2. Dynamical model of a multi-machine power system

Now we consider a power system made of n generators. Under some standard assumptions, the motion of the interconnected generators can be described by the classical model with flux decay dynamics. The generator is modeled by the voltage behind direct axis transient reactance. The angle of the voltage coincides with the mechanical angle relative to the synchronous rotating frame. The network has been reduced to internal bus representation. The dynamical model of the i-th machine is represented by (Bergen, 1986, Pai et al., 1989):

$$\begin{aligned} \dot{\delta}_i &= \omega_i - \omega_0 \\ \dot{\omega}_i &= \frac{1}{2H_i}\left(-D_i\left(\omega_i - \omega_0\right) + \omega_0\left(P_{m_i} - P_{e_i}\right)\right) \\ \dot{E}'_{q_i} &= \frac{1}{T'_{d_i}}\left(E_{f_i} - E_{q_i}\right) \end{aligned} \tag{1}$$

where

$$\begin{aligned} P_{e_i} &= E'_{q_i}\sum_{j=1, i\neq j}^{n} E'_{q_j} B_{ij}\sin\left(\delta_i - \delta_j\right) \\ E_{q_i} &= E'_{q_i} - \left(X_{d_i} - X'_{d_i}\right)\sum_{j=1, i\neq j}^{n} E'_{q_j} B_{ij}\cos\left(\delta_i - \delta_j\right) \end{aligned}$$

and $\delta_i(t)$ is the power angle of the i-th generator, $\omega_i(t)$ represents the relative speed, $E'_{q_i}(t)$ is the transient EMF in the quadrature axis

We consider that the $E_{f_i}(t)$, $i = 1, ..., n$ are the control inputs and the $\delta_i(t)$ are measurable outputs, together with the P_{e_i} and V_{t_i}, where V_{t_i} represents the terminal voltage at generator i. The P_{m_i} are supposed to be constant (standard assumption).

Then, the state representation of a n-machine power system is given by

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \qquad i = 1, ..., n \\ \dot{x}_{i2} &= -a_i x_{i2} + b_i - c_i x_{i3}\sum_{j=1}^{n} x_{j3} B_{ij}\sin(x_{i1} - x_{j1}) \\ \dot{x}_{i3} &= -e_i x_{i3} + d_i\sum_{j=1}^{n} x_{j3} B_{ij}\cos(x_{i1} - x_{j1}) + u_i \end{aligned} \tag{2}$$

where $a_i = D_i/2H_i$, $b_i = (\omega_o/2H_i)P_{m_i}$, $c_i = (\omega_o/2H_i)$, $d_i = (X_{d_i} - X'_{d_i})/T'_{d_i}$, $e_i = 1/T'_{d_i}$, are the systems parameters, $[x_{i1}, x_{i2}, x_{i3}]^T = \left[\delta_i(t), \omega_i(t), E'_{qi}(t)\right]^T$ represents the state vector, and the control input is given by $u_i = (1/T'_{d_i})k_{ci}u_{fi(t)}$

$$f_i(x) = \begin{bmatrix} x_{i2} \\ -a_i x_{i2} + b_i - c_i x_{i3}\sum_{j=1}^{n} x_{j3} B_{ij}\sin(x_{i1} - x_{j1}) \\ -e_i x_{i3} + d_i\sum_{j=1}^{n} x_{j3} B_{ij}\cos(x_{i1} - x_{j1}) \end{bmatrix}$$

$$g_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

We will now present our controller design based on the idea of continuous sliding mode control. In the same time, we present a controller design based on passivity theory in order to compare the performances of these two methodologies.

3. A continuous sliding-mode controller design

We consider the class of affine nonlinear systems described in the state space by

$$\dot{x} = f(x) + g(x)u, \qquad x(t_0) = x_0, \tag{3}$$

where $t_0 \geq 0$, $x \in B_x \subset R^n$ is the *state vector*, $u \in R^r$ is the *control input vector* , f and g are assumed to be bounded with their components being smooth functions of x. B_x denotes a closed and bounded subset centered at the origin.

The continuous sliding-mode control for the system (3), is designed as follows. Consider the following $(n-r)$-dimensional nonlinear sliding surface defined by

$$\sigma(x - x^*) = (\sigma_1(x - x^*), ..., \sigma_r(x - x^*))^T = 0 \tag{4}$$

where x^* is equilibrium point and each function $\sigma_i : B_x \times B_x \to R$, $i = 1, ..., r$, is a C^1 function such that $\sigma_i(0) = 0$.

The so-called *equivalent control method* (see De Carlo, 1988, Utkin, 1992) is used to determine the system motion restricted to surface $\sigma(x - x^*) = 0$, leading to the *equivalent control*

$$u_e = -\left[\frac{\partial\sigma}{\partial x}g(x)\right]^{-1}\left[\frac{\partial\sigma}{\partial x}f(x)\right] \tag{5}$$

where the matrix $[\partial\sigma/\partial x]g(x)$ is assumed to be nonsingular for all $x, x^* \in B_x$.

In order to complete the control design an additional control term u_N is added to the control input:

$$u = u_e + u_N \tag{6}$$

where u_e is the equivalent control (5), which acts when the system is restricted to $\sigma(x - x^*) = 0$, while u_N acts when $\sigma(x - x^*) \neq 0$.

The control u_N is selected as

$$u_N = -[\frac{\partial\sigma}{\partial x}g(x)]^{-1}L\sigma(x - x^*) \tag{7}$$

where L is an $r \times r$ positive definite matrix.

We can easily check that the system trajectory $x(t)$ is such that the following stable ordinary differential equation

$$\dot{\sigma}(x - x^*) = -L\sigma(x - x^*) \tag{8}$$

is satisfied for all t. This means that the system trajectory reaches the sliding surface asymptotically, since $\sigma(x(t) - x^*) = e^{-L(t-t_0)}\sigma(x(t_0) - x^*)$, $\forall t_0 > 0$, then, $\sigma(x(t) - x^*) \to 0$, when $t \to +\infty$. In fact, the input-output behavior of the closed-loop system (with the output $y = \sigma(x(t) - x^*)$ is given by equation (8).