - \Delta_\theta e_{ap}(k+1) \\
&= A_o\overline{e}_{es}(k) + \frac{\tau}{\theta^n}h_1 + h_2
\end{aligned}$$

donde $\gamma_o = \tau\theta$ (ver (3.14)), $A_o = I + \gamma_o(A - KC)$ (ver (B.6)) y

$$h_1 = B\Psi_o\left(\Delta_\theta^{-1}\overline{e}_{es}(k), u(k)\right) \qquad\qquad h_2 = -\Delta_\theta e_{ap}(k+1) \tag{3.48}$$

Considere la siguiente función de Lyapunov

$$V(\overline{e}_{es}(k)) = \overline{e}_{es}^T(k)P_o\overline{e}_{es}(k) \tag{3.49}$$

entonces

$$\begin{aligned}
\Delta(V(\overline{e}_{es}(k))) &= \overline{e}_{es}^T(k+1)P_o\overline{e}_{es}(k+1) - \overline{e}_{es}^T(k)P_o\overline{e}_{es}(k) \\
&= \left[A_o\overline{e}_{es}(k) + \frac{\tau}{\theta^n}h_1 + h_2\right]^T P_o\left[A_o\overline{e}_{es}(k) + \frac{\tau}{\theta^n}h_1 + h_2\right] - \overline{e}_{es}^T(k)P_o\overline{e}_{es}(k) \\
&= \overline{e}_{es}^T(k)A_o^TP_oA_o\overline{e}_{es}(k) + \overline{e}_{es}^T(k)A_o^TP_o\left(\frac{\tau}{\theta^n}h_1 + h_2\right) \\
&\quad + \left(\frac{\tau}{\theta^n}h_1 + h_2\right)^T P_oA_o\overline{e}_{es}(k) + \left(\frac{\tau}{\theta^n}h_1 + h_2\right)^T P_o\left(\frac{\tau}{\theta^n}h_1 + h_2\right) \\
&\quad -\overline{e}_{es}^T(k)P_o\overline{e}_{es}(k) \\
&= \overline{e}_{es}^T(k)\left[A_o^TP_oA_o - P_o\right]\overline{e}_{es}(k) + 2\overline{e}_{es}^T(k)A_o^TP_o\left(\frac{\tau}{\theta^n}h_1 + h_2\right) \\
&\quad + \left(\frac{\tau}{\theta^n}h_1 + h_2\right)^T P_o\left(\frac{\tau}{\theta^n}h_1 + h_2\right)
\end{aligned}$$

y a partir de la declaración 2 (véase anexo B) y las propiedades de γ_o (ver (3.16))

$$\begin{aligned}
\Delta(V(\overline{e}_{es}(k))) &= \overline{e}_{es}^T(k)\left[-\gamma_oP_o - \gamma_o(1-\gamma_o)^nC^TC\right]\overline{e}_{es}(k) + 2\overline{e}_{es}^T(k)A_o^TP_o\left(\frac{\tau}{\theta^n}h_1 + h_2\right) \\
&\quad + \left(\frac{\tau}{\theta^n}h_1 + h_2\right)^T P_o\left(\frac{\tau}{\theta^n}h_1 + h_2\right) \\
&\leq -\gamma_o\overline{e}_{es}^T(k)P_o\overline{e}_{es}(k) + 2\overline{e}_{es}^T(k)A_o^TP_o\left(\frac{\tau}{\theta^n}h_1 + h_2\right) \\
&\quad + \left(\frac{\tau}{\theta^n}h_1 + h_2\right)^T P_o\left(\frac{\tau}{\theta^n}h_1 + h_2\right) \\
&\leq -\gamma_o\|\overline{e}_{es}(k)\|_{P_o}^2 + 2\|A_o\overline{e}_{es}(k)\|_{P_o}\left\|\frac{\tau}{\theta^n}h_1 + h_2\right\|_{P_o} + \left\|\frac{\tau}{\theta^n}h_1 + h_2\right\|_{P_o}^2
\end{aligned}$$