# On discrete-time output-feedback control of feedback linearizable systems

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#### Abstract

In this paper, we present a control-observer scheme for discrete-time nonlinear systems. A controller and an observer are proposed for a class of discrete-time nonlinear systems. The results obtained are applied to a flexible robot in order to illustrate the proposed scheme.

Keywords: Discrete controller, Euler discretization, Nonlinear Observer, Flexible robot.

## 1. Introduction

Motivated by the recent advances in digital technology, discrete-time nonlinear systems control theory is receiving an increasing attention in different aspects of control and dynamic systems theory originally developed for continuous-time systems. Such is the case of feedback linearization (see e.g. [1, 10, 11]), passivity-based (cf. [3]), backstepping (cf. [7]). See also [4].

The present paper deals with the problem of observer-based output feedback stabilization of Euler approximate discrete-time systems under the standing assumption that the continuous-time system is feedback linearizable. In particular, we will propose a control scheme which relies on the ability to make that the closed loop system has a cascaded structure. Earlier contributions in this direction include [2].

Our main contribution is an observer-based controller which ensures a form of exponential stability which has a uniform bound on the overshoot of the systems response and a convergence rate which is linear in

the sampling period. This specific form of stability is important since only then, one can guarantee that the exact discrete-time and in its turn, the sampled-data systems have certain stability properties. See [8, 9].

## 2. Problem statement

Notation. Given any symmetric positive definite matrices P, Q we will denote by  $||x||_P^2 := x^T P x$  for any  $x \in \mathbb{R}^n$  and use the constants  $c_1$ ,  $c_2$  in the relation  $c_1||x||_P \le ||x||_Q \le c_2||x||_P$ . We will use c for a generic positive constant, i.e., we will write with an abuse of notation,  $c+c=c^2=c$ . We denote by  $\xi(k)$  the solution of the difference equation  $\xi(k+1)=F_r(k,\xi(k))$  with initial conditions  $k_0 \ge 0$  and  $\xi_0 = \xi(k_0)$ .

We consider feedback linearizable (in continuous time) nonlinear affine systems. We are concerned by the output feedback problem of the Euler discretization of nonlinear systems in the normal form, i.e., we are interested in designing an observer and an output-feedback controller for the Euler-based system

$$\begin{cases} x(k+1) = A_{\tau}x(k) + \tau B \left\{ \alpha(x(k)) + \beta(x(k))u(k) \right\} \\ y = Cx(k) = x_1(k) \end{cases}$$
where  $A_{\tau} = (I_n + \tau A) = \begin{pmatrix} 1 & \tau & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \tau \\ 0 & 0 & \cdots & 1 \end{pmatrix},$ 

$$A = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}; \qquad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}.$$

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