where x, u, y classically denote the state, the input and the measured output vectors respectively, and θ some vector of unknown parameters. A, B, C, Φ are assumed to be known matrices of appropriate dimensions, continuous and uniformly bounded in time.
The main result of [21] can be summarized as follows:
If the following assumptions hold,

(A1) There exists a bounded time-varying matrix $K(t)$ such that: $\dot{\eta}(t) = (A(t) - K(t)C(t))\eta(t)$ is exponentially stable.

(A2) The solution $\Lambda(t)$ of $\dot{\Lambda}(t) = [A(t) - K(t)C(t)]\Lambda(t) + \Phi(t)$ is persistently exciting in the sense that there exist α, β, T such that:

$$\alpha I \leq \int_t^{t+T} \Lambda(\tau)^T C^T \Sigma(\tau) C(\tau) \Lambda(\tau) d\tau \leq \beta I \quad (2)$$

for some bounded positive definite matrix Σ.

Then, the system (3) below is an exponential observer for system (1), in the sense that for any set of initial conditions, $\hat{x}(t) - x(t)$ and $\hat{\theta}(t) - \theta$ exponentially decay to zero:

$$\begin{array}{rcl} \dot{\Lambda}(t) &=& [A(t) - K(t)C(t)]\Lambda(t) + \Phi(t) \\ \dot{\hat{x}}(t) &=& A(t)\hat{x}(t) + B(t)u(t) + \Phi(t)\hat{\theta}(t) \\ &+& [K(t) + \Lambda(t)\Gamma\Lambda^T(t)C^T(t)\Sigma(t)].[y(t) - C(t)\hat{x}(t)] \\ \dot{\hat{\theta}}(t) &=& \Gamma\Lambda^T(t)C^T(t)\Sigma(t)[y(t) - C(t)\hat{x}(t)] \end{array} \quad (3)$$

Taking advantage of classical recursive least square algorithms, an adaptation law for the parameter gain Γ of the above observer can obviously be obtained as follows (*e.g.* as in [20]):

$$\dot{\Gamma}(t) = -\Gamma(t)\Lambda^T(t)C^T(t)\Sigma(t)C(t)\Lambda(t)\Gamma(t) + \lambda\Gamma(t), \quad (4)$$

for $\lambda > 0$.
Our purpose here is to discuss such a design at the light of available results on observers for state-affine systems [13], [4].

B. *Kalman-like interpretation of the adaptive observer*

Let us first recall the result on state observer design for so-called state-affine systems of the following form [13]:

$$\begin{array}{rcl} \dot{x} &=& A(u,y)x + \varphi(u,y) \\ y &=& Cx \end{array} \quad (5)$$

where the components of matrix $A(u,y)$ and vector $\varphi(u,y)$ are continuous functions depending on u and y, uniformly bounded.
The result is as follows:
If the input is persistently exciting, in the sense that there exist α, β, T such that:

$$\alpha I \leq \int_t^{t+T} \Psi_u(t,\tau)^T C^T \Sigma(\tau) C \Psi_u(t,\tau) d\tau \leq \beta I, \quad (6)$$

where Ψ_u denotes the transition matrix for the system $\dot{x} = A(u,y)x$, $y = Cx$, and Σ some positive definite bounded matrix.
Then, an exponential observer for system (5) is given by:

$$\begin{array}{rcl} \dot{\hat{x}} &=& A(u,y)\hat{x} + \varphi(u,y) + S^{-1}C^T\Sigma(y - C\hat{x}) \quad (7) \\ \hat{y} &=& C\hat{x} \end{array}$$

where S is the solution of the equation:

$$\dot{S} = -\rho S - A(u,y)^T S - SA(u,y) + C^T\Sigma C, \; S(0) > 0 \quad (8)$$

for some positive constant ρ sufficiently large.
Defining indeed the estimation error as $e = \hat{x} - x$, the error system is given by:

$$\dot{e} = \left\{A(u,y) - S^{-1}C^T\Sigma C\right\} e \quad (9)$$

and from (6), $V(e) = e^T S e$ is a Lyapunov function for this system satisfying $\dot{V} \leq -\rho V$ [13].

Now, in the case of a system affine in the state and depending on unknown parameters in an affine way, a model can be given as follows:

$$\begin{array}{rcl} \dot{x} &=& A(u,y)x + \varphi(u,y) + \Phi(u,y)\theta \\ y &=& Cx \end{array} \quad (10)$$

where Φ satisfies the same properties as A, φ.
Assuming excitation condition (6) for state estimation on the one hand, and some additional one of the form (2) with $K = S^{-1}C^T$ and S as in (8) for parameter estimation on the other hand, an adaptive observer can be proposed as follows (where S_θ corresponds to Γ^{-1} of (4)):

$$\begin{array}{rcll} \dot{\hat{x}} &=& A(u,y)\hat{x} + \varphi(u,y) + \Phi(u,y)\hat{\theta} & (11) \\ && + \left\{\Lambda S_\theta^{-1}\Lambda^T C^T + S_x^{-1}C^T\right\}\Sigma(y - C\hat{x}) & (12) \\ \dot{\hat{\theta}} &=& S_\theta^{-1}\Lambda^T C^T \Sigma(y - C\hat{x}) & (13) \\ \dot{\Lambda} &=& \left\{A(u,y) - S_x^{-1}C^T C\right\}\Lambda + \Phi(u,y) & (14) \\ \dot{S}_x &=& -\rho_x S_x - A(u,y)^T S_x - S_x A(u,y) + C^T\Sigma C & (15) \\ \dot{S}_\theta &=& -\rho_\theta S_\theta + \Lambda^T C^T \Sigma C \Lambda, \; S_x(0) > 0, S_\theta(0) > 0 & (16) \end{array}$$

where ρ_x and ρ_θ are sufficiently large positive constants (and Σ is as in (2)).
With $e_x := \hat{x} - x$ and $\epsilon_\theta := \hat{\theta} - \theta$, we indeed get:

$$\begin{array}{rcl} \dot{e}_x &=& \left\{A(u,y) - \Lambda S_\theta^{-1}\Lambda^T C^T\Sigma C - S_x^{-1}C^T\Sigma C\right\} e_x \\ && + \Phi(u,y)\epsilon_\theta \\ \dot{\epsilon}_\theta &=& -S_\theta^{-1}\Lambda^T C^T \Sigma C e_x \end{array}$$

and following the same idea as in [21], the transformation:

$$\epsilon_x = e_x - \Lambda\epsilon_\theta,$$