5.2 Hamiltonian Control design

Now we design a control law based on passivity theory and energy function. The system is described in a Hamiltonian representation providing that the stability of the system can be guaranteed.

Consider system (2) and the following energy function

$$H = \sum_{j=1}^{n=3} \begin{pmatrix} \frac{1}{2c_i}x_{i2}^2 - \frac{b_i}{c_i}x_{i1} + \frac{e_i}{2d_i}x_{i3}^2 \\ -\frac{1}{2}x_{i3}\sum_{j=1}^{n=3} x_{j3}B_{ij}\cos(x_{i1}-x_{j1}) \end{pmatrix} \tag{18}$$

It follows that the system dynamics can be written as a generalized Hamiltonian control system with dissipation according to what follows

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix} \frac{\partial H}{\partial x_i} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_i \tag{19}$$

where

$$x_i = \mathrm{col}(x_{i1}, x_{i2}, x_{i3}),\ \mathcal{J}_i(x) = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{R}_i(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix}, g_i(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Let $(x_{i1}^*,\ x_{i2}^*, x_{i3}^*)$ be the equilibrium point of (2), obtained from the following equations

$$x_{i2}^* = 0$$

$$-a_i x_{i2}^* + b_i - c_i x_{i3}^* \sum_{j=1}^{n=3} x_{j3}^* B_{ij}\sin(x_{i1}^* - x_{j1}^*) = 0$$

$$-e_i x_{i3}^* + d_i \sum_{j=1}^{n} x_{j3}^* B_{ij}\cos(x_{i1}^* - x_{j1}^*) + \tilde{u}_i = 0 \tag{20}$$

Defining the constant excitation control $\tilde{u}_i$, it follows that

$$\tilde{u}_i = e_i x_{i3}^* - d_i \sum_{j=1}^{n=3} x_{j3}^* B_{ij}\cos(x_{i1}^* - x_{j1}^*). \tag{21}$$

Now, defining the energy function which includes the equilibrium point of the following form

$$\begin{aligned} H_e &= \sum_{j=1}^{n=3} \left(\frac{1}{2c_i}x_{i2}^2 - \frac{b_i}{c_i}(x_{i1} - x_{i1}^*) + \frac{e_i}{2d_i}(x_{i3} - x_{i3}^*)^2 \right) \\ &+ \sum_{i=1}^{n=3} \begin{pmatrix} x_{i3}\sum_{j=1}^{n} x_{j3}B_{ij}\cos(x_{i1}-x_{j1}) \\ +x_{i3}\sum_{j=1}^{n} x_{j3}^* B_{ij}\cos(x_{i1}^* - x_{j1}^*) \end{pmatrix} \end{aligned}$$

Then, system (19) can be represented by the Hamiltonian system with dissipation as

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix} \frac{\partial H_e}{\partial x_i} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v_i.$$

Since H_e is bounded from below, because of $x_{i1} \in [-\pi, \pi]$, and $\forall l > 0$ the set $\{x : H_e(x) \le l\}$ is compact. Thus $H_e(x)$ has a strict local minimum at $(x_{i1}^*,\ x_{i2}^*, x_{i3}^*)$.

Then, a control law which stabilizes the multi-machine power system is given by

$$u_i = \tilde{u}_i + v_i.$$

where

$$\begin{aligned} v_i &= -f_i g_i^T \frac{\partial H_e}{\partial x_i} \\ &= -f_i \begin{pmatrix} -\sum_{j=1}^{n=3} B_{ij} \begin{bmatrix} x_{j3}\cos(x_{i1} - x_{j1}) \\ -x_{j3}^*\cos(x_{i1}^* - x_{j1}^*) \end{bmatrix} \\ +\frac{e_i}{d_i}(x_{i3} - x_{i3}^*) \end{pmatrix} \\ &= -f_i \left\{ \begin{matrix} I_{d_i} + \frac{e_i}{d_i}x_{i3} \\ +\frac{1}{d_i}\begin{pmatrix} d_i \sum_{j=1}^{n=3} B_{ij}x_{j3}^* \\ \cos(x_{i1}^* - x_{j1}^*) - e_i x_{i3}^* \end{pmatrix} \end{matrix} \right\} \\ &= -f_i \left\{ I_{d_i} + \frac{e_i}{d_i}x_{i3} - \frac{1}{d_i}\tilde{u}_i \right\} \end{aligned}$$

where $\tilde{u}_i = e_i x_{i3}^* - d_i \sum_{j=1}^{n=3} x_{j3}^* B_{ij}\cos(x_{i1}^* - x_{j1}^*)$. Next, using $E_{q_i} = E'_{q_i} + (X_{d_i} - X'_{d_i})\, I_{q_i}$, and $d_i = (X_{d_i} - X'_{d_i})/T'_{d_i}$, $e_i = 1/T'_{d_i}$, it follows that $\frac{e_i}{d_i} = \frac{1}{(X_{d_i} - X'_{d_i})}$. Finally, the controller can be expressed only in terms of local measurable signals:

$$\begin{aligned} u &= \tilde{u}_i - f_i \left\{ \frac{1}{(X_{d_i} - X'_{d_i})}E_{q_i} - \frac{1}{d_i}\tilde{u}_i \right\} \\ &= \tilde{u}_i + \frac{f_i}{d_i}\tilde{u}_i - \frac{f_i}{(X_{d_i} - X'_{d_i})}\left(V_{t_i} + \frac{Q_{e_i}X_{d_i}}{V_{t_i}} \right) \end{aligned}$$

where $E_{q_i} = V_{t_i} + \frac{Q_{e_i}X_{d_i}}{V_{t_i}}$. Consequently, the resulting controller is a decentralized static output feedback.

6. Simulation results

The effectiveness of the here-proposed sliding-mode controller design has been validated through computer simulations.

The numerical values of the generator parameters (in per unit) were $D_1 = 5$, $D_2 = 3$, $X'_{d_1} = 0.252$, $X'_{d_2} = 0.319$, $X_{d_1} = 1.863$, $X_{d_2} = 2.36$, $H_1 = 1$, $H_2 = 2$, $T'_{d_1} = 6.9$, $T'_{d_2} = 7.96$, $E_{f_1} = 1.3$, $P_{m_1} = 0.35$, $P_{m_2} = 0.35$ and $\omega_s = 377$, $B_{12} = 0.56$, $B_{13} = 0.53$, $B_{23} = 0.6$,

With this parameter choice, the stable equilibrium state of the generator is

$$\begin{aligned} x_{11}^* = 0.6654, \quad x_{12}^* = 0, \quad x_{13}^* = 1.03 \\ x_{12}^* = 0.6425, \quad x_{22}^* = 0, \quad x_{23}^* = 1.01 \end{aligned}$$

The initial value of the states variables are

$$\begin{aligned} x_{11}(0) = 0.8, \quad x_{12}(0) = 0.3, \quad x_{13}(0) = 1.5 \\ x_{12}(0) = 0.5, \quad x_{22}(0) = -0.3, \quad x_{23}(0) = 0.5 \end{aligned}$$

The controller parameters are chosen as follows