where $\delta = \angle E'_q - \angle V$ is the generator rotor angle referred to the infinite bus (also called power angle), $\omega = \dot{\delta}$ is the rotor angular speed and E'_q is the stator voltage which is proportional to flux linkages. M is the per unit inertia constant, T_m is the constant mechanical power supplied by the turbine, and T'_{do} is the transient open circuit time constant. $X_d = x_d + x_L$ is the augmented reactance, where x_d is the direct axis reactance and x_L is the line reactance, X'_d is the transient augmented reactance and V is the infinite bus voltage which is fixed. P_g is the generated power while E_{fd} is the stator equivalent voltage given by field voltage v_f.

$$P_g = \frac{1}{X'_d} E'_q V \sin(\delta) + \frac{1}{2}\left(\frac{1}{X_q} - \frac{1}{X'_d}\right) V^2 \sin(2\delta),$$
$$E_{fd} = \frac{\omega_s M_f}{\sqrt{2} R_f} v_f$$

where v_f is the scaled field excitation voltage, x'_d is the transient direct axis reactance, x_q is the quadrature axis reactance, M_f is the mutual inductance between stator coils and R_f is the field resistance. We only consider the case where the dynamics of the damper windings are neglected, *i.e.* $D = 0$.

For a given constant field voltage $E_{fd} = E^*_{fd}$, the generator possesses two equilibrium points - one stable and one unstable. Throughout this work, the analysis and design are made around the stable equilibrium point eventhough similar analysis can be made around the unstable equilibrium point. Setting $(\delta^*, \omega^*, E'^*_q)$ as the stable equilibrium point of (5.1), then the system, represented in terms of the deviations variables $\Delta\delta = \delta - \delta^*, \Delta\omega = \omega - \omega^*, \Delta E'_q = E'_q - E'^*_q, u = E_{fd} - E^*_{fd}$ and of the following constants $m_1 = \frac{T_m}{M}$, $m_2 = \frac{-V}{MX'_d}$, $m_3 = \frac{V^2}{M}\left(\frac{1}{X'_d} - \frac{1}{X_q}\right)$, $m_4 = -\frac{X_d}{T'_{do}X'_d}$, $m_5 = -\left(\frac{X'_d - X_d}{T'_{do}X'_d}\right)V$, $m_6 = \frac{1}{T'_{do}}$, is given by

$$\begin{aligned}
\frac{d\Delta\delta}{dt} &= \Delta\omega \\
\frac{d\Delta\omega}{dt} &= m_1 + \left\{m_2(\Delta E'_q + E'^*_q) + m_3\cos(\delta)\right\}\sin(\delta) \\
\frac{d\Delta E'_q}{dt} &= m_4(\Delta E'_q + E'^*_q) + m_5\cos(\delta) + m_6(u + E^*_{fd})
\end{aligned} \tag{5.2}$$

where $\delta = \Delta\delta + \delta^*$. Defining the following change of variable $x_1 = \Delta\delta$, $x_2 = \Delta\omega$, $x_3 = \Delta E'_q$, and applying the methodology given in section 2, it follows that the Euler approximate model of the synchronous generator is given by

$$x(k+1) = \mathcal{F}_\tau(x(k)) + \mathcal{G}_\tau(x(k))u(k) \tag{5.3}$$

where $\mathcal{F}_\tau(x(k)) = x(k) +$

$$+\tau\begin{pmatrix} x_2(k) \\ m_1 + \left\{m_2(x_3(k) + E'^*_q) + m_3\cos(\tilde{x}_1)\right\}\sin(\tilde{x}_1) \\ m_4(x_3(k) + E'^*_q) + m_5\cos(\tilde{x}_1(k)) + m_6 E^*_{fd}\end{pmatrix}$$

$$\mathcal{G}_\tau(x(k)) = \tau\begin{pmatrix} 0 \\ 0 \\ m_6\end{pmatrix}, \qquad \tilde{x}_1 = x_1(k) + \delta^*.$$

A. Control law design.

In order to regulate the power angle of the generator (5.3), the following switching function was chosen

$$\begin{aligned}
\sigma(k) &= \mathcal{S}^T\left(x(k) - x_{ref}(k)\right) \qquad (5.4)\\
&= \mathcal{S}_1\left(x_1(k) - x_{1ref}(k)\right) + \mathcal{S}_2 x_2(k) + \mathcal{S}_3 x_3(k)
\end{aligned}$$

where $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ are constants that are chosen to satisfy the sliding condition and $x_{1ref}(k)$ is a constant reference signal.

Then the control law is given by

$$u(k) = u_e(k) + \Delta u(k).$$

B. Observer design

Consider the following change of coordinates $x_1 = \Delta\delta$, $x_2 = \Delta\omega$, $x_3 = m_1 + \left\{m_2(\Delta E'_q + E'^*_q) + m_3\cos(\delta)\right\}\sin(\delta)$. Taking the Euler discretization , we obtain

$$\begin{aligned}
x_1(k+1) &= x_1(k) + \tau x_2(k) \\
x_2(k+1) &= x_2(k) + \tau x_3(k) \\
x_3(k+1) &= x_3(k) + \tau\{m_4(\Delta E'_q(k) + E'^*_q) \\
&\qquad + m_5\cos(\delta) + m_6(u(k) + E^*_{fd})\}
\end{aligned} \tag{5.5}$$

where $m_4 E'^*_q + m_5\cos(\delta^*) + m_6 E^*_{fd} = 0$.

The dynamic system described in the new coordinates has the following structure

$$\begin{aligned}
x(k+1) &= A_\tau x(k) + \tau B\left\{\alpha(x(k)) + \beta(x(k))u(k)\right\} \\
y(k) &= Cx(k)
\end{aligned} \tag{5.6}$$

where $\alpha(x(k))$ and $\beta(x(k))$, in the original coordinates, are given by

$$\begin{aligned}
\alpha(x(k)) &= m_2\sin(\delta(k) + \tau\Delta\omega(k))E'^*_q + \Delta E'_q(k) \\
&\quad + \tau m_4(\Delta E'_q(k) + E'^*_q) + \tau m_5\cos(\delta(k)) \\
&\quad + m_3\cos(\delta(k) + \tau\Delta\omega(k))\sin(\delta(k) + \tau\Delta\omega(k)) \\
&\quad - \left(m_2(\Delta E'_q(k) + E'^*_q) + m_3\cos(\delta(k)\right)\sin(\delta(k)) \\
\beta(x(k)) &= \tau m_2 m_6\sin(\delta(k) + \tau\Delta\omega(k))
\end{aligned}$$

Then, an observer for system (5.6) is of the form

$$\begin{aligned}
z(k+1) &= A_\tau z(k) + \tau B\left\{\alpha(z(k)) + \beta(z(k))u(k)\right\} \\
&\quad + \tau\Delta_\theta^{-1}K\left[y(k) - \hat{y}(k)\right].
\end{aligned}$$

where $K = col\left(\ C^1_3,\ \ C^2_3,\ \ C^3_3\ \right) = col\left(\ 3,\ \ 3,\ \ 1\ \right)$, and the observer gain is given by

$$\tau\Delta_\theta^{-1}K = col\left(\ 3\tau\theta,\ \ 3\tau\theta^2,\ \ \tau\theta^3\ \right).$$

C. Simulation results.

The simulations were done considering the following nominal values of the generator's parameters (per unit) $T_m = 1$; $M = 0.033$; $\omega_s = 1$; $T'_{do} = 0.033$; $X_q = X_d = 0.9$; $X'_d = 0.3$; $V = 1.0$. Furthermore, the stable equilibrium point was obtained from (5.1) for a stator equivalent field equivalent voltage $E^*_{fd} = 1.1773$: $\delta^* = 0.870204$, $\omega^* = 1$, $E'^*_q = 0.822213$. The control and observer parameters are chosen as $\mathcal{S}_1 = 5$, $\mathcal{S}_2 = 2, \mathcal{S}_3 = 2$, $\eta = 0.1$, $\theta = 0.8$ and τ was chosen as $\tau = 0.01$.

Simulations were performed with the proposed discrete-time controller-observer scheme. First, in order to illustrate the performance of the observer, we consider the open-loop case. For this set of simulations, the