**Theorem 1:** *Consider the class of systems defined by (14). Assume that the system is zero-state detectable and that the generalized Hamiltonian has a strict local minimum. Then it follows that $x^*$ is a Lyapunov stable equilibrium point of the unforced dynamics. Moreover, the following output feedback*

$$u = -Fy = -Fg^T(x)\frac{\partial H^T}{\partial x} \tag{16}$$

*renders the equilibrium point asymptotically stable.*

## 5. Application to a multi-machine power system

A three-machine power system is now introduced to demonstrate the effectiveness of the continuous sliding mode controller. In this system, generator 3 is considered as an infinite bus, then generator 3 is used as the reference, *i. e.* $\left(E'_{q3} = const = 1\angle 0^\circ\right)$

The system has the following state-space representation

$$\begin{aligned} \dot{x}_{11} &= x_{12},\ \dot{x}_{21} = x_{22} \qquad (17)\\ \dot{x}_{12} &= \frac{1}{2H_1}\left[-D_1 x_{12} + \omega_0\left(P_{m_1} - P_{e_1}\right)\right]\\ \dot{x}_{22} &= \frac{1}{2H_2}\left[-D_2 x_{22} + \omega_0\left(P_{m_2} - P_{e_2}\right)\right]\\ \dot{x}_{13} &= \frac{1}{T'_{d_1}}\left(E_{f_1} - E_{q_1}\right),\ \dot{x}_{23} = \frac{1}{T'_{d_2}}\left(E_{f_2} - E_{q_2}\right) \end{aligned}$$

where $x_{11} = \delta_1$, $x_{21} = \delta_2$, $x_{12} = \omega_1$, $x_{22} = \omega_2$, $x_{13} = E'_{q_1}$, $x_{23} = E'_{q_2}$

### *5.1 Sliding-mode control design*

In this paper we introduce two continuous sliding mode controllers corresponding to two particular choices of the sliding surface:

*Sliding-Mode Control 1*

We consider the following nonlinear switching surface defined by

$$\sigma(x,x^*) = (\sigma_1(x,x^*), \sigma_2(x,x^*))^T = 0$$

where

$$\sigma_i(x,x^*) = s_{i1}(x_{i1} - x^*_{i1}) + s_{i2}(x_{i2} - x^*_{i2}) + s_{i3}(x_{i3} - x^*_{i3})$$

for $i = 1,2$ and $x^*_i = (x^*_{i1}, x^*_{i2}, x^*_{i3})$, for $i = 1,2$; is an equilibrium point of system (17).

Then, the *equivalent control* is given by

$$\begin{aligned} u_{e_i} &= -\left[\frac{\partial \sigma_i}{\partial x_i} g_i(x)\right]^{-1}\left[\frac{\partial \sigma_i}{\partial x_i} f_i(x)\right]\\ &= -\frac{1}{s_{i3}}\left\{\begin{array}{c} s_{i1}x_{i2} + s_{i3}\left(\begin{array}{c} -e_i x_{i3} + d_i \sum\limits_{j=1}^{n} x_{j3} \\ B_{ij}\cos(x_{i1} - x_{j1}) \end{array}\right) \\ +s_{i2}\left(\begin{array}{c} -a_i x_{i2} + b_i - c_i x_{i3} \\ \sum\limits_{j=1}^{n} x_{j3}B_{ij}\sin(x_{i1} - x_{j1}) \end{array}\right)\end{array}\right\} \end{aligned}$$

On the other hand, the control $u_{N_i}$ is selected as

$$\begin{aligned} u_{N_i} &= -\left[\frac{\partial \sigma_i}{\partial x_i} g_i(x)\right]^{-1} L_i \sigma_i(x,x^*)\\ &= -\frac{L_i}{s_{i3}}\left\{\begin{array}{c} s_{i1}(x_{i1} - x^*_{i1}) + s_{i2}(x_{i2} - x^*_{i2}) \\ +s_{i3}(x_{i3} - x^*_{i3}) \end{array}\right\} \end{aligned}$$

*Sliding-Mode Control 2*

Now, let us consider the following nonlinear switching surface given by

$$\sigma_i(x,x^*) = s_{i1}\tilde{x}_{i1} + s_{i2}\dot{\tilde{x}}_{i1} + s_{i3}\ddot{\tilde{x}}_{i1}$$

where $\tilde{x}_{i1} = x_{i1} - x^*_{i1}$.

This is equivalent to

$$\begin{aligned} \sigma_i(x,x^*) &= s_{i1}(x_{i1} - x^*_{i1}) + s_{i2}x_{i2}\\ &\quad + s_{i3}\left(\begin{array}{c} -a_i x_{i2} + b_i - c_i x_{i3} \\ \sum\limits_{j=1}^{n=3} x_{j3}B_{ij}\sin(x_{i1} - x_{j1}) \end{array}\right) \end{aligned}$$

where

$$\frac{\partial \sigma_i}{\partial x_i} g_i(x) = -s_{i3}c_i \sum_{j=1}^{n=3} x_{j3}B_{ij}\sin(x_{i1} - x_{j1})$$

for all $x_i \in B_{x_i}$.

**Remarks:**

1. The coefficients $s_{ij}$, $j = 1,2,3$ of sliding mode controller 1 must be chosen in order Assumption 1 is verified.

2. We can notice that the sliding surface of controller 2 differs from the surface of controller 1 by only one term: $x_{i3} - x^*_{i3}$ is replaced by $\ddot{\tilde{x}}_{i1}$. In this case, when the $s_{i1}$ are some positive constants, the sliding surfaces can be viewed as some stable second-order ordinary differential equations in the power angle $\delta_i$, ensuring convergence of the power angles to their equilibrium values, when the system trajectory remains on the sliding surface.

3. Furthermore the equivalent control $u_e$ can be viewed as an output linearizing controller rendering the system dynamics equivalent to the linear dynamics

   $$\dot{\sigma}_i(x,x^*) = s_{i1}\dot{\tilde{x}}_{i1} + s_{i2}\ddot{\tilde{x}}_{i1} + s_{i3}\dddot{\tilde{x}}_{i1} = 0$$

   The relative degree of each output (power angle) is equal to 3, thus the system has no zero dynamics in this case. Furthermore, stability can be stated by using stability analysis arguments (Lasalle theorem (Khalil, 1996)) apart from Lyapunov function candidate $V(x - x^*) = \frac{1}{2}\sigma^T(x - x^*)\sigma(x - x^*)$.