On the basis of the continuous sliding-mode control described above, the resulting the composite control is given by

$$u = -\left[\frac{\partial\sigma}{\partial x}g(x)\right]^{-1}\left[\frac{\partial\sigma}{\partial x}f(x) + L\sigma(x-x^*)\right]. \quad (9)$$

When the composite control (9) is applied to (3), one obtains the closed-loop nonlinear system

$$\overset{\bullet}{x} = f_e(x,x^*) + p(x,x^*) \quad (10)$$

where
$f_e(x,x^*) = \left\{I_{n\times n} - g(x)\left[\frac{\partial\sigma}{\partial x}g(x)\right]^{-1}\left(\frac{\partial\sigma}{\partial x}\right)\right\}f(x)$.
and
$p(x,x^*) = -g(x)[\frac{\partial\sigma}{\partial x}g(x)]^{-1}L\sigma(x-x^*)$.

Now, in order to study the stability properties of the closed-loop system, we introduce the following assumption.

Assumption 1. *The equilibrium point x^* of $\overset{\bullet}{x} = f_e(x,x^*)$ is locally exponentially stable.*

By use of Lyapunov's converse theorem (see Khalil, 1996), Assumption 1 ensures the existence of a Lyapunov function $V(e)$ with $e = x - x^*$ which satisfies the following inequalities

$$\left\|\frac{\partial V(e)}{\partial e}\right\| \le \alpha_4\|e\|, \quad \alpha_1\|e\|^2 \le V(e) \le \alpha_2\|e\|^2$$

$$\frac{\partial V(e)}{\partial e}\left\{f_e(e+x^*,x^*) + p(e+x^*,x^*)\right\} \le -\alpha_3\|e\|^2 \quad (11)$$

for some positive constants $\alpha_1, \alpha_2, \alpha_3$ and α_4.

Let consider $V(e)$ as a Lyapunov function candidate to investigate the stability of the origin $e = 0$ as an equilibrium point for the system (10). From both Assumption 1 and equation (11), the time derivative of V along the trajectories of (10) satisfies

$$\overset{\bullet}{V}(e) \le -c_3\|e\|^2 \quad (12)$$

then the system (10) is exponentially stable.

The Lyapunov function candidate V is instrumental to investigate the stability properties of the closed-loop system obtained when the composite control u is used. Then the following proposition can be stated.

Proposition 1: *Consider the nonlinear system (3) for which a composite control (5), (6), (7) is designed such that Assumption 1 is satisfied. Then, the closed-loop nonlinear system (10) is locally exponentially stable.*

4. Hamiltonian controller design

Now we derive an excitation controller using the methodology based on the notions of energy function and port-controlled Hamiltonian systems (PCHS).

We onsider the following affine nonlinear system

$$\begin{aligned}\overset{\bullet}{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned} \quad (13)$$

where $x \in \mathbb{R}^n$ is the state vector of the system, $u \in \mathbb{R}^m$ is the control vector and $y \in \mathbb{R}^p$ is the output vector. In this paper we are interested in the class of systems that can be equivalently represented in a Hamiltonian form with dissipative terms in the following way

$$\begin{aligned}\overset{\bullet}{x} &= (\mathcal{J}(x) - \mathcal{R}(x))\frac{\partial H^T}{\partial x} + g(x)u \\ y &= g^T(x)\frac{\partial H^T}{\partial x}\end{aligned} \quad (14)$$

where x , u, y are the energy variables, $H(x_1,...,x_n)$: $\mathbb{R}^n \to \mathbb{R}$ represents the total stored energy and the interconnection structure is captured in the $n\times n$ matrix $\mathcal{J}(x)$ and the $n \times m$ matrix $g(x)$. The matrix $\mathcal{J}(x)$ is skew-symmetric, *i.e.*

$$\mathcal{J}(x) = -\mathcal{J}^T(x), \quad \forall x \in \mathbb{R}^n$$

and $\mathcal{R}(x)$ is a non-negative symmetric matrix depending on x, *i.e.*

$$\mathcal{R}(x) = \mathcal{R}^T(x) \ge 0, \quad \forall x \in \mathbb{R}^n.$$

The main advantage of this kind of representation is that the total energy function can be considered as a Lyapunov function. Moreover, from (14), we obtain the power-balance equation

$$\frac{dH}{dt} = -\frac{\partial H}{\partial x}\mathcal{R}(x)\frac{\partial H^T}{\partial x} + u^T y$$

with $u^T y$ the power externally supplied to the system and $-\frac{\partial H}{\partial x}\mathcal{R}(x)\frac{\partial H^T}{\partial x}$ representing the energy-dissipation due to the resistive elements. As it is well known (see Maschke et al., 1998), the equality above establishes the passivity properties of the system in the following sense.

Definition 1: *System (13) is passive with respect the output $y = h(x)$ if there exists a smooth non-negative function $H(x)$, such that $H(0) = 0$ and the following inequality holds*

$$H(x(t)) - H(x(0)) \le \int_0^t u(s)y(s)ds. \quad (15)$$

If in addition, the system satisfies the detectability properties stated in the next definition

Definition 2: *The system (13) is zero-state detectable if $u(t) = 0$, $y(t) = 0$ $\forall t \ge 0$, implies that $\lim_{t\to\infty} x(t) = 0$.*

Then it is possible to formulate the following result, that is fundamental concerning the stability properties of the considered class of systems.