We assume that $\beta(x) \neq 0$ for all x.

We address this control problem by designing an exponentially stabilizing observer-based controller for the approximated linearizable system (2.1). More precisely, we will design an observer-based controller which guarantees the following property for the closed-loop system:

Definition 1. (Uniform Exponential Stability) The origin of the system $\xi(k+1) = F_\tau(k, \xi(k))$ is said to be uniformly exponentially stable if there exist r, $\tau_{\max}$, $\kappa > 0$ and for each $\tau \in (0, \tau_{\max}]$, $\lambda_\tau > 0$ such that,

$$\|\xi(k_0)\| \leq r \quad \Rightarrow \quad \|\xi(k)\| \leq \kappa \|\xi_\circ\| e^{-\lambda_\tau (k-k_\circ)} \qquad (2.2)$$

$\forall k \geq k_\circ$. If furthermore (2.2) holds for all $\xi(k_0) \in R^n$ then, the origin is said to be uniformly globally exponentially stable.

The property defined above is probably the most useful for discrete-time systems since it imposes a bound on the overshoots which is uniform in the initial conditions and the sampling time. Moreover, in the particular case when λ_τ is proportional to τ, this property guarantees that the *exact* discrete-time model is (globally) asymptotically practically stable. Roughly speaking, this means that the solutions tend to an arbitrarily small ball whose size is independent of τ and can be made smaller as $\tau_{\max}$ becomes smaller. See [8] for precise definitions and the only formal framework we are aware of, which establishes asymptotic practical stability of exact discrete-time nonlinear systems based on uniform (practical) asymptotic stability of approximate discrete-time systems.

3. Observer-based control

3.1. Control design

Consider the system (2.1) under the action of the static feedback-linearizing control law,

$$u = \beta^{-1}(x)\left[v(x) - \alpha(x)\right], \qquad (3.1)$$

where $\alpha(x)$ and $\beta(x)$ are assumed to be known, $\beta(x) \neq 0$ for all $x \in \mathbb{R}^n$, and the external control input $v(x)$ is defined as

$$v(x) = -F\Omega_\rho x \qquad (3.2)$$

where the matrices $F \in \mathbb{R}^{1\times n}$ and $\Omega_\rho \in \mathbb{R}^{n\times n}$ are given by

$$\begin{aligned} \Omega_\rho &= diag(\rho^n, \dots, \rho), \\ F &= \begin{pmatrix} C_n^0 & \cdots & C_n^{n-1} \end{pmatrix}, \end{aligned} \qquad \rho \geq 1, \quad (3.3)$$

with $C_n^p = \frac{n!}{(n-p)!p!}$. Then, the resulting closed-loop system is

$$x(k+1) = (A_\tau - \tau BF\Omega_\rho)\, x(k). \qquad (3.4)$$

The following result is useful to establish our main result.

Lemma 1. *There exists $\tau_{\max} > 0$ sufficiently small such that the system (2.1) in closed-loop with (3.1),(3.2), (3.3) is uniformly globally exponentially stable with λ_τ proportional to $\tau \in (0, \tau_{\max})$ and for all $\rho > 0$ such that $\rho\tau_{\max} \in (0,1)$.*

The proof of this Lemma is based on the following statement and is omitted here for lack of space.

Claim 1 ([6]). *Let $A_c = I + \gamma_c(A - BF)$ where the matrices A and B are in the usual Brunovsky controllable form, and F and C_n^p are defined as (3.3). Then, for every $\gamma_c \in (0,1)$, the unique symmetric positive definite matrix P_c satisfying the algebraic equation*

$$A_c^T P_c A_c - P_c = -\gamma_c P_c - \gamma_c(1-\gamma_c)^n F^T F$$

is given by $P_c = N^T N$, where $N = \Lambda_c E_c$, $\Lambda_c = diag(1, (1-\gamma_c)^{\frac{1}{2}}, \dots, (1-\gamma_c)^{\frac{n-1}{2}})$ and, letting i and j denote the rows and columns of E_c respectively, the elements of E_c are $E_c(i,j) = C_{n-i}^{j-i}$ for $j \geq i$ and $E_c(i,j) = 0$ otherwise.

3.2. Observer design

In this section we introduce an observer for the class of systems *(2.1)* which belongs to the class of systems with a triangular structure. This property of the nonlinearity is important because it ensures the uniform observability of the system.

An observer for the transformed system (2.1) is given by

$$\begin{aligned} z(k+1) &= A_\tau z(k) + \tau B\left[\alpha(z(k)) + \beta(z(k))u(k)\right] \\ &\quad + \tau\Delta_\theta^{-1} K\left[y(k) - \hat{y}(k)\right] \end{aligned} \qquad (3.5)$$

where

$$\begin{aligned} \Delta_\theta &= diag\begin{pmatrix} \frac{1}{\theta} & \cdots & \frac{1}{\theta^n} \end{pmatrix} \quad \text{for} \quad \theta \geq 1, \qquad (3.6) \\ K &= col\begin{pmatrix} C_n^1 & \cdots & C_n^n \end{pmatrix} \quad \text{with} \quad C_n^p = \frac{n!}{(n-p)!p!}. \end{aligned}$$

The term $\tau\Delta_\theta^{-1}K$ represents the observer gain.

Defining the estimation error as $e = z - x$, it follows that the dynamics of the estimation error is of the form

$$\begin{aligned} e(k+1) &= \left\{A_\tau - \tau\Delta_\theta^{-1}KC\right\} e(k) \qquad (3.7) \\ &\quad + \tau B\Psi_o^\tau(e(k), x(k), u(k)) \end{aligned}$$

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